

# Game On: Social Networks and Markets

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Extended slide deck - ASSA deck will be abbreviated

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# Game on!

- ▶ **Social networks have influenced equity trading since**
  - ▶ its beginning in the 17th century
  - ▶ trading clubs connected to Amsterdam Stock Exchange ([De la Vega \(1688\)](#))
  - ▶ coffee houses where London Stock Exchange operated ([Standage \(2006\)](#))
- ▶ **Now social media make**
  - ▶ social networks larger and more observable to researchers
  - ▶ example: GameStop



- ▶ **This paper**
  - ▶ **Model:** people trade based on what they learn from social network
  - ▶ **Application:** GameStop and beyond

# Main results

## ▶ Closed-form

### ▶ beliefs:

- ▶ network spillover effects
- ▶ convergence to mix of rational and fanatic views
- ▶ thought leaders and influencers matter

### ▶ prices:

- ▶ social network effects
- ▶ bubbles
- ▶ excess volatility
- ▶ price momentum and fundamental momentum (e.g., PEAD)
- ▶ long-run reversal

### ▶ portfolios:

- ▶ differ across investors
- ▶ bursts of high volume

## ▶ Helps explain

- ▶ GameStop
- ▶ the anatomy of historical bubbles ([Kindleberger \(2000\)](#), [Shleifer \(2000\)](#))
- ▶ financial markets more broadly (momentum, reversal, etc.)

## Related literature

## ► Theory

- ▶ DeGroot (1974) model and persuasion bias, DeMarzo et al. (2003)  
→ **Introduce rational agents+asset markets into DeGroot model**
- ▶ Networks, surveys by Jackson (2010) and Golub and Sadler (2016)
- ▶ Cascades of defaults, survey by Jackson and Pernoud (2020)
- ▶ Information percolation (Duffie et al. (2009)) with social transmission bias (Hirshleifer (2020)) → **Add network effects**
- ▶ Behavioral finance, survey by Barberis and Thaler (2003)  
→ **New theory of momentum, reversal, and excess volatility, and network price dynamics (distinguishing feature cf. other theories)**

- Empirical finance literature: social networks affect

- ▶ housing market expectations and prices, based on Facebook data (Bailey et al. (2018))
  - ▶ local bias and firm values (Kuchler et al. (2020))
  - ▶ equity market participation of retail investors (Hong et al. (2004), Brown et al. (2008), Kaustia and Knüpfer (2012))
  - ▶ portfolios of money managers (Hong et al. (2005))
- **Model these phenomena + new predictions: influencers, thought leaders, network as driver of anomalies and volume**

# Outline of talk

- ▶ **Model**
- ▶ Solution and results
- ▶ Case study of GameStop

## Model: assets and investors

- ▶ **Stock trades at discrete times,  $t = 0, 1, 2, \dots$**

- ▶ Supply of shares  $s$
- ▶ Payoff:  $v + u_\tau \in \mathbb{R}$  at revelation time  $\tau \sim \text{geometric}(\pi)$ 
  - ▶  $u_t$  observable random walk with  $\text{Var}(\Delta u_t) = \sigma_u^2$
  - ▶  $v$  unobserved; each time  $t$ ,  $v$   $\begin{cases} \text{is revealed with probability } \pi \\ \text{remains unknown with prob. } 1 - \pi \end{cases}$
- ▶ Price:

$$price(t) = p(t)1_{(t < \tau)} + [v + u(t)]1_{(t > \tau)}$$

- ▶  $N$  investors

- ▶ Each investor  $i$  receives a signal at time 0:  $x_i(0) = v_i$ , where
  - ▶  $v = \sum_{i=1}^N \kappa_i v_i$
  - ▶ known weights sum to one,  $\sum_{i=1}^N \kappa_i = 1$
- ▶ Four types:
  1. Naive (boundedly rational, persuasion bias)
  2. Fanatic
  3. Rational short-term investors
  4. Long-term investors

# Model: learning in a social network

- ▶ **Naive learning in a social network:**

$$x_i(t) = A_i x(t-1)$$

where weights sum to one,  $\sum_j A_{ij} = 1$

- ▶ **Example**

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \end{pmatrix} = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ * & * & * & * & * \end{pmatrix} \begin{pmatrix} x_1(t-1) \\ x_2(t-1) \\ x_3(t-1) \\ x_4(t-1) \\ x_5(t-1) \end{pmatrix}$$





