The Fiscal Theory of the Price Level with a Bubble

Markus Brunnermeier
Princeton University

Sebastian Merkel
University of Exeter

Yuliy Sannikov
Stanford University

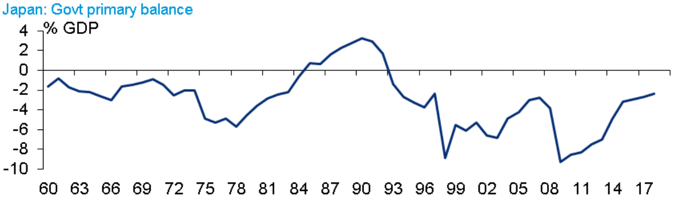
January 2022

Motivation

- Different monetary theories emphasize different roles of money and equilibrium equations
- Fiscal Theory of the Price Level (FTPL):
    - broad money (including nom. bonds) as a store of value
    - value of government debt given by discounted stream of future primary surpluses

$$\frac{\mathcal{B}_t+\mathcal{M}_t}{\mathcal{P}_t} = \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\left(T_s - G_s\right) ds\right] \quad \left[+ \quad \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\Delta i_s \frac{\mathcal{M}_s}{\mathcal{P}_s} ds\right]\right]$$

- The Japan critique:



- Broader question: can a country permanently run primary deficits?

Deriving the Key Equation of the FTPL

- Nominal government flow budget constraint

$$\left(\mu_t^{\mathcal{B}}\mathcal{B}_t + \mathcal{P}_t T_t\right) dt = \left(i_t \mathcal{B}_t + \mathcal{P}_t G_t\right) dt$$

- Multiply by nominal SDF $\xi_t/\mathcal{P}_t$, integrate from t to T, take expectations and limit $T \to \infty$

$$\frac{\mathcal{B}_t}{\mathcal{P}_t} = \underbrace{\mathbb{E}_t\left[\int_t^\infty \frac{\xi_s}{\xi_t}\left(T_s - G_s\right) ds\right]}_{\text{PV of primary surpluses}} + \underbrace{\lim_{T\to\infty} \mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T}{\mathcal{P}_T}\right]}_{\text{bubble}}$$

- Bubble term?
  - in literature: invoke private-sector transversality condition to conclude $\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T}{\mathcal{P}_T}\right] \to 0$
  - this paper: environments in which the previous argument fails

When Can a Bubble Exist?

- Assume stationary debt-to-GDP ratio and no aggregate risk
  - $\frac{\mathcal{B}_T}{\mathcal{P}_T} = \frac{\mathcal{B}_t}{\mathcal{P}_t} e^{g(T-t)}$
  - $\frac{\xi_T}{\xi_t} \propto e^{-r^f(T-t)}$

- Then $\mathbb{E}_t\left[\frac{\xi_T}{\xi_t}\frac{\mathcal{B}_T}{\mathcal{P}_T}\right] \to 0 \Leftrightarrow r^f > g$
  - thus: bubble can exist $\Leftrightarrow r^f \leq g$
  - more generally: $r^b \leq g$ with $r^b =$ risk-adjusted discount rate for gov. debt

When Can a Bubble Exist?

- Assume stationary debt-to-GDP ratio and no aggregate risk
  - $\frac{\mathcal{B}_T}{\mathcal{P}_T} = \frac{\mathcal{B}_t}{\mathcal{P}_t} e^{g(T-t)}$
  - $\frac{\xi_T}{\xi_t} \propto e^{-r^f(T-t)}$
- Then $\mathbb{E}_t \left[\frac{\xi_T}{\xi_t} \frac{\mathcal{B}_T}{\mathcal{P}_T} \right] \to 0 \Leftrightarrow r^f > g$
  - thus: bubble can exist $\Leftrightarrow r^f \leq g$
  - more generally: $r^b \leq g$ with $r^b =$ risk-adjusted discount rate for gov. debt
- For log utility and balanced growth path

$$r^f = \rho + \mu^c - (\sigma^c)^2, \qquad g = \mu^C$$

  Two examples for $r^f \leq g$ with long-lived agents:
  1. perpetual youth: $\mu^c < \mu^C$ due to population growth
  2. uninsurable idiosyncratic risk: large σ^c offsets ρ even if $\mu^c = \mu^C$

