Household Savings and Monetary Policy under Individual and Aggregate Stochastic Volatility

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Motivation

• Modern macroeconomics:
  • Questions:
    • heterogeneity in income or productivity
    • assets with differing liquidity (machines, liquid bonds)
    • aggregate (and idiosyncratic) risk and uncertainty
    • redistribution
  • Tools:
    • global solutions
• These are usually done in isolation.
• This paper: We do all in one framework.
HANK Model

• Households
  • Two assets: bonds (liquid) and machines (illiquid)
  • Two (occasionally binding) borrowing constraints
  • Idiosyncratic shocks to productivity level (risk)
  • Aggregate shock to productivity variance (uncertainty)
  • Sticky wages
• Firms
  • CRS with machines and labor
  • Aggregate shocks to TFP level (risk)
  • Aggregate shocks to TFP variance (uncertainty)
• Government
  • Fiscal policy (progressive income taxation as in Heathcote, Storesletten, and Violante 2017)
  • Monetary policy (Taylor rule with ZLB)
Risk and Uncertainty

• Household productivity: \( e^{\left(\eta_{\ell,t}(j) - \frac{\bar{\sigma}_\ell^2}{1 - (\rho\sigma_{\ell})^2}\right)} \)

Individual risk: \( \eta_{\ell,t}(j) = \rho\eta_{\ell,t-1}(j) + \exp\left(\sigma_{\ell,t-1} - \frac{1}{2} \frac{\sigma_{\ell}^2}{1 - (\rho\sigma_{\ell})^2}\right) \bar{\sigma}_{\ell}\epsilon_{\ell,t}(j) \)

Individual uncertainty: \( \sigma_{\ell,t} = \rho^{\sigma_{\ell}} \sigma_{\ell,t-1} + \sigma_{\ell} \epsilon_{\sigma_{\ell},t} \)

• Aggregate TFP: \( e^{\left(\eta_{\theta,t} \frac{\bar{\sigma}_\theta^2}{1 - (\rho\sigma_{\theta})^2}\right)} \)

TFP risk: \( \eta_{\theta,t} = \rho^{\theta} \eta_{\theta,t-1} + \exp\left(\sigma_{\theta,t-1} - \frac{1}{2} \frac{\sigma_{\theta}^2}{1 - (\rho\sigma_{\theta})^2}\right) \bar{\sigma}_{\theta}\epsilon_{\theta,t} \)

TFP uncertainty: \( \sigma_{\theta,t} = \rho^{\sigma_{\theta}} \sigma_{\theta,t-1} + \sigma_{\theta} \epsilon_{\sigma_{\theta},t} \)

where \( \epsilon_{\ell,t}, \epsilon_{\sigma_{\ell},t}, \epsilon_{\theta,t}, \epsilon_{\sigma_{\theta},t} \sim \mathcal{N}(0, 1) \)
Related Literature

- Uncertainty shocks
- HANK
- Numerical solutions to heterogeneous agent models
Model Generated Statistics

- Global solutions following L. Maliar, S. Maliar, and Winant 2021

<table>
<thead>
<tr>
<th></th>
<th>Wealth Gini</th>
<th>Consumption Gini</th>
<th>Net Income Gini</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.78</td>
<td>0.36</td>
<td>0.43</td>
</tr>
<tr>
<td>Model</td>
<td>0.66</td>
<td>0.276</td>
<td>0.360</td>
</tr>
<tr>
<td>95% CI</td>
<td>(0.655, 0.684)</td>
<td>(0.274, 0.277)</td>
<td>(0.359, 0.361)</td>
</tr>
</tbody>
</table>

Table: Data from Krueger, Mitman, and Perri 2016
Generalized Impulse Response

- Koop, Pesaran, and Potter 1996
- TFP uncertainty

\[ \sigma_{\theta,t} = \rho^{\sigma_{\theta}} \sigma_{\theta,t-1} + \sigma_{\sigma_{\theta}} (\varepsilon_{\sigma_{\theta},t + 1}) \]

- Individual uncertainty

\[ \sigma_{\ell,t} = \rho^{\sigma_{\ell}} \sigma_{\ell,t-1} + \sigma_{\sigma_{\ell}} (\varepsilon_{\sigma_{\ell},t + 1}) \]

- 1 standard deviation innovation
- 100 initial conditions
- 100 draws of innovations for each initial condition
- Time period: 1 quarter
- 200 agents
Average Labor Earnings

