Candidate Info

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Subfields: Economics of Information, Game Theory, Micro Theory, Applied Theory and Strategy

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Job Preferences: Private sector, Government, Academia (US east coast preferred)

Persuasion Through Trial Design

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Introduction

- In Bayesian persuasion models, a sender commits to an information structure.
- Ex: Pharma firms conduct studies to earn FDA approval.
  - Hypothesis test has type I error rate α, type II error rate β, target significance level p^*
  - Chance of significant result maximized when $\alpha = p^*$, $\beta = 0$
- In the real world, the sender may have limited control.

Introduction

- My model: a researcher wants to persuade a policymaker to adopt her treatment.
- She can only control the number of iid subjects to enroll in a trial.
    - Under pre-registration, the researcher commits to sample size ex ante.
    - Under sequential sampling, the researcher observes each subject outcome before deciding whether to enroll the next subject.

Introduction

- Pre-registration is common in medicine using sites like clinicaltrials.gov.
- Even in fields without formal pre-registration, researchers often commit to sample sizes at outset of trials.
- Sequential sampling can provide one avenue for experimenter bias.
- To quantify this, compare against the Bayesian persuasion benchmark.

Research Question + Preview of Results

What outcomes can be induced under pre-registration / sequential sampling?

- As subject outcomes become arbitrarily uninformative...
  - under SS, researcher payoff approaches first-best, and policymaker payoff approaches first-worst.
  - under PR, optimal trial approaches full revelation, and policymaker payoff approaches first-best.
- However, when subject outcomes are highly informative, the policymaker may prefer sequential sampling.

Related Literature and Contribution

- Bayesian persuasion: Kamenica and Gentzkow (2011), many others
  - My model explores which BP outcomes can be induced in a simple model of trial design.
- Bayesian persuasion through sequential sampling: Brocas and Carillo (2007), Morris and Strack (2019), Henry and Ottaviani (2019)
  - My paper is the first to study how the set of inducible outcomes differs under pre-registration.

Model

- State is $\omega \in \{0, 1\}$, with $Pr(\omega = 1) = \mu \in (0, 1)$
- Policymaker must choose $a \in \{0, 1\}$ and earns payoff $u(a, \omega)$, parameterized below
- He maximizes EU, chooses action 1 when $Pr(\omega = 1|\cdot) \geqslant z$

$u(a, \omega)$	$a = 0$	$a = 1$
$\omega = 0$	0	$-z$
$\omega = 1$	0	$1 - z$

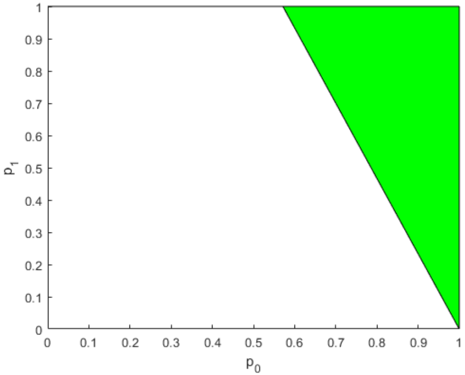
for some $z \in (\mu, 1)$

Model

- Researcher earns utility $v(a) = a$
- Before policymaker acts, researcher conducts public trial to maximize her EU
- Characterize trials by induced action distributions $p = (p_0, p_1) = (Pr(a = 0|\omega = 0), Pr(a = 1|\omega = 1))$
  - Researcher EU: $V(p) = \mu p_1 + (1 - \mu)(1 - p_0)$
  - Policymaker EU: $U(p) = \mu(1 - z)p_1 - (1 - \mu)z(1 - p_0)$

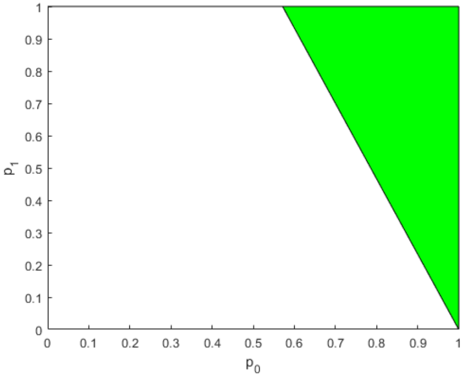
Bayesian Persuasion

- p_0 = prob of rejecting bad treatment, p_1 = prob of adopting good treatment
- Bayesian persuasion can induce any (p_0, p_1) subject to incentive-compatibility: $\frac{\mu p_1}{\mu p_1 + (1-\mu)(1-p_0)} \geqslant z$



Bayesian Persuasion

- Policymaker's favorite point is upper-right hand corner, full revelation
- Researcher's favorite point is upper-left corner, inequality binds, and $V^{BP} = \mu + \frac{\mu(1-z)}{z}$, $U^{BP} = 0$



Model

- Now suppose researcher can only control size of trial
- Trial consists of some number of treated subjects, under either *pre-registration* or *sequential sampling* regime
- Each subject either improves (s_1) or not (s_0), with distribution

