(In)efficiency in Information Acquisition and Aggregation through Prices*

Alessandro Pavan† Savitar Sundaresan ‡ Xavier Vives §

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Abstract

The paper studies the interaction between traders' acquisition of private information and the aggregation of information in financial markets. We consider a canonical market microstructure in which partially-informed traders compete in schedules and prices partially aggregate the traders' private information. Before submitting their demand schedules, traders acquire information about the long-term profitability of the traded asset. We show that, when the errors in the traders' signals are correlated, policies that induce the traders to submit the efficient schedules when the traders' private information is exogenous do not necessarily induce them to collect the efficient amount of private information. In particular, we identify conditions under which such policies induce over-investment (alternatively, under-investment) in information acquisition, relative to what is efficient. We find that, as information technology reduces the cost of acquiring information, the economy eventually moves to a regime with excessive information acquisition. Finally, we show that, generically, there exist no policy based on the price of the financial asset and the volume of individual trades that implements efficiency in both information acquisition and trading. Such an impossibility result, however, turns into a possibility result when taxes can be made contingent on the aggregate volume of trade, or when the acquired information is verifiable.

Keywords: information acquisition, aggregation through prices, information externalities, team efficiency.

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†Northwestern University. Email: alepavan@northwestern.edu

‡Imperial College London. Email: s.sundaresan@imperial.ac.uk

§IESE Business School. Email: xvives@iese.edu

1 Introduction

Improvements in information technology are reducing the cost of acquiring and processing information. This cost reduction naturally raises the question of whether such improvements are likely to contribute to higher welfare or benefit a few at the expenses of society at large. Such concerns are at the heart of various policy proposals to "put sand in the wheels" of financial markets as a way of limiting speculative trading facilitated by asymmetric information (the Tobin tax being a prominent example).

In this paper, we present a tractable framework to study both the positive and normative issues of interest. In particular, we characterize the sources of inefficiency in the collection of private information prior to trading and relate them to possible inefficiencies in the limit orders that traders submit in financial markets, given available information. We first show that, in the absence of policy interventions, the equilibrium usage of information is inefficient. That is, the limit orders that traders submit in equilibrium fail to maximize welfare, given the dispersion of private information. However, the inefficiency can be corrected with appropriate (non-linear) taxes/subsides on the trades. We then show that, when the traders' private information is endogenous, policies that induce efficiency in the usage of information (equivalently, that induce the traders to submit the efficient limit orders) need not induce the traders to collect information efficiently. We identify conditions under which traders over-invest (alternatively, under-invest) in information acquisition. Finally, we show that, generically, there exist no tax/subsidies measurable in the price of the financial asset and in the volume of individual trades that induce efficiency in both information acquisition and trading. Such an impossibility result, however, turns into a possibility result when taxes can be made contingent on the aggregate volume of trade, or when the acquisition of information is verifiable.

Our model is a canonical linear-quadratic-Gaussian financial microstructure à la Grossman and Stiglitz 1980 in which a unit-mass continuum of traders compete by submitting a collection of generalized limit orders (equivalently, a demand schedule). The traders face uncertainty about the asset's value, as well as the value that other investors in the market (noisy traders, high-frequency traders, hedge-fund managers, and the like) assign to the asset. Before submitting their generalized limit orders, each trader receives a private signal about the asset's value whose noise is endogenous and correlated across traders. Such a correlation may originate, for example, in the traders paying attention to common sources of information, which have source-specific noise. Importantly, such a correlation, in addition to being realistic, has major implications for the (in)efficiency of the equilibrium acquisition and usage of information, as we discuss further on.[1]

[1]Primarily for tractability, the literature typically assumes instead that the noise in the agents' signals is

Our first main result is that, except in very special cases, absent policy interventions, the market does not use the information it collects efficiently. As in Vives 2017, the inefficiency originates in the interaction between two externalities. First, traders do not account for how their orders affect the co-movement between the equilibrium market-clearing price (and hence the equilibrium asset allocations) and the various fundamental shocks that are responsible for different agents' payoffs (a pecuniary externality). Second, traders do not account for the fact that a collective change in limit orders may induce a change in the information contained in the equilibrium price, which in turn affects other agents' ability to align their trades with the asset's value (a learning externality).

The pecuniary externality makes the traders overreact to their private information, whereas the learning externality makes them under-react to it. The knife-edge case in which the two externalities cancel each other out obtains when the equilibrium demand schedules are perfectly inelastic (such as when traders are restricted to submitting market orders). When the equilibrium schedules are downward sloping, the pecuniary externality dominates and the equilibrium trades feature excessive sensitivity to the traders' private information. When, instead, the equilibrium schedules are upward sloping, the learning externality dominates and the sensitivity of the equilibrium limit orders to the traders' private information is inefficiently low. Interestingly, as the precision of the traders' private information grows (for example, due to a reduction in the cost of acquiring information facilitated by technological progress), the pecuniary externality gains weight in relation to the learning externality.[2] We show that, no matter whether traders over- or under-respond to their private information, the aforementioned inefficiencies in the equilibrium usage of information can always be corrected using a (non-linear) tax-subsidy contingent on both the equilibrium price of the asset and the individual volume of trade. More precisely, a linear-quadratic tax on the volume of trade paired with an ad-valorem tax on the dollar amount traded induces the traders to submit the efficient limit orders.

Our second main result is that inducing the traders to trade efficiently does not guarantee that they invest efficiently in acquiring private information prior to trading. In particular, suppose that the planner could enforce the efficient usage of information by constraining the traders to submit the efficient demand schedules. The traders would then over-invest in information acquisition when the efficient schedules are downward sloping, and under-invest in information acquisition when they are upward sloping. In other words, when the pecuniary

iid across agents.

[2]Provided that the noise in the traders' information is not too large, when the precision of the traders' information is relatively low, the learning externality dominates and the demand schedules are upward sloping, whereas the opposite is true (i.e., the pecuniary externality dominates and the demand schedules are downward sloping) for high levels of precision.

externality prevails in the usage of information, so that the traders over-respond to their private information, forcing the traders to trade efficiently induces them to over-invest in the acquisition of private information. When, instead, the learning externality prevails, in the absence of any policy intervention, traders under-respond to private information. In this case, forcing them to trade efficiently would induce them to under-invest in the acquisition of private information. The inefficiencies in the collection of information thus parallel those in the usage of information. Importantly, these results hinge on the noise in the agents' information being correlated among traders. If such noise were uncorrelated, holding fixed the efficient demand schedules, the only effect of an increase in the precision of the traders' private information on welfare would be through the reduction in the dispersion of individual trades around the average trade. However, when the traders submit the efficient limit orders, the private and the social value of reducing such a dispersion coincide, in which case efficiency in the usage of information implies efficiency in the acquisition of information.

We also show that, if traders could be trusted to submit the efficient demand schedules, then an ad-valorem tax on the dollar amount traded would induce the efficient collection of private information.

Next, we show that, absent any policy intervention, as information technology makes the collection of information cheaper, the economy eventually enters into a regime of over-investment in information acquisition and excessive sensitivity of the equilibrium trades to private information. In other words, the secular trend of improvement in information technology may have the undesirable effect of enticing over-investment in information acquisition and over-reaction to it in the trading of financial assets.[3] Hence, perhaps surprising, putting "sand in the wheels" of financial markets can be particularly valuable precisely when the cost of information acquisition is low.

We show that, generically, there do not exist taxes/subsides contingent on the price of the asset and on the volume of individual trades that induce efficiency in both the collection and the usage of information. This impossibility result, however, can be overturned by conditioning the tax/subsidy on the aggregate volume of trade or, when feasible, directly on the information acquired by the traders. Conditioning the marginal tax rate on the aggregate volume of trade provides the planner with flexibility on the way it realigns the private incentives for trading with the social ones. The extra flexibility in turn permits it to also realign the private benefits of more accurate private information to their social counterparts. Alternatively, when

[3]See, for example, Nordhaus 2015 on the sharp decline in the cost of computation (and therefore of information processing). See also Gao and Huang 2020 and Goldstein, S. Yang, and Zuo 2020 for the effects of the dissemination of corporate disclosures over the internet on the production of information by corporate outsiders.

individual investments in information are verifiable (as when the traders purchase information from known sources), efficiency in trading can be induced with standard taxes that depend only on the price paid and on the individual volume of trade, whereas efficiency in information acquisition can be controlled separately through an additional tax/subsidy that depends on the amount of information purchased.

Related Literature

The paper is related to several strands of the literature. The first strand is the literature investigating the sources of inefficiency in the equilibrium usage of private information when information is exogenous. See, among others, Vives 1988, Angeletos and Pavan 2007, Amador and Weill 2012, Myatt and Wallace 2012, and Vives 2017. Among these works, the closest to ours is Vives 2017 who also studies inefficiency in information aggregation through prices when traders submit demand schedules, but in a setting in which the traders' private information is exogenous and the noise in the agents' signals is uncorrelated.

The second strand is the literature on information acquisition in financial markets. See Diamond and Verrecchia 1981 and Verrecchia 1982 for earlier contributions. More recently, Peress 2010 examines the trade-off between risk sharing and information production, whereas Manzano and Vives 2011 study information acquisition in markets with correlated noise, while Kacperczyk, Van Nieuwerburgh, and Veldkamp 2016 study information acquisition in markets with multiple risky assets. Dávila and Parlatore 2019 study the effect of trading costs on information aggregation and acquisition. None of these papers, however, studies inefficiencies in information acquisition and how the latter relate to inefficiencies in trading.[4]

In particular, Vives 1988 shows that, in a Cournot economy in which a continuum of privately-informed traders with conditionally independent signals submit market orders, both the decentralized acquisition of information and the equilibrium trades are efficient. In the present paper, we show that the same result extends to economies in which the information collected in equilibrium is subject to correlated noise, provided that the traders are restricted to submitting market orders instead of richer supply/demand functions. When traders submit market orders, neither the pecuniary externality nor the learning externality of conditioning on prices is present and efficiency obtains.

Colombo, Femminis, and Pavan 2014 show that efficiency in actions does not imply effi-

[4]See also the literature on the Grossman-Stiglitz paradox, namely on the (lack of) incentives to acquire information when prices are fully revealing (see Grossman and Stiglitz 1980, and Vives 2014 for a potential resolution of the paradox). Related is also the literature on strategic complementarity/substitutability in information acquisition (see, among others, Ganguli and L. Yang 2009, Hellwig and Veldkamp 2009, Manzano and Vives 2011, Myatt and Wallace 2012, and Pavan and Tirole 2021).

ciency in information acquisition when payoffs depend on the dispersion of individual actions around the average action. In the present paper, we consider a richer setting in which agents compete in schedules and where information is partially aggregated in the equilibrium price. We show that, even in the absence of externalities from the dispersion of individual actions around the mean action, efficiency in information usage does not imply efficiency in information acquisition when the noise in the agents' signals is correlated. As anticipated above, that noise is correlated is important. As shown in Vives 2017, when the noise in the agents' signals is independent across agents, policies that correct inefficiencies in the usage of information induce efficiency in the acquisition of information, despite the (imperfect) aggregation of information made possible by the limit orders. Efficiency in the usage of information also implies efficiency in information acquisition in the macro business-cycle economies considered in Angeletos, Iovino, and La'O 2020. In these economies, prices imperfectly aggregate information, as in our paper, but agents have access to complete markets that fully insure them against any idiosyncratic consumption risk. In contrast, in our economy, traders consume the returns to their own investments and policies that correct inefficiencies in the usage of information need not induce efficiency in the collection of information.[5]

The third strand is the recent literature analyzing the impact of technological progress on the collection of information and its usage in financial markets. Farboodi, Matray, and Veldkamp 2018 show that the growth of big data, combined with the size distribution of firms, can lead to a decline in price informativeness for smaller firms. Peress 2005 shows that a declining cost of information collection is outweighed by a parallel decline in the cost of entry to financial markets and the interaction between the two can explain several empirical anomalies. Malikov 2019 shows that falling information costs can actually contribute to a rise in passive investment by reducing the cost of, and therefore the returns to, stock picking. Several papers (see, among others, Azarmsa 2019, Mihet 2018, and Kacperczyk, Nosal, and Stevens 2019) show that technological progress that facilitates the collection of information can lead to increasing levels of inequality. Unlike most of the work in this literature, we focus on the normative implications of technological improvements in the collection of information.

A fourth strand is the literature building on Tobin 1978's proposal to put sand-in-the-wheels on foreign exchange transactions as a way to curb volatility and speculation. Similar interventions have been advocated for financial markets (e.g. Stiglitz 1989 and L. H. Summers

[5] A similar conclusion holds in Colombo, Femminis, and Pavan 2021. That paper considers an economy in which agents can perfectly insure against any idiosyncratic consumption risk, but where production is affected by investment spillovers. They show, among other things, that familiar taxes-subsidies linear in revenues that correct for market power induce efficiency in production but not in the acquisition of information. Instead, more sophisticated Pigouvian taxes where the marginal rates depend on aggregate output can induce efficiency in both the usage and the acquisition of information.

and V. P. Summers 1989). High volumes of speculation (particularly in the short-term) and/or "noise trading" are typically assumed to be detrimental to welfare. However, some theoretical work shows that a tax on financial transactions may increase price volatility and lower depth and welfare (see, among others, Kupiec 1996, Sørensen 2017, and Song and Zhang 2005). Subrahmanyam 1998 and Dow and Rahi 2000 show that a quadratic transaction tax may have ambiguous effects on speculators' profits and on the welfare of other traders. Umlauf 1993, Colliard and Hoffmann 2017, and Deng, Liu, and Wei 2018 document a negative impact of transaction taxes on trading volume and an ambiguous impact of the same taxes on price volatility and depth. Using transaction data from the Italian Stock Exchange, Cipriani, Guarino, and Uthemann 2019 estimate a model with price elastic informed and noise traders to asses the effects of a transaction tax on informed and noise traders. They find that the tax reduces trading activity and price volatility, but also price informativeness for most stocks. In the present paper, we identify the structure of taxes that induce efficiency in both the usage and the acquisition of information.

Finally, in this paper, we assume that higher investments in information acquisition can reduce the agents' exposure to correlated noise in information. Recent work by Woodford 2012a, Woodford 2012b, and Nimark and Sundaresan 2019 shows that rational inattention can also explain the agents' exposure to correlated noise and that the equilibrium of a rationally-inattentive economy shares several features with those of an economy in which the agents' use of information is "biased" in the sense of prospect theory. Particularly related in this respect is Frydman and Jin 2020. That paper demonstrates how rational inattention can lead to endogenous bias in valuation, and that the noise in perception is closely linked to the bias in perception. Our paper shares with this literature the property that investments in information acquisition also affect the agents' exposure to correlated noise, something that, from the perspective of an outside observer, may look like a bias in decision making.

Organization. The rest of the paper is organized as follows. Section 2 describes the model. Section 3 compares the equilibrium to the efficient usage of information, identifies the sources of the inefficiency, and shows how certain taxes/subsidies may restore efficiency in trade. Section 4 identifies inefficiencies in information acquisition and discusses possible policy corrections. Section 5 concludes. All proofs are in the Appendix at the end of the document.

2 Model

In this section, we describe the trading environment, the equilibrium choice of demand schedules, and the the traders' information-acquisition problem.

2.1 Trading environment

The market is populated by a continuum of traders, indexed by $i \in [0,1]$, facing a continuum of liquidity suppliers trading a homogenous and perfectly divisible asset. Let x_i denote the quantity of the asset demanded by trader i and $\tilde{x} = \int_0^1 x_i di$ the traders' aggregate demand. The asset demanded by the traders is provided by liquidity suppliers according to the elastic aggregate inverse supply rule

$$p = \alpha - u + \beta \tilde{x}.$$

Such a rule is micro-founded as arising from an aggregate cost for the representative liquidity supplier equal to

$$(\alpha - u)\,\tilde{x} + \beta \frac{\tilde{x}^2}{2},$$

where α and β are positive scalars, and where $u \sim N(0, \sigma_u^2)$. The term $\alpha - u$ proxies for the opportunity cost for the representative supplier of unloading the asset. The term $\beta \tilde{x}^2/2$, instead, is a quadratic trading cost that proxies for the supplier's risk aversion, or, more generally, for all sort of possible limits to arbitrage opportunities.

Each trader i's payoff from purchasing x_i units of the asset at price p is given by

$$\pi_i \equiv (\theta - p)\,x_i - \lambda \frac{x_i^2}{2},$$

where λ is a positive scalar, and where $\theta \sim N(0, \sigma_\theta^2)$. The term θ proxies for the traders' gross common value from purchasing the asset, whereas the term $\lambda \frac{x_i^2}{2}$ is a quadratic trading cost whose role parallels the corresponding term in the representative supplier's payoff.

To simplify the derivation of the equilibrium formulas, we assume that the variables θ and u are independently distributed. The results, however, extend to the case where they are imperfectly correlated. For notational purposes, given any Gaussian random variable h with variance σ_h^2, we denote by $\tau_h \equiv 1/\sigma_h^2$ the variable's precision.

2.2 Information

For simplicity, we assume that the representative supplier knows u (alternatively, we interpret $\alpha - u$ as the investor's expected opportunity cost from unloading the asset). The traders, instead, do not know θ. They privately collect information about θ prior to submitting their demand schedules, but also condition the latter on the information that the market-clearing price contains about θ (that is, they account for the fact that the equilibrium price imperfectly aggregates the traders' dispersed information about θ).

Formally, we assume that each trader observes a (possibly large) collection of Gaussian signals about θ differing in their noises and in the extent to which such noises are correlated among traders. Such signals are summarized in a uni-dimensional statistic

$$s_i \equiv \theta + \epsilon_i$$

where

$$\epsilon_i \equiv f(y_i)(\eta + e_i)$$

is a combination of idiosyncratic and correlated noise. Precisely, the noise variable $\eta \sim N(0, \sigma_\eta^2)$ is perfectly correlated among the traders and can be thought of as originating in the imperfect information contained in the common sources the traders attain to (as, e.g., in Myatt and Wallace 2012). The variables $e_i \sim N(0, \sigma_e^2)$, instead, are i.i.d. among the traders and can be interpreted as idiosyncratic disturbances, or individual mistakes in interpreting available information. The variables $(\theta, u, \eta, (e_i)_{i\in[0,1]})$ are jointly independent. The total noise in trader i's information ϵ_i depends on (η, e_i) but also on the trader's investment in information acquisition, $y_i \in \mathbb{R}_+$. Depending on the context, the latter can be interpreted as the amount of information acquired by the individual on a market, or by the attention allocated to exogenous sources of information. The cost of y_i is given by a differentiable function $\mathcal{C}(y_i)$, with $\mathcal{C}'(y_i), \mathcal{C}''(y_i) > 0$ for all $y_i > 0$.

The idea behind the above information structure is that traders learn from a variety of (Gaussian) signals of different precision, whose noise is imperfectly correlated among the traders. It is well known that, in Gaussian environments, each trader's beliefs about θ is summarized by a sufficient statistic whose noise is a combination of a perfectly correlated term and a perfectly idiosyncratic one. That is, any symmetric cross-sectional distribution of Gaussian posteriors can be generated by giving each agent a perfectly private and a perfectly public signal. Because, in this environment, the traders condition on the price, they do not need to form expectations about the aggregate action in the market. As a result, any such pair of signals can in turn be summarized in a one-dimensional statistic of the form $s_i = \theta + k_\eta \eta + k_e e_i$, for some scalars k_η, k_e. For our purposes, we assume that these coefficients are functions of the traders' information acquisition policy. More acquisition reduces the traders' exposure to both types of noise. For tractability, we assume that the marginal effect of more acquisition on the reduction of the influence of both noises is the same, with the function f taking the form $f(y) = y^{-1/2}$. Such an assumption allows us to express the precision of the combined noise term ϵ as

$$\tau_\epsilon(y) \equiv \frac{y\tau_e\tau_\eta}{\tau_e + \tau_\eta}.$$

The analysis is facilitated by the above structure. However, the key insights extend to richer specifications in which the traders' information acquisition activity $y_i = (y_i^\eta, y_i^e)$ is multi-dimensional, with y_i^η and y_i^e parametrizing the traders' exposure to common and idiosyncratic noise, respectively.

2.3 Timing

At $t = 0$, traders simultaneously make their information acquisition decisions $(y_i)_{i\in[0,1]}$. A $t = 1$, traders privately observe their signals $(s_i)_{i\in[0,1]}$. At $t = 2$, both the traders and the representative supplier simultaneously submit their schedules. At $t = 3$, the market clears, the equilibrium price is formed, the equilibrium trades are implemented, and payoffs are realized.