Average Labor Earnings = \( w_t H_t \int e^{(\eta_{t,\ell}(j)-\frac{\sigma_{\ell}^2}{1-(\rho_{\ell})^2})} dj \)
Net Income Gini

Net Income = \left( \frac{R_{t-1}}{\pi_t} - 1 \right) b_{t-1}(j) + [r^k_t - d] k_{t-1}(j) + \tau_t(j) + \tau_1 \left[ w_t H_t \exp \left( \eta_{\ell, t}(j) - \frac{1}{2} \frac{\bar{\sigma}^2_{\ell}}{1 - (\rho_{\ell})^2} \right) \right]^{1 - \tau_2}
Conclusion

• Response to individual uncertainty shocks is much larger than the response to TFP uncertainty shocks
• Individual uncertainty shocks lead to a persistent increase in wealth following an initial reduction.
• Individual uncertainty shocks increase income inequality.
• **Future versions:** Correlation between individual and aggregate uncertainty.
Thank you!
Appendix
Model and Calibration
Household Problem

\[
\max_{c_t, i_t, b_t, k_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t(j)^{1-\gamma} - 1}{1 - \gamma} \\
\text{s.t.} \quad c_t(j) + i_t(j) + b_t(j) + \Psi(i_t(j), k_{t-1}(j)) = \\
\frac{R_{t-1}}{\pi_t} b_{t-1}(j) + \tau_1 \left[ w_t H_t \exp \left( \eta_{\ell, t}(j) - \frac{1}{2} \frac{\sigma_{\ell}^2}{1 - (\rho_{\ell})^2} \right) \right]^{1-\tau_2} + \tau_t(j) \\
k_t(j) = \left[ 1 - d + r_t^k \right] k_{t-1}(j) + i_t(j) \\
k_t(j) \geq 0 \quad b_t(j) \geq b
\]

• $\Psi(\cdot, \cdot)$: adjustment cost on machines and shares
• $\tau_t(j)$: transfers
Labor Union

\[
\max_{W_t(m)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \int \left( \frac{c_t(j)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{h_t(j)^{1+\vartheta}}{1+\vartheta} \right) dj - \frac{\mu_w}{1-\mu_w} \frac{1}{2\kappa_w} \left[ \log \left( \frac{W_t(m)}{W_{t-1}(m)} \frac{1}{\pi^*} \right) \right]^2 \right]
\]

s.t. 
\[
H_t(m) = \left( \frac{W_t(m)}{W_t} \right)^{\mu_w-1} H_t
\]

\[
h_t(j) = \int \left( \frac{W_t(m)}{W_t} \right)^{\mu_w-1} H_t dm
\]
Labor Union (Continued)

\[
\log \left( \frac{\pi_t}{\pi^*} \right) = \kappa_w \left( \psi H_t^{1+\vartheta} - \mu_w (1 - \tau_2) Z_t \tilde{\Lambda}_t \right) + \beta E_t \log \left( \frac{\pi_t + 1}{\pi^*} \right)
\]

\[
Z_t \equiv \tau_0 \left( w_t L_t \right)^{(1-\tau_2)} \int_0^1 (\exp \left( \eta_{\ell,t} (j) - \frac{1}{2} \frac{\sigma_\ell^2}{1 - (\rho_\ell)^2} \right))^{(1-\tau_2)} dj
\]

\[
\tilde{\Lambda}_t \equiv \int_0^1 \frac{1}{\int_0^1 (\exp \left( \eta_{\ell,t} (j) - \frac{1}{2} \frac{\sigma_\ell^2}{1 - (\rho_\ell)^2} \right))^{(1-\tau_2)} dj} dj.
\]
Firms

- Production function

\[ Y_t = \bar{A} \exp \left( \eta_{t-1} - \frac{\bar{\sigma}_t^2}{1 - (\rho_t^2)} \right) K_{t-1}^{\alpha} H_t^{1 - \alpha} \]
Central Bank

- Taylor rule subject to ZLB

\[ R_t \equiv \max\{1.0, \left( R_{t-1} \right)^{\mu} \left( \frac{\pi_t}{\pi_*} \right)^{\phi_{\pi}} \left( \frac{Y_t}{Y_*} \right)^{\phi_Y} \}^{1-\mu} \exp \left( \eta_{R,t} - \frac{1}{2} \frac{\sigma_R^2}{1 - (\rho_R)^2} \right) \]