	$s = s_0$	$s = s_1$
$\omega = 0$	ρ	$1 - \rho$
$\omega = 1$	$1 - \rho$	ρ

for some $\rho \in (.5, 1)$

Trial Design

- Likelihood of seeing good outcome in bad state is same as seeing bad outcome in good state
- Successes and failures "cancel out"
- Instead of # of successes x, look at difference $d = x - (n - x)$

Trial Design

- When d positive, posterior given by $Pr(\omega = 1|d) = \frac{\mu\rho^d}{\mu\rho^d+(1-\mu)(1-\rho)^d}$

- When d negative, posterior given by $Pr(\omega = 1|d) = \frac{\mu(1-\rho)^{-d}}{\mu(1-\rho)^{-d}+(1-\mu)\rho^{-d}}$

- Policymaker adopts if $d \geqslant d^* = \left\lceil \frac{\ln(\frac{1-\mu}{\mu}\frac{z}{1-z})}{\ln(\frac{\rho}{1-\rho})} \right\rceil \geqslant 0$

Sequential Sampling

- Under sequential sampling, researcher's choice is over stopping rules

- Each stopping rule T is associated with some induced $p(T)$

- Researcher's optimal stopping rule T^*: stop enrolling subjects iff $Pr(\omega = 1|s_1, ...) \geqslant z$, that is iff $d \geqslant d^*$

Sequential Sampling

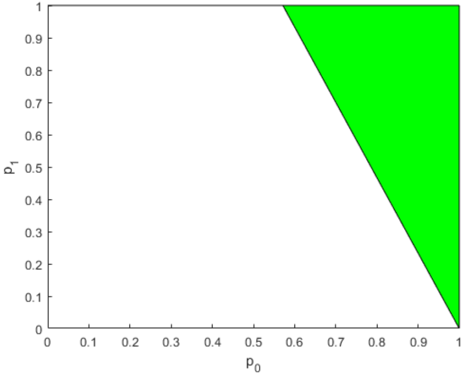
Proposition

Under sequential sampling, as $\rho \rightarrow_{+} .5$,
$V(p(T^*)) \rightarrow \mu + \frac{\mu(1-z)}{z} = V^{BP}$

- Define $\hat{z}(d^*)$ to be posterior belief after seeing difference d^* starting from prior μ
- Brocas and Carillo (2007): optimal stopping rule yields researcher payoff $\mu + \frac{\mu(1-\hat{z}(d^*))}{\hat{z}(d^*)}$
- $z \leqslant \hat{z}(d^*) \leqslant \frac{z\rho}{z\rho+(1-z)(1-\rho)}$ implies $z \leqslant \lim_{\rho \rightarrow_{+} .5} \hat{z}(d^*) \leqslant z$

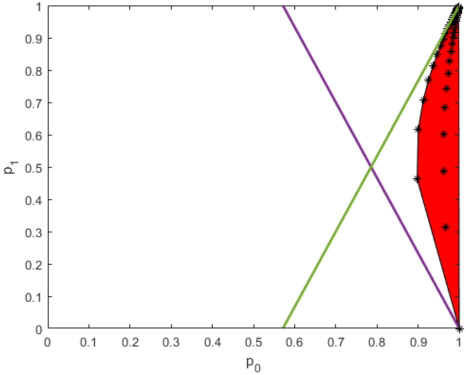
Bayesian Persuasion

- Corollary: researcher's trial approaches upper-left corner
- Policymaker payoff approaches 0
- Sender can approach any BP outcome



Pre-registration

- Under pre-registration, researcher chooses sample size $n \in \{0, 1, ..., \infty\}$, and can randomize.
- Each choice of n induces some $p(n)$.
- Below: $\mu = .3$, $z = .5$, $\rho = .68(d^* = 2)$



Pre-Registration

Proposition

Under pre-registration, for any $n(\rho)$, as $\rho \to_+ .5$, $p_0(n(\rho)) \to 1$

- From Hoeffding's inequality:
  $1 - p_0 = Pr(d \geqslant d^* | \omega = 0, n) \leqslant e^{-2(\frac{1}{2} + \frac{d^*}{2n} - (1-\rho))^2 n}$
- From first derivative: bound gets tighter as n increases
- When $n = d^*$, bound approaches 0 as $\rho \to_+ .5$

