An Explanation of Real US Interest Rates with an Exchange Economy

Matching evidence on US real bond rates

Tamas Csabafi, Max Gillman

University of Missouri-St. Louis

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1 University of Missouri - St. Louis, Department of Economics, University of Missouri - St. Louis, 1 Uninversity Boulevard, St. Louis, MO 63121; gillmanm@umsl.edu
Explanation of real 3-month US Tbill interest rates, 1975Q1-2020Q4. Offers solution to puzzle:

- Standard asset pricing fundamentals cannot explain rates.
- Model adds in a premium that can be viewed as reflecting liquidity.
- Explains business cycle filtered real ex-post interest rates, & output, labor.
- Explains level of real interest rates by adding back HP filtered trend.
- 3 shock construction methods used to back out shock.
- Robustness & sensitivity analysis, eg. with money-only economy.
- Iterative convergence shocks: best correlation, rel. volatility near one.
Motivation: Liquidity premium focus: post-2008 & Covid

Many Empirical and Theoretic explanations of real US government bond liquidity premium

- Liquidity premiums in wide array of monetary policy models.
- Think of as widening spread between equity return and bond return.
- Paper asks if augmented standard monetary real business cycle model can explain real bond interest rates, with small set of shocks:
  - to goods sector productivity, bank sector productivity,
  - money supply growth rate.
- In exchange economy, with money supply causing inflation,
  - with bank sector producing alternative to money: "exchange credit."
- Explain also negative real interest rates as central bank "fix" rates.
Results with this Approach

- Model can explain rates accurately only if endogenous velocity;
- consumer earns interest on deposits in banks to avoid inflation tax.
- Bank-based exchange means: eg. credit card paid off end-period.
- Money is used at point of purchase: eg. debit card, currency.
- Historical inflation (not iid): "downward sloping" money demand.
- Optimal inflation tax avoidance induces asset pricing wedge:
  - Expected change in the "user cost" of exchange.
- Results advance monetary real bus. cycle real bond interest rates
  - even during negative real interest rates & historically 1975-2020.
Specific Results

- Show both HP filtered real bond rate; **Level** of rate in model & data.
- High correlation model predicted and data;
- higher correlation, better volatility with HP trend added in: Level.
- HP trend has content: inflation not random process; eg. War, Crises.
- Relative volatility near 1: stand. dev. of model divided by sd of data.
- Iterative convergence Shock construction method best (Benk);
- also Bayesian, Solow.
Three Alternative Methods to Back-out Shocks

- Solow residual is used for bank, goods sector productivity shocks,
- plus actual M1 money supply growth rate for money supply shock.
- Standard Bayesian estimation of all three shocks.
- "Benk" method extends Nolan-Thoenissen 2009;
  - backs historical shock sequences using time series data
  - & full DSGE solution for decision variables;
- each variable solution depends on state variable & 3 shocks.
Iterative convergence enables matching variance-covariance matrices

- Pagan-Wickens 2019: DSGE model consistency
- with covariance matrix of shocks.
- Iterative convergence algorithm:
- same matrix in calibration, & in backed-out shocks.
- Shocks inputted back into model to give model predictions of real rate historically; compared to data.
- Overidentify shocks; estimate shocks using maximum likelihood estimation procedure.
- Solow method fairs poorly;
- both Bayes & Benk methods work well.
User cost premium, compared to standard asset pricing kernel

- Large literature: user cost = interest differential:
- Special case: cash-only economy gives fundamentals, user cost $R$.
- General user cost: $R - R^d$: nom.rate $R$ minus dividend yield $R^d$.
- **Ratio of Expected change in $R - R^d$ to exp. ch in $R$**: is *Asset Pricing Wedge* as Liquidity Premium.
- Other wedges based on putting liquidity services in Utility function, but without explaining historical data.
Intuition

- Model: real business cycle (RBC) bank sector production function supported empirically in microeconomic banking literature.
- User cost equals per unit cost of banking that avoids inflation tax plus per unit inflation tax paid.
- Allows us only goods and leisure in utility function;
- explain historical real bond rates: econometrically constructed shocks, without ambiguous utility parameters or transaction cost functions.
Model: Cash-in-Advance extended exchange, endog. velocity.

Show Results in Figures and Tables.

Robustness: Special CIA case; iid shocks,

Model extension with Interest on Reserves.

Include Mixed model of both Benk shocks for productivity;

+ money supply growth rate.

Mixed model does almost as well as Benk model.

Discussion of policy, model, negative interest rates worldwide.

