Imperfect Exchange Rate Expectations

Giacomo Candian[1] Pierre De Leo[2]

[1]HEC Montreal

[2]University of Maryland

Imperfect exchange rate expectations

Exchange rates are forward-looking & carry predictable excess returns

High interest rate currencies depreciate too little in short run

(UIP puzzle, Fama 84)

. . . and depreciate too much in the medium run

(UIP puzzle reversal, Engel 16, Valchev 20)

Imperfect exchange rate expectations

Exchange rates are forward-looking & carry predictable excess returns

High interest rate currencies depreciate too little in short run

(UIP puzzle, Fama 84)

...and depreciate too much in the medium run

(UIP puzzle reversal, Engel 16, Valchev 20)

Do agents' expectational errors explain exchange rate dynamics?

- Use survey data to inform theories of expectation formation

(Coibion & Gorodnichenko 15, Angeletos, Huo & Sastry 20)

Imperfect exchange rate expectations

Exchange rates are forward-looking & carry predictable excess returns

High interest rate currencies depreciate too little in short run

(UIP puzzle, Fama 84)

. . . and depreciate too much in the medium run

(UIP puzzle reversal, Engel 16, Valchev 20)

Do agents' expectational errors explain exchange rate dynamics?

- Use survey data to inform theories of expectation formation

(Coibion & Gorodnichenko 15, Angeletos, Huo & Sastry 20)

Is there a common thread across macro and exchange rate expectations?

This paper

1. FX expectations display both under and over reaction
    - Isolate forecast errors from excess returns in UIP regressions
    - Under reaction & over reaction account for UIP puzzle & reversal

This paper

1. FX expectations display both under and over reaction
   - Isolate forecast errors from excess returns in UIP regressions
   - Under reaction & over reaction account for UIP puzzle & reversal
2. Identify belief distortions consistent with under and over reaction
   - Simple model with biased beliefs on interest rate differential
   - Validate mechanism using interest rate expectations

This paper

1. FX expectations display both under and over reaction
    - Isolate forecast errors from excess returns in UIP regressions
    - Under reaction & over reaction account for UIP puzzle & reversal
2. Identify belief distortions consistent with under and over reaction
    - Simple model with biased beliefs on interest rate differential
    - Validate mechanism using interest rate expectations
3. Develop GE model consistent with macro & FX forecast data
    - Belief distortions on TFP and demand shocks
        - disciplined by macro survey forecast data (output & inflation)
    - Reproduces several open-economy stylized facts
        - UIP puzzle & reversal, Excess volatility, Backus-Smith, PPP, etc
    - Rationalizes switch in UIP deviations post global financial crisis
        - Forecast errors are endogenous to monetary policy

Related literature

Empirics:

- UIP regressions: Fama 84, Hassan & Mano 15, Engel 16, Valchev 20
- Survey FX expectations: Frankel & Froot 87 89, Bacchetta, Mertens & Van Wincoop 09, Stavrakeva & Tang 20a, 20b, Frankel & Chinn 19, Kalemli-Ozcan & Varela 20
- Survey macro expectations: Coibion & Gorodnichenko 15, Kohlhas & Walther 20, Bordalo, Gennaioli, Ma & Schleifer 20, Angeletos, Huo & Sastry 20

Theory:

- FX puzzles under FIRE: Engel 16, Valchev 20, Bacchetta & Van Wincoop 20, Chernov & Creal 20, Dahlquist & Panasse 20, Itskhoki & Mukhin 19 20
- FX puzzles w/o FIRE: Gourinchas & Tornell 06, Bacchetta & Van Wincoop 06, Burnside, Han, Hirshleifer, & Wang 11, Ilut 12, Yu 13, Stavrakeva & Tang 20a, Bunsupha 18
- PE theories of reversal in forecast errors: Valente, Vasudevan & Wu (2021), Molavi, Tahbaz-Salehi, Vedolin (2021)

Excess return predictability

Excess return predictability

Excess returns predictability regression:

$$\underbrace{\Delta s_{t+k} - (r_{t+k-1} - r^{\star}_{t+k-1})}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \alpha_k + \beta_k (r_t - r^{\star}_t) + \varepsilon_{t+k}$$

Excess return predictability

Excess returns predictability regression:

$$\underbrace{\Delta s_{t+k} - (r_{t+k-1} - r^{\star}_{t+k-1})}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \alpha_k + \beta_k(r_t - r^{\star}_t) + \varepsilon_{t+k}$$

Under UIP & RE:

$$\beta_k \equiv \frac{\text{Cov}\left(\Lambda_{t+k}, r_t - r^{\star}_t\right)}{\text{Var}(r_t - r^{\star}_t)} = 0 \qquad \forall k \geq 1$$

Excess return predictability

Excess returns predictability regression:

$$\underbrace{\Delta s_{t+k} - (r_{t+k-1} - r^{\star}_{t+k-1})}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \alpha_k + \beta_k(r_t - r^{\star}_t) + \varepsilon_{t+k}$$

Under UIP & RE:

$$\beta_k \equiv \frac{\text{Cov}\left(\Lambda_{t+k}, r_t - r^{\star}_t\right)}{\text{Var}(r_t - r^{\star}_t)} = 0 \qquad \forall k \geq 1$$

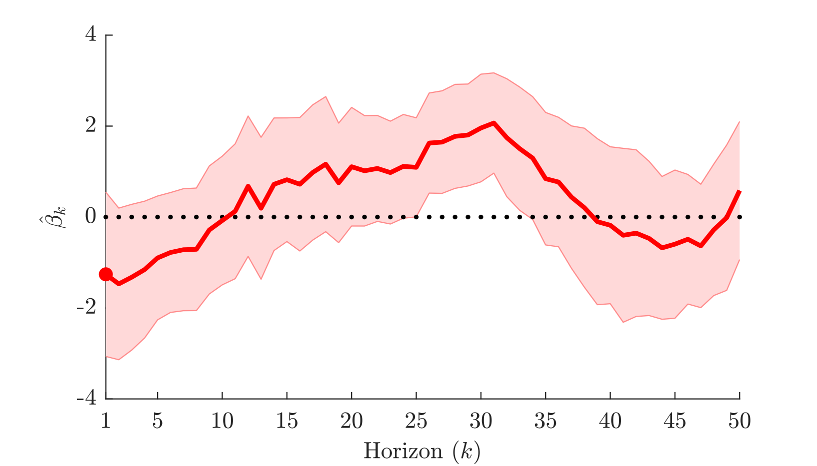
In the data:

$\hat{\beta}_k < 0$ at short k: UIP puzzle (Fama 84)

$\hat{\beta}_k > 0$ at medium k: UIP puzzle reversal (Engel 16, Valchev 20)

Excess return predictability

$$\Delta s_{t+k} - (r_{t+k-1} - r^\star_{t+k-1}) = \alpha_k + \beta_k (r_t - r^\star_t) + \varepsilon_{t+k}$$



Exchange rates: CAD, DKK, DEM-EUR, JPY, NOK, SEK, CHF, GBP

Time periods: 1990:m1-2007:m12

Full sample

Decomposition of excess returns

Decompose excess returns: (Frankel & Froot 89)

$$\underbrace{\Delta s_{t+k} - \left(r_{t+k-1} - r^{\star}_{t+k-1}\right)}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \underbrace{\left(s_{t+k} - E^{s}_{t+k-1} s_{t+k}\right)}_{\text{Forecast error}} - \underbrace{\xi_{t+k-1}}_{\substack{\text{Expected return} \\ E^{s}_{t+k-1}\Lambda_{t+k}}}$$

Decomposition of excess returns

Decompose excess returns: (Frankel & Froot 89)

$$\underbrace{\Delta s_{t+k} - (r_{t+k-1} - r^{\star}_{t+k-1})}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \underbrace{\left(s_{t+k} - E^{s}_{t+k-1} s_{t+k}\right)}_{\text{Forecast error}} - \underbrace{\xi_{t+k-1}}_{\substack{\text{Expected return} \\ E^{s}_{t+k-1}\Lambda_{t+k}}}$$

Decompose β_ks:

$$\underbrace{\frac{\mathrm{Cov}\left(\Lambda_{t+k}, r_t - r^{\star}_t\right)}{\mathrm{Var}(r_t - r^{\star}_t)}}_{\beta_k} = \underbrace{\frac{\mathrm{Cov}\left(s_{t+k} - E^{s}_{t+k-1} s_{t+k}, r_t - r^{\star}_t\right)}{\mathrm{Var}(r_t - r^{\star}_t)}}_{\beta^{s}_k} - \underbrace{\frac{\mathrm{Cov}\left(\xi_{t+k-1}, r_t - r^{\star}_t\right)}{\mathrm{Var}(r_t - r^{\star}_t)}}_{\beta^{\xi}_k}$$

Decomposition of excess returns

Decompose excess returns: (Frankel & Froot 89)

$$\underbrace{\Delta s_{t+k} - (r_{t+k-1} - r^\star_{t+k-1})}_{\substack{\text{Excess foreign-currency return} \\ \Lambda_{t+k}}} = \underbrace{\left(s_{t+k} - E^s_{t+k-1} s_{t+k}\right)}_{\text{Forecast error}} - \underbrace{\xi_{t+k-1}}_{\substack{\text{Expected return} \\ E^s_{t+k-1}\Lambda_{t+k}}}$$