- Monetary policy shock

\[ \eta_{R,t} = \rho_R \eta_{R,t-1} + \sigma_R \varepsilon_{R,t}, \quad \varepsilon_{R,t} \sim \mathcal{N}(0,1) \]
Market Clearing

- Market clearing
\[ \int_{0}^{1} b_t (j) \, dj = 0 \]

\[ C_t + K_t - (1 - d) K_{t-1} + \int_{0}^{1} AC(i) \, di = Y_t \]

- \( \int_{0}^{1} AC(i) \, di = \int_{0}^{1} \Psi (i_t (j), k_{t-1} (j)) \, dj \) : aggregate cost of adjustment
- \( C_t = \int_{0}^{1} c_t (j) \, dj \)
- \( K_{t-1} = \int_{0}^{1} k_{t-1} (j) \, dj \)
## Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 2.0$</td>
<td>Risk aversion</td>
<td>standard</td>
</tr>
<tr>
<td>$\beta = 0.975$</td>
<td>Discount factor</td>
<td>standard</td>
</tr>
<tr>
<td>$d = 0.0135$</td>
<td>Depreciation rate</td>
<td>standard</td>
</tr>
<tr>
<td>$\Gamma_2 = 1.1686$</td>
<td>Illiquid asset adjustment cost</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_3 = 2.0$</td>
<td>Illiquid asset adjustment cost</td>
<td></td>
</tr>
<tr>
<td>$\xi = 0.0$</td>
<td>Illiquid asset adjustment cost</td>
<td></td>
</tr>
<tr>
<td>$\varepsilon = 0.25$</td>
<td>Illiquid asset adjustment cost</td>
<td></td>
</tr>
<tr>
<td>$\bar{b} = -0.1$</td>
<td>Liquid asset borrowing constraint</td>
<td>75% of people have liquid assets (Kaplan, Violante, and Weidner 2014)</td>
</tr>
<tr>
<td>$\tau_1 = 0.8$</td>
<td>Tax function parameter</td>
<td>Heathcote, Storesletten, and Violante 2017</td>
</tr>
<tr>
<td>$\tau_2 = 0.181$</td>
<td>Tax function parameter</td>
<td>Heathcote, Storesletten, and Violante 2017</td>
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</tbody>
</table>

### Household

### Labor Union

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vartheta = 1.0$</td>
<td>Labor supply elasticity</td>
<td>standard</td>
</tr>
<tr>
<td>$\psi = 0.8796$</td>
<td>Disutility of labor shift</td>
<td>$H = 1$ in model without agg risk</td>
</tr>
<tr>
<td>$\mu_w = 1.1$</td>
<td>Elasticity of substitution among goods</td>
<td>profits share of 10%</td>
</tr>
<tr>
<td>$\kappa_w = 0.15$</td>
<td>Slope of wage Phillips curve</td>
<td>Auclert et al. 2021</td>
</tr>
</tbody>
</table>