Outline

1. Two Models with a Bubble
   - Baseline Example: Perpetual Youth
   - Alternative Example: Uninsurable Idiosyncratic Risk
   - Bubble Existence and Transversality
2. The Bubble as a Fiscal Resource
   - “Mining the Bubble”
   - Bubble Mining and Inflation
   - Optimal Bubble Mining
3. Price Level Determination [in Paper]

Outline

1. Two Models with a Bubble
   - Baseline Example: Perpetual Youth
   - Alternative Example: Uninsurable Idiosyncratic Risk
   - Bubble Existence and Transversality
2. The Bubble as a Fiscal Resource
   - "Mining the Bubble"
   - Bubble Mining and Inflation
   - Optimal Bubble Mining
3. Price Level Determination [in Paper]

Perpetual Youth: Model Setup

- Continuous time, infinite horizon, one consumption good
- Growing continuum of (infinitely-lived) agents
  - endowed with human capital at birth that depreciates over time
  - can trade government bonds
- Government
  - exogenous spending
  - taxes output
  - issues (nominal) bonds
- Financial friction: no trade with yet unborn generations (alleviated by bonds)

Perpetual Youth: Model Setup – Some Formal Details

- Popuation L_t grows at fixed rate $g > 0$
- Preferences ($i \in [0, L_t]$ agent index):

$$\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \log c_s^i dt\right]$$

- Each agent has human capital endowment k_t^i ($k_{t_0^i}^i = 1$ at birth):
  - output flow: $ak_t^i dt$
  - output tax by government: $\tau a k_t^i dt$
  - constant depreciation: $dk_t^i = -\delta k_t^i dt$
- Real bond holdings b_t^i satisfy:

$$db_t^i = \left(r_t^f b_t^i + (1-\tau)ak_t^i - c_t^i\right) dt$$

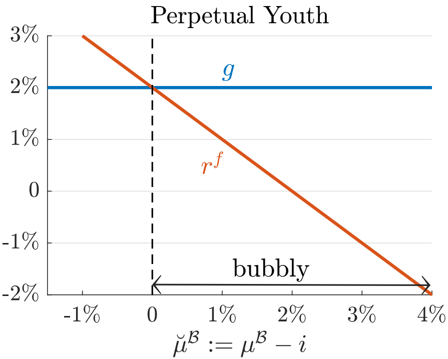
- Government:
  - budget constraint

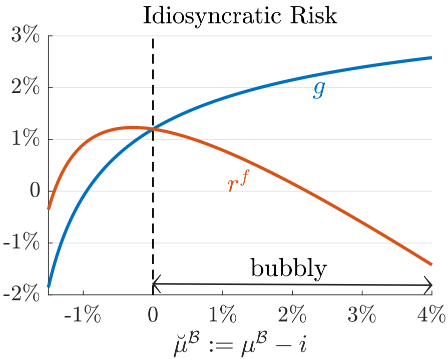
$$\overbrace{i\mathcal{B}_t}^{\text{interest payments}} = \overbrace{\mathcal{P}_t \underbrace{(\tau a - \mathfrak{g})}_{=:s} K_t}^{\text{prim. surpluses}} + \overbrace{\mu^{\mathcal{B}} \mathcal{B}_t}^{\text{bond issuance}}$$

Idiosyncratic Risk: Model Setup (Changes relative to Perpetual Youth)