Decompose β_ks:

$$\underbrace{\frac{\text{Cov}\left(\Lambda_{t+k}, r_t - r^\star_t\right)}{\text{Var}(r_t - r^\star_t)}}_{\beta_k} = \underbrace{\frac{\text{Cov}\left(s_{t+k} - E^s_{t+k-1} s_{t+k}, r_t - r^\star_t\right)}{\text{Var}(r_t - r^\star_t)}}_{\beta^s_k} - \underbrace{\frac{\text{Cov}\left(\xi_{t+k-1}, r_t - r^\star_t\right)}{\text{Var}(r_t - r^\star_t)}}_{\beta^\xi_k}$$

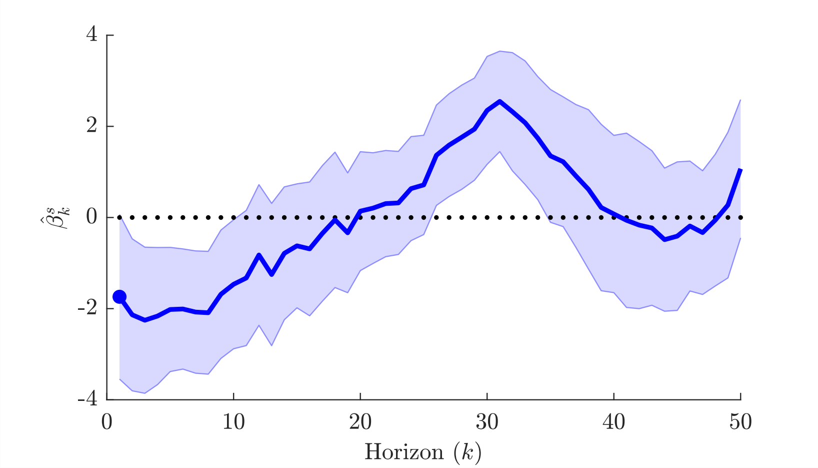
Predictability regression on forecast errors:

$$\underbrace{\left(s_{t+k} - E^s_{t+k-1} s_{t+k}\right)}_{\text{Forecast error}} = \alpha^s_k + \beta^s_k (r_t - r^\star_t) + \nu_{t+k}$$

Data: 3m exchange rate consensus forecast from *Consensus Economics*

Forecast error predictability

$$s_{t+k} - E^s_{t+k-1} s_{t+k} = \alpha_k + \beta^s_k (r_t - r^\star_t) + \varepsilon_{t+k}$$



Exchange rates: CAD, DKK, DEM-EUR, JPY, NOK, SEK, CHF, GBP

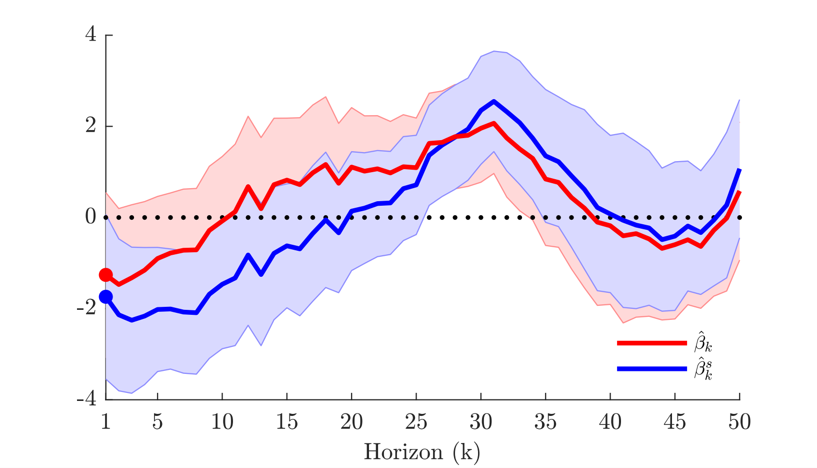
β^s by country

Time periods: 1990:m1-2007:m12 Source: *Consensus Economics*

Full sample

Excess return vs. Forecast error predictability

$$s_{t+k} - E^s_{t+k-1} s_{t+k} = \alpha_k + \beta^s_k (r_t - r^\star_t) + \varepsilon_{t+k}$$



Full sample

Implications for behavioral theories of exchange rates

Model: Interest rate process

Standard foreign-exchange no-arbitrage condition:

$$E_t^s s_{t+1} - s_t = \underbrace{r_t - r_t^\star}_{x_t} - \xi_t$$

Model: Interest rate process

Standard foreign-exchange no-arbitrage condition:

$$E_t^s s_{t+1} - s_t = \underbrace{r_t - r_t^\star}_{x_t} - \xi_t$$

Assume x_t follows an AR(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t \qquad\qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Model: Interest rate process

Standard foreign-exchange no-arbitrage condition:

$$E_t^s s_{t+1} - s_t = \underbrace{r_t - r_t^\star}_{x_t} - \xi_t$$

Assume x_t follows an AR(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Agents observe x_t. Agents' beliefs about the process for x_t:

$$x_t = z_t + \nu_t \qquad \nu_t \sim \mathcal{N}(0, \tilde{\sigma}_\nu^2)$$

$$z_t = \rho z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Two sources of belief distortion:

1. $\tilde{\sigma}_\nu^2 > 0$: agents perceive x_t to be more transitory than it actually is
2. $\tilde{\rho} > \rho$: agents perceive ε_t to be too persistent

Model: Interest rate process

Standard foreign-exchange no-arbitrage condition:

$$E_t^s s_{t+1} - s_t = \underbrace{r_t - r_t^\star}_{x_t} - \xi_t$$

Assume x_t follows an AR(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Agents observe x_t. Agents' beliefs about the process for x_t:

$$x_t = z_t + \nu_t \qquad \nu_t \sim \mathcal{N}(0, \tilde{\sigma}_\nu^2)$$

$$z_t = \rho z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Two sources of belief distortion:

1. $\tilde{\sigma}_\nu^2 > 0$: agents perceive x_t to be more transitory than it actually is
2. $\tilde{\rho} > \rho$: agents perceive ε_t to be too persistent

Model: Interest rate process

Standard foreign-exchange no-arbitrage condition:

$$E_t^s s_{t+1} - s_t = \underbrace{r_t - r_t^\star}_{x_t} - \xi_t$$

Assume x_t follows an AR(1) process:

$$x_t = \rho x_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Agents observe x_t. Agents' beliefs about the process for x_t:

$$x_t = z_t + \nu_t \qquad \nu_t \sim \mathcal{N}(0, \tilde{\sigma}_\nu^2)$$

$$z_t = \tilde{\rho} z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

Two sources of belief distortion:

1. $\tilde{\sigma}_\nu^2 > 0$: agents perceive x_t to be more transitory than it actually is
2. $\tilde{\rho} > \rho$: agents perceive ε_t to be too persistent

Evolution of beliefs

Subjective expectations about the interest differential evolve according to:

$$E_t^s x_{t+1} = (1-\kappa)\tilde{\rho} E_{t-1}^s x_t + \kappa \tilde{\rho} x_t$$

The Kalman gain κ and the conditional variance Σ at steady state:

$$\kappa = \frac{\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2}{\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2 + \tilde{\sigma}_\nu^2}; \qquad \Sigma = (1-\kappa)(\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2)$$

Note: $\tilde{\sigma}_\nu^2 = 0 \Rightarrow \kappa = 1$

Kalman filter

Equilibrium exchange rate

Equilibrium exchange rate

$$s_t = -\sum_{j=0}^{\infty} E_t^s x_{t+j} + \sum_{j=0}^{\infty} E_t^s \xi_{t+j} + \lim_{T\to\infty} E_t^s s_{t+T}$$

Equilibrium exchange rate

Equilibrium exchange rate

$$s_t = -\sum_{j=0}^{\infty} E_t^s x_{t+j} + \sum_{j=0}^{\infty} E_t^s \xi_{t+j} + \lim_{T\to\infty} E_t^s s_{t+T}$$

Simplifying assumptions (relax both in GE model)

1. $\xi_t = 0$, $\forall t$: no expected ex. ret. (e.g. risk premia, convenience yields)
2. $\lim_{T\to\infty} E_t^s s_{t+T} = 0$: stationary & correctly perceived long-run level

