Credit Constraints and Quantitative Easing

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Motivation

- Studying the effectiveness of Quantitative Easing (QE).

- Since the start of the Covid-19 pandemic, the Federal Reserve has implemented QE policies.

- The pandemic has disproportionately affected low(er)-income households.
A New Keynesian model with borrowing constraints on one set of households ("impatient/constrained").

Agents: unconstrained Households, constrained Households, intermediate-good firms, final-good firms, a government and a monetary authority.

QE is introduced using the portfolio rebalancing channel.

Characterizing the effect of QE on output analytically and numerically.
The Portfolio Rebalancing Channel

- Assets of different maturities are imperfect substitutes.
- If assets are imperfect substitutes, changes in the demand for one asset would change the relative price of that asset. As such, the Central Bank can use QE asset purchases to alter the relative price of long-term assets, thereby changing the yields of these assets.
- This could lead to effects on real economic activity.
Credit constraints make QE less effective.
Patient (Unconstrained) Households

The problem of the representative household:

$$\max_{\{c_P,t,n_P,t,b_s^t,b_L^t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t^P \left( \frac{c_{P,t}^{1-\sigma}}{1 - \sigma} - \frac{n_{P,t}^{1+\nu}}{1 + \nu} \right)$$  \hspace{1cm} (1)

subject to the budget constraint:

$$c_{P,t} + \frac{b_s^t}{R_s^t P_t} + \frac{b_L^t}{R_L^t P_t} \left[ 1 + \frac{\psi}{2} \left( \phi \frac{b_s^t}{b_L^t} - 1 \right)^2 \right] \leq w_{P,t} n_{P,t} + \frac{b_s^{t-1}}{P_t} + \frac{b_L^{t-1}}{P_t} - T_{P,t} + \Pi_{P,t}$$  \hspace{1cm} (2)

The solution gives:

$$\beta \mathbb{E}_t \left( \frac{c_{P,t+1}}{c_P,t} \right)^{-\sigma} = \mathbb{E}_t \pi_{t+1} \left[ \frac{1}{R_s^t} + \frac{\psi \phi}{R_L^t} \left( \phi \frac{b_s^t}{b_L^t} - 1 \right) \right]$$  \hspace{1cm} (3)

$$\beta \mathbb{E}_t \left( \frac{c_{P,t+1}}{c_P,t} \right)^{-\sigma} = \mathbb{E}_t \pi_{t+1} \left[ 1 + \frac{\psi}{2} \left( \phi \frac{b_s^t}{b_L^t} - 1 \right)^2 - \psi \phi \left( \phi \frac{b_s^t}{b_L^t} - 1 \right) \frac{b_s^t}{b_L^t} \right]$$  \hspace{1cm} (4)

$$\chi n_{P,t} c_P^\sigma = w_{P,t}$$  \hspace{1cm} (5)
Credit Constrained (Hand-to-Mouth) Households

The problem of the representative household:

\[
\max_{\{c_{l,t}, n_{l,t}\}_{t=0}^\infty} \mathbb{E}_0 \sum_{t=0}^\infty \beta_t \left( \frac{c_{l,t}^{1-\sigma}}{1-\sigma} - \chi_{l,t} \frac{n_{l,t}^{1+\nu}}{1+\nu} \right) 
\]

subject to the following budget constraint:

\[
c_{l,t} \leq w_{l,t} n_{l,t} - T_{l,t} 
\]

Solving the household problem gives the following labor supply condition.

\[
\chi n_{l,t}^{\nu} c_{l,t}^{\sigma} = w_{l,t}. 
\]
The intermediate-good sector is perfectly competitive.

Each firm $j$ employs labor from both household sectors, and produces an intermediate good $x_{j,t}$.

Production function: $x_{j,t} = N_{P,j,t}^{1-\alpha} N_{l,j,t}^\alpha$.

Each period, a firm $j$ has the probability $1 - \eta$ of changing its price as in Calvo (1983, *JME*).

Labor demand conditions:

$$ (1 - \alpha) N_{P,t}^{-\alpha} N_{l,t}^\alpha mc_t = w_{P,t} $$  \hspace{1cm} (9) \\

$$ \alpha N_{P,t}^{1-\alpha} N_{l,t}^{\alpha-1} mc_t = w_{l,t} $$  \hspace{1cm} (10)
Final-Good Firms

- Monopolistically competitive.
- They transform intermediate goods into final goods.
- Demand for variety $j$:

$$y_{j,t} = \left( \frac{p_{j,t}}{p_t} \right)^{-\varepsilon} y_t = x_{j,t}$$  \hspace{1cm} (11)

- The New Keynesian Phillips Curve (in log deviations):

$$\hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa \hat{m}_c_t$$ \hspace{1cm} (12)
A representative perfectly competitive financial intermediary (FI) sells short term bonds (to the Ricardian agents) that pay $R^S_t$ in return.