2.4 Supply and demand schedules

As anticipated above, the representative supplier's (inverse) supply of the asset is given by $p = \alpha - u + \beta\tilde{x}$. Equivalently, his direct supply schedule is given by

$$\tilde{x} = \frac{1}{\beta}(p + u - \alpha).$$

Given her private information $I_i = (y_i, s_i)$, trader i's demand schedule maximizes, for each price p, the trader's expected payoff

$$\mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|I_i, p\right]$$

taking into account how the price $p = P(\theta, u, \eta)$ co-moves with the traders' fundamental value θ, the representative supplier's payoff shock u, and the common noise η in the traders' information. The solution to this problem is the demand schedule given by

$$X(p; I_i) = \frac{1}{\lambda}(\mathbb{E}[\theta|I_i, p] - p) \tag{1}$$

where $\mathbb{E}[\theta|I_i, p]$ denotes the trader's expectation of θ given the quality of the trader's information, as proxied by y_i, the realization s_i of the trader's private signal, and the price p.

2.5 Information acquisition

At $t = 0$, each trader $i \in [0, 1]$ selects y_i to maximize his expected profit

$$\mathbb{E}\left[\left(\theta - p - \frac{\lambda}{2}X(p; I_i)\right) X(p; I_i)|y_i\right] - \mathcal{C}(y_i)$$

where the expectation is over (s_i, θ, p), given y_i. Following the pertinent literature, we focus on equilibria and on team-efficient allocations (defined below) in which the market-clearing price $p = P(\theta, u, \eta)$ is an affine function of all aggregate variables (θ, u, η), and where all agents acquire information of the same quality (equivalently, pay the same attention to all relevant sources), and follow the same rule to map their information into the demand schedules.

3 Inefficiency in the Usage of Information

Fixing the precision of the traders' private information τ_ϵ (equivalently, their information acquisition activity $y_i = y$, all i), we start by solving for the traders' equilibrium demand schedules. We then compare the equilibrium demand schedules to their efficient counterparts (equivalently, to the decentralized efficient use of information), and discuss the nature of the inefficiency in the usage of information, and possible policies alleviating the inefficiency.

3.1 Equilibrium usage of information

In any symmetric equilibrium in which the price is an affine function of (θ, u, η), each trader's demand schedule is an affine function of her private signal s_i and the price p. That is,[6]

$$x_i = X(p; I_i) = a^* s_i + \hat{b}^* - \hat{c}^* p$$

[6]The reason why we are denoting the sensitivity $\hat{c}^*$ of the equilibrium demand schedules to the price and the constant term $\hat{b}^*$ in the equilibrium demand schedules with the ^ symbol is that, in the Appendix, we use the notation (a, b, c) to denote the sensitivity of the induced trades (the volume of the asset purchased/sold by each trader) to the traders' private information and the endogenous signal contained in the price. We do not use ^ for the sensitivity a^* of the equilibrium demand schedules to the traders' private information s_i because that sensitivity is the same no matter whether one looks at the submitted demand schedules or the at the induced trades.

for some scalars $(a^*, \hat{b}^*, \hat{c}^*)$ that depend on the exogenous parameters of the model, as well as on the quality of the agents' information $y_i = y$. Aggregating across traders, we then have that the aggregate demand is equal to

$$\tilde{x} = \int x_i di = a^*(\theta + f(y)\eta) + \hat{b}^* - \hat{c}^* p.$$

Notice that, given the general informational structure assumed above, although idiosyncratic errors in signals wash out in the aggregate demand,[7] the quality of the agents' information (parametrized by y) impacts the aggregate demand through its effect on the traders' exposure to common correlated noise η. Combining the above expression with the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$ from the representative liquidity supplier, we then have that the equilibrium price must satisfy

$$p = \frac{1}{1+\beta\hat{c}^*}\Big(\alpha - u + \beta\hat{b}^* + \beta a^*(\theta + f(y)\eta)\Big) = \frac{\alpha + \beta\hat{b}^*}{1+\beta\hat{c}^*} + \frac{\beta a^*}{1+\beta\hat{c}^*} z, \tag{2}$$

where

$$z = \theta + f(y)\eta - \frac{u}{\beta a^*}. \tag{3}$$

The information about θ contained in the market-clearing price is thus the same as the one contained in the endogenous public signal

$$z = \theta + \omega,$$

where

$$\omega = f(y)\eta - \frac{u}{\beta a^*}$$

is a combination of the common noise η in the traders' information and the payoff shock u to the representative supplier's schedule.

Note that, fixing y, given the sensitivity a^* of the traders' demand schedules to their private information s_i, the precision $\tau_\omega(a^*)$ of the noise ω in the endogenous signal z contained in the price is given by[8]

$$\tau_\omega(a^*) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta}. \tag{4}$$

[7] This is so since we make the convention that the analog of the strong law of large numbers holds for a continuum of independent random variables with uniformly bounded variances. The last property holds as long as the y_i's have a common lower bound strictly larger than 0.

[8] Because y is held fixed, to alleviate the notation, we drop the dependence of τ_ω on y and emphasize only its dependence on a.

When the price takes the form in (2), the conditional expectation of θ given s_i and p is given by

$$\mathbb{E}[\theta|s_i, p] = \mathbb{E}[\theta|s_i, z] = \gamma_1(\tau_\omega(a^*))s_i + \gamma_2(\tau_\omega(a^*))z \tag{5}$$

where, for any τ_ω, the functions γ_1 and γ_2 are given by[9]

$$\gamma_1(\tau_\omega) \equiv \frac{\tau_\epsilon y\tau_\eta\left(y\tau_\eta - \tau_\omega\right)}{y^2\tau_\eta^2(\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)} \tag{6}$$

and

$$\gamma_2(\tau_\omega) \equiv \frac{\tau_\omega y\tau_\eta\left(y\tau_\eta - \tau_\epsilon\right)}{y^2\tau_\eta^2(\tau_\omega + \tau_\epsilon + \tau_\theta) - \tau_\omega\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)}. \tag{7}$$

That is, each trader's expectation of θ is a weighted average of her private signal, s_i, and the endogenous public signal contained in the price, z. Note that, in the expressions above and throughout the rest of the section, we dropped the dependence of the precision $\tau_\epsilon(y)$ on y to ease the exposition. Using (1) and (5), we then have that the coefficients $(a^*, \hat{b}^*, \hat{c}^*)$ in the affine strategy describing the equilibrium demand schedules satisfy

$$a^* = \frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta\left(y\tau_\eta - \tau_\omega(a^*)\right)}{y^2\tau_\eta^2(\tau_\omega(a^*) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^*)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)}, \tag{8}$$

$\hat{c}^* = \hat{C}(a^*)$, and $\hat{b}^* = \hat{B}(a^*)$, where, for any a, the functions $\hat{C}$ and $\hat{B}$ are given by

$$\hat{C}(a) \equiv -\frac{\left(1 - \lambda a\frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a}{\beta(\beta+\lambda)a + \beta\left[\left(1 - \lambda a\frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a\right]}, \tag{9}$$

and

$$\hat{B}(a) \equiv \frac{\alpha}{\beta+\lambda}\left(\lambda\hat{C}(a) - 1\right). \tag{10}$$

From the properties of equation (8), we then have the following result:

Proposition 1. *Suppose $y_i = y$ for all i, with y fixed exogenously. There exits a unique symmetric equilibrium. The sensitivity of the traders' equilibrium demand schedules to their private information, a^*, is given by the unique real root to equation 8 and is such that $0 < a^* < \frac{1}{\lambda}$. Given a^*, the equilibrium values of the other two parameters $\hat{c}^*$ and $\hat{b}^*$ defining the equilibrium demand schedules are given by the functions (9) and (10).*

Fixing the quality of the traders' private information y, the equilibrium demand schedules

[9] Again, we drop the dependence of $\gamma_1(\tau_\omega(a^*; y); y)$ and $\gamma_2(\tau_\omega(a^*; y); y)$ on y to ease the notation, and only emphasize the dependence of these functions on the precision τ_ω of the endogenous signal z contained in the market-clearing price.

are thus obtained by solving a familiar fixed-point problem in which (a) the traders correctly account for the information contained in the market-clearing price, and (b) the latter is consistent with the submitted demand schedules. The novelty relative to previous work is the presence of common correlated error in the traders' information, η, which affects both the submitted schedules and the statistical properties of the market-clearing price.

3.2 Efficient Use of Information

We now isolate the inefficiencies in the submitted limit orders, by focusing again on the case in which the precision of the traders' private information is exogeneous.

3.2.1 Welfare losses

Ex-post total welfare is given by

$$W \equiv \int_0^1 \left(\theta x_i - \frac{\lambda}{2} x_i^2\right) di + \left(u - \alpha - \beta \frac{\tilde{x}}{2}\right) \tilde{x}.$$

The integral term is the total gross payoff that the traders derive from purchasing the asset. The remaining term is the gross payoff that the representative liquidity supplier derives from selling the asset. All payoffs are net of the money exchanged between the traders and the liquidity supplier. It is easy to see that, under complete information, the allocation (equivalently, the trades) that maximizes total surplus are given by $x_i = x^o$ for all i, with[10]

$$x^o \equiv \frac{\theta + u - \alpha}{\beta + \lambda}. \tag{11}$$

Under the first-best allocation, (ex-post) total welfare is then given by

$$W^o \equiv \left(\theta - \frac{\lambda}{2} x^o\right) x^o + \left(u - \alpha - \beta \frac{x^o}{2}\right) x^o = \left(\theta + u - \alpha - \frac{\beta + \lambda}{2} x^o\right) x^o.$$

Next, let

$$WL \equiv \mathbb{E}[W^o] - \mathbb{E}[W]$$

denote the (ex-ante) expected welfare losses that arise when the traders purchase the asset in quantities different from the first-best level, due to imperfect information. Under any strategy profile for the agents in which $X(p; I_i)$ is affine in s_i and p, the welfare losses can be expressed

[10] See the Appendix for the derivation.

as follows (the derivations are in the Appendix):

$$WL = \frac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2] + \lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2}. \tag{12}$$

The term $\mathbb{E}[(\tilde{x}-x^o)^2]$ captures the losses due to the discrepancy between the aggregate level of trade $\tilde{x}$ and its first-best counterpart, x^o. The term $\mathbb{E}[(x_i-\tilde{x})^2]$, instead, captures the losses due to the dispersion of the individual trades around the average level.

3.2.2 Team Problem

Consistently with the rest of the literature (see, among others, Vives 1988, Angeletos and Pavan 2007, Amador and Weill 2012, Myatt and Wallace 2012, and Vives 2017), we define the efficient use of information as the traders' strategy (that is, the collection of demand schedules) that minimizes the ex-ante welfare losses, subject to the constraint that the traders' demand schedules (equivalently, the trades) be affine in their private signal and the price. While the welfare definition accounts for the representative supplier's payoff, the solution to the above team problem treats the latter's behavior as given. This is for two reasons: (a) we are interested in isolating the inefficiencies that pertain to the traders' usage of information; (b) the analysis is motivated by markets in which policy interventions can manipulate the behavior of certain investors (the traders in our model) but not others (the liquidity suppliers in our model, who are meant to capture a variety of small investors, such as noisy traders and the like, whose behavior is believed to be difficulty to manipulate). Accordingly, $(a^T, \hat{b}^T, \hat{c}^T)$ identifies the efficient use of information if, whenever all traders submit the demand schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$, the welfare loses are as small as under any other affine schedule $x_i = a' s_i + \hat{b}' - \hat{c}' p$.[11] Paralleling the analysis of the equilibrium use of information in the previous subsection, we have that, when all traders submit the above demand schedules, the information contained in the market-clearing price is the same as the one in the endogenous signal

$$z = \theta + \omega$$

where the noise

$$\omega = f(y)\eta - \frac{u}{\beta a}$$

has the same structure as in the equilibrium usage of information.

Lemma 1. *For any sensitivity a of the demand schedules to the traders' private information,*

[11] Again, we use the symbol ˆ to distinguish the efficient demand schedules from the efficient trades.

the values of $\hat{c}$ and $\hat{b}$ in the demand schedules that minimize the welfare losses are given by the same functions (9) and (10) that define the equilibrium usage of information.

Using Lemma 1, we can express the welfare losses as a function of a and the precision

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_\eta \tau_u}{\beta^2 a^2 \tau_u + y\tau_\eta}.$$

of the endogenous signal z contained in the market-clearing price as follows (see the Appendix for the formal proof):[12]

$$WL(a,\tau_\omega(a)) = \frac{\left[\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta+\lambda\right)\tau_\omega(a)} + \frac{\lambda^2 a^2}{2\left(\beta+\lambda\right)y\tau_\eta} + \frac{\lambda a\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{\left(\beta+\lambda\right)y\tau_\eta}$$

$$+\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2\left(\beta+\lambda\right)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}. \tag{13}$$

The last term $\lambda a^2/y\tau_e$ in $WL(a,\tau_\omega(a))$ represents the welfare losses due to the dispersion of individual trades around the average trade. The other terms represent the losses due to the volatility of the aggregate volume of trade around its first-best level. Both losses are computed under the optimal choice of $\hat{c}$ and $\hat{b}$, using the result in Lemma 1.

The efficient level of a, which we denote by a^T, is thus the value of a that minimizes $WL(a,\tau_\omega(a))$. Now let

$$\begin{aligned}
\Delta(a) &\equiv -\frac{\tau_\epsilon \beta^2 y^4 \tau_\eta^4 \tau_u\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2\left(\beta^2 a^2\tau_u+y\tau_\eta\right)^2\left(\tau_\omega(a)+\tau_\theta\right)},\\
\Xi(a) &\equiv \frac{y\tau_\epsilon\tau_\eta^2\beta\left(\tau_\omega(a)+\tau_\theta\right)}{\lambda\tau_e}.
\end{aligned}$$

We then have the following result:

Proposition 2. *Suppose that $y_i = y$ for all i, with y fixed exogenously. The team problem has a unique solution. The efficient sensitivity a^T of the traders' demand schedules to their*

[12] Note that, given $(a,\tau_\theta,\tau_\eta,y,\tau_e)$, τ_u affects WL only through its effect on $\tau_\omega(a)$. Hence, holding $(a,\tau_\theta,\tau_\eta,y,\tau_e)$ fixed, changes in $\tau_\omega(a)$ can be thought of as originating in changes in τ_u.

private information is implicitly given by

$$a^T = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\omega(a^T))}{y^2\tau_\eta^2 (\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T)\tau_\epsilon (\tau_\theta + 2y\tau_\eta) + \Xi(a^T) + \Delta(a^T)} \tag{14}$$

and is such that $0 < a^T < \frac{1}{\lambda}$*. Given* a^T*, the other two parameters defining the efficient demand schedules,* $\hat{c}^T$ *and* $\hat{b}^T$*, are given by the same functions in (9) and (10) that describe the corresponding coefficients under the equilibrium usage of information.*

When, for any a, b and c are set optimally, the welfare losses $WL(a, \tau_\omega(a))$ are a convex function of a reaching a minimum at $a = a^T$, with $0 < a^T < \frac{1}{\lambda}$. Note that the equation that determines the value of a^T differs from the one that yields the equilibrium value of a^* only by the two terms $\Xi(a)$ and $\Delta(a)$ in the denominator of the right-hand side of (14). The first term, $\Xi(a)$, is a *pecuniary externality* that arises because the traders do not internalize that their demand schedules impact the co-movement between the market-clearing price and the aggregate shocks (u, θ, η), which, in turn, impacts the way the equilibrium trades co-move with these variables, the volatility of the aggregate volume of trade, and ultimately the payoffs of the other traders and of the representative liquidity supplier. Importantly, the pecuniary externality is independent of the informational content of the price. The term $\Xi(a)$ is always positive, thus contributing to an over-reaction of the equilibrium trades to private information. Essentially, traders respond strongly to their private information because they don't internalize the aggregate price movements that follow. The social planner, instead, internalizes such movements and demands that traders respond less to their private information.

The second term, $\Delta(a)$, is essentially a scaling of

$$\frac{\partial WL(a, \tau_\omega(a))}{\partial \tau_\omega(a)} \frac{\partial \tau_\omega(a)}{\partial a}.$$

Therefore, this term can be thought of as a proxy for the *information externality* that originates in the fact that traders do not internalize how their demand schedules impact the informativeness of the equilibrium price and hence the possibility for other traders to respond to the relevant shocks. This term is always negative thus contributing to under-reaction of the equilibrium demand schedules to private information. Essentially, traders do not consider that responding more to their private information leads to a more informative price and hence to more efficient trades. The social planner, instead, internalizes this effect and asks that the traders respond more to their private information.

The two externalities in turn reflect the two fundamental roles played by prices in financial markets: as a proxy for the cost of trading, and as a source of information about the relevant payoffs, as we will discuss in Subsection 3.2.4.

3.2.3 Fictitious Environment

To shed more light on the role of the above externalities, it is useful to consider a *fictitious environment* in which traders are naive in that they do not recognize the information contained in the market-clearing price. Such a benchmark is similar in spirit to the (fully) cursed equilibrium of Eyster and Rabin 2005. To facilitate the comparison to the true economy, further assume that, in this fictitious environment, each trader, in addition to receiving the private signal

$$s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv \epsilon_i}$$

as in the baseline model, also observes an exogenous public signal

$$z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv \zeta}$$

whose structure is the same as the one of the endogenous public signal contained in the market-clearing price, but with the endogenous noise $-u/\beta a$ replaced by the exogenous one χ, with the latter drawn from a Normal distribution with mean zero and variance τ_χ^{-1} independently of all other variables (this shock is the same for all traders).

As we show in the Appendix, in the cursed equilibrium of this fictitious economy, traders submit affine demand schedules $x_i = a^*_{exo} s_i + \hat{b}^*_{exo} + \hat{c}^*_{exo} z - \hat{d}^*_{exo} p$, where the sensitivity of the traders' demands to their private information is given by

$$a^*_{exo} \quad = \quad \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta \left(y\tau_\eta - \tau_\zeta\right)}{y^2 \tau_\eta^2 (\tau_\zeta + \tau_\epsilon + \tau_\theta) - \tau_\zeta \tau_\epsilon (\tau_\theta + 2y\tau_\eta)}. \tag{15}$$

Note that the formula in (15) is similar to the one in (8) in the baseline economy, except for the fact that the precision $\tau_\omega(a)$ of the endogenous public signal contained in the market-clearing price is replaced by the precision τ_ζ of the exogenous public signal about θ.

Now suppose that, in this fictitious economy, the planner can select a but, given the latter, is constrained to choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a^*_{exo} and $(\hat{b}^*_{exo}, \hat{c}^*_{exo}, \hat{d}^*_{exo})$ as in the cursed equilibrium. [13] The level of a that maximizes ex-ante welfare

[13] Whereas, in the baseline economy, choosing $\hat{b}^T$ and $\hat{c}^T$ to satisfy the same relationship between a^* and $(\hat{b}^*, \hat{c}^*)$ as in equilibrium is without loss of optimality for the planner, as we showed above, in the fictitious economy, this need not be true. However, imposing the restriction permits us to isolate the relevant effects. It

is then equal to

$$a^T_{exo} = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\zeta)}{y^2 \tau_\eta^2 (\tau_\zeta + \tau_\epsilon + \tau_\theta) - \tau_\zeta \tau_\epsilon (\tau_\theta + 2y\tau_\eta) + \frac{y\tau_\epsilon \tau_\eta^2 \beta(\tau_\zeta + \tau_\theta)}{\lambda \tau_e}}. \quad (16)$$

Again, the formula for a^T_{exo} is similar to the one for a^T in the baseline model, except for the fact that $\tau_\omega(a)$ is replaced by τ_ζ and the term $\Delta(a)$ in the denominator of the expression giving the socially-optimal level of a in the baseline model is equal to zero, reflecting the fact that the agents do lot learn from the price. Note that $y\tau_\epsilon\tau_\eta^2\beta(\tau_\zeta + \tau_\theta)/\lambda\tau_e$ has exactly the same form as the pecuniary externality $\Xi(a)$ in the baseline model. Hence, in this fictitious economy, the cursed-equilibrium demand schedules unambiguously feature an excessively high sensitivity to private information relative to the solution to the planner's problem: $a^*_{exo} > a^T_{exo}$. Furthermore, when the precision of the exogenous public signal in the cursed economy is the same as the precision of the endogenous public signal under the solution to the planner's problem in the baseline model (that is, when $\tau_\zeta = \tau_\omega(a^T)$), the values of a^T and a^T_{exo} are easily comparable and a^T_{exo} coincides with the solution to the equation $\partial WL(a^T, \tau_\omega(a^T))/\partial a = 0$. Relative to the solution to the planner's problem in the cursed economy, the planner in the true, baseline, economy recognize the value of increasing the precision of information contained in the price and thus demands that traders increase the sensitivity of their demand schedules to their private information ($a^T > a^T_{exo}$).