Equilibrium exchange rate

Equilibrium exchange rate

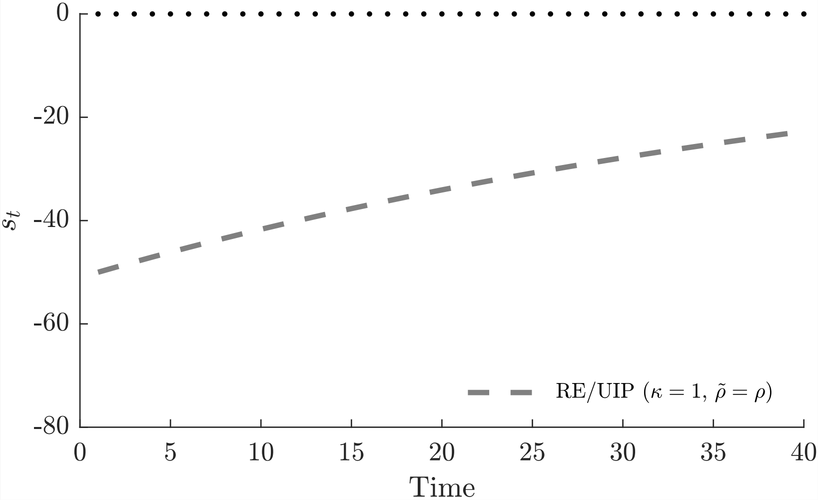
$$s_t = -\sum_{j=0}^{\infty} E_t^s x_{t+j} + \sum_{j=0}^{\infty} E_t^s \xi_{t+j} + \lim_{T\to\infty} E_t^s s_{t+T}$$

Simplifying assumptions (relax both in GE model)

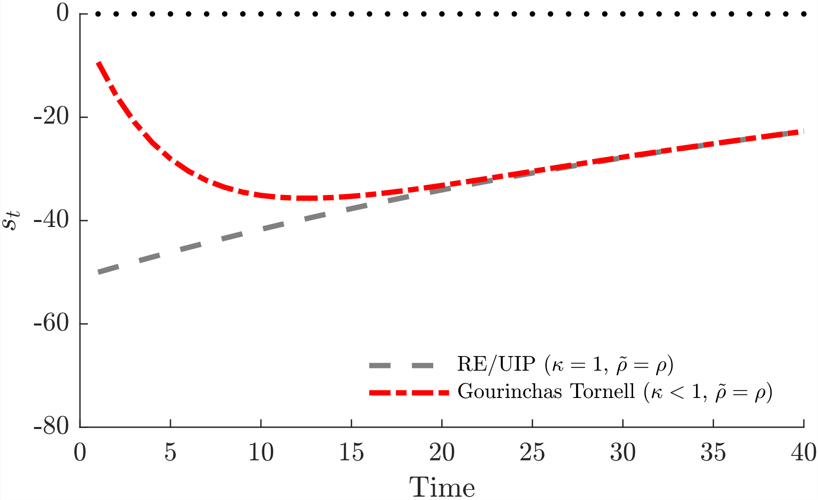
1. $\xi_t = 0, \forall t$: no expected ex. ret. (e.g. risk premia, convenience yields)
2. $\lim_{T\to\infty} E_t^s s_{t+T} = 0$: stationary & correctly perceived long-run level

$$\implies s_t = -x_t - \frac{\tilde{\rho}}{1-\rho}\kappa \sum_{i=0}^{\infty} [(1-\kappa)\tilde{\rho}]^i x_{t-i}$$

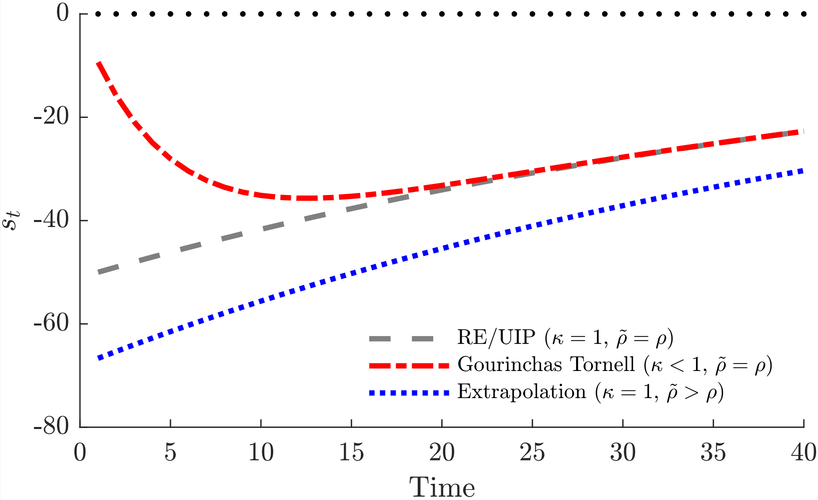
IRFs to an interest rate differential innovation



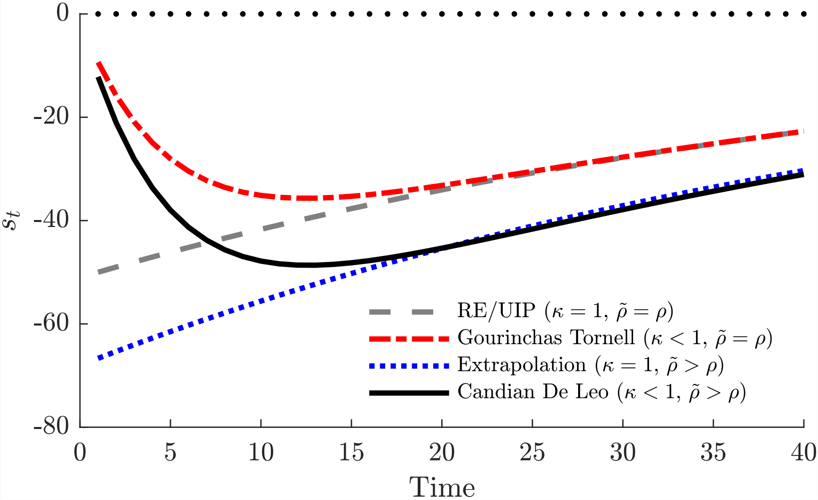
IRFs to an interest rate differential innovation



IRFs to an interest rate differential innovation

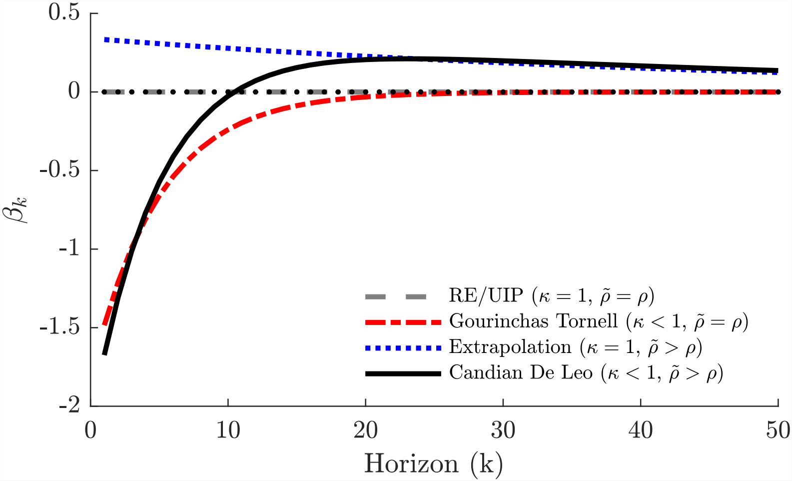


IRFs to an interest rate differential innovation



Excess returns regression

$$\Lambda_{t+k} = \alpha_k + \beta_k x_t + \varepsilon_{t+k}$$



Excess return predictability

Proposition

The following holds true for the excess return predictability coefficient β_k:

- *If $\tilde{\rho} \leq \rho$, then β_k is negative for all $k \geq 1$.*
- *If $\rho < \tilde{\rho} < \bar{\rho}(\kappa)$, then there exists a $\bar{k} > 1$ such that β_k is negative for $k < \bar{k}$ and positive for $k \geq \bar{k}$. β_k converges to zero as $k \to \infty$.*
- *If $\tilde{\rho} \geq \bar{\rho}(\kappa)$, then β_k is positive for all $k \geq 1$.*

Delayed overshooting & magnified adjustment Excess comovement

Testable implication

- Necessary condition for β_k reversal:
  Following an innovation in the interest rate differential, interest rate expectations initially under-react, while subsequently over-react.
- We verify this is a robust property of **interest rate expectations data**

Interest rate Expectations

Biased beliefs in GE

GE model: Baseline environment

Two-country New-Keynesian model Households Firms Market clearing

- Central banks target CPI inflation $r_t = \phi_\pi \pi_t \quad r_t^\star = \phi_\pi \pi_t^\star$
- Incomplete international asset markets $E_t^s \Delta s_{t+1} = r_t - r_t^\star - \xi_t$
- Households derive liquidity services from government bonds Liquidity
  - Endogenous time-varying expected excess returns ($\xi_t \propto -(r_t - r_t^\star)$)
    Nagel 16, Engel 16, Engel & Wu 20

GE model: Baseline environment

Two-country New-Keynesian model

Households Firms Market clearing

- Central banks target CPI inflation $r_t = \phi_\pi \pi_t \quad r_t^\star = \phi_\pi \pi_t^\star$
- Incomplete international asset markets $E_t^s \Delta s_{t+1} = r_t - r_t^\star - \xi_t$
- Households derive liquidity services from government bonds Liquidity
  - Endogenous time-varying expected excess returns ($\xi_t \propto -(r_t - r_t^\star)$)
    Nagel 16, Engel 16, Engel & Wu 20
- Belief distortions ($\tilde{\rho} > \rho$ and $\kappa < 1$) about: Shocks
  - TFP shocks
  - Demand (preference) shocks

$\Rightarrow$ Agents make forecast errors on endogenous variables
(e.g., interest rates, output, inflation, exchange rates, etc.)