Conclusion.
A Model with Exchange Credit

- \( u(c_t, x_t) = \frac{[c_t x_t^\psi]^{1-\theta}}{1-\theta} \). Goods \( c_t \), Leisure \( x_t \)
- \( 1 = x_t + l_{Gt} + l_{Qt} \). Time: labor in goods sector \( l_G \); bank \( l_Q \)
- \( w_t(l_{Gt} + l_{Qt}) + (1 + r_t - \delta_k) k_t \) income for labor and capital rental
- \( \bar{R}_t^d d_t \) yield on bank deposits \( \bar{R}_t^d \) is bank profit rate.
- \( p_{Qt} q_t \) consumer cost: exchange credit;
- \( p_{Qt} \) is real compet. price; \( q \) quant.
- \( w_t(l_{Gt} + l_{Qt}) + (1 + r_t - \delta_k) k_t + \bar{R}_t^d d_t - m_{t+1} (1 + \pi_{t+1}) + m_t - b_{t+1} (1 + \pi_{t+1}) + (1 + \bar{R}_t) b_t + \Gamma_t - c_t - p_{Qt} q_t \geq k_{t+1} \)
- Next period capital stock after investment in money, bonds, capital.
Exchange Technology and Deposit Constraint

- Exchange for consumption $c_t$ by real money + real lump-sum transfer, $m_t + \Gamma_t$, and by exchange credit $q_t$,
- as perfect substitutes in exchange.
- $m_t + \Gamma_t + q_t \geq c_t$.
- Consumption greater than or equal to deposited funds, $c_t \geq d_t$. 
exchange credit production is given by

\[ q_t = A_Q e^{v_t} (l_{Qt})^\gamma d_t^{1-\gamma}. \]

for \( \gamma \in (0, 1) \), marginal cost of \( \frac{q_t}{d_t} = A_Q e^{v_t} \left( \frac{l_{Qt}}{d_t} \right)^\gamma \)
endogenously upward sloping in contrast to eg. Berk-Green 2004;
lit. assumes exog. convex marginal cost of financial services.
marginal cost per unit is convex upward sloping for \( 0 < \gamma < 0.5 \),
concave for \( 1 > \gamma > 0.5 \), and \( J \)-shaped at \( \gamma = 0 \).
At \( \gamma = 1 \) : no well-defined equilibrium; eg King-Plosser 1984
Profit, \( \Pi_{Qt} \equiv p_{Qt} q_t - w_t l_{Qt} - \bar{R}_t^d d_t \), with respect to \( l_{Qt} \) and \( d_t \) :
\[ w_t = p_{Qt} \gamma A_Q e^{v_t} \left( \frac{l_{Qt}}{d_t} \right)^{\gamma-1} ; \bar{R}_t^d = p_{Qt} (1 - \gamma) A_Q e^{v_t} \left( \frac{l_{Qt}}{d_t} \right)^\gamma. \]
physical capital $k_t$ and labor time $l_{Gt}$ inputs to produce output $y_t$

$y_t = A_G e^{z_t} (l_{Gt})^\alpha k_t^{1-\alpha}$.

First order conditions: maximizing profit, $y_t - w_t l_{Gt} - r_t k_t$,

$w_t = \alpha A_G e^{z_t} \left( \frac{k_t}{l_{Gt}} \right)^{1-\alpha}; \quad r_t = (1 - \alpha) A_G e^{z_t} \left( \frac{k_t}{l_{Gt}} \right)^{-\alpha}$.

Government: Nominal transfer

$T_t = B_{t+1} - B_t (1 + \bar{R}_t) + M_{t+1} - M_t$.

From new govt bonds $B$ and new money $M$.

Real terms: $\Gamma_t \equiv \frac{T_t}{P_t}$;

$\Gamma_t = b_{t+1} (1 + \pi_{t+1}) - b_t (1 + \bar{R}_t) + m_{t+1} (1 + \pi_{t+1}) - m_t$.