### Firm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.325$</td>
<td>Capital share</td>
<td>standard</td>
</tr>
<tr>
<td>$\bar{A} = 0.4735$</td>
<td>Constant in production function</td>
<td>$Y = 1$ in model without agg risk</td>
</tr>
</tbody>
</table>
Calibration (cont.)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = 0.0$</td>
<td>Nominal rate persistence</td>
<td></td>
</tr>
<tr>
<td>$R_\pi = 1.0175$</td>
<td>Long run nominal rate</td>
<td></td>
</tr>
<tr>
<td>$Y_\pi = 1$</td>
<td>Long run output</td>
<td></td>
</tr>
<tr>
<td>$\pi_\pi = 1.005$</td>
<td>Inflation target</td>
<td></td>
</tr>
<tr>
<td>$\phi_\pi = 1.5$</td>
<td>MP response to inflation</td>
<td></td>
</tr>
<tr>
<td>$\phi_y = \frac{25}{4}$</td>
<td>MP response to output</td>
<td></td>
</tr>
<tr>
<td>$\rho_\ell = 0.966$</td>
<td>Persistence of idiosyncratic shocks</td>
<td>Auclert et al. 2021</td>
</tr>
<tr>
<td>$\sigma_\ell = 0.2379$</td>
<td>Standard deviation of idios.-level shocks (in the absence of uncertainty shocks)</td>
<td>Auclert et al. 2021</td>
</tr>
<tr>
<td>$\rho^{\sigma_\ell} = 0.84$</td>
<td>Persistence of idios.-volatility shocks</td>
<td>Based on Bayer et al. (2019)</td>
</tr>
<tr>
<td>$\sigma_{\sigma_\ell} = 0.02$</td>
<td>Standard deviation of idios.-volatility shocks</td>
<td>Based on Bayer et al. (2019)</td>
</tr>
<tr>
<td>$\rho^\theta = 0.9$</td>
<td>Persistence of TFP-level shocks</td>
<td>standard</td>
</tr>
<tr>
<td>$\sigma_\theta = 0.016$</td>
<td>Standard deviation of TFP-level shocks (in the absence of uncertainty shocks)</td>
<td>standard</td>
</tr>
<tr>
<td>$\rho^{\sigma_\theta} = 0.73$</td>
<td>Persistence of TFP-volatility shocks</td>
<td>Based on Fernandez-Villaverde et al. (2015)</td>
</tr>
<tr>
<td>$\sigma_{\sigma_\theta} = 0.04$</td>
<td>Standard deviation of TFP-volatility shocks</td>
<td>Based on Fernandez-Villaverde et al. (2015)</td>
</tr>
<tr>
<td>$\rho^R = 0.5$</td>
<td>Persistence of monetary-policy shocks</td>
<td>standard</td>
</tr>
<tr>
<td>$\sigma_R = 0.01$</td>
<td>Standard deviation of monetary-policy shocks</td>
<td>standard</td>
</tr>
<tr>
<td>Model and Calibration</td>
<td>Solution Algorithm</td>
<td>Related Literature</td>
</tr>
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**Solution Algorithm**
Deep Learning Analysis of Maliar, Maliar, Winant (2019)

1. **HANK model:**

\[
\begin{align*}
E_{\epsilon} [f_1 (X (s), \epsilon)] &= 0 \\
& \quad \ldots \\
E_{\epsilon} [f_n (X (s), \epsilon)] &= 0
\end{align*}
\]

s = state, X (s) = decision function, \( \epsilon \) = innovations.

2. Parameterize \( X (s) \approx \varphi (s; \theta) \) with a **deep neural network**.

3. Construct **objective function** for DL training

\[
\min_{\theta} (E_{\epsilon} [f_1 (\varphi (s; \theta), \epsilon)])^2 + \ldots + (E_{\epsilon} [f_n (\varphi (s; \theta), \epsilon)])^2 \rightarrow 0
\]

4. **All-in-one expectation** operator is a critical novelty:

\[
(E_{\epsilon} [f_j (\varphi (s; \theta), \epsilon)])^2 = E_{(\epsilon_1, \epsilon_2)} [f_j (\varphi (s; \theta), \epsilon_1) \cdot f_j (\varphi (s; \theta), \epsilon_2)]
\]

with \( \epsilon_1, \epsilon_2 = \) two independent draws.

4. **Stochastic gradient descent** for training (random grids)

5. Google **TensorFlow** platform
Solution Algorithm

- Use algorithm of Maliar, Maliar and Winant (2021)
- 13 agg. variables \( \{ C_t, H_t, K_t, I_t, Y_t, \pi_t, w_t, r^k_t, R_{t-1}, \eta_R, t, \eta_\theta, t, \sigma_\theta, t, \sigma_\ell, t \} \)
- 8 individual variables \( \{ c_t (j), k_t (j), i_t (j), b_t (j), q_t (j), \eta_\ell, t (j), \nu_t (j), \varphi_t (j) \} \)

\( \nu_t (j), \varphi_t (j) = \) Lagrange multipliers; \( q_t (j) = \) value of an additional unit of illiquid assets
- 5 aggregate state variables:

\[ \{ R_{t-1} \} \quad \{ \eta_R, t, \eta_\theta, t, \sigma_\theta, t, \sigma_\ell, t \} \]

endogenous \quad \text{exogenous} \quad \text{(1)}

- 3 individual state variables:

\[ \{ k_{t-1} (j), b_{t-1} (j) \} \quad \{ \eta_\ell, t (j) \} \]

endogenous \quad \text{exogenous} \quad \text{(2)}

- 3J + 5 dimensional state space, where \( J = \) number of agents
Neural Networks