3.2.4 Sign of externalities and slope of demand curves

We now return to the economy in which both the traders and the planner account for the information contained in the market-clearing price. Whether the sensitivity of the equilibrium demand schedules to the traders' private information is excessively high or excessively low (compared to the efficient level a^T) then depends on which of the two externalities described above prevails. Comparing (8) with (14), we have that the sign of $(a^* - a^T)$ equals the sign of $\Xi(a^T) + \Delta(a^T)$. When $\Xi(a^T) + \Delta(a^T) = 0$, the two externalities cancel each other out, the submitted schedules are inelastic (i.e., $\hat{c}^T = 0$) and $a^* = a^T$ (see Lemma 2 in the Appendix). When $\Xi(a^T) + \Delta(a^T) > 0$, the pecuniary externality dominates, $\hat{c}^T > 0$ (the efficient demand schedules are downward sloping) and the equilibrium features an excessive response to private information. When, instead, $\Xi(a^T) + \Delta(a^T) < 0$, the information externality dominates,

is also possible to show that, given a, a planner who expects p to be orthogonal to (θ, η), as do the traders in the cursed economy, optimally chooses $(\hat{b}, \hat{c}, \hat{d})$ to satisfy the same relationship between these coefficients and a as in the cursed equilibrium (the proof is available upon request).

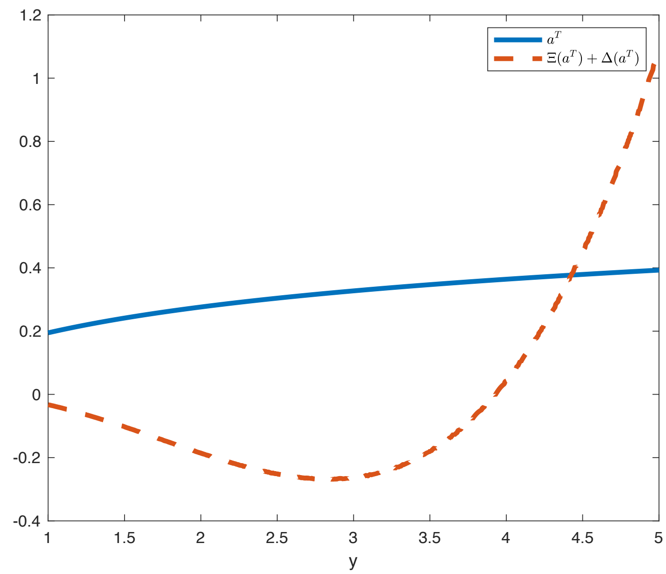

Figure 1: The blue solid line corresponds to a^T while the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

$\hat{c}^T < 0$ (the efficient demands are upward sloping) and the equilibrium response to private information is insufficiently low.

It is worth noting that if the traders were restricted to submitting market orders (like in a Cournot model), then the usage of information would be efficient since the two externalities would not be present (See Subsection 6.2 for a formal proof of this result).

Using simulations, it is possible to nail down the effect of variations in the quality y of the traders' private information on the two externalities and on the slope of the efficient demand schedules. Figure 1 depicts the sensitivity of the traders' efficient demand schedules to their private information a^T (solid blue curve) as well as the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ (dashed orange curve), as a function of the quality y of the traders' private information.

As y increases, the efficient response a^T to the traders' private information increases, reflecting the higher value of responding to more accurate private information. Furthermore, because both Ξ and Δ increase with a^T, a higher y contributes to a higher value of $\Xi(a^T) + \Delta(a^T)$ via the indirect effect that y has on the two externalities through a^T.

In addition, holding a^T fixed, we have that y has a direct effect on both $\Xi(a^T)$ and $\Delta(a^T)$. Whereas $\Xi(a^T)$ is increasing in y, $\Delta(a^T)$ is decreasing. Combining the direct with the indirect effects, we then have that $\Xi(a^T)$ unambiguously increases with y, whereas $\Delta(a^T)$ is non-monotonic in y. For small values of y, the sum of the two externalities is negative and

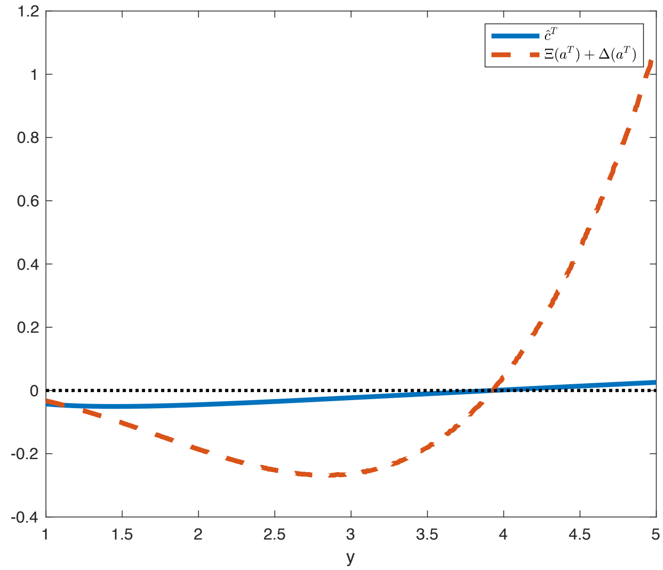

Figure 2: The blue solid line corresponds to $\hat{c}^T$ whereas the orange dashed line represents the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$. The two curves switch signs for the same value of y. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

decreasing in y, whereas, for sufficiently high values of y, the sum of the two externalities is positive and increasing in y, as can be seen from Figure 1.

Next, we turn to the relationship between the two externalities and the slope of the efficient demand schedules, $\hat{c}^T$. Figure 2 depicts the sensitivity $\hat{c}^T$ of the efficient demand schedules to the price (the solid blue curve) along with the sum of the two externalities $\Delta(a^T) + \Xi(a^T)$ (the orange dashed curve), as a function of the quality y of the traders' private information. The two curves switch sign for the same value of y. As explained above, when the traders possess high quality private information (high values of y), the marginal value of generating additional information through the price is low and the pecuniary externality dominates over the information externality, so that $\Xi(a^T) + \Delta(a^T)$ is positive and increasing in y. In this case, $\hat{c}^T$ is positive meaning that the efficient demand schedules are downwards sloping, as they would be in an economy in which the fundamental value of the asset θ is known to the traders. When, instead, the traders possess low quality private information, the information externality dominates over the pecuniary externality so that $\Xi(a^T) + \Delta(a^T)$ is negative and first decreasing and then increasing in y. In this case, $\hat{c}^T$ is negative meaning that the efficient demand schedules are downwards sloping, reflecting the high sensitivity of the traders' estimates of the fundamental value of the asset θ to the price, relatively to the sensitivity of the same estimates to their private information.

We conclude this subsection by highlighting the role that the common noise η in the traders' private information plays for the sign and magnitude of the two externalities identified above. Unsurprisingly, both a^* and a^T are increasing in τ_η reflecting the fact that responding to private information is more valuable (both for the traders and for the planner) when it is less affected by correlated noise and hence more precise. Similarly, holding a fixed, the precision $\tau_\omega(a)$ of the endogenous signal z contained in the market-clearing price naturally increases with τ_η, reflecting the fact that the noise in the traders' signals becomes less correlated when τ_η increases and, as a result, washes out more at the aggregate level, making the price more informative, for given demand schedules. Furthermore, fixing a^T, the absolute value of both $\Xi(a^T)$ and $\Delta(a^T)$ increases with τ_η, reflecting the larger role that either externality plays when the noise in private signals is less correlated. However, whereas the pecuniary externality $\Xi(a^T)$ increases with τ_η, the information externality $\Delta(a^T)$ decreases with it. Combining all of the above effects, we then have that the sum of the two externalities $\Xi(a^T) + \Delta(a^T)$ can be non-monotonic in τ_η, depending on the other parameters' values.

3.3 Policies inducing efficient trades

Next, we discuss policies that correct the inefficiencies in the usage of information identified in the previous subsections, once again holding fixed the quality of the traders' information y for the time being.

Proposition 3. *Suppose that $y_i = y$ for all i, with y fixed exogenously. The efficient use of information can be implemented with a policy that charges the traders a total tax bill equal to $T(x_i, p) = \frac{\delta}{2}x_i^2 - t_0 x_i + t p x_i$, where*

$$\delta = \frac{\lambda\left(\Xi(a^T) + \Delta(a^T)\right)}{y^2\tau_\eta^2(\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)},$$

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda}\left[\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] - \beta a^T}{\frac{\beta}{\beta+\lambda}\left[\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta} - \beta a^T\right] + \beta a^T},$$

and

$$t_0 = (1+t_p)\alpha - \frac{\alpha\left(\lambda + \delta + (1+t_p)\beta\right)}{\beta + \lambda}.$$

The efficient use of information can thus be induced through a combination of a linear-quadratic tax $\frac{\delta}{2}x_i^2 - t_0 x_i$ on the individual volume of trade (equivalently on the quantity of the asset purchased), along with a linear ad-valorem tax $t_p p x_i$. The role of δ is to manipulate the traders' adjustment cost (from λ to $\lambda + \delta$). This manipulation suffices to induce the traders to submit demand schedules whose sensitivity to their exogenous private information is equal to the efficient level a^T. The role of the linear ad-valorem tax is to guarantee that, once the sensitivity a coincides with the efficient level a^T, the sensitivity c of the equilibrium demand schedules to the price coincides with the efficient level $\hat{c}^T$. In the absence of such a correction, the traders fail to submit the efficient demand schedules even if they respond efficiently to their private information.

Finally, the role of the linear tax $t_0 x_i$ on the individual volume of trade is to guarantee that the fixed part b of the demand schedule also coincides with its efficient counterpart $\hat{b}^T$.

4 Inefficiency in information acquisition and policy corrections

We now investigate whether efficiency in trading (equivalently, in information usage) implies efficiency in the acquisition of private information. We start by considering the case where efficiency in usage is induced by controlling directly the agents' usage of information, that is, by imposing that the traders submit the efficient demand schedules $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$. We then consider the case where efficiency in usage is induced through the policy in Proposition 3. In both cases, we find that agents do not acquire information efficiently. We conclude by considering richer families of policy interventions which permits us to uncover both an impossibility and a couple of possibility results.

Let y^T denote the socially optimal precision of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the precision of private information is y^T. Next, let $\mathbb{E}[W^T; \bar{y}]$ denote ex-ante gross welfare when all traders acquire information of quality $\bar{y}$ but then submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Such a welfare function is gross of the costs of information acquisition. Finally, let $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ denote the ex-ante gross profit of a trader acquiring information of quality y_i when all other traders acquire information of quality $\bar{y}$, and all traders (including i) submit the efficient demand schedules for information of quality y^T (that is, the schedules corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ mentioned above). The payoff is again gross of the cost of information acquisition. We then have the following result:

Proposition 4. *Let y^T denote the socially optimal quality of private information and suppose that all traders are constrained to submit the efficient demand schedules for information of quality y^T (parametrized by $(a^T, \hat{b}^T, \hat{c}^T)$). When $\hat{c}^T > 0$ (i.e., when the pecuniary exter-*

nality dominates over the information externality so that the efficient demand schedules are downward sloping),

$$\left.\frac{\partial \mathbb{E}[\pi_i^T; y_i, \bar{y}]}{\partial y_i}\right|_{y_i=\bar{y}=y^T} > \left.\frac{\partial \mathbb{E}[W^T; \bar{y}]}{\partial \bar{y}}\right|_{\bar{y}=y^T}$$

whereas the opposite inequality holds when $\hat{c}^T < 0$ *(i.e., when the information externality dominates over the pecuniary externality and, as a result, the efficient demand schedules are upward sloping).*

Using the envelope theorem, we show in the Appendix that y^T solves the optimality condition

$$\left.\frac{\partial \mathbb{E}[W^T; \bar{y}]}{\partial \bar{y}}\right|_{\bar{y}=y^T} = \mathcal{C}'(y^T). \tag{17}$$

Because $\mathbb{E}[\pi_i^T; y_i, \bar{y}]$ is strictly concave in y_i, the result in the proposition implies that, when $\hat{c}^T > 0$ (i.e., when the pecuniary externality dominates over the information externality and the efficient demand schedules are downward sloping), a trader who expects all other traders to acquire information of quality y^T has incentives to acquire information of quality higher than the efficient level y^T. Recall from the previous section that, in such situations, in the absence of policy interventions, agents over-respond to private information. The result in the last proposition then implies that, forcing a trader to respond less to his private information comes with the undesirable effect of inducing him to over-invest in information acquisition.

When, instead, $\hat{c}^T < 0$ (i.e., when the information externality dominates over the pecuniary externality and the efficient demand schedules are upward sloping), in the absence of policy interventions, the trader would under-respond to private information. The result in the last proposition then implies that, in such a situation, forcing the trader to trade efficiently would induce him to under-invest in information acquisition.

In the special case in which $\hat{c}^T = 0$ (that is, when the efficient demand schedules are inelastic and hence can be implemented with market orders), in the absence of policy interventions, a trader endowed with information of quality y^T would trade efficiently. In this case, when information is endogenous, the trader would acquire information of efficient quality y^T, even in the absence of policy interventions.

The next result shows that the properties of individual best responses discussed above are inherited by the equilibrium choices.

Proposition 5. *Let* y^T *denote the socially optimal quality of private information and suppose that all traders are constrained to submit the efficient demand schedules for information of quality* y^T *(parametrized by* $(a^T, \hat{b}^T, \hat{c}^T)$*). When* $\hat{c}^T > 0$ *(i.e., when the efficient demand schedules are downward-sloping), the quality of private information acquired in equilibrium is* *higher than y^T, whereas the opposite is true when $\hat{c}^T < 0$ (i.e., when the efficient demand schedules are upward-sloping).*

Note that the last two results hinge on $\tau_\eta \in (0, +\infty)$. When $\tau_\eta = 0$, the noise in the agents' private signals is infinite, making the signals worthless both for the individual traders and for the planner. When, instead, $\tau_\eta \to +\infty$, the correlated noise in the agents' private signals disappears, in which case the aggregate volume of trade becomes invariant to the quality of the traders' private information. In this case, holding the demand schedules fixed, the only effect of an increase in the quality of the traders' private information on welfare is through the reduction in the dispersion of individual trades around the aggregate trade. Because this effect is weighted equally by the planner and by the individual traders, the private and the social value of information coincide in this case, which guarantees that traders acquire information of efficient quality.

The above propositions thus help clarifying that the results obtained in the previous literature establishing that efficiency in trade implies efficiency in information acquisition hinge on the simplifying assumption that the noise in the traders' information is uncorrelated and unaffected by individual investments in information acquisition.

When, instead, the noise in the traders' information is correlated and endogenous, policies that force the traders to submit the efficient limit orders may induce the traders to over-investing (alternatively, under-investing) in information acquisition. As explained above, this is because traders fail to internalize both how their private information affects the covariance of the aggregate volume of trade with all the relevant payoff shocks and the ability of other traders to learn from prices.

Recall that, for small y, the information externality dominates over the pecuniary externality and demand schedules are upward sloping whereas, for large y, the pecuniary externality dominates and the demand schedules are downward sloping. The above results thus also imply that, when traders are constrained to trade efficiently, as technological progress makes information cheaper (that is, the cost of information acquisition decreases), the economy eventually enters into a regime of over-investment in information acquisition. We conjecture that the following result is also true but did not establish it formally:

Conjecture: *When $\hat{c}^T > 0$ (downward-sloping efficient demand schedules), the equilibrium in the absence of policy interventions features excessive acquisition and excessive usage of private information, i.e., $y^* > y^T$ and $a^* > a^T$, whereas the opposite is true (i.e., $y^* < y^T$ and $a^* a^T$ when $\hat{c}^T < 0$ (upward-sloping efficient demand schedules).*

We now address the question of whether efficiency in information acquisition can be induced through an appropriate design of fiscal policy, in a world where the traders can be forced to submit the efficient demand schedules (we address the more relevant question of whether efficiency in both acquisition and trade can be induced through an appropriate fiscal policy at the end of the section).

Proposition 6. *Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the quality of information is y^T. Suppose all traders are constrained to submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$ but can choose the quality of private information. The traders can be induced to acquire information of quality y^T by charging them a tax bill $T(p, x_i) = \hat{t}_p p x_i$ with*

$$\hat{t}_p = \frac{\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T}{\beta a^T},$$

where γ_2 is the function defined in (7).

That is, if traders can be trusted to trade efficiently, then efficiency in information acquisition can be induced through a simple ad valorem tax/subsidy on traders' total expenditure px_i (equivalently, with a familiar proportional tax/subsidy on the price of the asset, similar to those often discussed in the policy debate). The result, however, hinges on the traders being forced to trade efficiently (that is, on them submitting the efficient demand schedules for quality of information y^T).

We now address the more relevant question of whether efficiency in both information acquisition and information usage can be induced through an appropriate design of the fiscal policy. In the previous section, we showed that, when the quality of information is exogenous, efficiency in information usage can be induced through a combination of a linear-quadratic tax on the individual volume of trade paired with a tax/subsidy on the price (both rebated in a lump-sum manner, if desired). Based on other results in the literature, one may conjecture that the same policy mix also induces efficiency in information acquisition. The next result shows that this is not the case. If the planner were to use the tax $T(p, x_i)$ in Proposition 3 (applied to $\bar{y} = y^T$) that induces the traders to submit the efficient demand schedules when y_i is exogenously fixed at $y_i = y^T$ for all i, then, when the quality of private information is endogenous, the traders would acquire information of quality other than y^T and then submit demand schedules other than the efficient ones. More generally, the next proposition shows that there exists no policy measurable in the individual volume of trade and in the price of the financial asset that induces efficiency in both trading and information acquisition.

Proposition 7. *Generically (i.e., with the exception of a set of parameters of zero Lebesgue measure), there exists no policy $T(x_i, p)$ that induces efficiency in both information acquisition and information usage.*

The result is established in the Appendix by showing that any policy that induces the traders to submit the efficient demand schedules once they collect the efficient amount of private information y^T must coincide with the one in Proposition 3 (applied to $\bar{y} = y^T$), except for terms that play no role for incentives. However, any such a policy induces the traders to misperceive the marginal value of their private information (around the efficient level y^T) and hence to collect an inefficient amount of private information.

To induce efficiency in both information acquisition and trading, the planner must thus resort to un-orthodox policies where the tax bill is contingent on information other than the individual volume of trade x_i and the price of the financial asset p. Our next result shows that efficiency in both information acquisition and trade can be induced by conditioning the marginal tax rate on the aggregate volume of trade:

Proposition 8. *There exists a (linear-quadratic) tax*

$$T^*(x_i, \tilde{x}, p) = \frac{\delta^*}{2} x_i^2 + (t_{\tilde{x}}^* \tilde{x} - t_0^*)\, x_i + t_p^* p x_i$$

that induces efficiency in both information acquisition and trade.

As anticipated above, the contingency $t_{\tilde{x}}^* \tilde{x}$ of the marginal tax rate $\partial T^*(x_i, \tilde{x}, p)/\partial x_i$ on the aggregate volume of trade $\tilde{x}$ is essential to induce efficiency in both acquisition and trade. As we show in the Appendix, with the above policy, the planner can equalize the expected marginal tax rate

$$\frac{\partial}{\partial x_i} \mathbb{E}[T^*(x_i, \tilde{x}, p) | x_i, p; y_i, y^T] \Big|_{x_i = a^T s_i + \hat{b}^T - \hat{c}^T p;\, y_i = y^T}$$

of each individual who acquires information of quality $y_i = y^T$ and then trades efficiently with the discrepancy

$$\mathbb{E}[\theta | x_i, p; y_i, y^T] - p - \lambda x_i \big|_{x_i = a^T s_i + \hat{b}^T - \hat{c}^T p;\, y_i, y^T}$$

between the marginal benefit and the marginal cost of expanding the trades around the efficient level $x_i = a^T s_i + \hat{b}^T - \hat{c}^T p$. Eliminating such a discrepancy is essential to induce efficiency in trades. Importantly, the new contingency provides the planner with flexibility on how to eliminate such a discrepancy (when, instead, the policy depends only on x_i and p, there exists a unique way of eliminating such a discrepancy, as shown in the proof of Proposition 7). This extra flexibility in turn can be used to realign the marginal private value of more precise private information to its social counterpart, something that is not possible when the policy depends only on x_i and p. In the proof in the Appendix, we also show that the policy that implements efficiency in both information acquisition and trade is in fact unique up to terms that do not matter for incentives.