Full Proof

Pre-Registration

- As $p_0 \to 1$, researcher can do no better than full revelation

- This uniquely maximizes policymaker welfare

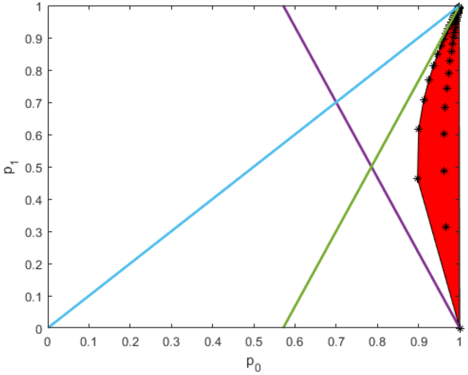
Pre-Registration

Proposition

Under pre-registration, $p_1(n) \leqslant p_0(n)$ for all n

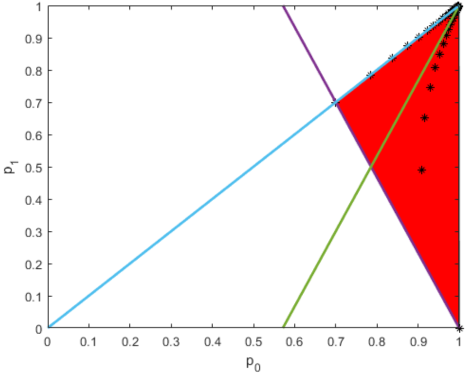
Proof:

- $p_1 = Pr(d \geqslant d^*|n, \omega = 1) = Pr(d \leqslant -d^*|n, \omega = 0) \leqslant p_0$



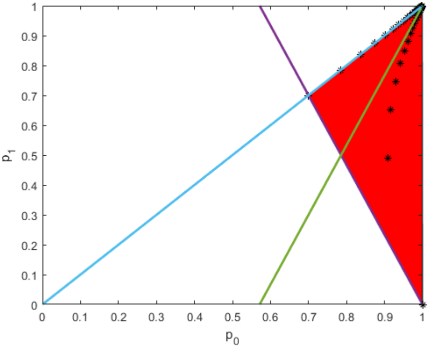
Pre-Registration

- Bound attainable when $\rho^* = \frac{\frac{z(1-\mu)}{(1-z)\mu}}{1+\frac{z(1-\mu)}{(1-z)\mu}}$, seeing $d = 1$ makes policymaker indifferent between actions
- If $\rho \geqslant \rho^*$ and $n = 1$, $p_0 = \rho = p_1$



Pre-Registration

- Researcher's optimal p determined by indifference curve
  - When $\mu < .5$ and $\rho > \rho^*$, $n = 1$ is optimal for researcher
  - When $\mu > .5$, full revelation is optimal for researcher $\forall \rho$
- Different bias levels lead to different slopes of (linear) IC



Discussion

- When $\rho > \rho^*$ and $\mu < .5$, policymaker prefers sequential sampling
  - Under pre-registration, researcher will choose $n = 1$
  - Under sequential sampling, audience may see more info

Conclusion

- Requiring researcher to commit to a sample size can greatly affect inducible outcomes and policymaker welfare.

- Policymaker prefers pre-registration when ρ small or $\mu > .5$, prefers sequential sampling when ρ large and $\mu < .5$.

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Proposition: Under PR, for any $n(\rho)$, as $\rho \rightarrow_{+} .5$, $p_0(n(\rho)) \rightarrow 1$

- Hoeffding's inequality states that if $X_i \in [0,1]$ independent $\forall i$, then $Pr(\sum_i X_i - E[\sum_i X_i] \geqslant t * n) \leqslant e^{-2nt^2}$

- Define $X_i = 1$ if subject i improves, $X_i = 0$ otherwise

- $Pr(d \geqslant d^* | n, \omega) = Pr(x \geqslant \frac{n+d^*}{2} | n, \omega)$, where $x = \sum_i X_i$

- Write $1 - p_0 = Pr(x \geqslant \frac{n+d^*}{2} | \omega = 0) = Pr(x - n(1-\rho) \geqslant \frac{n+d^*}{2} - n(1-\rho) | \omega = 0) = Pr(x - n(1-\rho) \geqslant \frac{\frac{n+d^*}{2} - n(1-\rho)}{n} * n | \omega = 0) \leqslant e^{-2(\frac{1}{2} + \frac{d^*}{2n} - (1-\rho))^2 n}$

Back

Proof of Proposition

- Have $1 - p_0 \leqslant e^{-2(\frac{1}{2}+\frac{d^*}{2n}-(1-\rho))^2 n}$

- $\frac{d}{dn}[-2(\frac{1}{2}+\frac{d^*}{2n}-(1-\rho))^2 n] = -2(\frac{1}{2}+\frac{d^*}{2n}-(1-\rho))[\frac{1}{2}-(1-\rho)]$

- Since $(1-\rho) < \frac{1}{2}$, derivative < 0, and $[-2(\frac{1}{2}+\frac{d^*}{2n}-(1-\rho))^2 n] < 0$

Proof of Proposition

- When $n = d^*$, bound becomes $1 - p_0 \leqslant e^{-2(\frac{1}{2}+\frac{1}{2}-(1-\rho))^2 d*}$
- Approaches 0 as $\rho \rightarrow_+ .5$
- For larger n the bound is tighter; smaller n never chosen
- Hence regardless of choice of n, $1 - p_0 \rightarrow 0$ as $\rho \rightarrow_+ .5$.