- Fixed continuum of agents
  - operate physical capital subject to idiosyncratic risk, AK production technology
  - can increase capital by physical reinvestment
  - can trade capital and government bonds
- Financial friction: incomplete markets:
  - agents cannot trade idiosyncratic risk
- Why is second example interesting?
  - capital asset & return on capital $>$ growth rate g (as in Blanchard (2019), Reis (2021))
  - endogeneous growth rate g affected by gov. policy
  - markets incomplete even with bubble, richer welfare implications

r^f versus g for Different Policies (Monetary Steady State)





Bubble and Transversality

- In both models, long-lived agents have transversality conditions (TVCs)

$$\lim_{T \to \infty} \mathbb{E}\left[\xi_T^i b_T^i\right] = 0$$

- Why do TVCs not rule out bubbles?
  - TVCs: limit of $\mathbb{E}\left[\xi_T^i b_T^i\right]$ (*individual* bond wealth)
  - bubble: limit of $\mathbb{E}\left[\xi_T^i \mathcal{B}_T / \mathcal{P}_T\right]$ (*aggregate* bond wealth)

  $\Rightarrow$ bubble consistent with TCVs if individual and aggregate bond wealth differ

Bubble and Transversality

- In both models, long-lived agents have transversality conditions (TVCs)

$$\lim_{T \to \infty} \mathbb{E}\left[\xi_T^i b_T^i\right] = 0$$

- Why do TVCs not rule out bubbles?
  - TVCs: limit of $\mathbb{E}\left[\xi_T^i b_T^i\right]$ (*individual* bond wealth)
  - bubble: limit of $\mathbb{E}\left[\xi_T^i \mathcal{B}_T / \mathcal{P}_T\right]$ (*aggregate* bond wealth)

  $\Rightarrow$ bubble consistent with TCVs if individual and aggregate bond wealth differ

- Properties of b_T^i and $\mathcal{B}_T^i / \mathcal{P}_T$ differ when there are beneficial *equilibrium trades*
  1. perpetual youth: bonds allow intergenerational resource transfers
     - individuals pass bonds on to newborn generations

     $\rightarrow$ b_T^i grows at lower rate than aggregate bond wealth
  2. idiosyncratic risk: bonds are safe assets, allow for self-insurance
     - individuals trade bonds to rebalance portfolios in response to idiosyncratic shocks

     $\rightarrow$ b_T^i is stochastic, aggregate bond wealth deterministic

Digression: Dynamic Trading Perspective

- Alternative debt valuation approach (from our paper "Debt as Safe Asset"):
  - price actual cash flows from individual portfolios, including trading cash flows
  - then aggregate over all agents to obtain total value of debt
  - because individual TVCs hold, this yields "bubble-free" valuation formula

Digression: Dynamic Trading Perspective

- Alternative debt valuation approach (from our paper "Debt as Safe Asset"):
  - price actual cash flows from individual portfolios, including trading cash flows
  - then aggregate over all agents to obtain total value of debt
  - because individual TVCs hold, this yields "bubble-free" valuation formula
- Resulting valuation formulas with "service flow terms"
  - perpetual youth:

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int_0^{L_0} \xi_t^i \beta_t^i di\right)}_{=\xi_t^{**}} sK_t dt\right] + \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int_0^{L_0} \xi_t^i \beta_t^i di\right)}_{=\xi_t^{**}} \frac{C_t^0 - (S_t^0 + (1-\tau)aK_t^0)}{\mathcal{B}_t^0/\mathcal{P}_t} \frac{\mathcal{B}_t}{\mathcal{P}_t} dt\right]$$

  - idiosyncratic risk:

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int \xi_t^i \beta_t^i di\right)}_{=\xi_t^{**}} sK_t dt\right] + \mathbb{E}\left[\int_0^\infty \underbrace{\left(\int \xi_t^i \beta_t^i di\right)}_{=\xi_t^{**}} (\tilde{\sigma}^c)^2 \frac{\mathcal{B}_t}{\mathcal{P}_t} dt\right]$$

- Note: discount rate implied by ξ^{**} different from m (Reis, 2021)