Our final result shows that, when the acquisition of information is verifiable (e.g., when traders acquire information of verifiable quality from known sources), the contingency of the policy on the aggregate volume of trade can be dispensed with and replaced with a dependency of the policy on the expenditure on information acquisition.

Proposition 9. *Let y^T denote the socially optimal quality of private information and $(a^T, \hat{b}^T, \hat{c}^T)$ the coefficients describing the efficient demand schedules when the quality of information is y^T. Suppose that the acquisition of private information is verifiable and let T^{tot} denote the policy defined by*

$$T^{tot}(x_i, p, y_i) = \frac{\delta}{2}x_i^2 - t_0 x_i + p t_p x_i - A y_i$$

where (δ, t_p, t_0) are as in Proposition 3 and

$$A = -\frac{\left(a^T\right)^2}{2\tau_\eta \left(y^T\right)^2}\left[\frac{\beta(\beta+\lambda)\hat{c}^T}{\left(1+\beta\hat{c}^T\right)^2} + \frac{\beta t_p + \delta}{1+\beta\hat{c}^T}\right] - \frac{\delta\left(a^T\right)^2}{2\tau_e \left(y^T\right)^2}.$$

The above policy induces efficiency in both information acquisition and information usage.

Simulations show that $A < 0$ when $\hat{c}^T > 0$ and $A > 0$ when $\hat{c}^T < 0$. That is, expenses on information acquisition are taxed when the efficient demand schedules are downward sloping and subsidized when they are upward sloping, reflecting the fact that, under the policy of Proposition 3, agents over-invest in information acquisition in the former case and underinvest in the latter.

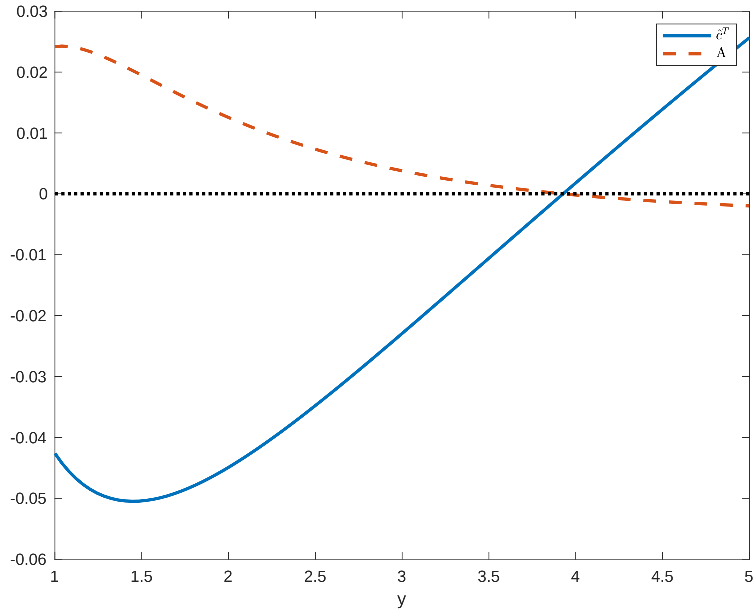

Figure 3: The blue solid line corresponds to $\hat{c}^T$ whereas the orange dashed line represents the subsidy/tax A on information acquisition (scaled by a factor of 10 for readability). The two curves switch signs for the same value of y. The parameter values used for this simulation are $\lambda = \beta = \tau_e = \tau_\eta = \tau_\theta = 1$, $\tau_u = 30$, and $1 \leq y \leq 5$.

5 Conclusions

We relate the sources of inefficiency in trading in financial markets to those in the collection of private information. We show that, when the private information the traders collect prior to submitting their limit orders is exogenous, inefficiency in trading can be corrected with a an appropriate combination of ad-valorem taxes with non-linear subsidies/taxes on the volume of individual trades. When information is endogenous, instead, and the noise in information is correlated among traders, any policy measurable in the price of the financial asset and in the volume of individual trades that induces efficiency in trading fails to induce efficiency in information acquisition. However, the above negative result can be turned into a positive one by conditioning the marginal tax rates on the aggregate volume of trade or, when feasible, by subsidizing the acquisition of private information when the latter is verifiable.

A key driver for the identified inefficiencies is the correlation in the noise in the traders' private information. In future work, it would be interesting to extend the analysis to a broader class of economies in which financial decisions interact with real decisions and in which agents trade multiple assets over multiple periods.

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6 Appendix

6.1 Omitted Proofs and Extended Derivations

Proof of Proposition 1.

As explained in the main text, when the traders submit affine demand schedules with parameters $(a, \hat{b}, \hat{c})$, the equilibrium price is equal to

$$p = \frac{\alpha + \beta \hat{b}}{1 + \beta \hat{c}} + \frac{\beta a}{1 + \beta \hat{c}} \left(\theta + f(y)\eta - \frac{u}{\beta a} \right).$$

The information about θ contained in the equilibrium price is thus the same as the one contained in a public signal

$$z = \theta + \omega,$$

with noise

$$\omega \equiv f(y)\eta - \frac{u}{\beta a}$$

of precision[14]

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{(\beta^2 a^2 \tau_u + y \tau_\eta)}.$$

In turn, this implies that the equilibrium trades $x_i = a s_i + \hat{b} - \hat{c} p$ are affine functions of the traders' exogenous private information s_i and the endogenous public information z contained in the price. That is, when the endogenous public information contained in the price is equivalent to z, a trader with private signal s_i purchases an amount of the asset equal to

$$x_i = a s_i + b + c z$$

where

$$b = \hat{b} - \hat{c} \frac{\alpha + \beta \hat{b}}{1 + \beta \hat{c}} \tag{18}$$

and

$$c = -\frac{\beta a \hat{c}}{1 + \beta \hat{c}}. \tag{19}$$

For each vector $(a, \hat{b}, \hat{c})$ describing the traders' demand schedules, there thus exists a unique vector (a, b, c) describing the traders' equilibrium trades as a function of their (exogenous) private information, s_i, and (endogenous) public information, z, and vice versa. Hereafter, we find it more convenient to characterize the equilibrium use of information in terms of the vector (a, b, c) describing the equilibrium trades. When the individual trades are given by $x_i = a s_i + b + c z$, the aggregate trade is equal to

$$\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b + cz.$$

[14] To derive $\tau_\omega(a)$ we use the fact that $f(y) = 1/\sqrt{y}$.

Using the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$, we thus have that

$$\tilde{x} = a(z + \frac{u}{\beta a}) + b + cz = (a+c)z + \frac{u}{\beta} + b.$$

Using the expression for the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$ from the representative investor, we then have that the equilibrium price can be expressed as a function of a and the endogenous public signal z as follows:

$$p = \alpha + \beta b + \beta(a+c)z. \tag{20}$$

Next, observe that

$$\begin{aligned}
\mathbb{E}[\theta|I_i, p] = \mathbb{E}[\theta|s_i, z] &= \begin{bmatrix} Cov(\theta, s_i) & Cov(\theta, z) \end{bmatrix} \begin{bmatrix} Var(s_i) & Cov(s_i, z) \\ Cov(s_i, z) & Var(z) \end{bmatrix}^{-1} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix} \\
&= \begin{bmatrix} \sigma_\theta^2 & \sigma_\theta^2 \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 + \sigma_\epsilon^2 & \sigma_\theta^2 + f(y)^2\sigma_\eta^2 \\ \sigma_\theta^2 + f(y)^2\sigma_\eta^2 & \sigma_\theta^2 + \sigma_\omega^2(a) \end{bmatrix}^{-1} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix},
\end{aligned}$$

where $\sigma_\theta^2 \equiv \tau_\theta^{-1}$, $\sigma_\omega^2(a) \equiv \tau_\omega(a)^{-1}$, $\sigma_\eta^2 \equiv \tau_\eta^{-1}$, and $\sigma_\epsilon^2 \equiv \tau_\epsilon^{-1}$. Substituting for the inverse of the variance-covariance matrix, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i, z] &= \frac{1}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2} \times \\
&\quad \begin{bmatrix} \sigma_\theta^2 & \sigma_\theta^2 \end{bmatrix} \begin{bmatrix} \sigma_\theta^2 + \sigma_\omega^2(a) & -(\sigma_\theta^2 + f(y)^2\sigma_\eta^2) \\ -(\sigma_\theta^2 + f(y)^2\sigma_\eta^2) & \sigma_\theta^2 + \sigma_\epsilon^2 \end{bmatrix} \begin{bmatrix} s_i - \mathbb{E}[s_i] \\ z - \mathbb{E}[z] \end{bmatrix}.
\end{aligned}$$

Expanding the quadratic form, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i, z] &= \frac{\sigma_\theta^2\left(\sigma_\omega^2(a) - f(y)^2\sigma_\eta^2\right)}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2}(s_i - \mathbb{E}[s_i]) \\
&\quad + \frac{\sigma_\theta^2\left(\sigma_\epsilon^2 - f(y)^2\sigma_\eta^2\right)}{(\sigma_\theta^2 + \sigma_\epsilon^2)(\sigma_\theta^2 + \sigma_\omega^2(a)) - (\sigma_\theta^2 + f(y)^2\sigma_\eta^2)^2}(z - \mathbb{E}[z]).
\end{aligned}$$

Simplifying, and using the fact that $\sigma_\theta^2 \equiv \tau_\theta^{-1}$, $\sigma_\omega^2(a) \equiv \tau_\omega(a)^{-1}$, $\sigma_\eta^2 \equiv \tau_\eta^{-1}$, $\sigma_\epsilon^2 \equiv \tau_\epsilon^{-1}$, and $f(y) = 1/\sqrt{y}$, we have that

$$\begin{aligned}
\mathbb{E}[\theta|s_i, z] &= \frac{\frac{1}{\tau_\theta}\left(\frac{1}{\tau_\omega(a)} - \frac{1}{y\tau_\eta}\right)}{(\frac{1}{\tau_\theta} + \frac{1}{\tau_\epsilon})(\frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)}) - (\frac{1}{\tau_\theta} + \frac{1}{y\tau_\eta})^2}(s_i - \mathbb{E}[s_i]) \\
&\quad + \frac{\frac{1}{\tau_\theta}\left(\frac{1}{\tau_\epsilon} - \frac{1}{y\tau_\eta}\right)}{(\frac{1}{\tau_\theta} + \frac{1}{\tau_\epsilon})(\frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)}) - (\frac{1}{\tau_\theta} + \frac{1}{y\tau_\eta})^2}(z - \mathbb{E}[z]),
\end{aligned}$$

from which we obtain that

$$\mathbb{E}[\theta|s_i,z] = \frac{y^2\tau_\eta^2\tau_\epsilon\left(1-\frac{\tau_\omega(a)}{y\tau_\eta}\right)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\omega(a)+\tau_\theta)-\tau_\epsilon\tau_\omega(a)(\tau_\theta+2y\tau_\eta)}(s_i-\mathbb{E}[s_i]) + \frac{y^2\tau_\eta^2\tau_\omega(a)\left(1-\frac{\tau_\epsilon}{y\tau_\eta}\right)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\omega(a)+\tau_\theta)-\tau_\epsilon\tau_\omega(a)(\tau_\theta+2y\tau_\eta)}(z-\mathbb{E}[z]).$$

Finally, using the fact that $\mathbb{E}[s_i]=\mathbb{E}[z]=0$, we have that

$$\mathbb{E}[\theta|s_i,z] = \gamma_1(\tau_\omega(a))s_i+\gamma_2(\tau_\omega(a))z$$

where[15]

$$\gamma_1(\tau_\omega(a)) \equiv \frac{\tau_\epsilon y\tau_\eta\left(y\tau_\eta-\tau_\omega(a)\right)}{y^2\tau_\eta^2\left(\tau_\omega(a)+\tau_\epsilon+\tau_\theta\right)-\tau_\omega(a)\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)} \tag{21}$$

and

$$\gamma_2(\tau_\omega(a)) \equiv \frac{\tau_\omega(a)\left(y^2\tau_\eta^2-\tau_\epsilon y\tau_\eta\right)}{y^2\tau_\eta^2\left(\tau_\omega(a)+\tau_\epsilon+\tau_\theta\right)-\tau_\omega(a)\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)} = \left(1-\gamma_1(\tau_\omega(a))\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}. \tag{22}$$

Now recall that the equilibrium trades must satisfy

$$x_i = \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i,z]-p\right).$$

Using the fact that $p=\alpha+\beta b+\beta(a+c)z$, and the characterization of $\mathbb{E}[\theta|s_i,z]$ from above, we thus have that

$$x_i = \frac{1}{\lambda}\left[\gamma_1(\tau_\omega(a))s_i-(\alpha+\beta b)+\left(\gamma_2(\tau_\omega(a))-\beta(a+c)\right)z\right].$$

We conclude that (1) the sensitivity of the equilibrium trades to private information must satisfy

$$a=\frac{\gamma_1(\tau_\omega(a))}{\lambda}, \tag{23}$$

(2) the sensitivity of the equilibrium trades to the endogenous public information must satisfy

$$c=\frac{1}{\lambda}\left(\gamma_2(\tau_\omega(a))-\beta(a+c)\right),$$

and (3) the constant b in the equilibrium trades must satisfy

$$b=-\frac{\alpha+\beta b}{\lambda}. \tag{24}$$

Replacing the expression for $\gamma_1(\tau_\omega(a))$ in (21) into (23), we thus conclude that the sensitivity a^* of the equilibrium demand schedules to the traders' private information must solve equation (8) in the proposition.

Using (22), along with the fact that $\gamma_1(\tau_\omega(a))=\lambda a$, in turn we have that

[15] Consistently with the rest of the analysis above, because y is held constant, we drop it from the arguments of γ_1 and γ_2 to ease the notation.

$$c=\frac{1}{\lambda}\left[\left(1-\lambda a\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\beta(a+c)\right]$$

from which we obtain that the sensitivity of the equilibrium trades to the endogenous public signal must satisfy

$$c=\frac{1}{\beta+\lambda}\left[\left(1-\lambda a\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\beta a\right]. \tag{25}$$

Using (24), in turn we have that the constant b in the equilibrium trades is given by

$$b=-\frac{\alpha}{\beta+\lambda}. \tag{26}$$

Finally, inverting the relationship between c and $\hat{c}$ and b and $\hat{b}$ in (19) and (18), we obtain that the other two parameters defining the equilibrium demand schedules are given by

$$\hat{c}^*=-\frac{c}{\beta\left(a^*+c\right)}=-\frac{\frac{1}{\beta+\lambda}\left(\gamma_2(\tau_\omega(a^*))-\beta a^*\right)}{\beta a^*+\beta\frac{1}{\beta+\lambda}\left(\gamma_2(\tau_\omega(a^*))-\beta a^*\right)}$$

and

$$\hat{b}^*=\left(1+\beta\hat{c}^*\right)b+\alpha\hat{c}^*=-\left(1+\beta\hat{c}^*\right)\frac{\alpha}{\beta+\lambda}+\alpha\hat{c}^*=\frac{\alpha}{\beta+\lambda}\left(\lambda\hat{c}^*-1\right).$$

Replacing

$$\gamma_2(\tau_\omega(a^*))=\left(1-\gamma_1(\tau_\omega(a^*))\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\omega(a^*)+\tau_\theta}=\left(1-\lambda a^*\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\omega(a^*)+\tau_\theta}$$

into the above expressions, we thus conclude that, given a^*, the values of $\hat{c}^*$ and $\hat{b}^*$ are given by the functions (9) and (10), as claimed in the proposition.

To complete the proof, it thus suffices to show that equation (8) admits a unique solution and is such that $0<a^*<\frac{1}{\lambda}$. To see this, use the fact that $\tau_\omega(a)$ is given by the function

$$\tau_\omega(a)\equiv\frac{\beta^2a^2y\tau_\eta\tau_u}{\beta^2a^2\tau_u+y\tau_\eta}$$

along with the fact that $\gamma_1(\tau_\omega(a))$ is given by the function in (21) to rewrite equation (8) as follows:

$$a=\frac{1}{\lambda}\frac{\tau_\epsilon y^2\tau_\eta^2(\beta^2a^2\tau_u+y\tau_\eta)-\beta^2a^2y\tau_u\tau_\eta\tau_\epsilon y\tau_\eta}{y^2\tau_\eta^2\left(\beta^2a^2y\tau_u\tau_\eta+\left(\tau_\epsilon+\tau_\theta\right)\left(\beta^2a^2\tau_u+y\tau_\eta\right)\right)-\beta^2a^2y\tau_u\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)}. \tag{27}$$

We thus have that a^* must solve the following cubic equation

$$0=\lambda\beta^2\tau_ua^3\left[y^3\tau_\eta^3+y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right)-y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right]+\lambda ay^3\tau_\eta^3\left(\tau_\epsilon+\tau_\theta\right)-\tau_\epsilon y^3\tau_\eta^3. \tag{28}$$

Now note that, in a cubic equation of the form $Ax^3+Bx^2+Cx+D=0$, if

$$\Delta\equiv18ABCD-4B^3D+B^2C^2-4AC-27A^2D^2<0,$$

then the equation has a unique real root. In our case, $B=0$ and $C>0$ and, as a result, $\Delta=-4AC-27A^2D^2$. Furthermore (using the fact that $\tau_\epsilon\equiv\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}$), we have that

$$
\begin{aligned}
A &= \lambda\beta^2\tau_u\left(y^3\tau_\eta^3+y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right)-y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right)\propto y\tau_\eta\left(y^2\tau_\eta^2+y\tau_\eta\tau_\theta-\tau_\epsilon\tau_\theta-\tau_\epsilon y\tau_\eta\right)\\
&\propto (\tau_\theta+y\tau_\eta)(y\tau_\eta-\tau_\epsilon)\propto y\tau_\eta-\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\propto\frac{\tau_\eta}{\tau_e+\tau_\eta}>0.
\end{aligned}
$$

Therefore $\Delta<0$, and hence the above cubic equation has a unique real root. Furthermore, because D is negative, the unique real root is positive. Replacing $a=\frac{1}{\lambda}$ into the cubic equation, we have that

$$
\begin{aligned}
&\beta^2\tau_u\frac{1}{\lambda^2}\left(y^3\tau_\eta^3+y^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta\right)-y\tau_\eta\tau_\epsilon\left(\tau_\theta+2y\tau_\eta\right)\right)+y^3\tau_\eta^3\left(\tau_\epsilon+\tau_\theta\right)-\tau_\epsilon y^3\tau_\eta^3\\
=&\beta^2\tau_u\frac{y\tau_\eta}{\lambda^2}\left(y^2\tau_\eta^2+y\tau_\eta\tau_\theta-\tau_\epsilon\tau_\theta-\tau_\epsilon y\tau_\eta\right)+y^3\tau_\eta^3\tau_\theta>0.
\end{aligned}
$$

This implies that $0<a^*<\frac{1}{\lambda}$. Q.E.D.

Derivation of Condition 11.

Recall that $W=\int_0^1\left(\theta x_i-\frac{\lambda}{2}x_i^2\right)di-\left(\alpha-u+\beta\frac{\tilde{x}}{2}\right)\tilde{x}$. Because $\int_0^1\left(x_i^2\right)di>\left(\int_0^1 x_i di\right)^2$, we have that W is maximal when $x_i=x^o$ for all i, with

$$
x^o\equiv\frac{\theta-\alpha+u}{\beta+\lambda}.
$$

Q.E.D.

Derivation of Condition 12.

