# Prioritarianism, Inequality and Fractional Stochastic Dominance

Stéphane Mussard & Marc Dubois

January, 9th 2022 - Boston (virtual)

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# Introduction

The crisis of the "Gilets jaunes" in France, 2018

- Claims of the middle class:

$\hookrightarrow$ they support a large part of the taxes (carbon tax)

$\hookrightarrow$ they are poor workers: poverty line = 1113€

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# Introduction

- The economic analysis of the *Gilets jaunes* is a relative point of view: relative to the upper classes
- The gas price has been multiplied by 3 since 1990 in France
- The wealth tax was suppressed in 2018 (households with a capital greater than 800,000 €) : the fiscal gain was near €4.2 billions
- The new tax (real estate fortune) provides now a fiscal gain around €1.3 billion.

# Setup

- How to integrate a relative point of view in the economic analysis and in tax reforms?

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- How to integrate a relative point of view in the economic analysis and in tax reforms?
- Prioritarian point of view: A prioritarian social welfare function is defined as (Adler, 2012):

$$W(f) = \int_a^b g \circ u(y)\ f(y)\ dy \tag{1}$$

• $g$: point of view of the social planner i.e. value judgments: ethical transfer principles

• $u$: (utility function) behavior of the agents: risk aversion, inequality aversion (empirical phenomena)

# Setup

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- Utility transfers rather than income transfers (Adler, 2012).
- Utility increment $T^1(\alpha, u(y), \delta)$: For all $f \in \mathscr{F}$ and for all $y \in \Omega$, a probability measure $h(y)$ is obtained from $f(y)$ by a utility increment whenever

$$h(y) = \begin{cases} f(y) - \alpha & \text{at point } u(y) \ , \\ f(y) + \alpha & \text{at point } u(y) + \delta \ , \\ f(y) & \text{elsewhere.} \end{cases}$$

# Setup

## Definition

A utility transfer of order 2 is the sum of an increment and a decrement (a Pigou-Dalton transfer or rich-to-poor transfers):

$$T^2(\alpha, u(y), \delta) = T^1(\alpha, u(y), \delta) - T^1(\alpha, u(y) + \delta, \delta)$$

## Definition

A mean-preserving utility transfer of order $s+1$ is given by:

$$T^{s+1}(\alpha, u(y), \delta) := T^s(\alpha, u(y), \delta) - T^s(\alpha, u(y) + \delta, \delta), \;\; s \in \mathbb{N}$$

# Setup

- The social welfare variation issued from a utility transfer of order $s+1$:

$$\Delta W(T^{s+1}(\alpha, u(y), \delta)) := W(f + T^{s+1}(\alpha, u(y), \delta)) - W(f)$$

**Definition**

***Generalized Utility Transfer Principle of order*** $s+1$. For all $f \in \mathscr{F}$, for all $y \in \Omega$ and $s \in \mathbb{N}$, a social welfare function respects the generalized utility transfer principle, if

$$\Delta W(T^{s+1}(\alpha, u(y), \delta)) \geq 0 \qquad (\mathrm{UTP}^{s+1})$$

# Prioritarian fractional dominance

- If a utility transfer of order 2 is not sufficient for a welfare improvement, a transfer of higher order is investigated

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- If a utility transfer of order 2 is not sufficient for a welfare improvement, a transfer of higher order is investigated
- This reasoning is only partial because there are other possible transfers between order 2 and 3, *i.e.* transfers of order $c + 2$ of fractional parameter $c \in (0, 1]$ or transfers of order $c + s$

**Definition**

***Generalized Utility Transfer Principle of order*** $c + s$. For all $f \in \mathscr{F}$, for all $y \in \Omega$, $s \in \mathbb{N}$ and $c \in (0, 1]$, a social welfare function respects the generalized utility transfer principle of order $c + s$, if

$$\Delta W(T^s(\alpha, u(y), \delta)) \geq \frac{c}{(1-c)\delta + c} \Delta W(T^s(\alpha, u(y) + \delta, \delta)) \tag{UTP$^{c+s}$}$$

# Prioritarian fractional dominance

- Suppose that the transfer is normalized such that $\delta = 1$, and that the parameter of fractional dominance $c = 0.5$, then

$$\Delta W(T^s(\alpha, u(y), \delta = 1)) \geq 0.5 \Delta W(T^s(\alpha, u(y) + \delta, \delta = 1))$$

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- **A relative point of view:** The social planner performs the utility transfer of order $0.5 + s$ whenever the social welfare variation at the bottom of the distribution is at least higher than 50% of the social welfare variation at the top