Outline

Debt Valuation with a Bubble

- Primary surplus $sK_t = (\tau a - \mathfrak{g})K_t$
- Debt valuation equation ($K_0 \equiv 1$):

$$\frac{\mathcal{B}_0}{\mathcal{P}_0} = \lim_{T\to\infty}\left(\underbrace{\int_0^T e^{-(r^f-g)t} s dt}_{=:PVS_{0,T}} + e^{-(r^f-g)T}\frac{\mathcal{B}_0}{\mathcal{P}_0}\right)$$

- risk-free rate $r^f = g - \breve{\mu}^{\mathcal{B}}$

$s > 0$	$s = 0$	$s < 0$
$r^f > g$	$r^f = g$	$r^f < g$
$PVS > 0$	$PVS = 0$	$PVS < 0$
no bubble	bubble > 0	bubble > 0

$\longrightarrow \breve{\mu}^{\mathcal{B}}$

"Mining the Fiscal Bubble"

$s > 0$	$s = 0$	$s < 0$	
$r^f > g$	$r^f = g$	$r^f < g$	$\breve{\mu}^{\mathcal{B}}$
$PVS > 0$	$PVS = 0$	$PVS < 0$	
no bubble	bubble > 0	bubble > 0	

In all three cases, the bubble – or its mere possibility – grants government some leeway:

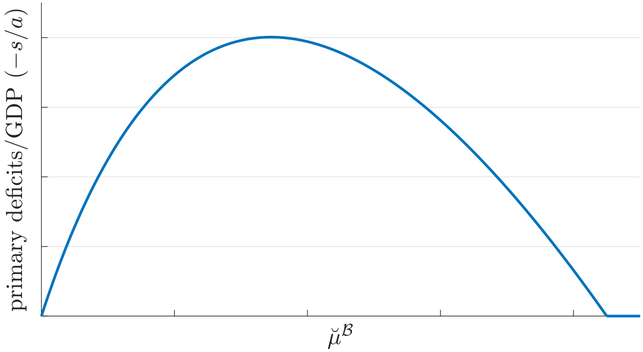
- $s < 0$: perpetual deficits are funded out of the bubble, never have to raise taxes ("bubble mining")
- $s = 0$: government debt enjoys positive value despite zero surpluses (debt "backed" by the bubble)
- $s > 0$: no equilibrium bubble, yet possibility of bubble makes debt more sustainable

  unexpected (persistent) drop in surpluses below zero

  $\Rightarrow$ bubble emerges instead of collapse of the value of debt

Bubble Mining Laffer Curve

- Primary deficit $= \breve{\mu}^{\mathcal{B}} \mathcal{B}_t / \mathcal{P}_t$
- Increasing $\breve{\mu}^{\mathcal{B}}$ dilutes bondholder claims and reduces "tax base" $\mathcal{B}_t / \mathcal{P}_t$



Is Bubble Mining Inflationary?

- Inflation

$$\pi = \mu^{\mathcal{B}} - g = \breve{\mu}^{\mathcal{B}} + i - g$$

- For given i and g: larger bubble mining $\breve{\mu}^{\mathcal{B}}$ is inflationary
- But:
  - If nom. interest rate i is unconstrained: can also raise $\breve{\mu}^{\mathcal{B}}$ through lower i
  - If growth g is endogeneous (idiosyncratic risk example):
    - bubble mining makes bonds less attractive $\rightarrow$ portfolio reallocation to capital
    - larger physical investment raises g and lowers inflation

$\Rightarrow$ Inflation-neutral increase in $\breve{\mu}^{\mathcal{B}}$ possible using combination of higher debt growth and lower interest rates