Recall that $WL=\mathbb{E}[W^o]-\mathbb{E}[W]$. Using the fact that

$$
W^o=\theta x^o-\frac{\lambda}{2}(x^o)^2-\left(\alpha-u+\beta\frac{x^o}{2}\right)x^o
$$

along with the fact that $x^o=\frac{\theta-\alpha+u}{\beta+\lambda}$, we then have that

$$
W^o=\theta x^o-\frac{\lambda}{2}(x^o)^2-\left(\alpha-u+\beta\frac{x^o}{2}\right)x^o=\frac{\beta+\lambda}{2}(x^o)^2.
$$

It follows that

$$
\begin{aligned}
WL &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right]-\mathbb{E}\left[\int_0^1\left(\theta x_i-\frac{\lambda}{2}x_i^2\right)di-\left(\alpha-u+\beta\frac{\tilde{x}}{2}\right)\tilde{x}\right]\\
&= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right]-\mathbb{E}\left[\left(\theta-\alpha+u\right)\tilde{x}-\beta\frac{\tilde{x}^2}{2}-\frac{\lambda}{2}\int_0^1 x_i^2 di\right].
\end{aligned}
$$

Using again the characterization of the FB allocation, $x^o=\frac{\theta-\alpha+u}{\beta+\lambda}$, and the fact that

$$
\mathbb{E}\left[\int_0^1 x_i^2 di\right]=\mathbb{E}\left[\mathbb{E}[x_i^2|\tilde{x}]\right]
$$

we have that

$$\begin{aligned} WL &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] - \frac{1}{2}\mathbb{E}\left[2\left(\beta+\lambda\right)\tilde{x}x^o - \beta\tilde{x}^2 - \lambda\int_0^1 x_i^2 di\right] \\ &= \frac{\beta+\lambda}{2}\mathbb{E}\left[(x^o)^2\right] + \frac{1}{2}\mathbb{E}\left[(\beta+\lambda)\tilde{x}^2 - 2x^o\tilde{x}(\beta+\lambda) - \lambda\tilde{x}^2 + \lambda\mathbb{E}[x_i^2|\tilde{x}]\right] \\ &= \frac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2] + \lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2}. \end{aligned}$$

Q.E.D.

Proof of Lemma 1.

The same arguments as in the proof of Proposition 1 imply that, when the traders submit demand schedules of the form $x_i = as_i + \hat{b} - \hat{c}p$, for some $(a, \hat{b}, \hat{c})$, the trades induced by market clearing can be expressed as a function of the endogenous public information z generated by the market-clearing price by letting $x_i = as_i + b + cz$ where

$$z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$$

is the endogenous information about θ contained in the equilibrium price, and where the noise in the endogenous signal has precision

$$\tau_\omega(a) \equiv \frac{\beta^2 a^2 y \tau_u \tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta}.$$

Furthermore, the values of b and c are given by (18) and (19). Using the above representation, we have that the aggregate volume of trade when the demand schedules are given by $(a, \hat{b}, \hat{c})$ is given by $\tilde{x} = a(\theta + f(y)\eta) + b + cz$ and hence ex-ante welfare is given by

$$\mathbb{E}[W] = \mathbb{E}\left[\left(\theta-\alpha+u\right)\left(a(\theta+f(y)\eta)+b+cz\right) - \beta\frac{(a(\theta+f(y)\eta)+b+cz)^2}{2} - \int_0^1 \frac{\lambda}{2}\left(as_i+b+cz\right)^2 di\right].$$

Note that

$$\frac{\partial\mathbb{E}[W]}{\partial b} = \mathbb{E}\left[\left(\theta-\alpha+u\right) - \beta\left(a(\theta+f(y)\eta)+b+cz\right) - \lambda\left(as+b+cz\right)\right] = -\alpha-(\beta+\lambda)b,$$

$$\frac{\partial^2\mathbb{E}[W]}{\partial b^2} = -(\beta+\lambda) < 0,$$

$$\frac{\partial\mathbb{E}[W]}{\partial c} = \mathbb{E}\left[z\left(\theta-\alpha+u\right) - \beta\left(a(\theta+f(y)\eta)+b+cz\right)z - \lambda z\left(as+b+cz\right)\right],$$

$$\frac{\partial^2\mathbb{E}[W]}{\partial c^2} = \mathbb{E}\left[-\beta z^2 - \lambda z^2\right] < 0,$$

and $\partial^2\mathbb{E}[W]/\partial c\partial b = 0$. Hence $\mathbb{E}[W]$ is concave in b and c . For any a, the optimal values of b and c are thus given by the FOCs $\partial\mathbb{E}[W]/\partial b = 0$ and $\partial\mathbb{E}[W]/\partial c = 0$ from which we obtain that

$$b = -\frac{\alpha}{\beta+\lambda}$$

and

$$\mathbb{E}\left[z\left(\theta+u\right) - \beta\left(a(\theta+f(y)\eta)\right)z - \beta cz^2 - \lambda azs - \lambda cz^2\right] = 0.$$

The last condition can be rewritten as

$$Cov\left[(\theta + u - \beta a(\theta + f(y)\eta)), z\right] - (\beta + \lambda) cVar(z) - \lambda a Cov(z, s) = 0$$

from which we obtain that

$$c \;=\; \frac{Cov\left[(\theta + u - \beta a(\theta + f(y)\eta)), z\right]}{(\beta + \lambda) Var(z)} - \frac{\lambda a Cov(z, s)}{(\beta + \lambda) Var(z)}.$$

Using the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$ and $s = \theta + \frac{1}{\sqrt{y}}(\eta + e)$, we have that

$$Var(z) = \frac{1}{\tau_\theta} + \frac{1}{\tau_\omega(a)} = \sigma_\theta^2 + \sigma_\omega^2(a),$$

where $\sigma_\theta^2 = 1/\tau_\theta$ and $\sigma_\omega^2(a) = 1/\tau_\omega(a)$. Furthermore,

$$\begin{aligned}
Cov\left[(\theta + u - \beta a(\theta + f(y)\eta)), z\right] &= Cov\left[(\theta + u - \beta a(\theta + f(y)\eta)), \theta + f(y)\eta - \frac{u}{\beta a}\right] \\
&= Cov\left[\theta(1 - \beta a), \theta\right] + Cov\left[u, -\frac{u}{\beta a}\right] - Cov\left[\beta a f(y)\eta, f(y)\eta\right] \\
&= (1 - \beta a)\sigma_\theta^2 - \frac{\sigma_u^2}{\beta a} - \beta a f(y)^2 \sigma_\eta^2,
\end{aligned}$$

and

$$Cov\left[z, s\right] = \sigma_\theta^2 + f(y)^2 \sigma_\eta^2.$$

Hence,

$$\begin{aligned}
c &= \frac{(1 - \beta a)\sigma_\theta^2 - \frac{\sigma_u^2}{\beta a} - \beta a f(y)^2 \sigma_\eta^2}{(\beta + \lambda)\left(\sigma_\theta^2 + \sigma_\omega^2(a)\right)} - \frac{\lambda a(\sigma_\theta^2 + f(y)^2\sigma_\eta^2)}{(\beta + \lambda)\left(\sigma_\theta^2 + \sigma_\omega^2(a)\right)} \\
&= \frac{1}{\beta + \lambda}\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} - \beta a\right].
\end{aligned}$$

We conclude that, given a, the optimal values for c and b are given by the same functions in (25) and (26) that characterize the parameters c and b as a function of a under the equilibrium usage of information. To go from the optimal trades to the demand schedules that implement them, it then suffices to use the functions defined by (18) and (19). We thus conclude that, for any choice of a^T, the optimal values of $\hat{c}^T$ and $\hat{b}^T$ are given by the functions (9) and (10), as claimed. Q.E.D.

Derivation of Condition 13.

As shown above, the welfare losses can be expressed as

$$WL = \frac{\beta + \lambda}{2}\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2],$$

where x^0 is given by (11). We have also shown above that, for any vector $(a, \hat{b}, \hat{c})$ describing the demand schedules, there exists a unique vector (a, b, c) describing the induced trades $x_i = as_i + b + cz$ at the market-clearing price, and vice versa, where $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$ is the endogenous signal contained in the market-clearing price. This also means, when the traders submit the demand schedules corresponding to the vector $(a, \hat{b}, \hat{c})$, the aggregate volume of trade at the market-clearing price can be expressed as a function of (θ, η, z) as follows: $\tilde{x} = a(\theta + f(y)\eta) + b + cz$. Therefore, the dispersion of individual trades around the aggregate trade can be expressed as

$$\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = \frac{a^2}{y\tau_e}.$$

Next, use the fact that, for any a, the optimal values of c and b are given by (25) and (26), along with the fact that $z \equiv \theta + f(y)\eta - \frac{u}{\beta a}$, and the fact that $f(y) = 1/\sqrt{y}$, to obtain that

$$\begin{aligned} \tilde{x} &= a(\theta + f(y)\eta) + b + cz \\ &= \frac{\lambda a(\theta + f(y)\eta) + u - \alpha + \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} z}{\beta + \lambda}. \end{aligned}$$

Combining the expression for $\tilde{x}$ derived above with the expression for x^0 in (11), we have that

$$\mathbb{E}[(\tilde{x} - x^o)^2] = \mathbb{E}\left[\left(\frac{\lambda a(\theta + f(y)\eta) + u - \alpha + \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} z}{\beta + \lambda} - \frac{\theta - \alpha + u}{\beta + \lambda}\right)^2\right].$$

Simplifying, we have that

$$\mathbb{E}[(\tilde{x} - x^o)^2] = \mathbb{E}\left[\left(\frac{\lambda a f(y)\eta}{\beta+\lambda} + \frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}(z-\theta)}{\beta+\lambda} - \frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\theta}{\beta+\lambda}\right)^2\right].$$

Using the fact that $f(y) = 1/\sqrt{y}$, and that $\mathbb{E}[\omega\theta] = \mathbb{E}[\eta\theta] = 0$, we then have that

$$\begin{aligned} \mathbb{E}[(\tilde{x} - x^o)^2] &= \frac{\left(\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)^2}{(\beta+\lambda)^2 \tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{(\beta+\lambda)^2 y\tau_\eta} \\ &+ \frac{\left(1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)^2}{(\beta+\lambda)^2 \tau_\theta}. \end{aligned}$$

Replacing the expressions for $\mathbb{E}[(x_i - \tilde{x})^2]$ and $\mathbb{E}[(\tilde{x} - x^o)^2]$ derived above into the formula for the welfare losses, we then have that, for any a, when $\hat{b}$ and $\hat{c}$ are set optimally, the welfare losses can be expressed as

$$\begin{aligned} WL(a, \tau_\omega(a)) &= \frac{\left[\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\omega(a)} + \frac{\lambda^2 a^2 + 2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{2(\beta+\lambda) y\tau_\eta} \\ &+ \frac{\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\theta} + \frac{\lambda a^2}{2y\tau_e}. \end{aligned}$$

as claimed in the main text. Q.E.D.

Proof of Proposition 2.

As shown above, once b and c are set optimally as a function of a to minimize the welfare losses, the latter can be expressed as a function of a and $\tau_\omega(a)$, with the formula for $WL(a,\tau_\omega(a))$ given by (13), with $\tau_\omega(a)=\frac{\beta^2a^2\tau_u\tau_\eta y}{\beta^2a^2\tau_u+y\tau_\eta}$. The socially optimal level of a is thus the one that minimizes $WL(a,\tau_\omega(a))$ and is given by the FOC

$$\frac{dWL(a,\tau_\omega(a))}{da} = \frac{\partial WL(a,\tau_\omega(a))}{\partial a} + \frac{\partial WL(a,\tau_\omega(a)))}{\partial \tau_\omega(a)}\frac{\partial \tau_\omega(a)}{\partial a} = 0.$$

Note that

$$\begin{aligned}\frac{\partial WL(a,\tau_\omega(a))}{\partial a} = & -\frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)} \\ & +\frac{\lambda^2 a+\lambda\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\lambda^2 a\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{(\beta+\lambda)\,y\tau_\eta} \\ & +\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}+\frac{\lambda a}{y\tau_e}.\end{aligned}$$

Next note that

$$\begin{aligned}\frac{\partial WL(a,\tau_\omega(a))}{\partial \tau_\omega(a)} = & \frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{2(\beta+\lambda)}\frac{\tau_\theta-\tau_\omega(a)}{(\tau_\omega(a)+\tau_\theta)^3}+\frac{\lambda a\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)}{(\beta+\lambda)\,y\tau_\eta}\frac{\tau_\theta}{(\tau_\omega(a)+\tau_\theta)^2} \\ & -\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]}{(\beta+\lambda)\,\tau_\theta}\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\theta}{(\tau_\omega(a)+\tau_\theta)^2}.\end{aligned}$$

Finally note that

$$\frac{\partial \tau_\omega(a)}{\partial a} = \frac{2\beta^2 a y^2\tau_\eta^2\tau_u}{(\beta^2a^2\tau_u+y\tau_\eta)^2}.$$

Using the above expressions we obtain that

$$\begin{aligned}\frac{dWL(a,\tau_\omega(a))}{da} = & -\frac{\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)}+\frac{\lambda a}{y\tau_e}+L(a) \\ & +\frac{\lambda^2 a+\lambda\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}-\lambda^2 a\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}}{(\beta+\lambda)\,y\tau_\eta} \\ & +\frac{\left[1-\lambda a-\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right]\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}\end{aligned}$$

where

$$L(a) \equiv \frac{\beta^2 a y^2 \tau_\eta^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)^2} \left\{ \frac{\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{(\beta + \lambda)} \frac{\tau_\theta - \tau_\omega(a)}{\left(\tau_\omega(a) + \tau_\theta\right)^3} + \frac{2\lambda a \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)}{(\beta + \lambda)\, y\tau_\eta} \frac{\tau_\theta}{\left(\tau_\omega(a) + \tau_\theta\right)^2} \right.$$

$$\left. - \frac{2\left[1 - \lambda a - \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta}\right]}{(\beta + \lambda)\, \tau_\theta} \left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right) \frac{\tau_\theta}{\left(\tau_\omega(a) + \tau_\theta\right)^2} \right\}.$$

Hence, the first-order-condition $dWL(a, \tau_\omega(a))/da = 0$ is equivalent to

$$\begin{aligned} 0 \;\; = \;\; & \lambda a \tau_\epsilon \left((y\tau_\eta + \tau_\theta)^2 \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} \right) + \lambda a y \tau_\eta \tau_\epsilon \left(\tau_\omega(a) + \tau_\theta\right) - 2\lambda a \tau_\epsilon \left(y\tau_\eta + \tau_\theta\right) \tau_\omega(a) \\ & + \lambda a \tau_\epsilon \frac{(\tau_\omega(a) + \tau_\theta)}{\tau_\theta} \left(y\tau_\eta - \left(y\tau_\eta + \tau_\theta\right) \frac{\tau_\omega(a)}{\tau_\omega(a) + \tau_\theta} \right)^2 + \lambda a y \tau_\eta \tau_\epsilon \frac{y\tau_\eta \left(\tau_\omega(a) + \tau_\theta\right)(\beta + \lambda)}{\lambda y \tau_e} \\ & + y\tau_\eta \tau_\epsilon \frac{(\beta + \lambda)\left(\tau_\omega(a) + \tau_\theta\right) y\tau_\eta L(a)}{\lambda} - y\tau_\eta \tau_\epsilon \left(y\tau_\eta - \tau_\omega(a)\right) \end{aligned}$$

from which we obtain that

$$\begin{aligned} y\tau_\eta \tau_\epsilon \left(y\tau_\eta - \tau_\omega(a)\right) \;\; = \;\; & \lambda a \left\{ y^2 \tau_\eta^2 \tau_\epsilon - \tau_\omega(a)\tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right) + \left(\tau_\omega(a) + \tau_\theta\right) y^2 \tau_\eta^2 \right. \\ & \left. + y\tau_\eta \tau_\epsilon \frac{y\tau_\eta \left(\tau_\omega(a) + \tau_\theta\right) \beta}{\lambda y \tau_e} + y\tau_\eta \tau_\epsilon \frac{(\beta + \lambda)\left(\tau_\omega(a) + \tau_\theta\right) y\tau_\eta L(a)}{\lambda^2 a} \right\}. \end{aligned}$$

Letting

$$\Delta(a) \equiv -\tau_\epsilon \frac{y^2 \tau_\eta^2}{\lambda^2 a} \frac{\beta^2 a y^2 \tau_\eta^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)^2} \frac{\left(1 - \lambda a - \lambda a \frac{\tau_\theta}{y\tau_\eta}\right)^2}{(\tau_\omega(a) + \tau_\theta)} < 0$$

$$\Xi(a) \equiv \frac{\tau_\epsilon y \tau_\eta^2 \left(\tau_\omega(a) + \tau_\theta\right) \beta}{\lambda \tau_e} > 0,$$

we conclude that a^T must solve

$$a = \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\omega(a))}{y^2 \tau_\eta^2 \left(\tau_\epsilon + \tau_\theta + \tau_\omega(a)\right) - \tau_\omega(a)\tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right) + \Xi(a) + \Delta(a)}.$$

It is straightforward to verify that

$$\begin{aligned} \left. \frac{dWL(a, \tau_\omega(a))}{da} \right|_{a = \frac{1}{\lambda}} = \; & \frac{\lambda \tau_\theta}{(\beta + \lambda)\, y\tau_\eta (\tau_\omega(a) + \tau_\theta)} \frac{y\tau_\eta}{\beta^2 a^2 \tau_u + y\tau_\eta} \\ & \left(1 - \frac{\beta^2 a^2 \tau_u}{(\beta^2 a^2 \tau_u + y\tau_\eta)} \times \frac{\tau_\theta}{(\tau_\omega(a) + \tau_\theta)} \right) + \frac{\lambda a}{y\tau_e} > 0 \end{aligned}$$

and that

$$\left.\frac{dWL(a,\tau_\omega(a))}{da}\right|_{a=0} = \frac{\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\left(-\lambda\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\omega(a)} + \frac{\lambda\left(\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,y\tau_\eta}$$
$$+\frac{\left(1-\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)\left(-\lambda+\lambda\left(\frac{y\tau_\eta+\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta}\right)}{(\beta+\lambda)\,\tau_\theta}$$
$$\propto \frac{\tau_\omega(a)}{y\tau_\eta}-1 = -\frac{y\tau_\eta}{\beta^2a^2\tau_u+y\tau_\eta} < 0,$$

which implies that $0 < a^T < \frac{1}{\lambda}$, as claimed in the proposition. Q.E.D.

Derivation of (15) and (16).

In the cursed economy, each trader receives a private signal $s_i = \theta + \underbrace{f(y)\eta + f(y)e_i}_{\equiv\epsilon_i}$ and a public signal $z = \theta + \underbrace{f(y)\eta + \chi}_{\equiv\zeta}$, and believes p to be orthogonal to (θ,η).

Following steps similar to those leading to Proposition 1, we have that

$$\mathbb{E}[\theta|s_i,z] = \gamma_1 s_i + \gamma_2 z,$$

where

$$\gamma_1 \equiv \frac{\tau_\epsilon y\tau_\eta\,(y\tau_\eta-\tau_\zeta)}{y^2\tau_\eta^2(\tau_\zeta+\tau_\epsilon+\tau_\theta)-\tau_\zeta\tau_\epsilon(\tau_\theta+2y\tau_\eta)}$$

and

$$\gamma_2 \equiv \frac{y\tau_\eta\tau_\zeta\,(y\tau_\eta-\tau_\epsilon)}{y^2\tau_\eta^2(\tau_\zeta+\tau_\epsilon+\tau_\theta)-\tau_\epsilon\tau_\zeta(\tau_\theta+2y\tau_\eta)}$$
$$=\left(1-\gamma_1\frac{\tau_\theta+y\tau_\eta}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}.$$

Observe that the cursed-equilibrium demand schedules must satisfy

$$x_i = \frac{1}{\lambda}\left(\mathbb{E}[\theta|s_i,z]-p\right). \tag{29}$$

Now let $x_i = a^*_{exo}s_i + \hat{b}^*_{exo} + \hat{c}^*_{exo}z - \hat{d}^*_{exo}p$ denote the cursed-equilibrium demand schedules. From the derivations above, we have that

$$a^*_{exo} = \frac{\gamma_1}{\lambda},$$
$$\hat{b}^*_{exo} = 0,$$
$$\hat{c}^*_{exo} = \frac{\gamma_2}{\lambda},$$

and

$$\hat{d}^*_{exo} = \frac{1}{\lambda}.$$

Using the formula for γ_1, we have that the formula for a^*_{exo} is equivalent to the one in (15) in the main text.

