Motivation

- Misspecification of asset pricing models typically confined to mean-variance analysis.
- Example: Any proposed SDF (M) needs to overcome the Hansen-Jagannathan (HJ) bound
  \[ \sigma(M) \geq \mathbb{E}[R] - R_f \]
  \[ \frac{R_f}{\sigma(R)} \]
- Consider the consumption based SDF \( M = \beta g \gamma \), where \( g \) and \( \gamma \) denote consumption growth and risk-aversion respectively.
- This model is misspecified since \( \sigma(g) \) is low in the data.
- Question: Can we use other statistics than mean and variance that gives more insight in determinants of misspecification?

A new bound

- Let \( F(x) = P(R \leq x) \) denote the CDF of the return DGP.
- Similarly, \( \tilde{F}(x) = \mathbb{I}(R \leq x) \), is the CDF of the risk-neutral distribution. Define \( Q_{\tau} \), the risk-neutral quantile function \( F(Q_{\tau}) = \tau, \forall \tau \in (0,1) \).
- Consider the ordinal dominance curve \( \phi(x) = F(Q_{\tau}) \).
- We then obtain a new bound on the SDF volatility for all \( \tau \in (0.1) \).

**Theorem 0.1 (Quantile bound).**

\[ \sigma(M) \geq \frac{\tau - \phi(\tau)}{R_f(\sqrt{\phi(\tau)} - (1 - \phi(\tau))} \]

\( \text{(1)} \)

**Conditional quantile premium**

- We also consider the conditional difference \( Q_{\tau} - \hat{Q}_{\tau} \).
- Von-Mises approximation yields \( \hat{Q}_{\tau} = \hat{F}_{\tau} + \tau \hat{F}_{\tau} \).
- Building on Chabi-Yo and Loudis (2020), we bound \( \tau - \hat{F}_{\tau} \), where \( \hat{F}_{\tau} \) is inferred at time \( t \) from option data.
- In spirit of Martin (2017), we test tightness of the bound, using quantile regression \( \hat{Q}_{\tau} = \beta(c\tau) + \beta(t\tau) \cdot \left( \hat{F}_{\tau} + \tau \hat{F}_{\tau} \right) \).

**Maturity:**

\( \tau = 0.01 \) \quad \( \hat{\beta}(\tau) \approx 0.06 \) \quad \( \hat{\beta}(\tau) \approx 0.97 \) \quad \( \hat{\beta}(\tau) \approx 0.97 \) \quad \( \text{Wald test} \) \quad \( R^2(\tau | \%) \) \quad \( R^2(\tau | \%) \)

\( \tau = 0.05 \) \quad \( \hat{\beta}(\tau) \approx 0.20 \) \quad \( \hat{\beta}(\tau) \approx 0.90 \) \quad \( \hat{\beta}(\tau) \approx 0.41 \) \quad \( \text{Wald test} \) \quad \( R^2(\tau | \%) \) \quad \( R^2(\tau | \%) \)

\( \tau = 0.1 \) \quad \( \hat{\beta}(\tau) \approx 0.17 \) \quad \( \hat{\beta}(\tau) \approx 0.83 \) \quad \( \hat{\beta}(\tau) \approx 0.46 \) \quad \( \text{Wald test} \) \quad \( R^2(\tau | \%) \) \quad \( R^2(\tau | \%) \)

\( \tau = 0.2 \) \quad \( \hat{\beta}(\tau) \approx 0.21 \) \quad \( \hat{\beta}(\tau) \approx 0.79 \) \quad \( \hat{\beta}(\tau) \approx 0.54 \) \quad \( \text{Wald test} \) \quad \( R^2(\tau | \%) \) \quad \( R^2(\tau | \%) \)

- Conclusion: \( \hat{Q}_{\tau} = \hat{Q}_{\tau} + \hat{F}_{\tau} \) is a good approximation of latent \( Q_{\tau} \).

**Bounds for S&P500 data**

- Shape of quantile bound roughly similar to disaster risk model.
- More formal testing shows that quantile bound is significantly higher than HJ bound.
- The LRR model can only reconcile this for very high levels of risk-aversion (\( \gamma \geq 90 \)).
- Intuition: disaster risk induces a peak in quantile bound for small \( \tau \).

Disaster and long-run risk models

- The quantile bound (1) is quite different depending on the asset pricing model. We consider two models:
  - (i) Disaster risk model (Backus et al., 2011):
    \[ M = \beta g \gamma, \] where \( \log \log g = \epsilon + \eta \) and \( \epsilon \sim N(\mu, \sigma^2), \eta(j = j) \sim N(j \theta, j^2 \nu) \).
  - (ii) Long-run risk (LRR) model (Bansal et al., 2012):
    \[ \log M_{t+1} = \text{Constant} - \log g_{t+1} + \left( \theta - 1 \right) \log R_{t+1}. \]

Here, \( R_{t+1} \) is the return on the consumption asset. Both \( g_{t+1}, R_{t+1} \) are conditionally lognormal.

- We compare the quantile bound and HJ bound using the model calibration from Backus et al. (2011) and Bansal et al. (2012).
- The figure shows that the quantile bound can be stronger than the HJ bound in disaster risk model, but not in LRR model.

\( \hat{Q}_{\tau, \tau} \) and \( \hat{Q}_{\tau, \tau} - \hat{Q}_{\tau, \tau} \) for \( \tau = 0.05 \)

**References**