**Lemma**

*Let $s \in \mathbb{N}$ and $c = 1$, $(UTP)^{s+1} \iff (UTP)^{c+s}$*

- The class of prioritarian social welfare functions:

$$\mathcal{G}_{c+s} := \left\{ W(f) \in \mathscr{C}^s \;\middle|\; \begin{array}{l} W \text{ satisfies } (\text{UTP})^{\ell}, \forall \ell = 1, \dots, s \\ W \text{ satisfies } (\text{UTP})^{c+s}, \;\; c \in (0,1] \end{array} \right\}$$

## Definition

***Dominance criteria:***

- Stochastic dominance of order $s$: Distribution function $F$ dominates Distribution function $G$ by $s$th degree stochastic dominance, if $F^{s-1}(x) \geq G^{s-1}(x)$ for all $x \in [a, b]$, with $F^{s-1}(x) = \int_a^x F^{s-2}(u)du$ and $F^1(x)$ being $F$ itself.
- Fractional stochastic dominance of order $s$: Distribution function $F$ dominates Distribution function $G$ by $c + s$th degree stochastic dominance, *i.e.* $F$ $(c+s)$SD $G$, if $W(f) \geq W(g)$.

# Prioritarian fractional dominance

- Huang et al. (2020) employ an expponential function to make an interpolation between the order $s$ and $s+1$:

$$h_c(x) := \begin{cases} \exp[(\frac{1}{c} - 1)x], \text{ if } 0 < c < 1 \\ 1, \text{ if } c = 1. \end{cases} \tag{2}$$

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- The equivalence relation between social welfare improvement and fractional dominance is the following:

**Theorem**

*For all $W \in \mathcal{G}_{c+s}$, and for $c \in (0,1]$, the two following statements are equivalent:*
*(i) $F\ (c+s)SD\ G$*
*(ii.a) $\int_a^y [F^s(t) - G^s(t)] dh_c(u(t)) \geq 0,\ \forall y \in [a,b]$ and,*
*(ii.b) $F^\ell(b) - G^\ell(b) \geq 0,\ \ \forall \ell = 2, \ldots, s$*

# Prioritarian fractional dominance

- Huang et al. (2020) suggest the following class of utility functions, depending on $h_c^{(1)}(x)$, that we rewrite in a prioritarian layout:

$$\mathcal{U}^*_{c+s} := \left\{ W(f) \in \mathscr{C}^s \;\middle|\; \begin{array}{l} (-1)^{(\ell+1)} g^{(\ell)} \geq 0, \ell = 1, \ldots, s \\ (-1)^{(s+1)} g^{(s)} e^{-(\frac{1}{c}-1)u} \text{nonincreasing on} [a, b \end{array} \right.$$

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- Our approach is equivalent to that of Huang et al. (2020):

**Lemma**

*For $s \in \{1, 2, \ldots\}$ and $c \in (0, 1]$, $\mathcal{U}^*_{c+s} = \mathcal{G}_{c+s}$*

# Application to optimal taxation

- Following Makdissi and Wodon (2002) and Duclos, Makdissi and Wodon (2008):

**Definition**

The first-order consumption dominance curve, $CD_k^1(y) = x_k(y)\, f(y)/X_k$, is the consumption of good $k$ for an individual whose current income is $y$ divided by the aggregate consumption of good $k$. The $CD$-curve, $CD_k^s(y) = \int_a^y CD_k^{s-1}(t)\, dt$, is given for all integers $s \in \{2, 3, 4, \ldots\}$.

# Application to optimal taxation

- Equivalence relation between social welfare improvement issued from an indirect marginal tax reform and $CD$-curves dominance.

## Theorem

*A revenue-neutral marginal tax reform* $dt_j = -\gamma \left( \frac{X_i}{X_j} \right) dt_i > 0$ *implies* $dW(f) \geq 0$ *for all* $W(f) \in \mathcal{G}_{s,c}$ *if, and only if, for* $c \in (0,1]$

$$\int_0^y \left[ CD_i^s(t) - \gamma CD_j^s(t) \right] dh_c(u(t)) \geq 0, \ \forall y \in \mathbb{R}_+, \ \ s \in \{1,2,3,\ldots\}$$

# Illustration on the French survey of budget families

- French survey of the budget families (42,900 individuals)

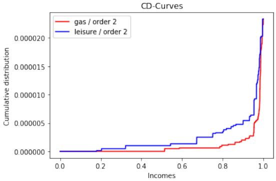



Fig.1a: $s = 2$ (gas / leisure)

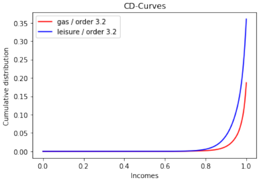



Fig.1b: $c + s = 3.2$ (gas / leisure)