GE model: Calibration approach

1. Calibrate standard open-economy parameters to conventional values
   + Shocks relative volatility to match $\rho(\Delta q_t, \Delta(c_t - c_t^\star)) \approx -0.20$

GE model: Calibration approach

1. Calibrate standard open-economy parameters to conventional values
   + Shocks relative volatility to match $\rho(\Delta q_t, \Delta(c_t - c_t^\star)) \approx -0.20$
2. Calibrate belief distortions to match moments of *macro* forecast errors
   2.1 Forecast revisions predict subsequent forecast errors ($\beta^{CG} > 0$)

   (Coibion & Gorodnichenko 15)

$$x_{t+3} - E_t^s x_{t+3} = \alpha^{CG} + \beta^{CG}(E_t^s x_{t+3} - E_{t-1}^s x_{t+3}) + \varepsilon_{t+3}^{CG}.$$

   2.2 Current outcomes predict subsequent forecast errors ($\beta^{KW} < 0$)

   (Kohlhas & Walther 18)

$$x_{t+3} - E_t^s x_{t+3} = \alpha^{KW} + \beta^{KW} x_t + \varepsilon_{t+3}^{KW}.$$

   We choose $x = y^{US}, \pi^{US}$ (cf Angeletos, Huo & Sastry 20)

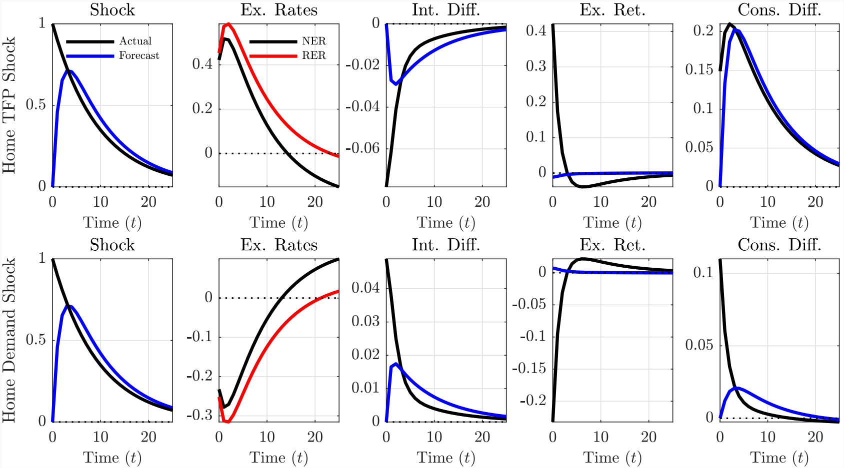
Baseline calibration

Parameter	Interpretation	Value	Parameter	Interpretation	Value
A. Preferences and Technology			ϕ_π	Monetary policy inflation response	2.50
β	Discount factor	0.99	α	Elasticity of liquidity function	0.15
σ	Risk aversion	5.00			
φ	Inverse Frisch elasticity	1.00	*B. Shocks and Beliefs*		
λ_p	1- Probability of price reset	0.75	σ_ζ/σ_a	Relative std of shocks	2.25
γ	Openness	0.05	ρ_x	Shocks persistence	0.90
θ	Trade elasticity	1.50	$\tilde{\rho}_x$	Shocks perceived persistence	0.98
δ	Intermediation cost	0.001	κ_x	Kalman gain	0.47

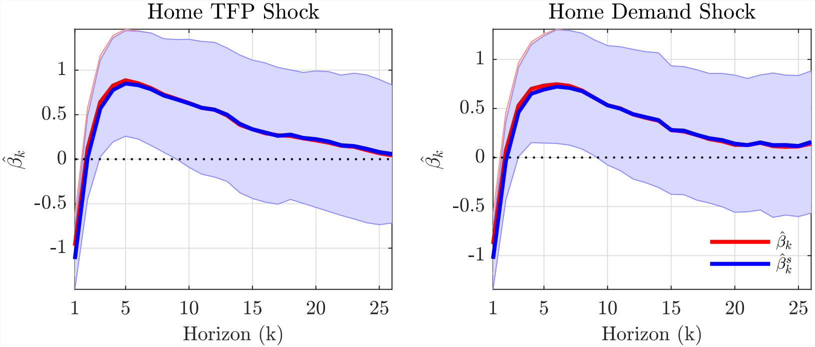
	Data	Model		Data	Model
A. Coibion-Gorodnichenko Regressions			*B. Kohlhas-Walther Regressions*		
β^{CG} Inflation	0.60 (0.53)	0.80 (0.92)	β^{KW} Inflation	-0.13 (0.08)	-0.03 (0.08)
β^{CG} Output	0.66 (0.20)	0.66 (0.24)	β^{KW} Output	-0.08 (0.05)	-0.13 (0.07)
β^{CG} Interest rate	0.64 (0.17)	0.80 (0.92)	β^{KW} Interest rate	-0.09 (0.03)	-0.03 (0.08)

Forecast errors and revisions

IRFs to a Home TFP improvement



Predictability Regressions in GE



$$\Lambda_{t+k} = \alpha_k + \beta_k(r_t - r_t^\star) + \varepsilon_{t+k}$$

Decomposition

Exchange rate dynamics in GE

Moments	Data	Model $\alpha = 0$	Model $\alpha = 0.15$
A. Exchange Rates and Macro Variables			
$\rho(\Delta s_t)$	0.16	0.29	0.28
$\sigma(\Delta s_t)/\sigma(\Delta y_t)$	6.26	3.05	3.12
$\rho(q_t)$	0.95	0.95	0.94
$\rho(\Delta q_t, \Delta s_t)$	0.99	1.00	1.00
$\rho(\Delta q_t, \Delta(c_t - c_t^\star))$	-0.17	-0.19	-0.19
$\sigma(r_t - r_t^\star)/\sigma(\Delta s_t)$	0.12	0.31	0.27
$\rho(r_t - r_t^\star)$	0.77	0.72	0.71
$\rho(\Delta c_t, \Delta c_t^\star)$	0.18	-0.05	-0.06

Moments	Data	Model $\alpha = 0$	Model $\alpha = 0.15$
B. Excess Returns Predictability			
β_1	-1.23 (0.90)	-0.81 (0.16)	-0.88 (0.20)
β_1^s	-1.74 (0.92)	-0.81 (0.16)	-1.04 (0.20)
β_1^ξ	-0.46 (0.26)	0.00 (0.00)	-0.15 (0.00)
Fama R^2	0.02	0.06	0.05
$\sum_k \beta_k^q$	> 0	> 0	> 0
C. Stavrakeva-Tang Regressions			
β^{ST}	0.17 (0.02)	0.47 (0.08)	0.48 (0.08)

The New Fama Puzzle

- Fama β turns positive after 2008 crisis ("The New Fama Puzzle")
  - Due to switch of covariance bwn forecast errors & interest differentials

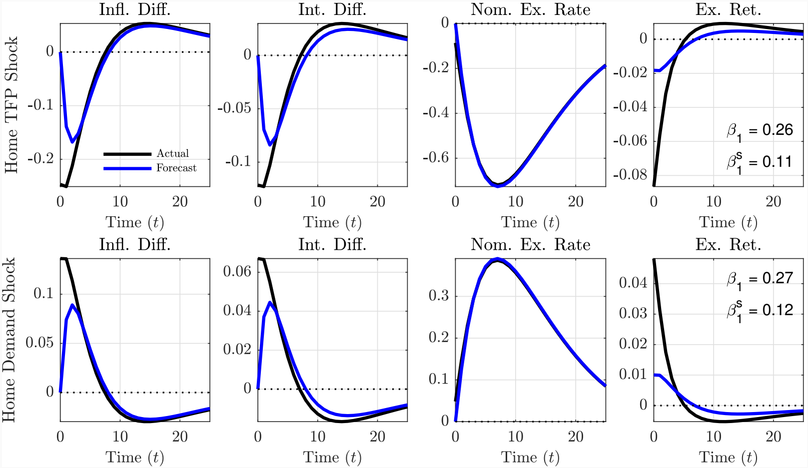
  Bussiere et al. 18

- In GE, UIP violations depend on:
  (a) agents' biases
  (b) structural relationships implied by the model

$\Rightarrow$ UIP violations are endogenous to monetary policy regime

- Our GE model can rationalize New Fama Puzzle
  (β_1^s's differ across policy regimes)
  - Interpret post-2008 passive monetary/active fiscal policy regime
    Bianchi & Melosi 17

The New Fama Puzzle: IRFs under passive monetary policy



The New Fama Puzzle: Fama β across policy regimes

	Data		Model		
Pre-2008 β_1	-1.26 (0.90)	AM/PF	-0.88 (0.20)	-0.88 (0.20)	-0.88 (0.20)
Post-2008 β_1	1.93 (2.30)	PM/AF	0.26 (0.02)	0.55 (0.17)	0.56 (0.17)
Tax rule			$\rho_\tau = 0$ $\delta_y = 0$	$\rho_\tau = 0$ $\delta_y = 0.25$	$\rho_\tau = 0.45$ $\delta_y = 0.25$

