Discordant Relaxations of Misspecified Models

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December 20, 2021

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• Misspecification in partially identified models can lead to spuriously informative bounds

• Problem of Inference: misleading CI.

This paper:

• Problem of Identification: misleading outer set
Often in practice, outer set $\tilde{\Theta}$ instead of $\Theta_I$ is estimated:

- Blundell, Gosling, Ichimura and Meghir (2007)
- Ciliberto and Tamer (2009), Ciliberto, Murry and Tamer (2018)
- Aucejo, Bugni and Hotz (2017)
- Dickstein and Morales (2018)
- Chesher and Rosen (2020)
- ...
Summary of Main Results

• Negative Result:
  outer sets can be misleading / discordant if based on nonsharp id. restrictions.

• Positive Result:
  outer sets may not be discordant under some conditions.

• Related Result:
  ideas on how to summarize discordant results from different assumptions.
Discordant Outer Sets: an Example
Example: Intersection Bounds

\[ \mathbb{E}[Y|Z] \leq \theta \leq \mathbb{E}[\bar{Y}|Z] \quad a.s. \]

Example: heterogeneous treatment model

\[(Y_0, Y_1)\] potential outcome
\[D\] binary treatment
\[Y = Y_D\] observed outcome
\[Z\] instrument
\[Y_d \quad y \mathbb{1}(D \neq d) + Y \mathbb{1}(D = d)\]
\[\bar{Y}_d \quad \bar{y} \mathbb{1}(D \neq d) + Y \mathbb{1}(D = d)\]
\[\theta_d \quad \mathbb{E}[Y_d]\]
\[\mathbb{E}[Y_d|Z] = \mathbb{E}[Y_d]\] mean independence
Example: Intersection Bounds

\[ \mathbb{E}[Y|Z] \leq \theta \leq \mathbb{E}[\overline{Y}|Z] \] a.s.

Then,

\[ \theta \in [\gamma, \overline{\gamma}] := \left[ \sup_z \mathbb{E}[Y|Z = z], \inf_z \mathbb{E}[\overline{Y}|Z = z] \right] \]

it implies for any \( h(\cdot) \geq 0, \)

\[ \mathbb{E} [h(Z)(Y - \theta)] \leq 0, \quad \mathbb{E} [h(Z)(\theta - \overline{Y})] \leq 0 \]

its id. set \( \Theta_I(h). \)
Recall $\gamma = \sup_z \mathbb{E}[Y|Z = z]$ \quad \overline{\gamma} = \inf_z \mathbb{E}[\overline{Y}|Z = z]$. 

- when $\gamma \leq \overline{\gamma}$, \quad $[\gamma, \overline{\gamma}] \subseteq \Theta_I(h)$

- when $\gamma > \overline{\gamma}$, \quad $[\gamma, \overline{\gamma}] = \emptyset$, \quad what is $\Theta_I(h)$?
Theorem

Suppose $\mathbb{E}[Y|Z] \leq \mathbb{E}[\overline{Y}|Z]$ a.s.

When full model refuted, i.e. when $\gamma > \overline{\gamma}$ and $[\gamma, \overline{\gamma}] = \emptyset$,

$$\forall \theta \in (\overline{\gamma}, \gamma), \exists h \geq 0, \text{ s.t. } \Theta_I(h) = \{\theta\}$$

**Takeaway**: an outer set can be very tight but misleading
Here, when the model is refuted, outer sets are discordant: there exist $h_1, h_2$ such that $\Theta_I(h_1) \neq \emptyset$, $\Theta_I(h_2) \neq \emptyset$, 

$$\Theta_I(h_1) \cap \Theta_I(h_2) = \emptyset$$
Here, when the model is refuted, outer sets are discordant: there exist \( h_1, h_2 \) such that \( \Theta_I(h_1) \neq \emptyset, \Theta_I(h_2) \neq \emptyset \),

\[
\Theta_I(h_1) \cap \Theta_I(h_2) = \emptyset
\]

In general,

nonsharp id. restrictions \( \Rightarrow \) discordant outer sets

examples including

- Artstein inequality on random set and Choquet capacity
- conditional moment inequality models
Compatible Outer Sets: an Example
Example: Adaptive Monotone IV

Potential Outcome Model:

\[ D \]  binary treatment: college education or not

\[ Z \]  instrument: max. years of education of parents

\[ Y_{dz} \]  potential outcome: potential wage

\[ Y = Y_{DZ} \]  observed outcome: observed wage

\[ \theta \]  ATE

Assume mean independence: \( \forall (d, z), \mathbb{E}[Y_{dz}|Z] = \mathbb{E}[Y_{dz}] \).
exclusion restriction in Manski (1990) $a_{\text{exclude}}$:

instruments $z$ has no impact on potential outcome $Y_{dz}$
Assumptions

Adaptive Monotone IV assumption $a_k$:

$Y_{dz}$ weakly increases with $z$ then remain flat after $z \geq k$
Outer Sets and Minimum Relaxation

Note that

- assumptions are nested,

\[ a_{\text{exclude}} \iff a_1 \Rightarrow a_2 \Rightarrow \cdots \Rightarrow a_K \]

- id. set \( \Theta_I(a_k) \) is an outer set of the id. set \( \Theta_I(a_{\text{exclude}}) \)

However,

- outer sets are compatible even if model \( a_{\text{exclude}} \) is refuted

  \[ \text{either } \Theta_I(a_k) \subseteq \Theta_I(a_{k'}) \text{ or } \Theta_I(a_{k'}) \subseteq \Theta_I(a_k) \]

- Importantly, data reveals the unique minimum relaxation needed to restore data-consistency.
In our paper,

- we derived the iff condition for the uniqueness of minimum data-consistent relaxation

- we also discussed what conclusion could be drawn if minimum data-consistent relaxations are not unique.