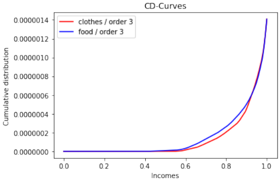
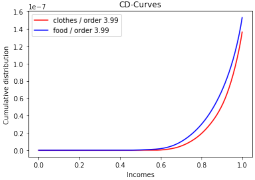



Fig.1c: $s = 3$ (clothes / food)




$c + s = 3.99$ (clothes / food)

# Illustration on the French survey of budget families

- Table 1 provides the minimal order of dominance $c+s$ for a given $\gamma$, *i.e.*,

$$c+s^{\star} := \arg\min_{c+s} \int_0^y \left[CD_i^s(t) - \gamma CD_j^s(t)\right] dh_c(t) \geq 0,\ \forall y \in \mathbb{R}_+$$

| | Gas | Leisure | Health | Transport | Clothes | Hairdressing |
|---|---|---|---|---|---|---|
| Food | $> 4.68$ | $< 2.24$ | $> 2.24$ | $< 3.69$ | $> 3.56$ | $< 2.17$ |
| Gas | | $< 2.16$ | $> 3.40$ | $> 2.78$ | $> 5.74$ | $< 2.13$ |
| Leisure | | | $> 2.21$ | $> 2.46$ | $> 2.33$ | $< 2.16$ |
| Health | | | | $< 2.19$ | $< 2.21$ | $< 2.17$ |
| Transport | | | | | $> 2.67$ | $< 2.16$ |
| Clothes | | | | | | $< 2.17$ |

Table: $c+s^{\star}$: Minimal fractional dominance order (Source EBF 2019)

- Duclos et al. (2008): a tax reform is welfare improving if the distributional benefit is no lower than the cost of funds:

$$\delta(y) := \frac{\int_0^y CD_i^s(t)\, dh_c(u(y))}{\int_0^y CD_j^s(t) dh_c(u(t))} \geq \gamma, \ \forall y \in \mathbb{R}_+$$

**Definition**

The critical ratio of marginal cost of funds:

$$\gamma_{ij}^s := \inf\{\delta(y) \mid \forall y \in \mathbb{R}_+\}$$

**Definition**

The minimal discrete dominance order $s$ such that the critical ratio of marginal cost of funds of the reform is no lower than than $\gamma$:

$$s^\star = \arg\min_s \gamma_{ij}^s \ \text{ s.t. } \ \gamma_{ij}^s \geq \gamma, \ \forall y \in \mathbb{R}_+$$

# Illustration on the French survey of budget families

- The difference between the critical ratio of marginal cost of funds issued from the minimal discrete dominance order and that issued from the fractional order of dominance is:

$$\Delta\gamma_{ij} := \gamma_{ij}^{s^\star} - \gamma_{ij}^{c+s^\star}$$

## Illustration on the French survey of budget families

- The difference between the critical ratio of marginal cost of funds issued from the minimal discrete dominance order and that issued from the fractional order of dominance is:

$$\Delta\gamma_{ij} := \gamma_{ij}^{s^\star} - \gamma_{ij}^{c+s^\star}$$

- $\Delta\gamma_{ij}$ = gain of efficiency when performing the indirect tax reform with fractional dominance

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- $\Delta\gamma_{ij}$ = decrease of the opportunity cost of performing a tax reform

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$$\Delta\gamma_{ij} := \gamma_{ij}^{s^\star} - \gamma_{ij}^{c+s^\star}$$

- $\Delta\gamma_{ij}$ = gain of efficiency when performing the indirect tax reform with fractional dominance
- $\Delta\gamma_{ij}$ = decrease of the opportunity cost of performing a tax reform
- $\Delta\gamma_{ij}$ = fiscal waste to be employed for other fiscal policies i.e. for the "gilet jaunes"

# Illustration on the French survey of budget families

$$\Delta\gamma_{ij} := \gamma_{ij}^{s^\star} - 1$$

| | Gas | Leisure | Health | Transport | Clothes | Hairdressing |
|---|---|---|---|---|---|---|
| Food | 0.08 | 0.59 | 0.53 | 0.16 | 0.12 | 4.28 |
| Gas | | 1.44 | 1.59 | 0.11 | 0.17 | 7.10 |
| Leisure | | | 1.45 | 0.38 | 0.59 | 2.17 |
| Health | | | | 0.77 | 0.54 | 7.11 |
| Transport | | | | | 0.16 | 3.57 |
| Clothes | | | | | | 4.28 |

Table: $\Delta\gamma_{ij}$ : Efficiency gain (Source EBF 2019)

# Conclusion

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- Fine grained dominance criterion for more efficiency in the redistribution
- This framework can be extended to Yaari's dual approach

Thanks for your attention!