Socially Optimal Bubble Mining

1. When would a (Ramsey-)planner controlling $(\tau, \breve{\mu}^{\mathcal{B}})$ choose bubble mining $\breve{\mu}^{\mathcal{B}} > 0$?
   - perpetual youth: never
     - allocation with stationary equilibrium bubble and $r^f = g$ is Pareto-optimal
     - "taxing" the only store of value by bubble mining is always Pareto-inferior
   - idiosyncratic risk: if idiosyncratic capital risk $\tilde{\sigma}$ is large
     - bubble improves idios. risk sharing, but crowds out growth-enhancing capital investments
     - bubble only partially completes markets, thus equilibrium may be constrained inefficient
     - for large $\tilde{\sigma}$, equilibrium bubble is "too large" relative to constrained efficient level

Socially Optimal Bubble Mining

1. When would a (Ramsey-)planner controlling $(\tau, \breve{\mu}^{\mathcal{B}})$ choose bubble mining $\breve{\mu}^{\mathcal{B}} > 0$?
   - perpetual youth: never
     - allocation with stationary equilibrium bubble and $r^f = g$ is Pareto-optimal
     - "taxing" the only store of value by bubble mining is always Pareto-inferior
   - idiosyncratic risk: if idiosyncratic capital risk $\tilde{\sigma}$ is large
     - bubble improves idios. risk sharing, but crowds out growth-enhancing capital investments
     - bubble only partially completes markets, thus equilibrium may be constrained inefficient
     - for large $\tilde{\sigma}$, equilibrium bubble is "too large" relative to constrained efficient level
2. Optimal bubble mining $\breve{\mu}^{\mathcal{B}}$ is independent of the need for government expenditures $\mathfrak{g}K_t$
   - Reason:
     - $\mathfrak{g} \uparrow \Rightarrow$ gov. must claim higher fraction of current output
     - taxing output (τ) is the most direct way to do so
     - bubble mining also distorts intergenerational resource transfer / capital-bond portfolio choice

   $\Rightarrow$ should rely on taxes, not bubble mining, as the marginal funding source for public expenditures

Outline

details

Conclusion

- Integrate the "missing" bubble term into the FTPL
- Two models with $r^f \leq g$ and bubbles
  1. perpetual youth: bubble facilitates inter-generational trade
  2. idiosyncratic risk: bubble facilitates risk sharing
- Transversality conditions do not rule out bubbles
  - because they apply to individual bond portfolios
  - while bubble is about aggregate bond wealth
- Government can mine the bubble for revenue (a form of seigniorage)
  - may not be (that) inflationary if growth-enhancing
  - but may also not be socially optimal
  - mining for the sole purpose of raising revenue never optimal
- Price level determination [in paper]
  - goods market clearing condition (through bubble wealth effect)
  - uniqueness: off-equilibrium tax backing

Determination of Price Level

Two questions:

1. What economic mechanism determines the price level?
   - FTPL intution still works with a bubble: wealth effect on gov. debt determines price level in the goods market
   - Once price level is determined, debt valuation equation determines the size of the bubble
2. Can fiscal policy resolve equilibrium multiplicity (FTPL as a selection device)?
   - two sources of multiplicity: (1) bubble multiplicity; (2) nominal indeterminacy
   - FTPL arguments can resolve both
   - off-equilibrium fiscal backing is sufficient
   - but requires credibility and fiscal capacity to promise off-equilibrium surpluses (otherwise: vulnerability to bubble crashes)

return

FTPL: Resolving Equilibrium Multiplicity

- If $\tau > 0$ along equilibrium path:
  - standard FTPL argument applies: unique $\mathcal{P}_t$ consistent with equilibrium, if surpluses (τ_s) do not react (too strongly) to the price level
  - but then $r^f > g$ and there is no bubble in equilibrium
- Resolving multiplicity with an equilibrium bubble:
  - more challenging: continuum of bubble values consistent with the same surplus path

    $\Rightarrow$ exogenous surplus sequence insufficient for uniqueness
  - contingent policy can select the bubble equilibrium
    - primary deficits on the equilibrium path (bubble mining)
    - switch to $\tau > 0$ if inflation breaks out

return