# Model: rational learning in a social network

## ► Time 1:

$$x_i(1) = x_r = E(v|x_1(0), \dots, x_N(0)) = (\kappa_1, \dots, \kappa_N)x(0)$$

Example:

$$\begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_5(1) \end{pmatrix} = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 20\% & 20\% & 20\% & 20\% & 20\% \end{pmatrix} \begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \\ x_5(0) \end{pmatrix}$$

## ► Time $t \geq 2$ :

$$x_i(t) = x_r = e_i' x(t-1)$$

Example:

$$\begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_f \\ x_r \end{pmatrix} = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(t-1) \\ x_2(t-1) \\ x_3(t-1) \\ x_f \\ x_r \end{pmatrix}$$

## Model: trading and equilibrium

- **Equilibrium price: supply equals demand**

$$s = \sum_{i=1}^N d_i(t)$$

- Demand from naive, fanatic, or rational long-term investor:

$$\max_{d_i} d_i E_t [x_i(t) + u(\tau) - p(t)] - \frac{1}{2w_i} \text{Var}_t [d_i(x_i(t) + u(\tau) - p(t))]$$

where  $w_i = wealth_i / \gamma_i$  is the absolute risk tolerance.

**Solution** using that  $\text{Var}_t(u_\tau) = \sigma_u^2 E(\tau - t) = \sigma_u^2 / \pi$

$$d_i(t) = \frac{\pi w_i}{\sigma_u^2} ( \underbrace{x_i(t) + u(t)}_{\text{fundamental value}} - p(t) )$$

- ▶ Demand from rational short-term investor

$$d_i(t) = \frac{w_i}{\sigma_u^2} \underbrace{((1 - \pi)E_t(p(t + 1)) + \pi(x_r + u(t)))}_{\text{expected price at time } t+1} - p(t)$$

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## Evolution of beliefs (warm-up: special cases)

- ▶ **Everyone is naive and connected** (DeGroot, DeMarzo et al.)

$$x(1) = Ax(0) \quad x(2) = Ax(1) = A^2x(0) \quad x(3) = Ax(2) = A^3x(0)$$

$$x(t) = A^t x(0) \rightarrow \begin{pmatrix} z_1 & z_2 & \dots & z_N \\ \vdots & \vdots & & \vdots \\ z_1 & z_2 & \dots & z_N \\ z_1 & z_2 & \dots & z_N \end{pmatrix} x(0) = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} z' x(0)$$

where eigenvector  $z'A = z'$  is related to Google's page rank

- ▶ **Rationality in the echo chamber: the stubbornness of truth**

$$\mathbf{x}(t) = A^{t-1} \mathbf{x}(1) \rightarrow \mathbf{1}_N e'_N \mathbf{x}(1) = \mathbf{1}_N \mathbf{x}_r$$

Cf. [Golub and Jackson \(2010\)](#): *all opinions in a large society converge to the truth  $\Leftrightarrow$  influence of the most influential agent vanishes*

- ▶ **One fanatic in the echo chamber**

$$x(t) = A^t x(0) \rightarrow 1_N e_N' x(0) = 1_N x_N(0)$$

# Beliefs with fanatics and rational (key framework)

## ► Time 2

$$\begin{pmatrix} x_1(2) \\ x_2(2) \\ x_3(2) \\ x_f \\ x_r \end{pmatrix} = A x(1) = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_r \end{pmatrix}$$

## Beliefs with fanatics and rational (key framework)

► Time 2

$$\begin{pmatrix} x_1(2) \\ x_2(2) \\ x_3(2) \\ x_f \\ x_r \end{pmatrix} = \textcolor{red}{A} x(1) = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_r \end{pmatrix}$$

► Time 3

$$\begin{pmatrix} x_1(3) \\ x_2(3) \\ x_3(3) \\ x_f \\ x_r \end{pmatrix} = A^2 x(1) = \begin{pmatrix} 49\% & 0 & 0 & 34\% & 17\% \\ 44\% & 16\% & 0 & 22\% & 18\% \\ 44\% & 0 & 16\% & 22\% & 18\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_r \end{pmatrix}$$