Tax rule:

$$\tilde{\tau}_t = \rho_\tau \tilde{\tau}_t + (1 - \rho_\tau) \left[\delta_b \tilde{\mathcal{B}}_{H,t-1} + \delta_y (\hat{y}_t - \hat{y}_t^n) \right]$$

Conclusions

Exchange rate expectations display both under and over reaction

- Account for UIP puzzle and reversal (Fama 84, Engel 16)
- Similar patterns in macro expectations (Angeletos et al. 20)

$\rightarrow$ provides new discipline on models of exchange rate expectations

Consistent with two-country model with biased beliefs

- Explains forecast error predictability of macro-variables & FX
- Explain many FX puzzles w/o relying on exogenous UIP shocks
- Explain switch in direction of UIP deviations post-2008

Backup slides

IRFs to interest rate innovation: Model prediction

In the simple model, exchange rate expectations under & over react because interest rate expectations under & over react to shocks

Define χ_k as the response of the 1-step-ahead **interest rate forecast error** to an innovation in the interest rate process:

$$\chi_k = \frac{\partial(x_{t+k} - E^s_{t+k-1}x_{t+k})}{\partial\varepsilon_t} \quad \forall k \geq 0.$$

IRFs to interest rate innovation: Model prediction

In the simple model, exchange rate expectations under & over react because interest rate expectations under & over react to shocks

Define χ_k as the response of the 1-step-ahead **interest rate forecast error** to an innovation in the interest rate process:

$$\chi_k = \frac{\partial(x_{t+k} - E^s_{t+k-1} x_{t+k})}{\partial \varepsilon_t} \quad \forall k \geq 0.$$

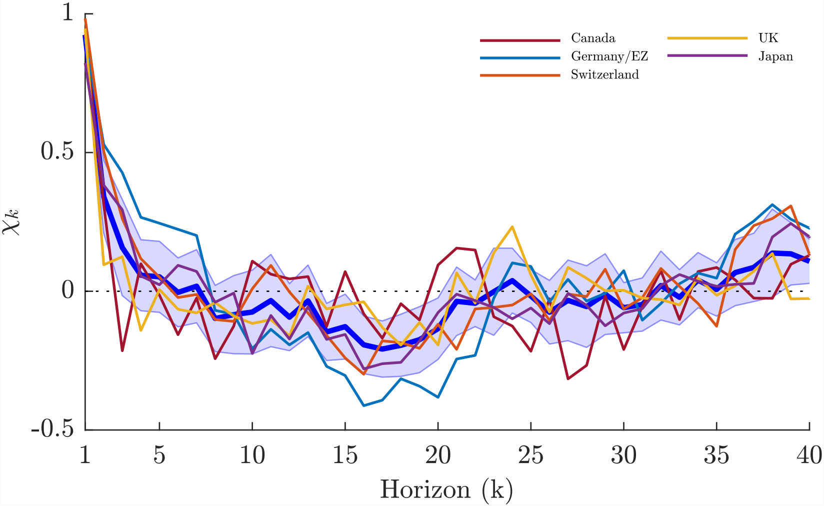
- **Testable implication:** β_k switch from negative to positive *only if* χ_k switch from positive to negative

Estimate χ_k by local projections (Jorda 05)

Model IRF of interest differential

IRFs to interest rate innovation

$$x_{t+k} - E_{t+k-1}x_{t+k} = \psi_k + \chi_k \hat{\varepsilon}_t + u_{t+k}$$



Time periods: 1990:m1-2007:m12 Source: *FX4Casts*

Full sample

Existing theories of behavioral expectations (III)

Diagnostic Expectations BGS 18, BGLS 19:

$$E_t^{\theta} x_{t+1} = E_t x_{t+1} + \theta [E_t x_{t+1} - E_{t-1} x_{t+1}]. \tag{1}$$

Lemma

Under diagnostic expectations of the form (1)

$$\beta_1 = \rho(1+\rho)\theta > 0$$
$$\beta_j = 0 \qquad \forall j > 1$$

Robust to adding recency effects.

Return

Evidence

	β_j	β^s_j	β^ξ_j
$j \leq H$	$\beta_j < 0$ Fama 84	$\beta^s_j < 0$ FF 89	$\beta^\xi_j \approx 0$
$j > H$	$\beta_j > 0$ Engel 16	$\beta^s_j > 0$ This paper	$\beta^\xi_j \approx 0$

$$\underbrace{\Delta s_{t+1} - (r_t - r^\star_t)}_{\text{excess returns } \Lambda_{t+1}} = \alpha_1 + \beta_1(r_t - r^\star_t)$$

$$\underbrace{\Delta s_{t+j+1} - (r_{t+j} - r^\star_{t+j})}_{\text{excess returns } \Lambda_{t+j+1}} = \alpha_j + \beta_j(r_t - r^\star_t)$$

$$\underbrace{\Delta s_{t+j+1} - (r_{t+j} - r^\star_{t+j})}_{\text{excess returns } \Lambda_{t+j+1}} = \underbrace{\left(s_{t+j+1} - E^s_{t+j} s_{t+j+1}\right)}_{\text{exp. errors}} - \xi_{t+j}$$

$$\underbrace{\left(s_{t+j+1} - E^s_{t+j} s_{t+j+1}\right)}_{\text{exp. errors}} = \alpha^s_j + \beta^s_j(r_t - r^\star_t)$$

Home Household

$$\max E_0^s \sum_{t=0}^{\infty}(\beta^t \zeta_t)\left[\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{L_t^{1+\varphi}}{1+\varphi}+\frac{1}{1-\gamma}\left(\frac{M_{H,t,}}{P_t}+\kappa\frac{B_{H,t}}{P_t}\right)^{1-\gamma}\right],$$

subject to

$$P_t C_t + M_{H,t} + B_{H,t+1} + B^m_{H,t+1} + S_t B^{*m}_{H,t+1} \leq$$
$$W_t L_t + M_{H,t-1} + R_{t-1} B_{H,t} + R^m_{t-1} B^m_{H,t} + S_t R^{*m}_{t-1} B^{*m}_{H,t} + \Pi_t - T_t.$$

Consumption basket:

$$C_t \equiv \left((1-\gamma)^{\frac{1}{\theta}}(C_{Ht})^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}(C_{Ft})^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \qquad \theta > 0, \gamma \in [0, 1/2)$$

Return

Home Firms

When the have a chance $(1-\lambda_p)$, set prices in domestic currency to maximize:

$$E_t^s \sum_{s=0}^{\infty} (\lambda_p)^s M_{t,t+s} \left[\tilde{P}_t(h) Y_{t+s}^d(h) - W_{t+s} L_{t+s}(h) \right],$$

subject to

$$Y_t(h) = A_t L_t(h),$$

$$Y_t^d(h) = \left(\frac{P_t(h)}{P_{Ht}} \right)^{-\nu} \left(\frac{P_{Ht}}{P_t} \right)^{-\theta} \left[(1-\gamma) C_t + \gamma \mathcal{Q}_t^{\theta} C_t^* \right],$$

Return

Shocks and Expectations

Each exogenous disturbance $x_t \in \{\log A_t, \log A_t^*, \log \zeta_t, \log \zeta_t^*\}$, follows:

$$x_t = \rho_x x_{t-1} + \varepsilon_t^x, \quad \varepsilon_t^x \sim \mathcal{N}(0, \sigma_x^2).$$

Households and firms observe the realizations of x_t but believe:

$$\begin{aligned} x_t &= z_t^x + \nu_t^x, & \nu_t^x &\sim \mathcal{N}(0, \tilde{\sigma}_{\nu,x}^2), \\ z_t^x &= \tilde{\rho}_x z_{t-1}^x + \varepsilon_t^x, & \varepsilon_t^x, &\sim \mathcal{N}(0, \sigma_x^2). \end{aligned}$$

Return

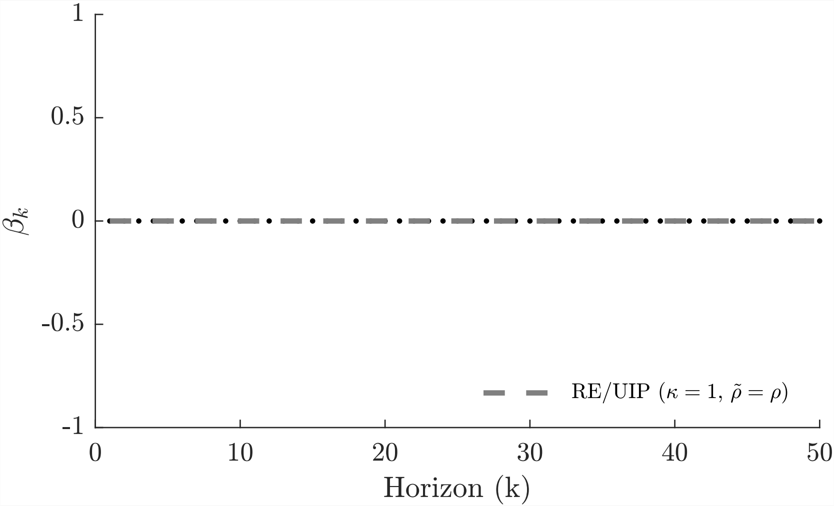
Existing theories of behavioral expectations (I)