Let $\sigma_t$ denote growth rate of money $\sigma: \Gamma_t = \sigma_t m_t = (\sigma + e^{u_t}) m_t$.
Equilibrium

- Shocks: matrix equation: $Z_t = \Phi_Z Z_{t-1} + \varepsilon_{Zt}$; shocks $Z_t = [z_t \ u_t \ \nu_t]'$
- [AR(1)] follow auto-regressive process of order one.
- Recursive representative consumer’s optimization problem is
- $V(s) = \max_{c,x,l_q,l_G,q,d,k',m',b'} \{ U(c, x) + \beta EV(s') \}$,
- subject to time, budget and deposit constraints.
Asset Pricing Liquidity Premium based on User Cost

- User cost enters the intratemporal and intertemporal margins.
- User cost is $\bar{R}_t - \bar{R}_t^d$:
- weighted average of cost per unit of goods from using exchange credit plus cost per unit of goods from using money, $\hat{m} = m + \Gamma$.
- with the weights in equilibrium being $\frac{q_t}{c_t}$ and $\frac{m_t}{c_t}$, $\frac{q_t}{c_t} + \frac{\hat{m}_t}{c_t} = 1$
- $\bar{R}_t - \bar{R}_t^d = \frac{q_t}{c_t} \gamma \bar{R}_t + \frac{\hat{m}_t}{c_t} \bar{R}_t$
- With $\frac{q_t}{c_t} \gamma \bar{R}_t = \frac{wL_t}{c_t}$, cost of labor used up in banking per unit of goods.
- $\frac{m_t}{c_t} \bar{R}_t$ is inflation tax paid to government.
- Average cost of exchange, $\bar{R}_t - \bar{R}_t^d$. 

Tamas Csabafi, Max Gillman (AEA Meetings An Explanation of Real US Interest Rates wit
• Intratemporal marginal rate of substitution between goods and leisure, 
  equals $\frac{1 + \bar{R}_t - \bar{R}^d_t}{w_t}$, ratio of the shadow price of goods of 1 plus 
  exchange cost $\bar{R}_t - \bar{R}^d_t$, to shadow price of leisure $w$.

• Intertemporal equilibrium conditions imply asset pricing kernel.

\[ 1 = \beta E_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^\theta \left( \frac{x_{t+1}}{x_t} \right) \psi(1-\theta) \left( \frac{1 + \bar{R}_{t+1}}{1 + \pi_{t+1}} - \frac{1 + \bar{R}_t - \bar{R}^d_t}{1 + \bar{R}_{t+1} - \bar{R}^d_{t+1}} \right) \right]. \]

• Without exchange credit ($A_Q = 0$), $\bar{R}_t - \bar{R}^d_t = \bar{R}_t$, no avoidance of inflation tax:

\[ 1 = \beta E_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^\theta \left( \frac{x_{t+1}}{x_t} \right) \psi(1-\theta) \left( \frac{1 + \bar{R}_t}{1 + \pi_{t+1}} \right) \right]. \]

• Special case is $1 = \beta E_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^\theta \left( \frac{x_{t+1}}{x_t} \right) \psi(1-\theta) \left( \frac{1 + \bar{R}_t}{1 + \pi_{t+1}} \right) \right]$.

• Write kernel as

\[ 1 = \beta E_t \left[ \left( \frac{c_t}{c_{t+1}} \right)^\theta \left( \frac{x_{t+1}}{x_t} \right) \psi(1-\theta) \left( \frac{1 + \bar{R}_t}{1 + \pi_{t+1}} \right) \right]. \]

• Wedge, added factor, is $\left( \frac{1 + \bar{R}_{t+1}}{1 + \bar{R}_t} - \frac{1 + \bar{R}_t - \bar{R}^d_t}{1 + \bar{R}_{t+1} - \bar{R}^d_{t+1}} \right)$ relative to fundamental kernel.
Random fluctuations in the expected user cost cause fluctuations in the real bond interest rate.

Eg. wedge captures how decrease in expected % change in user cost by more than a decrease in the expected % change in nominal rate $R$ causes current period real bond interest rate to fall.

Eg. should excess reserves be expected to rise more seigniorage transferred from Fed to banks through IOER policy, cost of avoiding inflation tax would be expected to fall, and real bond rate would fall as in an indirect liquidity effect.