## Beliefs with fanatics and rational (key framework)

► Time 11

$$\begin{pmatrix} x_1(11) \\ x_2(11) \\ x_3(11) \\ x_f \\ x_r \end{pmatrix} = \textcolor{red}{A}^{10} x(1) = \begin{pmatrix} 2.8\% & 0 & 0 & 64.8\% & 32.4\% \\ 3.8\% & .01\% & 0 & 58.6\% & 37.6\% \\ 3.8\% & 0 & .01\% & 58.6\% & 37.6\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_r \end{pmatrix}$$

► Time 101

$$\begin{pmatrix} x_1(101) \\ x_2(101) \\ x_3(101) \\ x_f \\ x_r \end{pmatrix} = A^{100} x(1) = \begin{pmatrix} 0\% & 0\% & 0 & 66.7\% & 33.3\% \\ 0\% & 0\% & 0 & 61.1\% & 38.9\% \\ 0\% & 0 & 0\% & 61.1\% & 38.9\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1(1) \\ x_2(1) \\ x_3(1) \\ x_f \\ x_r \end{pmatrix}$$

## Beliefs with fanatics and rational (key framework)

- ▶ **Hardheaded investors  $h$ :** collection of rational and fanatics
- ▶ **Transition matrix:**

$$A = \begin{bmatrix} A_{nn} & A_{nh} \\ 0 & I \end{bmatrix}$$

- ▶ **Assumption 1 (hardheaded-connected agents)** Any  $i$  influenced by a hardheaded  $j$ , either
  - ▶ directly, i.e.,  $A_{ij} > 0$ , or
  - ▶ indirectly, i.e.,  $A_{ik_1} > 0, A_{k_1k_2} > 0, \dots, A_{k_{\ell}j} > 0$

### Proposition (Network belief spillover and convergence)

*Naive agent views for  $t = 2, 3, \dots$*

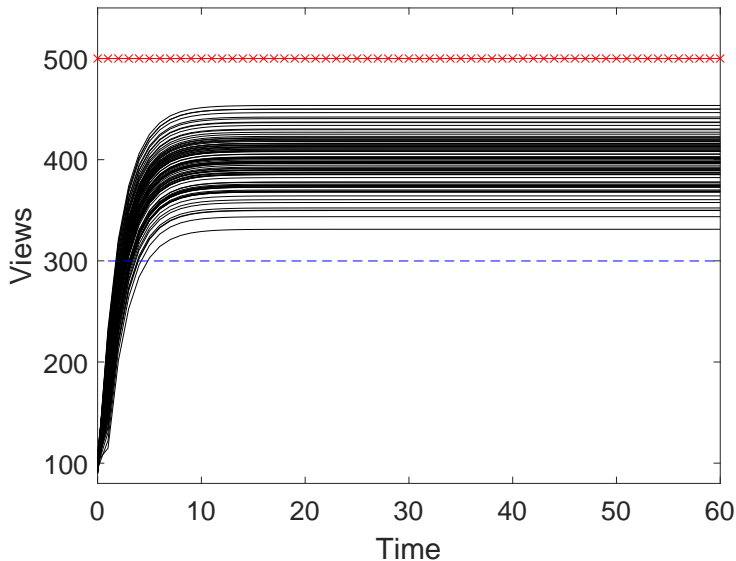
$$x_n(t) = A_{nn}^{t-1}x_n(1) + \sum_{k=0}^{t-2} A_{nn}^k A_{nh} x_h$$

*In the limits as  $t \rightarrow \infty$ , convex combination of fanatic and rational views*

$$x_n(t) \rightarrow (I - A_{nn})^{-1} A_{nh} x_h$$



## Numerical example: beliefs



# Influencers and thought leaders

- ▶ Fanatic has 57.8% **thought leadership**:  $\bar{x}(t) \rightarrow 57.8\%x_f + 42.2\%x_r$

## Influencers and thought leaders

- ▶ Fanatic has 57.8% **thought leadership**:  $\bar{x}(t) \rightarrow 57.8\%x_f + 42.2\%x_r$
- ▶ What determines thought leadership?
  - ▶ How **much people listen**
  - ▶ Their **influencer values**

$$57.8\% = \frac{66.7\% + 61.1\% + 61.1\% + 1 + 0}{5}$$

$$57.8\% = \begin{pmatrix} 1.56 & 0.33 & 0.33 \end{pmatrix} \begin{pmatrix} 20\% \\ 10\% \\ 10\% \end{pmatrix} + \frac{1}{5} \quad A = \begin{pmatrix} 70\% & 0 & 0 & 20\% & 10\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