Special case: Distorted Beliefs / Adaptive Expectations ($\tilde{\sigma}_\nu^2 > 0$ & $\tilde{\rho} = \rho$)

$$s_t - s_t^{RE} = \frac{1}{1-\rho}\Big(E_t x_{t+1} - E_t^s x_{t+1}\Big) = \frac{1}{1-\rho}\sum_{i=0}^{\infty}(\rho(1-\kappa))^{i+1}\varepsilon_{t-i}$$

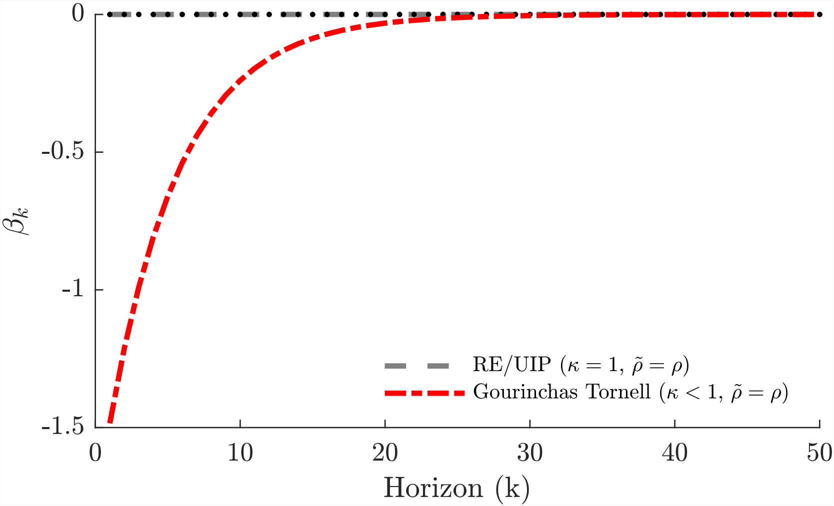
$$\beta_{j+1} = -[\rho(1-\kappa)]^j \frac{\rho(1-\rho(1-\kappa))(1-\kappa)(1+\rho)}{1-(1-\kappa)\rho^2} \leq 0 \qquad \forall j \geq 0$$

Implied βs



$$\Lambda_{t+j+1} = \alpha_j + \beta_j x_t + \varepsilon_{t+j+1}$$

Implied βs



$$\Lambda_{t+j+1} = \alpha_j + \beta_j x_t + \varepsilon_{t+j+1}$$

Existing theories of behavioral expectations (II)

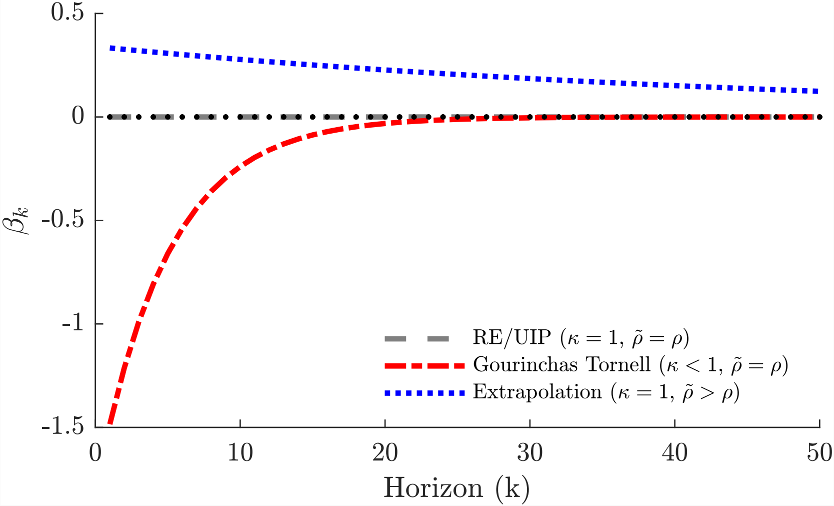
Special case: Pure Extrapolation Bias. ($\tilde{\sigma}_\nu^2 = 0$ & $\tilde{\rho} > \rho$)

$$s_t = -\frac{1}{1-\tilde{\rho}} x_t = \underbrace{\frac{1-\rho}{1-\tilde{\rho}}}_{>1} s_t^{RE}$$

$$\beta_j = \rho^j \frac{\tilde{\rho}-\rho}{1-\tilde{\rho}} > 0$$

Diagnostic Expectations

Implied βs



$$\Lambda_{t+j+1} = \alpha_j + \beta_j x_t + \varepsilon_{t+j+1}$$

Rational expectations equilibrium

Special case: Rational Expectations ($\tilde{\sigma}_\nu^2 = 0$ & $\tilde{\rho} = \rho$)

$$s_t^{RE} = -\frac{1}{1-\rho} x_t$$

Lemma

Under rational expectations, excess returns are unpredictable at all horizons:

$$\beta_j \equiv \frac{\mathrm{Cov}\left(\Lambda_{t+j+1}, x_t\right)}{\mathrm{Var(x_t)}} = 0 \qquad \forall j \geq 0.$$

UIP puzzle and reversal

Proposition

The Fama predictability coefficient in the model is:

$$\beta_1 = \frac{1-(1-\kappa)\tilde{\rho}}{1-\tilde{\rho}}\left(\frac{\tilde{\rho}\kappa}{1-(1-\kappa)\tilde{\rho}\rho} - \rho\right)$$

The Fama coefficient is negative as long as $\tilde{\rho} < \left[(1-\kappa)\rho + \frac{\kappa}{\rho}\right]^{-1} = \bar{\rho}$*.*
This condition is likely to be satisfied if $\tilde{\rho}$ *is not much larger than* ρ*.*

Delayed overshooting

Delayed overshooting with magnified adjustment

(Eichenbaum & Evans, 1995, Engel, 2016)

Proposition

There is delayed overshooting at horizon τ if and only if $\frac{\sigma_\nu^2}{\sigma_\varepsilon^2} > 0$ and:

$$\tau < \frac{\ln\left(\frac{1-\rho}{\kappa\tilde{\rho}(1-\tilde{\rho}(1-\kappa))}\left((1-\tilde{\rho})\left[1-(1-\kappa)\frac{\tilde{\rho}}{\rho}\right]+\kappa\tilde{\rho}\right)\right)}{\ln\left((1-\kappa)\frac{\tilde{\rho}}{\rho}\right)} = \bar{\tau}$$

Return

Engel (2016) puzzle

The Engel puzzle says that high interest rate currencies tend to be strong relative to the UIP exchange rate. Formally, this can be expressed as:

$$\text{Cov}(s_t - s_t^{IP}, x_t) < 0 \qquad \sum_{k=0}^{\infty} \beta_{k+1} > 0$$

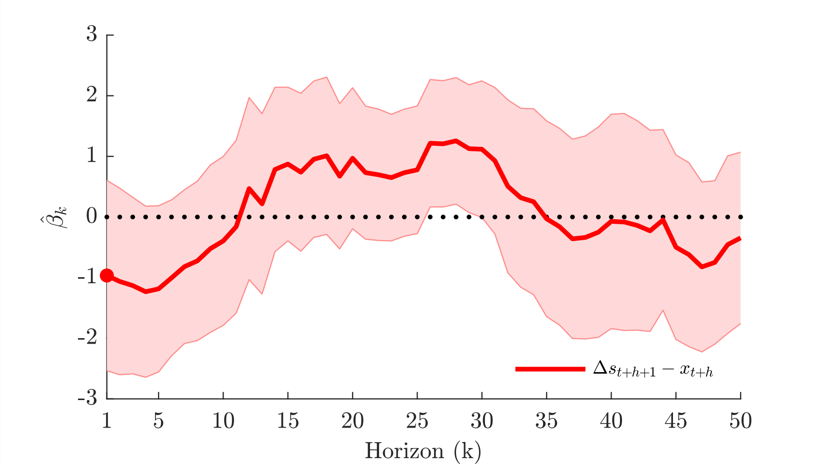
Proposition

The model with distorted beliefs and extrapolation explains the Engel puzzle as long as:

$$\tilde{\rho} - \rho > \frac{\tilde{\rho}\kappa(1-\rho)}{1-(1-\kappa)\tilde{\rho}} \left(\frac{(1-\kappa)\tilde{\rho}(\rho^{-1}-\rho)}{1-(1-\kappa)\tilde{\rho}\rho} \right) > 0. \tag{2}$$