Now suppose that the planner can select a but, given the latter, is constrained to choose $(\hat{b}, \hat{c}, \hat{d})$ to maintain the same relationship between a^*_{exo} and $(\hat{b}^*_{exo}, \hat{c}^*_{exo}, \hat{d}^*_{exo})$ as in the cursed equilibrium. Using the fact that

$$\gamma_2 = \left(1 - \gamma_1 \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta},$$

and the fact that $\gamma_1 = a^*_{exo}\lambda$, we have that, in the cursed equilibrium, the relationship between a^*_{exo} and $(\hat{b}^*_{exo}, \hat{c}^*_{exo}, \hat{d}^*_{exo})$ is given by

$$\hat{b}^*_{exo} = 0,$$

$$\hat{c}^*_{exo} = \frac{1}{\lambda}\left(1 - \lambda a^*_{exo} \frac{\tau_\theta + y\tau_\eta}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta},$$

and

$$\hat{d}^*_{exo} = \frac{1}{\lambda}.$$

The above properties imply that, in the cursed economy, for any choice of a, the planner is constrained to select demand schedules of the form

$$x_i = \frac{1}{\lambda}\left(\lambda a s_i + \left(1 - \frac{\lambda a\left(\tau_\theta + y\tau_\eta\right)}{y\tau_\eta}\right) \frac{\tau_\zeta}{\tau_\zeta + \tau_\theta} z - p\right). \tag{30}$$

The planner then chooses a to minimize the welfare losses

$$WL \quad = \quad \frac{(\beta + \lambda)\mathbb{E}[(\tilde{x} - x^o)^2] + \lambda \mathbb{E}[(x_i - \tilde{x})^2]}{2}$$

under the above demand schedules, taking into account the market-clearing condition.

Following steps similar to those in the baseline economy, and using the market-clearing condition, we have that, when the traders' demand schedules are given by (30),

$$\begin{aligned}
\frac{(\beta + \lambda)}{2}\mathbb{E}[(\tilde{x} - x^o)^2] \quad &= \quad \frac{\left(\left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}\right)^2}{\left(\beta + \lambda\right)^2 \tau_\zeta} + \frac{\lambda^2 a^2 + 2\lambda a\left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}}{\left(\beta + \lambda\right)^2 y\tau_\eta} \\
&+ \frac{\left(1 - \lambda a - \left(1 - \frac{\lambda a(y\tau_\eta + \tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta + \tau_\theta}\right)^2}{\left(\beta + \lambda\right)^2 \tau_\theta}
\end{aligned}$$

and

$$\frac{\lambda \mathbb{E}[(x_i - \tilde{x})^2]}{2} \quad = \quad \frac{\lambda a^2}{2y\tau_e}.$$

This means that, for any a, the welfare losses are equal to

$$
\begin{aligned}
WL &= \frac{\left[\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\zeta}+\frac{\lambda^2a^2+2\lambda a\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}}{2(\beta+\lambda)y\tau_\eta} \\
&+\frac{\left[1-\lambda a-\left(1-\frac{\lambda a(y\tau_\eta+\tau_\theta)}{y\tau_\eta}\right)\frac{\tau_\zeta}{\tau_\zeta+\tau_\theta}\right]^2}{2(\beta+\lambda)\tau_\theta}+\frac{\lambda a^2}{2y\tau_e}.
\end{aligned}
$$

Following steps similar to those in the proof of Proposition 2, we then have that the value of a that minimizes the above welfare losses is equal to

$$
a^T_{exo} = \frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\zeta)}{y^2\tau_\eta^2(\tau_\epsilon+\tau_\theta+\tau_\zeta)-\tau_\zeta\tau_\epsilon(\tau_\theta+2y\tau_\eta)+\frac{\tau_\epsilon y\tau_\eta^2(\tau_\zeta+\tau_\theta)\beta}{\lambda\tau_e}}
$$

as claimed in (16). Q.E.D.

Lemma 2. *$c^*=0$ if and only if $\Xi(a^*)+\Delta(a^*)=0$.*

Proof of Lemma 2. Recall that c^* is given by

$$
\begin{aligned}
c^* &= \frac{1}{\beta+\lambda}\left(\left(1-\lambda a^*-\lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\theta+\tau_\omega(a^*)}-\beta a^*\right) \\
&= \frac{1}{\beta+\lambda}\left(\gamma_2(a^*)-\beta a^*\right),
\end{aligned}
$$

whereas the externalities are given by

$$
\begin{aligned}
\Delta(a) &\equiv -\frac{\tau_\epsilon\beta^2ay^4\tau_\eta^4\tau_u\left(1-\lambda a-\lambda a\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2a(\beta^2a^2\tau_u+y\tau_\eta)^2(\tau_\omega(a)+\tau_\theta)}, \\
\Xi(a) &\equiv \frac{\tau_\epsilon y\tau_\eta^2(\tau_\omega(a)+\tau_\theta)\beta}{\lambda\tau_e}.
\end{aligned}
$$

We prove the lemma in two steps. First we show that, if $c^*=0$, then $\Xi(a^*)+\Delta(a^*)=0$. To see this, use the formula for c^* from above to verify that, when $c^*=0$, then $\beta a^*=\gamma_2(a^*)$. Using the fact that

$$
\begin{aligned}
a^* &= \frac{1}{\lambda}\frac{\tau_\epsilon y\tau_\eta(y\tau_\eta-\tau_\omega(a^*))}{y^2\tau_\eta^2(\tau_\omega(a^*)+\tau_\epsilon+\tau_\theta)-\tau_\omega(a^*)\tau_\epsilon(\tau_\theta+2y\tau_\eta)}, \\
\gamma_2(a^*) &= \frac{\tau_\omega(a^*)\left(y^2\tau_\eta^2-\tau_\epsilon y\tau_\eta\right)}{y^2\tau_\eta^2(\tau_\omega(a^*)+\tau_\epsilon+\tau_\theta)-\tau_\omega(a^*)\tau_\epsilon(\tau_\theta+2y\tau_\eta)}, \\
\tau_\epsilon &= \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}, \\
\tau_\omega(a^*) &= \frac{\beta^2a^{*2}y\tau_\eta\tau_u}{\beta^2a^{*2}\tau_u+y\tau_\eta},
\end{aligned}
$$

we then have that, when $c^* = 0$, then

$$\beta = \frac{\gamma_2(a^*)}{a^*} = \lambda \frac{\tau_\omega(a^*)\left(y^2\tau_\eta^2 - \tau_\epsilon y\tau_\eta\right)}{\tau_\epsilon y\tau_\eta\left(y\tau_\eta - \tau_\omega(a^*)\right)} = \lambda \frac{\tau_\omega(a^*)\left(y\tau_\eta - \frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\right)}{\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\left(y\tau_\eta - \tau_\omega(a^*)\right)}.$$

Using the formula for $\tau_\omega(a^*)$ we then have that

$$\beta = \lambda \frac{\frac{\beta^2 a^{*2} y\tau_\eta\tau_u}{\beta^2 a^{*2}\tau_u + y\tau_\eta}\left(\frac{y\tau_\eta^2}{\tau_e+\tau_\eta}\right)}{\frac{y\tau_e\tau_\eta}{\tau_e+\tau_\eta}\left(y\tau_\eta - \frac{\beta^2 a^{*2} y\tau_\eta\tau_u}{\beta^2 a^{*2}\tau_u + y\tau_\eta}\right)} = \lambda \frac{\beta^2 a^{*2} y\tau_\eta\tau_u\tau_\eta}{\tau_e\left(y^2\tau_\eta^2\right)} = \lambda \frac{\beta^2 a^{*2}\tau_u}{\tau_e y}$$

from which we obtain that

$$\beta = \frac{y\tau_e}{\lambda a^{*2}\tau_u}.$$

Furthermore, using the expression for c^* above, we have that, when $c^* = 0$,

$$\left(1 - \lambda a^* - \lambda a^* \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^*)}{\tau_\theta + \tau_\omega(a^*)} = \beta a^*.$$

Replacing the above expression into the formula for the two externalities, we thus have that

$$\Delta(a^*) + \Xi(a^*) = \frac{\tau_\epsilon y\tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{\lambda\tau_e} - \tau_\epsilon \frac{y^2\tau_\eta^2}{\lambda^2 a^*}\frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u}.$$

Using the expression for β above, we then have that

$$\begin{aligned}\Delta(a^*) + \Xi(a^*) &= \frac{\tau_\epsilon y\tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\frac{y\tau_e}{\lambda a^2\tau_u}}{\lambda\tau_e} - \frac{\tau_\epsilon y^2\tau_\eta^2}{\lambda^2 a^*}\frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u} \\ &= \frac{\tau_\epsilon y^2\tau_\eta^2}{\lambda^2 a^*}\left(\frac{\left(\tau_\omega(a^*) + \tau_\theta\right)}{a^*\tau_u} - \frac{\left(\tau_\theta + \tau_\omega(a^*)\right)}{a^*\tau_u}\right) = 0.\end{aligned}$$

Next, we prove the converse. We show that, if $\Delta(a^*) + \Xi(a^*) = 0$, then $c^* = 0$. To see this note that, when the sum of the two externalities is zero, then

$$\Delta(a^*) + \Xi(a^*) = \frac{\tau_\epsilon y\tau_\eta^2\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{\lambda\tau_e} - \frac{\tau_\epsilon y^4\tau_\eta^4\beta^2 a^*\tau_u\left(1 - \lambda a^* - \lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\lambda^2 a^*(\beta^2 a^{*2}\tau_u + y\tau_\eta)^2\left(\tau_\omega(a^*) + \tau_\theta\right)} = 0.$$

Using the various expressions above we then have that

$$0 = \frac{\left(\tau_\omega(a^*) + \tau_\theta\right)\beta}{y\tau_e} - \frac{1}{\lambda a^*}\frac{\tau_\omega(a^*)^2}{\beta^2 a^{*3}\tau_u}\frac{\left(1 - \lambda a^* - \lambda a^*\frac{\tau_\theta}{y\tau_\eta}\right)^2}{\tau_\omega(a^*) + \tau_\theta}$$

$$
\begin{aligned}
0 &= \frac{\beta a^*}{y\tau_e} - \frac{1}{\gamma_1(a^*)}\frac{\gamma_2(a^*)^2}{\beta^2 a^{*2}\tau_u} \\
\beta a^* &= \frac{\tau_\omega(a^*)\left(y\tau_\eta - \tau_\epsilon\right)}{\tau_\epsilon\left(y\tau_\eta - \tau_\omega(a^*)\right)}\frac{y\tau_e}{\beta^2 a^{*2}\tau_u}\gamma_2(a^*) \\
&= \frac{\beta^2 a^{*2}\tau_u}{\tau_e y}\frac{y\tau_e}{\beta^2 a^{*2}\tau_u}\gamma_2(a^*) \\
&= \gamma_2(a^*).
\end{aligned}
$$

Hence $\beta a^* = \gamma_2(a^*)$. But this means that $c^* = 0$. Q.E.D.

Proof of Proposition 3.

Under the proposed policy, each trader's demand schedule must satisfy the optimality condition

$$X_i(p; I_i) = \frac{1}{\lambda+\delta}\left(\mathbb{E}[\theta|I_i, p] - (1+t_p)p + t_0\right).$$

For any vector $(a, \hat{b}, \hat{c})$, when all traders submit affine demand schedules $x_i = as_i + \hat{b} - \hat{c}p$, the equilibrium price then continues to satisfy the same representation as in (2) but with $(a^*, \hat{b}^*, \hat{c}^*)$ replaced by $(a, \hat{b}, \hat{c})$. This also means that the equilibrium trades can be expressed as a function of the endogenous public signal z, as in the laissez-faire equilibrium with no policy. Letting $x_i = as_i + b + cz$ denote the trades generated by the demand schedules $x_i = as_i + \hat{b} - \hat{c}p$ (with z representing the endogenous public signal contained in the market-clearing price), we then have that the functions that map the coefficients $\hat{c}$ and $\hat{b}$ in the demand schedules into the coefficients c and b in the induced trades continue to be given by (9) and (10). Using the fact that $\mathbb{E}[\theta|s_i, z] = \gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z$, with the functions γ_1 and γ_2 as defined in (6) and (7), along with the fact that the market-clearing price satisfies $p = \alpha + \beta b + \beta(a+c)z$ as shown in (20), we then have that the equilibrium trades must satisfy

$$
\begin{aligned}
x_i &= \frac{1}{\lambda+\delta}\left[\gamma_1(\tau_\omega(a))s_i + \gamma_2(\tau_\omega(a))z - (1+t_p)\alpha - (1+t_p)\beta b - (1+t_p)\beta(a+c)z + t_0\right] \\
&= \frac{1}{\lambda+\delta}\left\{\gamma_1(\tau_\omega(a))s_i - (1+t_p)\left(\alpha + \beta b\right) + \left[\gamma_2(\tau_\omega(a)) - (1+t_p)\beta(a+c)\right]z + t_0\right\}.
\end{aligned}
$$

The sensitivity of the equilibrium trades to private information s_i under the proposed policy thus satisfies $a = \gamma_1(\tau_\omega(a))/(\lambda+\gamma)$. Using the formula for γ_1 in (6), we then have that the equilibrium a under the proposed policy is the unique solution to the following equation:

$$a = \frac{1}{\lambda+\delta}\frac{\tau_\epsilon y^2\tau_\eta^2 - \tau_\omega(a)\tau_\epsilon y\tau_\eta}{y^2\tau_\eta^2(\tau_\omega(a) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)}.$$

The equilibrium c, instead, is given by the unique solution to

$$c = \frac{1}{\lambda+\delta}\left[\gamma_2(\tau_\omega(a)) - (1+t_p)\beta(a+c)\right]$$

which is equal to

$$c = \frac{\gamma_2(\tau_\omega(a)) - (1+t_p)\beta a}{\lambda + \delta + (1+t_p)\beta}.$$

Finally, the equilibrium b is given by the unique solution to

$$b = \frac{-(1+t_p)\left(\alpha + \beta b\right) + t_0}{\lambda + \delta}$$

which is equal to

$$b = \frac{t_0 - (1+t_p)\alpha}{\lambda + \delta + (1+t_p)\beta}.$$

Now recall that the sensitivity a^T of the efficient trades to private information is given by the unique solution to

$$a \quad = \quad \frac{1}{\lambda} \frac{\tau_\epsilon y \tau_\eta (y\tau_\eta - \tau_\omega(a))}{y^2 \tau_\eta^2 \left(\tau_\epsilon + \tau_\theta + \tau_\omega(a)\right) - \tau_\omega(a)\tau_\epsilon \left(\tau_\theta + 2y\tau_\eta\right) + \Xi(a) + \Delta(a)}.$$

Therefore, the equilibrium value a under the proposed policy coincides with the efficient level a^T if and only if δ satisfies

$$\begin{aligned} &(\lambda+\delta)\left[y^2\tau_\eta^2\left(\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta\right) - \tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)\right] \\ &= \lambda\left[y^2\tau_\eta^2\left(\tau_\epsilon + \tau_\theta + \tau_\omega(a^T)\right) - \tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right) + \Xi(a^T) + \Delta(a^T)\right], \end{aligned}$$

from which we obtain that

$$\delta \quad = \quad \frac{\lambda\left(\Xi(a^T) + \Delta(a^T)\right)}{y^2\tau_\eta^2(\tau_\omega(a^T) + \tau_\epsilon + \tau_\theta) - \tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta + 2y\tau_\eta\right)}.$$

Now recall that, given a^T, the other two coefficients c^T and b^T describing the efficient trades are given by the functions in (25) and (26), implying that

$$c^T = \frac{1}{\beta+\lambda}\left(\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T) + \tau_\theta} - \beta a^T\right)$$

and

$$b^T = -\frac{\alpha}{\beta+\lambda}.$$

Hence, for the equilibrium levels of c and b under the proposed policy to coincide with the efficient levels it must be that

$$\frac{\gamma_2(\tau_\omega(a^T)) - (1+t_p)\beta a^T}{\lambda+\delta+(1+t_p)\beta} = \frac{1}{\beta+\lambda}\left(\left(1 - \lambda a^T - \lambda a^T \frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T) + \tau_\theta} - \beta a^T\right)$$

and

$$\frac{t_0 - (1+t_p)\alpha}{\lambda+\delta+(1+t_p)\beta} = -\frac{\alpha}{\beta+\lambda}$$

It is easy to see that the above two equations are satisfied when

$$t_p = \frac{\gamma_2(\tau_\omega(a^T)) - \frac{\lambda+\delta+\beta}{\beta+\lambda}\left[\left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a\right] - \beta a^T}{\beta\left\{\frac{1}{\beta+\lambda}\left[\left(1 - \lambda a - \lambda a\frac{\tau_\theta}{y\tau_\eta}\right)\frac{\tau_\omega(a)}{\tau_\omega(a)+\tau_\theta} - \beta a\right] + a^T\right\}}$$

and

$$t_0 = (1+t_p)\alpha - \frac{\alpha\left[\lambda+\delta+(1+t_p)\beta\right]}{\beta+\lambda}.$$

Q.E.D.

Proof of Proposition 4.

When all traders other than i acquire information of quality $\bar{y}$ and then submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, irrespectively of the information acquired by trader i and of the demand schedule submitted by the latter, the equilibrium price is given by

$$p(\theta, u, \eta; \bar{y}) = \alpha + \beta b^T + \beta(a^T + c^T) z(\theta, u, \eta; \bar{y})$$

where b^T and c^T are the coefficients obtained from $(a^T, \hat{b}^T, \hat{c}^T)$ using the functions (18) and (19), and where[16]

$$z(\theta, u, \eta; \bar{y}) \equiv \theta + f(\bar{y})\eta - u/\beta a^T.$$

Furthermore, the aggregate level of trade is equal

$$\tilde{X}(\theta, u, \eta; \bar{y}) = a^T[\theta + f(\bar{y})\eta] + b^T + c^T z(\theta, u, \eta; \bar{y})$$

whereas the equilibrium trade for agent i when he acquires information of quality y_i and then submits the demand schedule corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T z(\theta, u, \eta; \bar{y}).$$

It follows that, when all traders other than i acquire information of quality $\bar{y}$, trader i acquires information of quality y_i and all traders, including trader i, submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, trader i's ex-ante gross payoff is equal to

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}\left[(\theta - p(\theta, u, \eta; \bar{y}))\, X_i(\theta, u, \eta, e_i; \bar{y}, y_i) - \frac{\lambda}{2} X_i^2(\theta, u, \eta, e_i; \bar{y}, y_i)\right].$$

Using the fact that the market clearing price must also satisfy

$$p = \alpha - u + \beta \tilde{X}(\theta, u, \eta; \bar{y}),$$

we have that

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta, u, \eta; \bar{y})\right)\mathbb{E}[x_i|\theta, u, \eta; \bar{y}, y_i] - \frac{\lambda}{2}\mathbb{E}\left[x_i^2|\theta, u, \eta; \bar{y}, y_i\right]\right]$$

or, equivalently,

$$\mathbb{E}[\pi_i^T; \bar{y}, y_i] = \mathbb{E}_{\theta,u,\eta}\Big[\left(\theta - \alpha + u - \beta\tilde{X}(\theta, u, \eta; \bar{y})\right)\mathbb{E}[x_i|\theta, u, \eta; \bar{y}, y_i] - \tfrac{\lambda}{2} Var[x_i|\theta, \eta, u; \bar{y}, y_i]$$

$$-\tfrac{\lambda}{2}\left(\mathbb{E}[x_i|\theta, \eta, u; \bar{y}, y_i]\right)^2\Big],$$

[16] Observe that the functions (18) and (19) do not depend on y and hence c^T and b^T do not depend on y.

where

$$\mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i] \equiv \mathbb{E}[X_i(\theta,u,\eta,e_i;\bar{y},y_i)|\theta,u,\eta;\bar{y},y_i]$$

and

$$\mathbb{E}[x_i^2|\theta,u,\eta;\bar{y},y_i] \equiv \mathbb{E}\left[(X_i(\theta,u,\eta,e_i;\bar{y},y_i))^2|\theta,u,\eta;\bar{y},y_i\right]$$

and

$$Var[x_i|\theta,\eta,u;\bar{y},y_i] \equiv \mathbb{E}\left[(X_i(\theta,u,\eta,e_i;\bar{y},y_i) - \mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i])^2|\theta,u,\eta;\bar{y},y_i\right].$$

