ENDOGENOUS SPATIAL PRODUCTION NETWORKS
Quantitative Implications for Trade and Productivity

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Introduction
Heterogeneous Firms, Production Networks, and Trade

- Production is organized in large-scale firm-to-firm networks
  - firms are vastly heterogeneous in size, input sourcing and importance in network
  - firms’ outcomes are shaped by those of connected firms – suppliers and customers
  - supply chain networks span across space → trade costs affect network formation
  - production networks reorganize endogenously in response to shocks

- Objective
  - Design data generating process for large spatial supply chain networks
    - feasibly estimable weighted directed random graph model
  - Evaluate GE impact of micro- and macro- shocks to spatial network economy
    - e.g. firm-level distortions; market integration; technology improvements
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1. Document importance of endogenous networks in firm size heterogeneity
   - Indian firm network micro-data → choice of suppliers & intensity of use explain 80%

2. Develop tractable firm network formation model where firm heterogeneity → trade
   - rationalizes firm-to-firm network data and accommodates gravity relationships

3. Propose scalable framework for estimation + counterfactual analysis
   - maximum likelihood estimation + no simulation for counterfactuals

4. Evaluate impact of reducing inter-state border frictions by 10%
   - sizable district-level welfare gains [1%,8%]
   - > 1/2 changes in firms’ input sales from endogenous network changes
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# Related Literature

This paper: Firm-to-Firm Trade in Endogenous Production Networks

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Notation

- **Locations** indexed by $o, d \in \mathcal{J} \equiv \{1, \cdots, J\}$
  
  
  [o for origin, d for destination]

- **Firms** indexed by $s, b \in \mathcal{M} \equiv \{1, \cdots, M\}$
  
  
  [s for seller, b for buyer]
Data Sources

- Universe of intra-state firm-to-firm transactions
  [assembled from commercial tax authorities in 5 Indian states]
  - 141 districts:
    Gujarat (25), Maharashtra (35), Tamil Nadu (32), Odisha (30) and West Bengal (19)
  - 5 years: FY 2011-12 to 2015-16
  - 2.6 million firms and 103 million firm-to-firm connections

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Data
Firm-to-Firm Input-Output Matrix

- **Data** [value of goods sold by $s$ to $b$]
  \[ \text{sales}_{od}(s,b) \]

- **Cost Shares** [$b$’s intensity of use of $s$]
  \[ \pi_{od}(s,b) = \frac{\text{sales}_{od}(s,b)}{\text{input costs}_{d}(b)} \]
  \[ \text{input costs}_{d}(b) = \sum_{s} \text{sales}_{od}(s,b) \]

- **Intensity of Use**
  \[ \text{intensity of use}_{o}(s) = \sum_{b} \pi_{od}(s,b) \]
Empirical Regularities
Margins of Firms’ Sales

input sales\(_o(s) = N_o(s) \times \frac{\sum_b \pi_{od}(s,b)}{N_o(s)} \times \frac{\sum_b \pi_{od}(s,b) \times \text{input costs}_d(b)}{\sum_b \pi_{od}(s,b)} \]

[\# Customers] [Intensity per Customer] [Average Customer Size]

- Larger Indian firms (higher input sales)
  - tend to have more customers [35%]
  - tend to be used more intensively by customers [46%]
  - tend to have larger customers [19%]
Empirical Regularities
Upstream & Downstream Margins of Firms’ Sales

upstream margin $\approx 81\%$

$\text{#Customers } \times \text{Intensity per Customer } \times \text{Average Customer Size}$

downstream margin $\approx 19\%$

- **Downstream Margin** $\implies$ role of exogenous network linkages
  - choice of quantity to sell $\equiv$ downstream decision
  - downstream decision affects upstream firms $\implies$ demand shocks propagate upstream

- **Upstream Margin** [Intensity of Use] $\implies$ role of endogenous network formation
  - choice of suppliers and intensity of use $\equiv$ upstream decision
  - upstream decision affects downstream firms $\implies$ cost savings propagate downstream


Empirical Regularities
Upstream & Downstream Margins of Firms’ Sales

upstream margin≈81%

#Customers × Intensity per Customer × Average Customer Size

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Model
Overview

- Develop GE model of network formation between spatially distant firms
  - firms have multiple input requirements
  - randomly encounter potential input suppliers
  - select most cost-effective supplier for each requirement

- Low production cost firms end up larger because
  - find more customers
  - used more intensively by their customers
  - customers use cheaper inputs intensively $\rightarrow$ lower production costs
  - lower production costs $\rightarrow$ customers become larger themselves
Model Overview

- Develop GE model of network formation between spatially distant firms
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Production Function

\[ y_d(b) = z_d(b) \left( \frac{l_d(b)}{1 - \alpha_d} \right)^{1-\alpha_d} \left( \prod_{k=1}^{K_d(b)} \frac{m_d(b,k)^{1/\alpha_d}}{\alpha_d} \right)^{\alpha_d} \]

\[ m_d(b,k) = \sum_{s \in S_d(b)} m_{od}(s,b,k) \]