## Influencers and thought leaders

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- **Meaning of influencer values:** 1% increase in following leads to  $\bar{x}_n \rightarrow 59.3\%x_f + 40.7\%x_r$  an increase of 1.56% in thought leadership

$$59.3\% = (1.56 \quad 0.33 \quad 0.33) \begin{pmatrix} 21\% \\ 10\% \\ 10\% \end{pmatrix} + \frac{1}{5} \quad A = \begin{pmatrix} 70\% & 0 & 0 & 21\% & 9\% \\ 40\% & 40\% & 0 & 10\% & 10\% \\ 40\% & 0 & 40\% & 10\% & 10\% \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



## Influencers and thought leaders, continued

### Proposition (Influencers and thought leaders)

**A.** The thought leadership of any naïve agent is zero,  $\theta_i = 0$ , and the vector of thought leaderships of hardheaded agents,  $\theta_h$ , depends on how much attention they get from naïve agents,  $A_{nh}$ , and their influencer values,  $\mu$ :

$$\theta'_h = \mu' A_{nh} + \frac{w'_h}{w},$$

**B.** If naive agent  $i$  increases his following  $A_{ij}$  of hardheaded agent  $j$  by  $\epsilon$  at the expense of a lower following of other hardheaded agents, then

$$\frac{\partial \theta_j}{\partial \varepsilon} = \mu_i$$

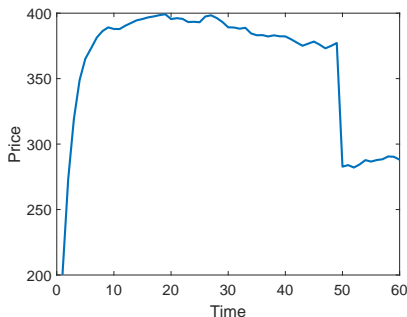
# Equilibrium price

- Equalizing supply and demand,  $s = \sum_{i=1}^N d_i(t)$ , yields

$$p(t) = (1 - c) \left[ \frac{w_n}{w} \bar{x}_n(t) + \frac{w_f}{w} \bar{x}_f + \frac{w_l + w_s}{w} x_r + u(t) - \frac{s\sigma_u^2}{\pi w} \right] + cE_t(p(t+1))$$

where  $c = \frac{1}{1 + \frac{w}{w_s} \frac{\pi}{1-\pi}} < 1$ . **Solution:** iterate forward to infinity

- Numerical example:



# Social network effect on price

## Proposition (Network effects on price)

*Equilibrium price*

$$p(t) = p_r(t) + p_n(t)$$

- ▶ *Rational part:*

$$p_r(t) = x_r + u(t) - \frac{s\sigma_u^2}{\pi w}.$$

- ▶ *Network part:*

$$p_n(t) = \frac{w_n \cdot (1 - c)}{w} \sum_{k=0}^{\infty} c^k (\bar{x}_n(t+k) - x_r) + \frac{w_f}{w} (\bar{x}_f - x_r)$$

As  $t \rightarrow \infty$ , network part converges to

$$p_n^{\infty} = \sum_{j=N_n+1}^{N_n+N_f} \theta_j (x_j - x_r)$$





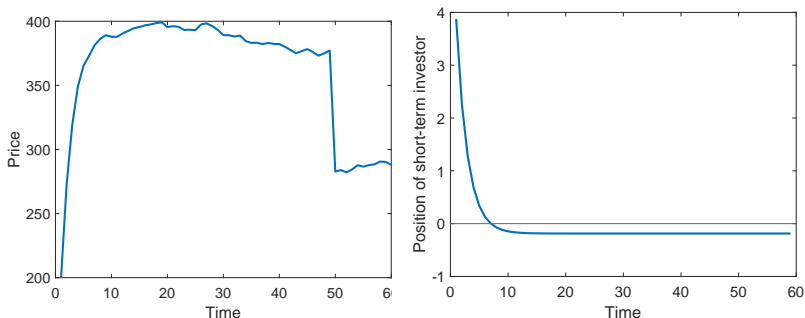
# Value and momentum

## ► Returns:

$$\begin{aligned} E_t(r(t+1)) &= (1 - \pi)E_t(\Delta p(t+1)) + \pi(x_r + u(t) - p(t)) \\ &= (1 - \pi) \underbrace{\Delta p_n(t+1)}_{\text{network momentum}} + \pi \underbrace{b(t)}_{\text{value}} \end{aligned}$$

network mom  $\cong$  price mom,  $\Delta p_n(t+1) \cong \Delta p_n(t) = \Delta p(t) - \Delta u(t)$