For any macroprudential policy that affects liquidity in bank system user cost $\bar{R}_{t+1} - \bar{R}_{t+1}^d$ can change, gives kernel explanatory power.
Figure: Benk (Solid), Bayes (Dots), Solow (Dashed) Shocks for TFP, Bank Productivity, and Money Supply Growth.
Figure: Key Variables: Benk and Bayesian Shock Implied Series vs. US Data.
<table>
<thead>
<tr>
<th></th>
<th>2nd</th>
<th>1975Q1-2020Q4</th>
<th></th>
<th>1975Q1-2007Q4</th>
<th></th>
<th>2008Q1-2020Q4</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moments</td>
<td>Solow</td>
<td>Benk</td>
<td>Bayes</td>
<td>Mixed</td>
<td>Solow</td>
<td>Benk</td>
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<td></td>
<td>R − π Corr.</td>
<td>-0.05</td>
<td>0.48</td>
<td>0.16</td>
<td>0.41</td>
<td>-0.11</td>
<td>0.59</td>
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<td></td>
<td>Rel. Vol.</td>
<td>15.8</td>
<td>1.09</td>
<td>1.85</td>
<td>1.33</td>
<td>13.7</td>
<td>1.08</td>
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<tr>
<td>R Corr.</td>
<td>-0.39</td>
<td>0.62</td>
<td>0.50</td>
<td>0.62</td>
<td>-0.44</td>
<td>0.61</td>
<td>0.49</td>
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<td>Rel. Vol.</td>
<td>47.0</td>
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<td>2.41</td>
<td>2.25</td>
<td>378</td>
<td>1.98</td>
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<tr>
<td>π Corr.</td>
<td>-0.34</td>
<td>0.73</td>
<td>0.34</td>
<td>0.77</td>
<td>-0.38</td>
<td>0.76</td>
<td>0.28</td>
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<tr>
<td></td>
<td>Rel. Vol.</td>
<td>57.7</td>
<td>1.85</td>
<td>1.24</td>
<td>1.99</td>
<td>51.3</td>
<td>1.97</td>
</tr>
<tr>
<td>y Corr.</td>
<td>0.97</td>
<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
<td>0.97</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.52</td>
<td>0.75</td>
<td>0.71</td>
<td>0.82</td>
<td>1.49</td>
<td>0.78</td>
</tr>
<tr>
<td>C Corr.</td>
<td>0.79</td>
<td>0.87</td>
<td>0.32</td>
<td>0.78</td>
<td>0.82</td>
<td>0.90</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.0</td>
<td>1.38</td>
<td>2.50</td>
<td>0.63</td>
<td>1.35</td>
<td>1.82</td>
</tr>
<tr>
<td>i Corr.</td>
<td>0.90</td>
<td>0.71</td>
<td>0.30</td>
<td>0.90</td>
<td>0.93</td>
<td>0.72</td>
<td>0.18</td>
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<tr>
<td></td>
<td>Rel. Vol.</td>
<td>2.21</td>
<td>0.89</td>
<td>1.85</td>
<td>1.12</td>
<td>2.14</td>
<td>0.98</td>
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<tr>
<td>I_G Corr.</td>
<td>0.29</td>
<td>0.60</td>
<td>0.05</td>
<td>0.51</td>
<td>0.41</td>
<td>0.72</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.24</td>
<td>0.88</td>
<td>1.44</td>
<td>0.58</td>
<td>1.15</td>
<td>0.84</td>
</tr>
</tbody>
</table>

**Table:** Second Moments of Model Implied Variables, US data.
Figure: HP Filtered Ex post Real Interest Rate (Benk) vs. US Data.
Figure: HP Filter Trend Component of the Ex-post Real Bond Interest Rate
Figure: Level of Ex-post Real Bond Interest Rate: Benk Method (black line) vs. US Data (dashed).
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Solow</td>
<td>Corr.</td>
<td>0.118</td>
<td>0.135</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>20.369</td>
<td>14.799</td>
<td>60.406</td>
</tr>
<tr>
<td>Benk</td>
<td>Corr.</td>
<td>0.859</td>
<td>0.874</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.039</td>
<td>1.0636</td>
<td>1.106</td>
</tr>
<tr>
<td>Bayes</td>
<td>Corr.</td>
<td>0.652</td>
<td>0.635</td>
<td>-0.147</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.354</td>
<td>1.406</td>
<td>1.495</td>
</tr>
<tr>
<td>Mixed</td>
<td>Corr.</td>
<td>0.815</td>
<td>0.835</td>
<td>0.217</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.136</td>
<td>1.169</td>
<td>1.334</td>
</tr>
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</table>