- \( \alpha_d \), materials share at \( d \)
- \( K_d(b) \), # tasks of \( b \)
- \( S_d(b) \), set of potential suppliers for \( b \)
Model

Marginal Cost

\[
\hat{c}_d(b) = \frac{\omega_d^{1-\alpha_d}}{z_d(b)} \times K_d(b) \prod_{k=1}^{\alpha_d} \left( \frac{p_d(b,k)}{K_d(b)} \right)
\]

effective price of task k for b

cost share of task k

Effective Price

\[
p_d(b,k) = \min_{s \in S_d(b)} \left\{ \frac{\tilde{m}_{od}(s,b,k)}{a_{od}(s,b,k)} \times \frac{\tau_{od}}{c_o(s)} \right\}
\]

markup
trade cost
seller MC
match productivity
Model
Functional Form Assumptions

\[ P(\text{s meets b}) = \frac{\lambda}{M} \quad \text{Bernoulli Encounters} \]

\[ P(a_{od}(s,b,k) \leq a) = \left(1 - \left(\frac{a}{a_0}\right)^{-\xi}\right) 1\{a > a_0\} \quad \text{Pareto Match Productivities} \]

\[ \tilde{m}_{od}(s,b,k) \sim \text{Limit Pricing} \]

\[ P(z_d(b) \leq z) = \exp\left(-T_d z^{-\theta}\right) 1\{z > 0\} \quad \theta > \zeta \quad \text{Fréchet Productivities} \]
Taking Model to Data
Network Formation → Quasi-Dynamic Programming

- **Recursive Problem**

\[ c_d(b) = \frac{w^{1-\alpha_d}}{z_d(b)} \times \prod_{k=1}^{K_d(b)} \min_{s \in S_d(b)} \left\{ \frac{\bar{m}_{od}(s,b,k)\tau_{od}}{a_{od}(s,b,k)} \times c_0(s) \right\} \]

- **Estimands** [exogenous: \( \tau_{od} \)| endogenous: \( c_d(b) \)]
  - very high-dimensional → full solution methods infeasible
  - interdependence in link formation → simulation burdensome

[Rust (1987), Anderson & van Wincoop (2003), Antràs & de Gortari (2020)]
Taking Model to Data
Quasi-Dynamic Programming \(\rightarrow\) Conditional Choice Probabilities

■ **Conditional Choice Probabilities**
[conditional on \(c_0(s)\), probability that \(s\) gets chosen for any task of any firm at \(d\)]

\[
\pi^0_{od}(s, -) = \frac{c_0(s)^{-\zeta} \tau^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{0'}(s')^{-\zeta} \tau_{0'd}^{-\zeta}}
\]

■ CCPs which depend on endogenous state \(\rightarrow\) sample analogs
[Hotz & Miller (1993) \(\rightarrow\) Menzel (2015)]
Conditional Choice Probabilities
[conditional on $c_o(s)$, probability that $s$ gets chosen for any task of any firm at $d$]

$$\pi_{od}^0(s, -) = \frac{c_o(s)^{-\zeta} \tau_{od}^{-\zeta}}{\sum_{s' \in M} c_{o'}(s')^{-\zeta} \tau_{o'd}^{-\zeta}}$$

CCPs which depend on endogenous state $\mapsto$ sample analogs
[Hotz & Miller (1993) $\mapsto$ Menzel (2015)]
Taking Model to Data
Conditional Choice Probabilities → Balls-and-Bins Model

symmetric + Cobb-Douglas tasks \(\implies\) task proportions = cost shares

\[
\pi_{od}(s,b) = 0.04, 0.5, 0.02, 0.3, 0.02, 0.08, 0.04
\]

\[
\pi_{od}(s,b) = 0, 0.5, 0, 0.25, 0, 0.25, 0
\]

all possible suppliers

suppliers of \(b\)

model

data

discrete # tasks \(\implies\) success probabilities [CCPs] = \(\mathbb{E}\) [task proportions] = \(\mathbb{E}\) cost shares
Estimation
Balls-and-Bins Model \rightarrow Multinomial Logit

- **Estimation Equation**

\[
\mathbb{E} [\pi_{od}(s,b)] = \pi^0_{od}(s,b) = \frac{c_o(s)^{-\zeta}}{\sum_{s' \in \mathcal{M}} c_{o'}(s')^{-\zeta}} \tau_{od}^{-\zeta} \tau_{o'd}^{-\zeta}
\]

- **Estimands**
  - marginal costs \( c_o(s)^{-\zeta} \equiv \text{firm fixed effects} \)
  - trade frictions \( \tau_{od}^{-\zeta} = \exp(X_{od}'\beta) \) \([X_{od} \equiv \text{distance, borders etc.}]

- natural choice since probability of sourcing adds to unity
  
Estimation
Multinomial Logit: Computational Issues

- generalized linear model + millions of fixed effects $\implies$
  - high-dimensional non-linear optimization $\rightarrow$ infeasible by Newton methods
  - incidental parameters bias in $\beta$