Return

Evidence from excess returns predictability regressions

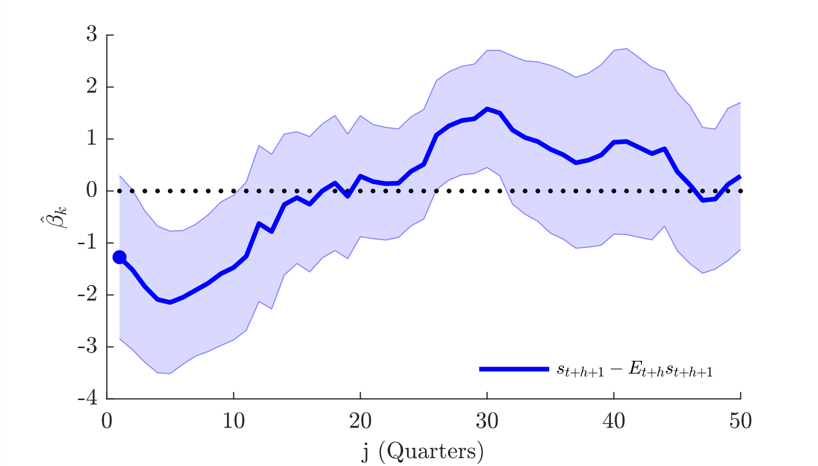


Exchange rates: CAD, DKK, DEM-EUR, JPY, NOK, SEK, CHF, GBP

Time periods: 1990:m1-2019:m7

Return

Forecast errors predictability



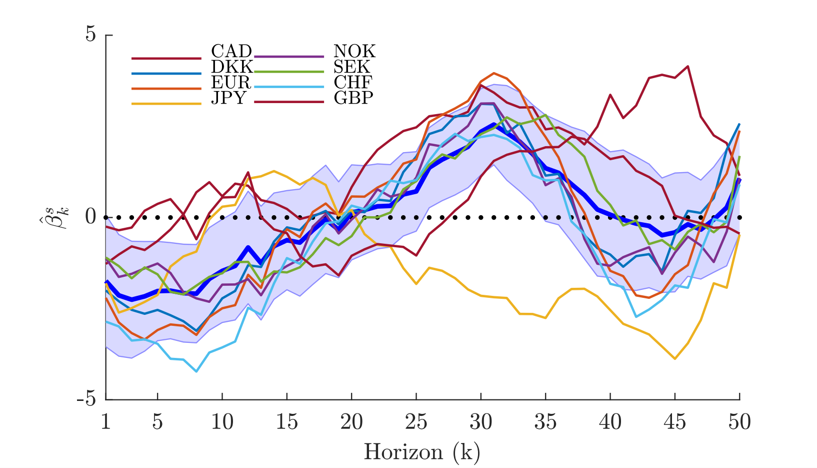
Exchange rates: CAD, DKK, DEM-EUR, JPY, NOK, SEK, CHF, GBP

Time periods: 1990:m1-2019:m7 Source: *Consensus Economics*

Return

Forecast error predictability

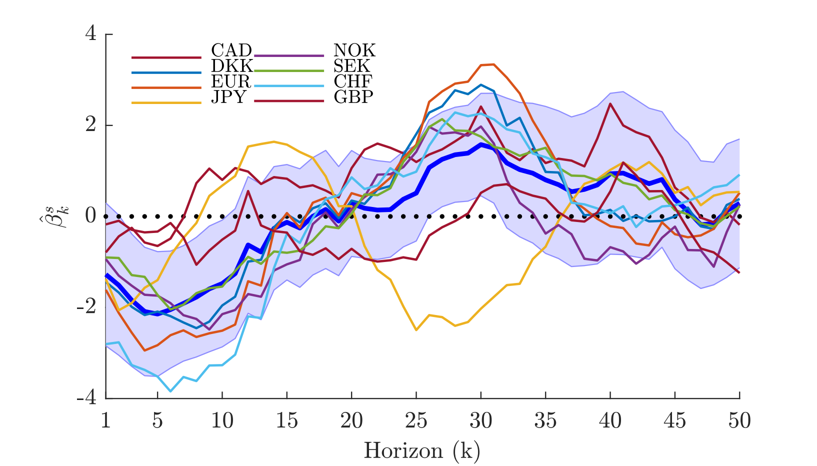
$$s_{t+k} - E^s_{t+k-1}s_{t+k} = \alpha_k + \beta^s_k(r_t - r^\star_t) + \varepsilon_{t+k}$$



blank

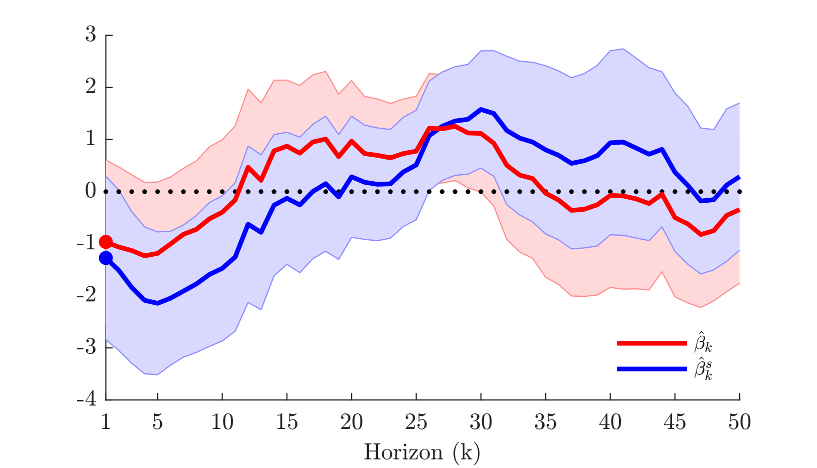
Full sample
Return

Forecast errors predictability



Return

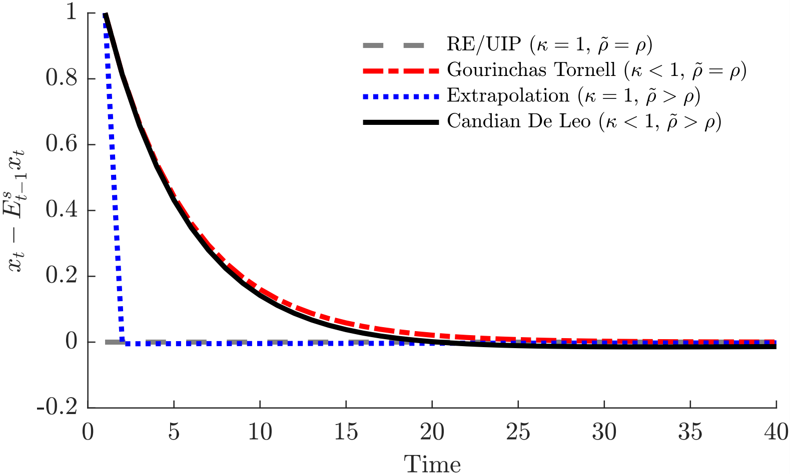
Excess returns vs. Forecast errors predictability



blank blank

Return

IRFs to interest rate innovation: Model



Return

Market Clearing and Equilibrium

1. Labor markets clear

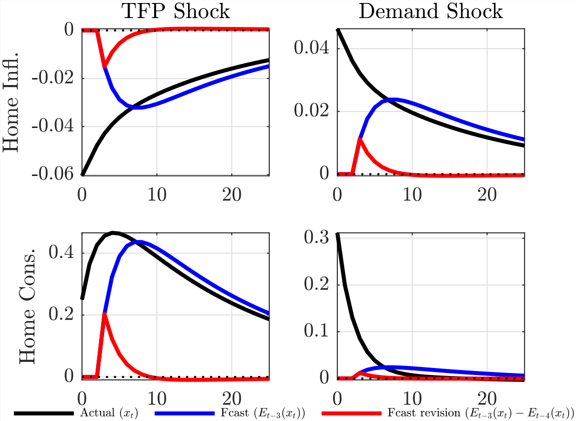
2. Goods market clear:

$$Y_{Ht} = \left(\frac{P_{Ht}}{P_t}\right)^{-\theta} \left[(1-\gamma)C_t + \gamma \mathcal{Q}_t^{\theta} C_t^*\right],$$
$$Y_{Ft} = \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\theta} \left[\gamma \mathcal{Q}_t^{-\theta} C_t + (1-\gamma) C_t^*\right].$$