Table: Correlation of Model Implied Unfiltered Real Interest Rate Series with US Data with relative standard deviation in parentheses.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T M B</td>
<td>81.68% 8.65% 0.49%</td>
<td>84.27% 5.09% 1.57%</td>
<td>90.04% 7.46% 7.06%</td>
</tr>
<tr>
<td>T B M</td>
<td>81.69% 0.49% 8.65%</td>
<td>84.28% 1.56% 5.10%</td>
<td>90.06% 7.10% 7.42%</td>
</tr>
<tr>
<td>B T M</td>
<td>70.34% 16.26% 8.24%</td>
<td>82.68% 5.33% 5.10%</td>
<td>31.55% 67.91% 7.17%</td>
</tr>
<tr>
<td>B M T</td>
<td>70.33% 8.24% 16.28%</td>
<td>80.64% 5.09% 5.37%</td>
<td>32.52% 7.20% 67.92%</td>
</tr>
<tr>
<td>M T B</td>
<td>73.88% 24.69% 6.84%</td>
<td>74.03% 6.96% 10.29%</td>
<td>38.81% 65.78% 0.22%</td>
</tr>
<tr>
<td>M B T</td>
<td>73.88% 6.83% 24.70%</td>
<td>74.07% 10.28% 7.01%</td>
<td>38.81% 0.22% 65.75%</td>
</tr>
<tr>
<td>Average</td>
<td>T B M</td>
<td>T B M</td>
<td>T B M</td>
</tr>
<tr>
<td></td>
<td>40.88% 25.89% 30.26%</td>
<td>32.20% 31.51% 28.08%</td>
<td>74.58% 12.95% 17.81%</td>
</tr>
</tbody>
</table>

**Table:** Variance Decomposition of the Real Interest Rate
### Robustness: Three Experiments, Ex-post Level of Real Bond Interest Rate

<table>
<thead>
<tr>
<th>Method (Model)</th>
<th>2nd Moments</th>
<th>1975Q1-2020Q4</th>
<th>1975Q1-2007Q4</th>
<th>2008Q1-2020Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Benk</td>
<td>Corr.</td>
<td>0.859</td>
<td>0.874</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.039</td>
<td>1.064</td>
<td>1.106</td>
</tr>
<tr>
<td>Benk CIA</td>
<td>Corr.</td>
<td>0.546</td>
<td>0.521</td>
<td>-0.072</td>
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<tr>
<td></td>
<td>Rel. Vol.</td>
<td>2.938</td>
<td>3.222</td>
<td>3.183</td>
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<tr>
<td>Benk i.i.d.</td>
<td>Corr.</td>
<td>0.736</td>
<td>0.758</td>
<td>-0.299</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.045</td>
<td>1.076</td>
<td>1.069</td>
</tr>
<tr>
<td>Benk: with IOER</td>
<td>Corr.</td>
<td>0.861</td>
<td>0.875</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>Rel. Vol.</td>
<td>1.037</td>
<td>1.055</td>
<td>1.106</td>
</tr>
</tbody>
</table>

**Table:** Robustness: 3 Experiments for Level of Real Bond Interest Rate
**Figure:** Post 2008Q1 Bank Shocks (Benk method) in Baseline (solid), IOER Extension (dotted), and with Real Interest Rate Data (dashed).
Figure: Goods TFP Productivity Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.
Figure: Bank Productivity Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.
Figure: Money Supply Shock vs. Model Predicted, Actual Data, Real T-bill Interest Rate.
Figure: Real S&P 500 Yield versus Bank Productivity Shock.
Discussion 1: Model explains Data

- Results: model explains real US bond interest rates historically including "unconventional" times, negative real bond rates.
- Criticism: bank production tech. is simplistic, not bank behavior; rebutted by empirical evidence gathered for some 35 years estimating bank production with labor, capital & deposited funds;
- 3rd input solves dilemma of lack of equilibrium w/o deposits input.
- Left out capital as input in bank production function to simplify, while still fitting real interest rate data.
Our inflation tax wedge is related to other asset pricing wedges.
Some posit wedge to be function of utility of holding credit good.
We substitute use of utility for positing wedge by instead 
building exchange technology: "cash" & credit goods perfect subst.
User cost: aver.cost of optimal mix money-exchange credit.
Mixed model results, combined with impulse responses, imply
money supply & bank productivity shocks are well identified.
Discussion 3: Price Theoretic Terms

- Ratio $\frac{\bar{R}^d}{\bar{R} - \bar{R}^d}$ is ratio of Red to Blue area with Marginal Cost MC
- Red: producer surplus $R^d$; Blue: User Cost $\bar{R} - \bar{R}^d$

Figure: $\frac{R^d}{\bar{R} - \bar{R}^d}$: Inflation Tax Avoided (Red) to Inflation Tax Paid (Blue).
Conclusion

- Inflation tax avoidance causes wedge that determines asset pricing because markets must anticipate changes in $R$ and $R^d$ to price bonds.
- Paper offers rational expectations explanation of real bond rates.
- Inflation is caused by money supply growth:
- Historical, 1975-2020, paper shows standard asset pricing puzzle can be solved by extending monetary real business cycle model through addition of exchange credit and endogenous velocity.
- Expected changes in user cost drive ex-post real bond interest rate, even into negative territory.