Using the fact that

$$\mathbb{E}[x_i|\theta,u,\eta;\bar{y},y_i] = a^T[\theta + f(y_i)\eta] + b^T + c^T z(\theta,u,\eta;\bar{y})$$

and

$$Var[x_i|\theta,\eta,u;\bar{y},y_i] = \frac{\left(a^T f(y_i)\right)^2}{\tau_e},$$

we have that

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i} &= \mathbb{E}_{\theta,\eta,u}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta,u,\eta;\bar{y})\right)a^T f'(y_i)\eta\right] - \lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i) \\
&\quad -\lambda\mathbb{E}_{\theta,\eta,u}\left[\left(a^T[\theta + f(y_i)\eta] + b^T + c^T z(\theta,u,\eta;\bar{y})\right)a^T f'(y_i)\eta\right] \\
&= -a^T\beta\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right]f'(y_i) - \lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i) \\
&\quad -\lambda\left(a^T\right)^2 f(y_i)f'(y_i)\frac{1}{\tau_\eta} - \lambda a^T c^T\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right]f'(y_i).
\end{aligned}$$

Using the fact that

$$\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta,u,\eta;\bar{y})\eta\right] = a^T f(\bar{y})\frac{1}{\tau_n} + c^T\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right]$$

and

$$\mathbb{E}_{\theta,\eta,u}\left[z(\theta,u,\eta;\bar{y})\eta\right] = f(\bar{y})\frac{1}{\tau_n},$$

we then have that

$$\begin{aligned}
\frac{\partial \mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i} &= -a^T\beta\left[a^T f(\bar{y})\frac{1}{\tau_n} + c^T f(\bar{y})\frac{1}{\tau_n}\right]f'(y_i) - \lambda\frac{\left(a^T\right)^2}{\tau_e}f(y_i)f'(y_i) \\
&\quad -\lambda\left(a^T\right)^2 f(y_i)f'(y_i)\frac{1}{\tau_\eta} - \lambda a^T c^T f(\bar{y})\frac{1}{\tau_n}f'(y_i), \qquad (31)
\end{aligned}$$

from which we obtain that

$$\begin{aligned}
\left.\frac{\partial \mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i}\right|_{y_i=\bar{y}} &= -a^T\beta\left[a^T f(\bar{y})\frac{1}{\tau_n} + c^T f(\bar{y})\frac{1}{\tau_n}\right]f'(\bar{y}) - \lambda\frac{\left(a^T\right)^2}{\tau_e}f(\bar{y})f'(\bar{y}) \\
&\quad -\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})\frac{1}{\tau_\eta} - \lambda a^T c^T f(\bar{y})\frac{1}{\tau_n}f'(\bar{y}) \\
&= -f(\bar{y})f'(\bar{y})a^T\left\{\lambda\frac{a^T}{\tau_e} + (\beta+\lambda)(a^T + c^T)\frac{1}{\tau_\eta}\right\}. \qquad (32)
\end{aligned}$$

Next, observe that, when trader i also acquires information of quality $\bar{y}$ and all traders submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, then

$$\mathbb{E}[\pi_i^T; \bar{y}, \bar{y}] = \mathbb{E}_{\theta,u,\eta}\left[\left(\theta - \alpha + u - \beta\tilde{X}(\theta, u, \eta; \bar{y})\right)\tilde{X}(\theta, u, \eta; \bar{y}) - \frac{\lambda}{2}\frac{\left(a^T f(\bar{y})\right)^2}{\tau_e} - \frac{\lambda}{2}\left(\tilde{X}(\theta, u, \eta; \bar{y})\right)^2\right].$$

The ex-ante payoff of the representative investor when all traders acquire information of quality $\bar{y}$ and submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$\begin{aligned}\mathbb{E}[\Pi; \bar{y}] &= \mathbb{E}_{\theta,u,\eta}\left[(p(\theta, u, \eta; \bar{y}) - \alpha + u)\,\tilde{X}(\theta, u, \eta; \bar{y}) - \frac{\beta}{2}\left(\tilde{X}(\theta, u, \eta; \bar{y})\right)^2\right] \\ &= \frac{\beta}{2}\mathbb{E}_{\theta,u,\eta}\left[\left(\tilde{X}(\theta, u, \eta; \bar{y})\right)^2\right],\end{aligned}$$

where we used the fact that $p(\theta, u, \eta; \bar{y}) = \alpha - u + \beta\tilde{X}(\theta, u, \eta; \bar{y})$. We thus have that, when all traders acquire information of quality $\bar{y}$ and submit the demand schedules corresponding to $(a^T, \hat{b}^T, \hat{c}^T)$, ex-ante welfare is given by

$$\begin{aligned}\mathbb{E}[W^T; \bar{y}] &= \mathbb{E}[\pi_i^T; \bar{y}, \bar{y}] + \mathbb{E}[\Pi; \bar{y}] \\ &= \mathbb{E}_{\theta,u,\eta}\left[(\theta - \alpha + u)\,\tilde{X}(\theta, u, \eta; \bar{y}) - \frac{\lambda}{2}\frac{\left(a^T f(\bar{y})\right)^2}{\tau_e} - \frac{\lambda + \beta}{2}\left(\tilde{X}(\theta, u, \eta; \bar{y})\right)^2\right].\end{aligned}$$

Hence,

$$\frac{d\mathbb{E}[W^T; \bar{y}]}{d\bar{y}} = \mathbb{E}_{\theta,\eta,u}\left[(\theta - \alpha + u)\frac{\partial\tilde{X}(\theta, u, \eta; \bar{y})}{\partial\bar{y}} - \frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda + \beta)\tilde{X}(\theta, u, \eta; \bar{y})\frac{\partial\tilde{X}(\theta, u, \eta; \bar{y})}{\partial\bar{y}}\right],$$

where

$$\frac{\partial\tilde{X}(\theta, u, \eta; \bar{y})}{\partial\bar{y}} = (a^T + c^T)f'(\bar{y})\eta.$$

It follows that

$$\frac{d\mathbb{E}[W^T; \bar{y}]}{d\bar{y}} = -\frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda + \beta)(a^T + c^T)f'(\bar{y})\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta, u, \eta; \bar{y})\eta\right].$$

Using the fact that

$$\mathbb{E}_{\theta,\eta,u}\left[\tilde{X}(\theta, u, \eta; \bar{y})\eta\right] = (a^T + c^T)f(\bar{y})\frac{1}{\tau_n},$$

we thus have that

$$\frac{d\mathbb{E}[W^T; \bar{y}]}{d\bar{y}} = -\frac{\lambda\left(a^T\right)^2 f(\bar{y})f'(\bar{y})}{\tau_e} - (\lambda + \beta)\left(a^T + c^T\right)^2 f'(\bar{y})f(\bar{y})\frac{1}{\tau_n}. \quad (33)$$

Comparing (32) with (33), we thus have that, when $c^T < 0$,

$$\left.\frac{\partial\mathbb{E}[\pi_i^T; \bar{y}, y_i]}{\partial y_i}\right|_{y_i = \bar{y}} > \frac{d\mathbb{E}[W^T; \bar{y}]}{d\bar{y}},$$

whereas the opposite inequality holds when $c^T > 0$. Finally, use Condition (19) to observe that $\hat{c}^T = -\frac{c^T}{\beta(a^T + c^T)}$ and Condition (25), along with the formula for $\tau_\omega(a)$, to observe that $a^T + c^T > 0$. Jointly, the last two conditions imply that $sgn(\hat{c}^T) = -sgn(c^T)$ thus completing the proof. Q.E.D.

Proof of Proposition 5.

We start by establishing the (global) concavity of $\mathbb{E}[\pi_i^T;\bar{y},y_i]$ and $\mathbb{E}[W^T;\bar{y}]$ in y_i and $\bar{y}$, respectively (Recall that the coefficients defining the traders' demand are kept constant in both of these functions at (a^T,b^T,c^T), where (a^T,b^T,c^T) is the vector defining the efficient trades when the quality of private information is y^T). Using (31), we have that

$$\begin{aligned}\frac{\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i^2} &= -a^T\beta f(\bar{y})\frac{1}{\tau_\eta}\left(a^T+c^T\right)f''(y_i)-\lambda\left(a^T\right)^2\left[\frac{1}{\tau_e}+\frac{1}{\tau_\eta}\right]\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right)\\ &\quad -\lambda a^T c^T f(\bar{y})\frac{1}{\tau_\eta}f''(y_i)\\ &= -a^T f(\bar{y})\frac{1}{\tau_\eta}\left[\beta\left(a^T+c^T\right)+\lambda c^T\right]f''(y_i)-\lambda\left(a^T\right)^2\left[\frac{1}{\tau_e}+\frac{1}{\tau_\eta}\right]\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right).\end{aligned}$$

Now observe that $f''(y_i)=3\sqrt{y_i}/4y_i^3>0$ and $\frac{\partial}{\partial y_i}\left(f(y_i)f'(y_i)\right)=1/y_i^3>0$. Hence,

$$\frac{\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]}{\partial y_i^2} = -\frac{a^T}{y_i^3\tau_\eta}\left[\frac{3\sqrt{y_i}}{4\sqrt{\bar{y}}}\left(\beta a^T+(\beta+\lambda)\,c^T\right)+\lambda a^T\frac{\tau_\eta+\tau_e}{\tau_e}\right].$$

Now recall that, irrespective of the sign of c^T, $a^T>0$ and $a^T+c^T>0$, where the last inequality is established in the proof of Proposition 4. Hence, when $c^T\geq 0$, for any $(\bar{y},y_i)$, $\partial^2\mathbb{E}[\pi_i^T;\bar{y},y_i]/\partial y_i^2<0$. To see that the same inequality holds when $c^T<0$, recall that

$$c^T=\frac{1}{\beta+\lambda}\left[\left(1-\lambda a^T-\lambda a^T\frac{\tau_\theta}{y^T\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta}-\beta a^T\right].$$

Hence,

$$\beta a^T+(\beta+\lambda)\,c^T=\left(1-\lambda a^T-\lambda a^T\frac{\tau_\theta}{y^T\tau_\eta}\right)\frac{\tau_\omega(a^T)}{\tau_\omega(a^T)+\tau_\theta},$$

which, using

$$\tau_\omega(a^T)=\frac{\beta^2\left(a^T\right)^2y^T\tau_\eta\tau_u}{\beta^2\left(a^T\right)^2\tau_u+y^T\tau_\eta},$$

we can rewrite as

$$\beta a^T+(\beta+\lambda)\,c^T=\left[\left(1-\lambda a^T\right)y^T\tau_\eta-\lambda a^T\tau_\theta\right]\frac{\beta^2\left(a^T\right)^2\tau_u}{\beta^2\left(a^T\right)^2\tau_u\left(y^T\tau_\eta+\tau_\theta\right)+y^T\tau_\eta\tau_\theta}.$$

Hence,

$$sgn\left(\beta a^T+(\beta+\lambda)\,c^T\right)=sgn\left(\left(1-\lambda a^T\right)y^T\tau_\eta-\lambda a^T\tau_\theta\right).$$

Now recall that

$$a^T=\frac{1}{\lambda}\frac{\tau_\epsilon y^T\tau_\eta(y^T\tau_\eta-\tau_\omega(a^T))}{\left(y^T\right)^2\tau_\eta^2\left(\tau_\epsilon+\tau_\theta+\tau_\omega(a^T)\right)-\tau_\omega(a^T)\tau_\epsilon\left(\tau_\theta+2y^T\tau_\eta\right)+\Xi(a^T)+\Delta(a^T)} \tag{34}$$

with $\tau_\epsilon=\left(y^T\tau_e\tau_\eta\right)/\left(\tau_e+\tau_\eta\right)$ and observe that the numerator in (34) is positive. Because $a^T>0$, as shown above, this means that the denominator in (34) is also positive. Using the fact that

$$\left(1-\lambda a^{T}\right) y^{T}\tau_{\eta}-\lambda a^{T}\tau_{\theta}=\frac{y^{T}\tau_{\eta}Q}{\left(y^{T}\right)^{2}\tau_{\eta}^{2}\left(\tau_{\epsilon}+\tau_{\theta}+\tau_{\omega}(a^{T})\right)-\tau_{\omega}(a^{T})\tau_{\epsilon}\left(\tau_{\theta}+2y^{T}\tau_{\eta}\right)+\Xi(a^{T})+\Delta(a^{T})}$$

where

$$Q=y^{T}\tau_{\eta}\left(y^{T}\tau_{\eta}-\tau_{\epsilon}\right)\left(\tau_{\theta}+\tau_{\omega}(a^{T})\right)+\Xi(a^{T})+\Delta(a^{T}),$$

we thus have that

$$sgn\left(\left(1-\lambda a^{T}\right) y^{T}\tau_{\eta}-\lambda a^{T}\tau_{\theta}\right)=sgn\left(Q\right).$$

Now, using the fact that $\tau_{\epsilon}=\left(y\tau_{e}\tau_{\eta}\right)/\left(\tau_{e}+\tau_{\eta}\right)$, we have that Q can be rewritten as

$$Q=\left(y^{T}\tau_{\eta}\right)^{2}\frac{\tau_{\eta}}{\tau_{e}+\tau_{\eta}}\left(\tau_{\theta}+\tau_{\omega}(a^{T})\right)+\Xi(a^{T})+\Delta(a^{T})$$

and hence $sgn\left(Q\right)>0$ if $\Xi(a^{T})+\Delta(a^{T})>0$. The latter property holds because, as explained in the main text, when $c^{T}<0$, then $\hat{c}^{T}>0$ in which case $\Xi(a^{T})+\Delta(a^{T})>0$. We conclude that, no matter the sign of c^{T}, for any $\bar{y}$, $\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]$ is strictly concave in y_{i}.

Next, consider the concavity of $\mathbb{E}[W^{T};\bar{y}]$ in $\bar{y}$. Using (33), we have that

$$\begin{aligned}\frac{d^{2}\mathbb{E}[W^{T};\bar{y}]}{d\bar{y}^{2}} &= -\left[\frac{\lambda\left(a^{T}\right)^{2}}{\tau_{e}}+\left(\lambda+\beta\right)\left(a^{T}+c^{T}\right)^{2}\frac{1}{\tau_{n}}\right]\frac{\partial}{\partial\bar{y}}\left(f(\bar{y})f'(\bar{y})\right)\\ &< 0,\end{aligned}$$

where again the inequality follows from the fact that $\frac{\partial}{\partial\bar{y}}\left(f(\bar{y})f'(\bar{y})\right)>0$. Hence $\mathbb{E}[W^{T};\bar{y}]$ is strictly concave in $\bar{y}$.

Because $\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]$ is strictly concave in y_{i}, in equilibrium, all traders acquire information of quality y^{*} such that

$$\left.\frac{\partial\mathbb{E}[\pi_{i}^{T};\bar{y},y_{i}]}{\partial y_{i}}\right|_{y_{i}=\bar{y}=y^{*}}=\mathcal{C}'(y^{*}).$$

Now recall that the socially optimal quality of information satisfies

$$\left.\frac{d\mathbb{E}[W^{T};\bar{y}]}{d\bar{y}}\right|_{\bar{y}=y^{T}}=\mathcal{C}'(y^{T}).$$

Because $\mathbb{E}[W^{T};\bar{y}]$ is strictly concave in $\bar{y}$, the result in Proposition 4, then imply that, when $\hat{c}^{T}<0$, $y^{T}>y^{*}$, whereas, when $\hat{c}^{T}>0$, $y^{T}<y^{*}$. Q.E.D.

Proof of Proposition 6.

Under the proposed policy, each trader i's ex-ante gross expected payoff when all traders other than i collect information of quality $\bar{y}$, trader i collects information of quality y_{i}, and all traders (including i) submit the efficient demand schedules $(a^T, \hat{b}^T, \hat{c}^T)$ is equal to

$$\begin{aligned}\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] &= \mathbb{E}\left[\theta x_i - (1+\hat{t}_p)px_i - \frac{\lambda}{2}x_i^2\right] \\ &= \mathbb{E}\left[\theta x_i - (1+\hat{t}_p)\left(\alpha - u + \beta\tilde{x}\right)x_i - \frac{\lambda}{2}x_i^2\right]\end{aligned}$$

with

$$x_i = X_i(\theta, u, \eta, e_i; \bar{y}, y_i) = a^T \underbrace{[\theta + f(y_i)e_i + f(y_i)\eta]}_{s_i} + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right),$$

$$p = P(\theta, u, \eta; \bar{y}) = \alpha - u + \beta X(\theta, u, \eta; \bar{y}),$$

and

$$\tilde{x} = X(\theta, u, \eta; \bar{y}) = a^T(\theta + f(\bar{y})\eta) + b^T + c^T\left(\theta + f(\bar{y})\eta - \frac{u}{\beta a^T}\right),$$

and where b^T and c^T are the coefficients describing the equilibrium trades obtained from $\hat{b}^T$ and $\hat{c}^T$ using (18) and (19). Hence,

$$\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] = N - \beta(a^T + c^T)a^T\frac{1+\hat{t}_p}{\sqrt{\bar{y}}\sqrt{y_i}\tau_\eta} - \frac{\lambda c^T a^T}{\sqrt{\bar{y}}\sqrt{y_i}\tau_\eta} - \frac{\lambda}{2}\frac{\left(a^T\right)^2}{y_i\tau_\eta} - \frac{\lambda}{2}\frac{\left(a^T\right)^2}{y_i\tau_e}$$

where N is a function of all variables that do not interact with y_i. It follows that

$$\frac{\partial\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p]}{\partial y_i} = \frac{\beta(1+\hat{t}_p)(a^T+c^T)a^T}{2\tau_\eta y_i\sqrt{\bar{y}y_i}} + \frac{\lambda a^T}{2\tau_\eta y_i\sqrt{y_i}}\left(\frac{a^T}{\sqrt{y_i}} + \frac{c^T}{\sqrt{\bar{y}}}\right) + \frac{\lambda\left(a^T\right)^2}{2y_i^2\tau_e}.$$

Because $\mathbb{E}[\pi_i^T(\bar{y}, y_i); \hat{t}_p] - \mathcal{C}(y_i)$ is concave in y_i, for $y_i = \bar{y} = y^T$ to be sustained in equilibrium it is both necessary and sufficient that

$$\frac{\partial\mathbb{E}[\pi_i^T(y^T, y^T); \hat{t}_p]}{\partial y_i} = \mathcal{C}'(y^T)$$

which is equivalent to

$$\frac{\left[\beta(1+\hat{t}_p)+\lambda\right](a^T+c^T)a^T}{2\tau_\eta} + \frac{\lambda\left(a^T\right)^2}{2\tau_e} = \mathcal{C}'(y^T)\left(y^T\right)^2.$$

Using the fact that y^T satisfies

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta} + \frac{\lambda\left(a^T\right)^2}{2\tau_e} = \mathcal{C}'(y^T)\left(y^T\right)^2,$$

we have that the proposed policy implements the efficient acquisition of private information when

$$\hat{t}_p = \frac{(\beta+\lambda)c^T}{\beta a^T}.$$

Using the fact that

$$c^T = \frac{1}{\beta+\lambda}\left(\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T\right)$$

we then have that the optimal $\hat{t}_p$ can be rewritten as

$$\hat{t}_p = \frac{\gamma_2\left(\tau_\omega(a^T)\right) - \beta a^T}{\beta a^T}$$

as claimed in the proposition. Q.E.D.

Proof of Proposition 7.

Assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Given any policy $T(x_i, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$V(y^T, y_i) \equiv \sup_{g(\cdot)} \left\{\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] - \mathcal{C}(y_i)\right\}$$

where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z, and where

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta\tilde{x})g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2\right] - \mathbb{E}\left[T\left(g(s_i, z), \alpha - u + \beta\tilde{x}\right)\right].$$

Note that the definition of $\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)]$ uses the fact that the market-clearing price is given by $p = \alpha - u + \beta\tilde{x}$ with

$$\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$$

where b^T and c^T are the coefficients describing the equilibrium trades obtained from $\hat{b}^T$ and $\hat{c}^T$ using (18) and (19), and where

$$z \equiv \theta + f(y^T)\eta - \frac{u}{\beta a^T}.$$

It also uses the fact that, when all other traders submit the efficient demand schedules, any demand schedule for trader i (that is, any mapping from (s_i, p) into x_i) can be expressed as a function $g(s_i, z)$ of (s_i, z).[17]

For the policy $T(x_i, p)$ to implement the efficient acquisition and usage of information, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that the equilibrium price can be expressed as $p = \alpha + \beta b^T + \beta(a^T + c^T)z$, and the fact that

$$\mathbb{E}\left[\theta | s_i, z\right] = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z,$$

we thus have that for the policy T to implement the efficient trades, it must be differentiable in x_i and satisfy

$$\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right)$$

$$-\tfrac{\partial}{\partial x}T\left(a^T s_i + b^T + c^T z, \alpha + \beta b^T + \beta(a^T + c^T)z\right) = 0$$

for all (s_i, z). Next, observe that, when the individual trades efficiently, so that, for any

[17] It suffices to use (20) to observe that $p = \alpha + \beta b^T + \beta(a^T + c^T)z$.