3. Asset markets clear.

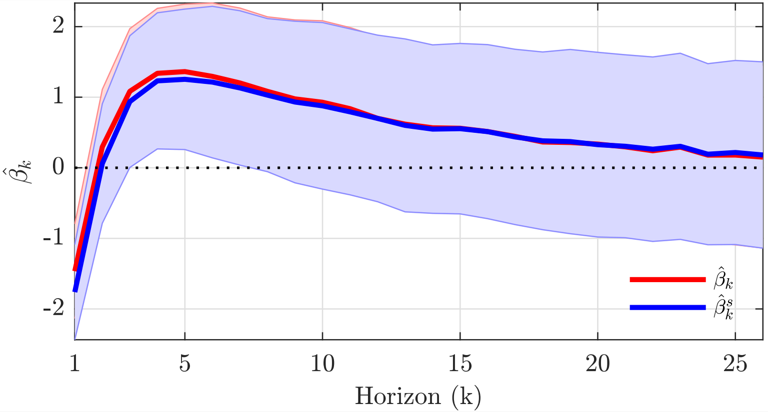
Return

IRFs of Forecasts and Revisions



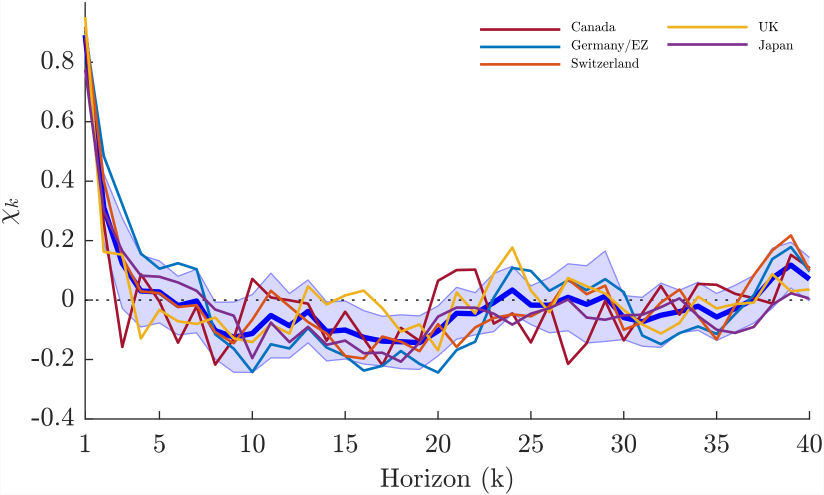
Return

Predictability Regressions in GE: Decomposition



Return

IRFs to interest rate innovation



Time periods: 1990:m1-2020:m6 Source: *FX4Casts* Return

Beta decomposition

Decompose β_ks:

$$\underbrace{\frac{\text{Cov}\left(\Lambda_{t+k}, r_t - r_t^\star\right)}{\text{Var}(r_t - r_t^\star)}}_{\beta_k} = \underbrace{\frac{\text{Cov}\left(s_{t+k} - E^s_{t+k-1} s_{t+k}, r_t - r_t^\star\right)}{\text{Var}(r_t - r_t^\star)}}_{\beta^s_k} - \underbrace{\frac{\text{Cov}\left(\xi_{t+k-1}, r_t - r_t^\star\right)}{\text{Var}(r_t - r_t^\star)}}_{\beta^s_\xi}$$

	β_1	β_1^s	β_1^ξ
Data	-1.26	-1.74	0.48
Model	-1.47	-1.76	0.29

Return

Kalman filter

Lemma

Subjective expectations about the interest differential evolve according to:

$$E_t^s x_{t+1} = (1-\kappa_t)\tilde{\rho} E_{t-1}^s x_t + \kappa_t \tilde{\rho} x_t,$$
$$\Sigma_t = (1-\kappa_t)(\tilde{\rho}^2 \Sigma_{t-1} + \sigma_\varepsilon^2),$$
$$\kappa_t = \frac{\tilde{\rho}^2 \Sigma_{t-1} + \sigma_\varepsilon^2}{\tilde{\rho}^2 \Sigma_{t-1} + \sigma_\varepsilon^2 + \tilde{\sigma}_\nu^2}$$

The Kalman gain κ_t and the conditional variance Σ_{t+1} eventually converge to their steady state values:

$$\kappa = \frac{\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2}{\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2 + \tilde{\sigma}_\nu^2}; \qquad \Sigma = (1-\kappa)(\tilde{\rho}^2 \Sigma + \sigma_\varepsilon^2)$$

Return

Households and liquidity serices (I)

The utility function of the representative household in country H is

$$E_0^s \sum_{t=0}^{\infty} (\beta^t \zeta_t) \left[\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} + \frac{1}{1-\gamma} (Q_t)^{1-\gamma} \right],$$

where

$$Q_t = \frac{M_{H,t,}}{P_t} + \kappa \frac{B_{H,t}}{P_t}$$

The Home budget constraint reads:

$$P_t C_t + M_{H,t} + B_{H,t+1} + B_{H,t+1}^m + S_t B_{H,t+1}^{*m} \leq$$
$$W_t L_t + M_{H,t-1} + R_{t-1} B_{H,t} + R_{t-1}^m B_{H,t}^m + S_t R_{t-1}^{*m} B_{H,t}^{*m} + \Pi_t - T_t.$$

Return

Households and liquidity serices (II)

The home household's FOCs wrt $M_{H,t}, B_{H,t}, B^m_{H,t}, {B^m_{H,t}}^\star$

$$
\begin{aligned}
1 &= \beta \mathbb{E}_t \left[\frac{\zeta_{t+1}}{\zeta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] + \frac{Q_t^{-\gamma}}{C_t^{-\sigma}}, \\
1 &= \beta R_t \mathbb{E}_t \left[\frac{\zeta_{t+1}}{\zeta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] + \kappa \frac{Q_t^{-\gamma}}{C_t^{-\sigma}}, \\
1 &= \beta R^m_t \mathbb{E}_t \left[\frac{\zeta_{t+1}}{\zeta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right], \\
1 &= \beta R^{*m}_t \mathbb{E}_t \left[\frac{\zeta_{t+1}}{\zeta_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right].
\end{aligned}
$$

The foreign households problem is symmetric. Return

Extension with endogenous liquidity premia

GE model extension:

- Households derive liquidity services from government bonds [Liquidity]
  - Endogenous time-varying expected excess returns ($\xi_t \propto -(r_t - r_t^\star)$)
    Nagel 16, Engel 16, Engel & Wu 20

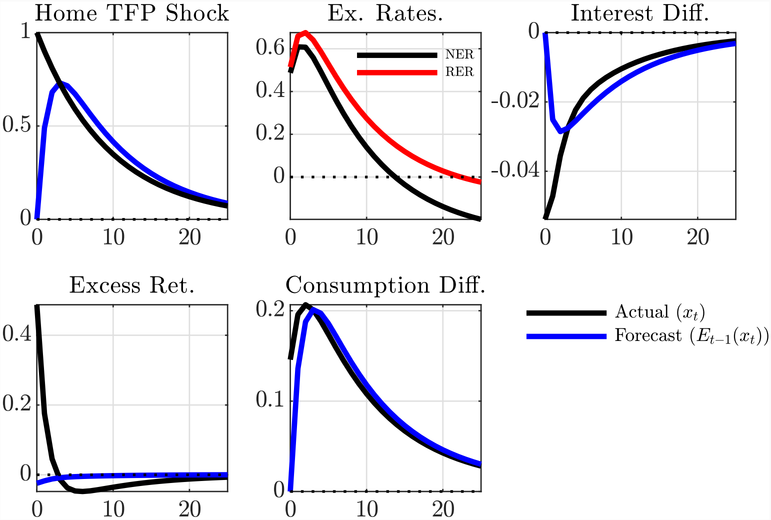
- Expectational errors on future path of liquidity premia [Extended Model: IRFs]
- Improves quantitative fit of the model [Moments I] [Moments II]
- Reproduces $\hat{\beta}$ and both its components ($\hat{\beta}^s$ & $\hat{\beta}^\xi$) [Fama β decomposition]

Forecast-error moments

	Data	Model	
		$\alpha = 0$	$\alpha = 0.15$
β^{CG} Inflation	0.61	0.54	0.59
β^{CG} Output	0.77	0.59	0.59
β^{CG} Interest rate	0.64	0.54	0.55
β^{KW} Inflation	-0.13	-0.09	-0.08
β^{KW} Output	-0.12	-0.13	-0.13
β^{KW} Interest rate	-0.07	-0.09	-0.08
β^{ST} Exchange rate	0.20	0.44	0.44

Forecast errors and revisions Return

IRFs to a Home TFP improvement



Return

Open-economy moments

	Data	Model	
		$\alpha = 0$	$\alpha = 0.15$
$\rho(\Delta s_t)$	0.30	0.31	0.30
$\sigma(\Delta s_t)/\sigma(\Delta y_t)$	5.20	3.26	3.59
$\rho(q_t)$	0.94	0.95	0.95
$\rho(\Delta q_t, \Delta s_t)$	0.99	0.99	1.00
$\rho(\Delta q_t, \Delta(c_t - c_t^\star))$	-0.40	-0.36	-0.38
Fama β	-1.23	-1.30	-1.47
Fama R^2	0.02	0.04	0.04
$\sigma(r_t - r_t^\star)/\sigma(\Delta q_t)$	0.16	0.30	0.22
$\rho(r_t - r_t^\star)$	0.90	0.77	0.76
$\rho(\Delta c_t, \Delta c_t^\star)$	0.30	-0.10	-0.12

Fama β decomposition Return

Households and liquidity serices (III)

The two relevant Euler equations in the model are:

$$0 = E_t\theta_{t+1} + r_t + \alpha r_t \qquad 0 = E_t\theta^*_{t+1} + r^*_t + \alpha r^*_t$$

where $\alpha = \beta\frac{\kappa}{1-\kappa} > 0$. .

UIP condition:

$$E_t\Delta s_{t+1} - (r^m_t - r^{m*}_t) = 0$$

Rearranging the above equations, one can show that:

$$E_t\Delta s_{t+1} - (r_t - r^*_t) = \alpha(r_t - r^*_t)$$

Return