- 2 neural networks (NN) with 4 hidden layers each and 128 neurons in each layer.
- Leaky relu as activation function. ADAM optimization algorithm. Batch size of 10.
- **Outputs of NNs:**
  - 1st NN \( \mathcal{N}^{agg} \): aggregate variables \( \{H_t, \pi_t\} \)
  - 2d NN \( \mathcal{N}^{indiv} \): individual variables \( \{\xi_t^k(j), \xi_t^c(j), \nu_t(j), \varphi_t(j)\} \)
    - \( \xi_t^a(j) \) = share of illiquid assets out of income net of consumption and the borrowing limit
    - \( \xi_t^k(j) \) = share of capital in illiquid assets
    - \( \nu_t(j), \varphi_t(j) \) = multipliers
- Need to approximate just six \( 3J + 5 \)-dimensional decision function to characterize the labor choices of all \( J \) agents.
Recovering Aggregate Variables

- Use weights of NNs to compute aggregate variables

\[ \mathcal{N}^{agg} (\Sigma) \rightarrow (H_t, \pi_t) \]

\[ k(j) \rightarrow K_t \]

\[ (H_t, K_t, \eta_{\theta, t}) \rightarrow (w_t, r^k_t, Y_t) \]

\[ (\pi_t, Y_t, R_{t-1}, \eta_{R, t}) \rightarrow R_t \]
Recovering Individual Variables

- NN for individuals

\[ \mathcal{N}^{\text{indiv}} (\Sigma) \rightarrow (\xi_t^k (j), \xi_t^c (j), \nu_t (j), \varphi_t (j)) \] (3)

- resources

\[ M_t (j) = \frac{R_{t-1}}{\pi_t} b_{t-1} (j) + \left[ 1 - d + r_t^k \right] k_{t-1} (j) + \tau_t (j) \]
\[ + \tau_1 \left[ w_t H_t \exp \left( \eta_{\ell, t} (j) - \frac{1}{2} \frac{\bar{\sigma}_{\ell}^2}{1 - (\rho_{\ell})^2} \right) \right]^{1-\tau_2} \] (4)

- consumption

\[ c_t (j) = \xi_t^c (M_t (j) - \bar{b}) \] (5)
Recovering Individual Variables (Continued)

- machines

\[ k_t (j) = \max \left( \xi_t^k (j) \cdot [M_t (j) - b - c_t (j)] , 0.0 \right) \] \hspace{1cm} (6)

- adjustment cost

\[ (k_t (j), k_{t-1} (j)) \rightarrow i_t (j) \rightarrow (\Psi_t (j), q_t (j)) \] \hspace{1cm} (7)

- bonds

\[ b (j) = \max \left( [M_t (j) - b - c_t (j) - k_t (j) - \Psi_t (j)] , b \right) \] \hspace{1cm} (8)
Relation to Literature about Uncertainty

1. **Aggregate uncertainty in RA models.**

2. **Idiosyncratic uncertainty on the production side.**
   - Assume representative household –uncertainty does not affect households of different income and wealth levels.

3. **Stochastic volatility in HA models.**
   - Bayer et al. (2019) and Schabb (2020).
Relation to HANK Literature

- **Aggregate MIT risk shocks + No uncertainty shocks**
  - Kaplan, Moll and Violante (2018), Alves, Kaplan, Moll and Violante (2020)
- **Aggregate MIT uncertainty shocks**
  - Bayer, Luetticke, Pham-Dao and Tjaden (2019) and Schabb (2020)

**This paper:** the first HANK model with both
- aggregate uncertainty shocks
- aggregate risk shocks
Relation to HA Computational Literature

Novel numerical methods for solving HANK models with distribution

- Based on **Reiter (2009)**:
  - *Idea:* local (perturbation) solutions at the aggregate level + Global solutions at the individual level

- Based on **Fernandez-Villaverde, Hurtado and Nuno (2020)**:
  - *Idea:* use neural networks to approximate aggregate law of motion (ALM)
  - ALM is approximated with a general function of distributional moments ⇒ Krusell and Smith (1998) type of algorithm
Uncertainty Shocks and Global Solutions
Global versus local solution methods

Solving models with uncertainty shocks

- Fernandez-Villaverde:
  - Perturbation solutions must be at least of order three
  \[ \Rightarrow \text{Volatility of shocks nontrivially enters decision rules} \]

- Groot (2020):
  - Even third-order perturbation methods may not be sufficient.

\[ \downarrow \]

Need global solutions to capture effects of volatility on decision rules
Approaches to Uncertainty Shocks in the Literature

- MIT aggregate shocks
- Low-order perturbation
- Reduced state space approximations

We address these problems with AI and deep learning (DL)

- Aggregate shocks in the solution procedure
- Global nonlinear solutions
- True state space