It gets liquidity $b^{CB}_t$ from the central bank in the form of long term assets with a return $R^{CB}_t$.

The FI buys long term bonds from the government that pay $R^L_{G,t}$ in return, and keeps reserves $q_t$ at the central bank that return $R^Q_t$.

The FI also faces adjustment costs of changing the mix of short term and long term assets.
The Financial Intermediary, Contd.

\[ \max_{\{c_t, I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{P,t}}{e^{-\sigma}} \right) \left\{ R^L_G b^L_G + R^Q_t q_t - (1 - \omega) R^S_t b^S_t - R^CB_t b^CB_t \left[ 1 + \frac{\Theta}{2} \left( \Gamma \frac{b^S_t}{b^CB_t} - 1 \right)^2 \right] \right\} \]  

subject to the following constraint:

\[ b^L_G, t + q_t \leq (1 - \omega) b^S_t + b^CB_t \]  

The solution to this problem gives:

\[ \Lambda_t = R^S_t + R^CB_t \Theta \Gamma \left( \Gamma \frac{b^S_t}{b^CB_t} - 1 \right) \]  

\[ \Lambda_t = R^CB_t \left\{ 1 - \Theta \Gamma \left( \Gamma \frac{b^S_t}{b^CB_t} - 1 \right) \frac{b^S_t}{b^CB_t} + \frac{\Theta}{2} \left( \Gamma \frac{b^S_t}{b^CB_t} - 1 \right)^2 \right\} \]
We let $b_{t}^{CB}$ transition as follows:

$$\ln\left(\frac{b_{t}^{CB}}{b}\right) = \rho_{M} \ln\left(\frac{b_{t-1}^{CB}}{b}\right) + \xi_{M,t}$$

(17)
The government budget constraint:

$$g_t + b^L_{G,t-1} + b^L_{t-1} = \frac{b^L_{G,t}}{R^L_{G,t}} + \frac{b^L_t}{R^L_t} + T_t$$ (18)
Market Clearing

- The resource constraint in the economy reads as follows:
  \[ y_t = \omega c_{l,t} + (1 - \omega) c_{p,t} + g_t \]  
  (19)

- Aggregate output in the economy is determined as
  \[ y_t = N_{P,t}^{1-\alpha} N_{l,t}^\alpha \]  
  (20)

- The financial market also clears:
  \[ b^L_{G,t} + q_t = (1 - \omega) b^S_t + b^{CB}_t \]  
  (21)

- The total demand for labor from both types of households and by all firms are \( \int_0^1 N_{l,j,t} dj = N_{l,t} \) and \( \int_0^1 N_{P,j,t} dj = N_{P,t} \). As such, the clearing conditions in the labor market are:
  \[ N_{l,t} = \omega n_{l,t} \]  
  (22)
  \[ N_{P,t} = (1 - \omega) n_{P,t} \]  
  (23)
We characterize the transmission of QE asset purchases in the economy by determining the response of output to a rise in $b_t^{CB}$.

We assume: $\hat{R}_t^{CB} = 0$.

Define:

$$\hat{y}_t = A_M \hat{b}_t^{CB} \quad (24)$$
The solution:

$$A_M = \frac{(1-\omega)(1-\beta\rho_M)(1-g)(1-\omega\sigma\Omega)\left(\frac{NM_1}{\Delta_1}\right)}{\sigma(1-\beta\rho_M)(1-\rho_M)(1-\omega\sigma\Omega)M_2 - \kappa(1-\omega)\rho_M[\sigma M_2 + (1-g)(\nu + \sigma\omega\Omega)M_1]} \quad (25)$$

If $\omega = 0$:

$$A_M = \frac{(1-g)\bar{cP}N_2}{\sigma\Delta_1} \quad (26)$$

where:

$$N_2 = \left\{ \frac{R^L X_2 + \psi\phi\gamma R^S X_1}{R^S R^L} - \frac{\psi\Sigma b^{CB}}{b^S} \right\}$$

If $\omega = 1$: $A_M = 0.$
Figure: Response of output to QE asset purchases (the value of $A_M$). The share of the HtM households in the economy, $\omega$, ranges from zero to the benchmark value of $1/3$. 
Figure: Impulse response of selected variables to a 1% increase in QE asset purchases. The first model ($\omega = 0$) abstracts from HtM households. The second model ($\omega = 1/3$) introduces HtM households.
Conclusions

Credit frictions actually reduce the effectiveness of quantitative easing (through the portfolio rebalancing channel).
Thank you!