- not a problem!
  - multinomial likelihood score equations coincide with Poisson likelihood
    [Baker (1994) $\rightarrow$ Taddy (2015)]
  - Poisson likelihood automatically satisfies adding up constraints
    [Fally (2015)]
  - Poisson likelihood $\implies$ no bias + fixed effects in closed-form
    [Hausman, Hall & Griliches (1984)]
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Firm Fixed Effects [low marginal costs $\iff$ high intensity of use]

$$\left( c_0(s)^{-\gamma} \right)^* = \sum_{b \in \mathcal{M}} \pi_{od}(s, b)$$
Origin-Destination Fixed Effects $\rightarrow$ Structural Gravity Specification

$$\left( \frac{\exp \left( \ln \left( c_{o}^{-\xi} \right) + X'_{od} \beta \right)}{\sum_{o'} \exp \left( \ln \left( c_{o'}^{-\xi} \right) + X'_{o'd} \beta \right)} \right)^* = \frac{1}{M_d} \sum_{b \in M_d} \left( \sum_{s \in M_o} \pi_{od} (s, b) \right)_{\text{total cost share of } b \text{ from } o}$$
Counterfactual Analysis

Large Network Approximation

- **Aggregate Trade Models + Exact Hat Algebra**

  \[
  \text{model degeneracy} \implies \text{model prediction} = \text{observed data}
  \]

- **Models with Large Networks and Granularity**

  \[
  \text{model non-degeneracy} \implies \text{model prediction(s)} \neq \text{observed data}
  \]

  - observed data $\rightarrow$ estimated model $\rightarrow$ \( \mathbb{E} \left[ \text{model predictions} \mid \text{initial state} \right] \)
  - counterfactual evaluation:

  \[
  \mathbb{E} \left[ \text{model predictions} \right] = \frac{\mathbb{E} \left[ \text{model predictions} \mid \text{counterfactual state} \right]}{\mathbb{E} \left[ \text{model predictions} \mid \text{initial state} \right]} 
  \]

  [Head & Mayer (2019), Dingel & Tintelnot (2020)]
Counterfactual Analysis

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  \[ \text{model non-degeneracy} \implies \text{model prediction(s)} \neq \text{observed data} \]

  - observed data \( \rightarrow \) estimated model \( \rightarrow \) \( \mathbb{E}[\text{model predictions} | \text{initial state}] \)
  - counterfactual evaluation:

  \[ \mathbb{E}[\text{model predictions}] = \frac{\mathbb{E}[\text{model predictions} | \text{counterfactual state}]}{\mathbb{E}[\text{model predictions} | \text{initial state}]} \]

  [Head & Mayer (2019), Dingel & Tintelnot (2020)]
Decline in Border Frictions
Counterfactual Experiment

- Trade across state borders subject to frictions
  - significant border effects in gravity regressions
  - sales taxes, border inspections, logistical delays etc.
  - $141 \times 141$ symmetric matrix of inter-district Head-Ries indices,
    \[
    \sqrt{\frac{sales_{od}sales_{do}}{sales_{oo}sales_{dd}}} \rightarrow
    \]

- 10% decline in trade costs between inter-state district pairs
Trade across state borders subject to frictions
- significant border effects in gravity regressions
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Decline in Border Frictions
Macro Outcomes: Aggregate Welfare Changes
## Decline in Border Frictions

### Micro Outcomes: Changes in Margins of Firms’ Sales, Shapley Decomposition

<table>
<thead>
<tr>
<th>State</th>
<th>Maharashtra (1)</th>
<th>Tamil Nadu (2)</th>
<th>Gujarat (3)</th>
<th>West Bengal (4)</th>
<th>Odisha (5)</th>
<th>All (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ% upstream margin</td>
<td>40.76%</td>
<td>40.81%</td>
<td>36.49%</td>
<td>39.44%</td>
<td>38.06%</td>
<td>55.69%</td>
</tr>
<tr>
<td>Δ% downstream margin</td>
<td>29.37%</td>
<td>34.14%</td>
<td>45.74%</td>
<td>31.44%</td>
<td>43.02%</td>
<td>33.45%</td>
</tr>
<tr>
<td>second order term</td>
<td>29.86%</td>
<td>25.04%</td>
<td>17.76%</td>
<td>29.14%</td>
<td>18.91%</td>
<td>10.85%</td>
</tr>
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</table>

\[
\frac{\Delta \text{Sales}}{\text{Sales}} \approx \frac{\Delta \text{Intensity of Use}}{\text{Intensity of Use}} + \frac{\Delta \text{Average Customer Size}}{\text{Average Customer Size}}
\]

\[
+ \frac{\Delta \text{Intensity of Use}}{\text{Intensity of Use}} \times \frac{\Delta \text{Average Customer Size}}{\text{Average Customer Size}}
\]

\[\text{upstream margin}\]

\[\text{downstream margin}\]
Conclusion

- Documented importance of endogenous networks towards firm heterogeneity
- Developed tractable model of endogenous spatial production networks
- Proposed scalable framework for structural estimation + counterfactual analysis
- Reducing border frictions
  - improves welfare across Indian districts in the range [1%, 8%]
  - > 1/2 firm-level changes from endogenous network changes
- Extensions:
  Supply Chain Dynamics, Search Frictions, Innovation Spillovers, Factor Market Frictions, Industry Dynamics
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