## ► Numerical example





# Spike in volume and excess price variation

## Proposition (Spike in volume and excess volatility)

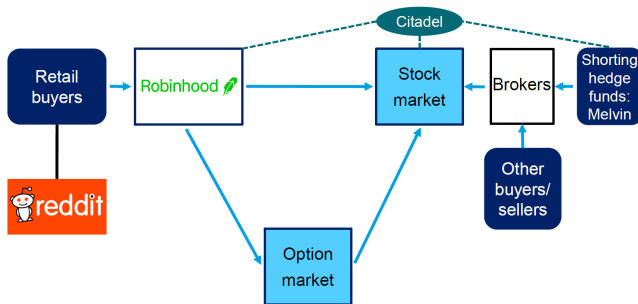
*With naive and fanatic agents,*

- ▶ *the trading volume is greater, but dies down over time*
- ▶ *the valuation ratio (price minus fundamental  $u(t)$ ) varies more*

# Outline of talk

- ▶ Model
- ▶ Solution and results
- ▶ **Case study of GameStop**

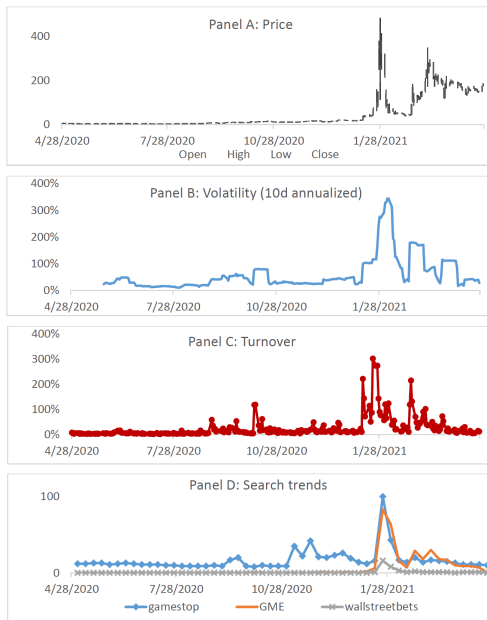
# Overview of the GameStop event



Further details, YouTube talk at **Markus' Academy, Princeton University**:

<https://bcf.princeton.edu/events/lasse-pedersen-on-gamestop-and-predatory-trading/>

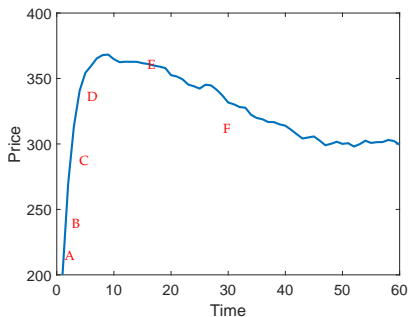
# GameStop: price, volatility, volume, public interest







# Conclusion: social network effects everywhere



Model elements	Bubble anatomy <sup>†</sup>	GameStop	Asset pricing
<b>A</b> Investors receive news, fanatics focus on one element	Initial displacement	Retail investors focus on plan to pivot online	Announcement effects, e.g. post-earnings drift
<b>B</b> Short-term investors bet on network spillovers	Speculation	Some institutional investors are long	Momentum and fundamental momentum
<b>C</b> Opinions spread through the network	Mania and emulation	More and more people hear about GameStop	Local bias and network spillover effects
<b>D</b> Influencers follow a fanatic	Authoritative blessing	Elon Musk tweets a link to Wall-StreetBets	Excess volatility
<b>E</b> Rational traders bet on reversal	Insiders sell out	Institutional investors sell	Value investing
<b>F</b> Fundamentals are revealed or fanatics gradually learn	Crash: revelation or revolution	Drops in January, March earnings announcement	Long-run reversal and value effect

<sup>†</sup>Stylized evolution of historical bubbles of [Kindleberger \(2000\)](#) and [Shleifer \(2000\)](#)

## References cited in slides

Brief list – numerous additional references cited in the actual paper

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