$p = \alpha + \beta b^T + \beta(a^T + c^T)z$, the quantity the individual buys is given by $x_i = a^T s_i + b^T + c^T z$, then

$$\begin{aligned}
\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right) \\
= \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right] \tfrac{x_i - b^T - c^T z}{a^T} \\
+ \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T\right] \left[\tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)}\right] \\
- \left(\alpha + \beta b^T + \lambda b^T\right) \\
= \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right] \tfrac{x - b^T}{a^T} \\
+ \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right) \tfrac{c^T}{a^T}\right] \left[\tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)}\right] \\
- \left(\alpha + \beta b^T + \lambda b^T\right).
\end{aligned}$$

Note that the term above is the discrepancy between the trader's marginal benefit and marginal cost of expanding his demand evaluated at the efficient trade. But this means that, for the policy $T(x,p)$ to implement the efficient use of information, it must be that $T(x,p)$ is a polynomial of second order of the form

$$T(x_i, p) = \frac{\delta}{2} x_i^2 + (t_p p - t_0)\, x_i + K(p), \tag{35}$$

for some vector (δ, t_p, t_0) and some function $K(p)$ which plays no role for incentives and which therefore we can disregard. In the proof of Proposition 3, we showed that there exists a unique vector (δ, t_p, t_0) that induces the traders to submit the efficient demand schedules when the precision of their private information is y^T (the vector in Proposition 3 applied to $y = y^T$). Thus if a policy T induces efficiency in both information acquisition and information usage, it must be of the form in (35), with (δ, t_p, t_0) as in Proposition 3 applied to $y = y^T$. When the policy takes this form, for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = as_i + b + cz$, for some (a,b,c), implying that

$$\begin{aligned}
\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] = \mathbb{E}\Bigg[(\theta + t_0)\,(as_i + b + cz) - \tfrac{\lambda+\delta}{2}\,(as_i + b + cz)^2 \\
-(1 + t_p)\left(\alpha - u + \beta\left[a^T(\theta + f(y^T)\eta) + b^T + c^T z\right]\right)(as_i + b + cz) \Bigg].
\end{aligned}$$

Letting M be a function of all variables that do not interact with y_i, we then have that, when $g(s_i, z) = as_i + b + cz$, for some (a,b,c),

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \quad = \quad M - \beta(1 + t_p)(a^T + c^T)a \frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{(\lambda + \delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta} - \frac{\lambda + \delta}{2}\frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2}\frac{a^2}{y_i \tau_e}.$$

Using the envelope theorem, we then have that

$$\frac{\partial V(y^T, y^T)}{\partial y_i} = \frac{[\beta(1+t_p)+\lambda+\delta]\,(a^T+c^T)a^T}{2\tau_\eta\,(y^T)^2} + \frac{(\lambda+\delta)\,(a^T)^2}{2\tau_e\,(y^T)^2} \quad - \quad \mathcal{C}'(y^T).$$

Note that in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to the efficient trades $a^T s_i + b^T + c^T z$. Recall that the efficient y^T is given by the solution to the following equation

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta\,(y^T)^2} + \frac{\lambda\,(a^T)^2}{2\tau_e\,(y^T)^2} \quad = \quad \mathcal{C}'(y^T).$$

Hence, for the policy of Proposition 3 (applied to $\bar{y} = y^T$) to implement the efficient acquisition of private information, it must be that

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{\tau_\eta} + \frac{\lambda\,(a^T)^2}{\tau_e} \quad = \quad \frac{[\beta(1+t_p)+\lambda+\delta]\,(a^T+c^T)a^T}{\tau_\eta} + \frac{(\lambda+\delta)\,(a^T)^2}{\tau_e}$$

or, equivalently,

$$(a^T+c^T)\tau_e\left[(\beta+\lambda)c^T - (\beta t_p+\delta)a^T\right] \quad = \quad \delta\,(a^T)^2\,\tau_\eta.$$

One can verify that the values of δ and t_p from Proposition 3 do not solve the above equation except for a non-generic set of parameters. Q.E.D.

Proof of Proposition 8.

As in the proof of Proposition 7, assume that all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, those corresponding to the coefficients $(a^T, \hat{b}^T, \hat{c}^T)$). Given any policy $T(x_i, \tilde{x}, p)$, the expected net payoff for trader i when he chooses information of quality y_i and then selects his demand schedule optimally is equal to

$$V(y^T, y_i) \equiv \sup\nolimits_{g(\cdot)}\left\{\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] - \mathcal{C}(y_i)\right\}$$

where $g : \mathbb{R}^2 \to \mathbb{R}$ is a generic function specifying the amount of shares $x_i = g(s_i, z)$ that the trader purchases as a function of s_i and z, with

$$z \equiv \theta + f(y^T)\eta - \frac{u}{\beta a^T}$$

and

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \equiv \mathbb{E}\left[\theta g(s_i, z) - (\alpha - u + \beta\tilde{x})g(s_i, z) - \tfrac{\lambda}{2}\left(g(s_i, z)\right)^2\right]$$

$$-\mathbb{E}\left[T\left(g(s_i, z), \tilde{x}, \alpha - u + \beta\tilde{x}\right)\right].$$

Note that, in writing $\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)]$, we use the fact that the market-clearing price is given by $p = \alpha - u + \beta\tilde{x}$ with

$$\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$$

where b^T and c^T are the coefficients describing the equilibrium trades obtained from $\hat{b}^T$ and $\hat{c}^T$ using (18) and (19). We also use the fact that, when all other traders submit the efficient demand schedules, any demand schedule for trader i (that is, any mapping from (s_i, p) into x_i) can be expressed as a function $g(s_i, z)$ of (s_i, z) by using (20) to express $p = \alpha + \beta b^T + \beta(a^T + c^T)z$ as an affine transformation of z.

For the policy $T(x_i, \tilde{x}, p)$ to implement efficiency in both information acquisition and usage, it must be that, when $y_i = y^T$, the function $g(\cdot)$ that maximizes the trader's payoff is equal to $g(s_i, z) = a^T s_i + b^T + c^T z$. Using the fact that the equilibrium price can be expressed as

$$p = \alpha - u + \beta\tilde{x} = \alpha + \beta b^T + \beta(a^T + c^T)z$$

and the fact that

$$\mathbb{E}\left[\theta | s_i, z; y_i, y^T\right]\Big|_{y_i = y^T} = \gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z,$$

we thus have that for the policy T to implement the efficient trades, it must be differentiable in x_i and, for all (s_i, z), must satisfy

$$\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right)$$

$$- \tfrac{\partial}{\partial x_i}\mathbb{E}\left[T\left(a^T s_i + b^T + c^T z, \tilde{x}, \alpha - u + \beta\tilde{x}\right)|s_i, z; y_i, y^T\right]\Big|_{y_i = y^T} = 0,$$

where

$$\tilde{x} = a^T(\theta + f(y^T)\eta) + b^T + c^T z$$

with

$$z \equiv \theta + f(y^T)\eta - \frac{u}{\beta a^T}.$$

Next recall from the proof of Proposition 7 that, when the individual trades efficiently,

$$\begin{aligned}\gamma_1(\tau_\omega(a^T))s_i + \gamma_2(\tau_\omega(a^T))z - \left[\alpha + \beta b^T + \beta(a^T + c^T)z\right] - \lambda\left(a^T s_i + b^T + c^T z\right) \\ = \left[\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right]\tfrac{x - b^T}{a^T} \\ + \left[\gamma_2(\tau_\omega(a^T)) - \beta(a^T + c^T) - \lambda c^T - \left(\gamma_1(\tau_\omega(a^T)) - \lambda a^T\right)\tfrac{c^T}{a^T}\right]\left[\tfrac{p - \alpha - \beta b^T}{\beta(a^T + c^T)}\right] \\ - \left(\alpha + \beta b^T + \lambda b^T\right).\end{aligned}$$

This means that, for the policy T to implement the efficient use of information, it must be that $T(x_i, \tilde{x}, p)$ is a polynomial of second order of the form

$$T(x_i, \tilde{x}, p) = \frac{\delta'}{2}x_i^2 + \left(pt'_p - t'_0 + t_{\tilde{x}}\tilde{x}\right)x_i + K'(\tilde{x}, p), \tag{36}$$

for some vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$, where $K'(\tilde{x}, p)$ is a function that does not depend on x_i, plays no role for incentives, and hence which we can disregard. Furthermore, under any such a policy,

$$\frac{\partial}{\partial x_i}\mathbb{E}\left[T\left(x_i,\tilde{x},p\right)|s_i,p;y_i,y^T\right]$$

$$=\delta' x_i + pt'_p - t'_0 + t_{\tilde{x}}\mathbb{E}\left[\tilde{x}|s_i,p;y_i,y^T\right]$$

$$=\delta' x_i + pt'_p - t'_0 + t_{\tilde{x}}\mathbb{E}\left[\frac{p-\alpha+u}{\beta}|s_i,p;y_i,y^T\right]$$

$$=\delta' x_i + pt'_p - t'_0 + \frac{t_{\tilde{x}}}{\beta}(p-\alpha) + \frac{t_{\tilde{x}}}{\beta}\mathbb{E}\left[u|s_i,p;y_i,y^T\right]$$

$$=\delta' x_i + pt'_p - t'_0 + \frac{t_{\tilde{x}}}{\beta}(p-\alpha) + \frac{t_{\tilde{x}}}{\beta}A^{\#}(y_i,y^T)s_i + \frac{t_{\tilde{x}}}{\beta}B^{\#}(y_i,y^T)p + \frac{t_{\tilde{x}}}{\beta}C^{\#}(y_i,y^T)$$

where we used the fact that $p=\alpha-u+\beta\tilde{x}$ and the fact that

$$\mathbb{E}\left[u|s_i,p;y_i,y^T\right] = A^{\#}(y_i,y^T)s_i + B^{\#}(y_i,y^T)p + C^{\#}(y_i,y^T)$$

where $A^{\#}(y_i,y^T)$, $B^{\#}(y_i,y^T)$, and $C^{\#}(y_i,y^T)$ are the coefficients of the projection of u on (s_i,p) when all agents other than i acquire information of quality y^T (and trade efficiently) whereas trader i acquires information of quality y_i.

When trader i too acquires information of quality $y_i=y^T$ and trades efficiently so that $x_i=a^T s_i + b^T + c^T z$ with $z=\frac{p-\alpha-\beta b^T}{\beta(a^T+c^T)}$,

$$\mathbb{E}\left[u|s_i,p;y_i,y^T\right] = A^{\#}(y^T,y^T)\frac{x_i-b^T-c^T\left(\frac{p-\alpha-\beta b^T}{\beta(a^T+c^T)}\right)}{a^T} + B^{\#}(y^T,y^T)p + C^{\#}(y^T,y^T)$$

$$=\frac{A^{\#}(y^T,y^T)}{a^T}x_i + \left[B^{\#}(y^T,y^T) - \frac{A^{\#}(y^T,y^T)c^T}{a^T\beta(a^T+c^T)}\right]p$$

$$+C^{\#}(y^T,y^T) - \frac{A^{\#}(y^T,y^T)b^T}{a^T} + \frac{A^{\#}(y^T,y^T)c^T(\alpha+\beta b^T)}{a^T\beta(a^T+c^T)}.$$

Then let

$$\hat{A}^{\#} \equiv \frac{A^{\#}(y^T,y^T)}{a^T}$$

$$\hat{B}^{\#} \equiv \left[B^{\#}(y^T,y^T) - \frac{A^{\#}(y^T,y^T)c^T}{a^T\beta(a^T+c^T)}\right]$$

$$\hat{C}^{\#} \equiv C^{\#}(y^T,y^T) - \frac{A^{\#}(y^T,y^T)b^T}{a^T} + \frac{A^{\#}(y^T,y^T)c^T(\alpha+\beta b^T)}{a^T\beta(a^T+c^T)}.$$

We thus have that, when trader i acquires information of quality $y_i=y^T$ and trades efficiently,

$$\frac{\partial}{\partial x_i}\mathbb{E}\left[T\left(x_i,\tilde{x},p\right)|s_i,p;y^T,y^T\right] = \delta x_i + t_p p - t_0$$

where

$$\delta = \delta' + \frac{t_{\tilde{x}}}{\beta}\hat{A}^{\#} \tag{37}$$

$$t_p = t'_p + t_{\tilde{x}} \frac{1 + \hat{B}^{\#}}{\beta} \tag{38}$$

$$t_0 = t'_0 + t_{\tilde{x}} \frac{\alpha}{\beta} - \frac{t_{\tilde{x}}}{\beta} \hat{C}^{\#}. \tag{39}$$

In the proof of Proposition 3, we showed that, when agents acquire information of quality y^T, for them to trade efficiently, the values of (δ, t_p, t_0) must coincide with those in Proposition 3 (applied to $y = y^T$). Thus, for the above policy to induce efficiency in both information acquisition and information usage, it must be that that the vector $(\delta', t'_p, t'_0, t_{\tilde{x}})$ satisfies Conditions (37)-(39) with (δ, t_p, t_0) given by the values determined in Proposition 3 applied to $y = y^T$. Note that, for any $t_{\tilde{x}}$, there exists unique values of (δ', t'_p, t'_0) that solve the above three conditions. Abusing notation, denote these values by $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$.

For any $t_{\tilde{x}}$, efficiency in trade requires equating the parameters (δ', t'_p, t'_0) to the unique solution $(\delta'(t_{\tilde{x}}), t'_p(t_{\tilde{x}}), t'_0(t_{\tilde{x}}))$ to Conditions (37)-(39) above.

Next, note that, when the policy takes the form in (36), for any y_i, the optimal choice of $g(\cdot)$ is affine and hence can be written as $g(s_i, z) = as_i + b + cz$, for some (a, b, c). This implies that

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] = \mathbb{E}\Bigg[(\theta + t'_0(t_{\tilde{x}}) - t_{\tilde{x}}\tilde{x}) (as_i + b + cz) - \tfrac{\lambda+\delta}{2} (as_i + b + cz)^2$$

$$-(1 + t'_p(t_{\tilde{x}})) \left(\alpha - u + \beta \left[a^T(\theta + f(y^T)\eta) + b^T + c^T z\right]\right) (as_i + b + cz) \Bigg].$$

Letting M be a function of all variables that do not interact with y_i, we then have that, when $g(s_i, z) = as_i + b + cz$, for some (a, b, c),

$$\mathbb{E}[\tilde{\pi}_i(y^T, y_i); g(\cdot)] \quad = \quad M - \left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}}))\right] \frac{a(a^T + c^T)}{\sqrt{y^T}\sqrt{y_i \tau_\eta}} - \frac{(\lambda + \delta)ca}{\sqrt{y^T}\sqrt{y_i \tau_\eta}} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_\eta} - \frac{\lambda + \delta}{2} \frac{a^2}{y_i \tau_e}.$$

Using the envelope theorem, we then have that

$$\frac{\partial V(y^T, y^T)}{\partial y_i} = \frac{\left[t_{\tilde{x}} + \beta(1 + t'_p(t_{\tilde{x}})) + \lambda + \delta\right] (a^T + c^T) a^T}{2\tau_\eta \left(y^T\right)^2} + \frac{(\lambda + \delta) \left(a^T\right)^2}{2\tau_e \left(y^T\right)^2} \quad - \quad \mathcal{C}'(y^T).$$

Once again, in writing the above derivative, we used the fact that, when $y_i = y^T$, the optimal demand schedule for trader i induces trades equal to the efficient trades $a^T s_i + b^T + c^T z$. Finally, recall that the efficient y^T is given by the solution to the following equation

$$\frac{(\beta + \lambda)(a^T + c^T)^2}{2\tau_\eta \left(y^T\right)^2} + \frac{\lambda \left(a^T\right)^2}{2\tau_e \left(y^T\right)^2} \quad = \quad \mathcal{C}'(y^T).$$

Hence, for the above policy to induce efficiency in information acquisition, it must be that

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{\tau_\eta}+\frac{\lambda\left(a^T\right)^2}{\tau_e} = \frac{\left[t_{\tilde{x}}+\beta(1+t'_p(t_{\tilde{x}}))+\lambda+\delta\right](a^T+c^T)a^T}{\tau_\eta}+\frac{(\lambda+\delta)\left(a^T\right)^2}{\tau_e} \quad (40)$$

Using (38), we have that

$$t'_p(t_{\tilde{x}})=t_p-t_{\tilde{x}}\frac{1+\hat{B}^{\#}}{\beta}$$

with t_p given by the unique value determined in Proposition 3 applied to $y=y^T$. Because the function $H:\mathbb{R}\to\mathbb{R}$ given by

$$H(t_{\tilde{x}})\equiv t_{\tilde{x}}+\beta t'_p(t_{\tilde{x}})=\beta t_p-t_{\tilde{x}}\hat{B}^{\#}$$

is linear, there exists a (unique) value of $t_{\tilde{x}}$ that solves (40).

We conclude that the policy in (36) with $t_{\tilde{x}}$ given by the unique solution to (40) and with (δ',t'_p,t'_0) given by the unique solution $(\delta'(t_{\tilde{x}}),t'_p(t_{\tilde{x}}),t'_0(t_{\tilde{x}}))$ to Conditions (37)-(39) induces efficiency in both information acquisition and information usage. Q.E.D.

Proof of Proposition 9.

Paralleling the derivations in the proof of Proposition 7, when the policy takes the proposed form and all traders other than i acquire information of quality y^T and then submit the efficient demand schedules (that is, the affine orders corresponding to the coefficients $(a^T,\hat{b}^T,\hat{c}^T)$ for quality of information y^T), the expected net payoff for trader i when he chooses information of quality y_i is maximized by submitting an affine demand schedule $x_i=as_i+\hat{b}-\hat{c}p$ which induces trades $x_i=as_i+b+cz$ that are affine in (s_i,z), where $z=\theta+f(y^T)\eta-u/\beta a^T$ is the endogenous signal contained in the market-clearing price.

Using this result, let

$$\hat{V}(y^T,y_i)\equiv\sup_{a,b,c}\left\{\mathbb{E}[\tilde{\pi}_i(y^T,y_i);a,b,c]-\mathcal{C}(y_i)+Ay_i\right\}$$

denote the maximal payoff that trader i can obtain by acquiring information of precision y_i when all other traders acquire information of precision y^T and then submit the efficient demand schedules for information of quality y^T. As shown in the proof of Proposition 7, the expected gross payoff that trader i obtains by inducing the affine trades $x_i=as_i+b+cz$ when he chooses information of quality y_i is equal to

$$\mathbb{E}[\tilde{\pi}_i(y^T,y_i);a,b,c]=M-\beta(1+t_p)(a+c)a\frac{1}{\sqrt{y^T}\sqrt{y_i}\tau_\eta}-\frac{(\lambda+\delta)ca}{\sqrt{y^T}\sqrt{y_i}\tau_\eta}-\frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_\eta}-\frac{\lambda+\delta}{2}\frac{a^2}{y_i\tau_e},$$

where M is a term collecting all variables that do not interact with y_i. Using the envelope theorem, we have that

$$\frac{\partial\hat{V}(y^T,y^T)}{\partial y_i}=\frac{\left[\beta(1+t_p)+\lambda+\delta\right](a^T+c^T)a^T}{2\tau_\eta\left(y^T\right)^2}+\frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2}-\mathcal{C}'(y^T)+A.$$

Again, in writing the above derivative we used the fact that, when $y_i=y^T$, the optimal demand schedule for trader i induces trades equal to $a^T s_i + b^T + c^T z$. Using the fact that y^T satisfies

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta\left(y^T\right)^2}+\frac{\lambda\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2} \quad = \quad \mathcal{C}'(y^T),$$

we thus have that the proposed policy induces the efficient acquisition of private information only if the following condition holds

$$\frac{(\beta+\lambda)(a^T+c^T)^2}{2\tau_\eta}+\frac{\lambda\left(a^T\right)^2}{2\tau_e} \quad = \quad \frac{\left(\beta(1+t_p)+\lambda+\delta\right)(a^T+c^T)a^T}{2\tau_\eta}+\frac{(\lambda+\delta)\left(a^T\right)^2}{2\tau_e}+A\left(y^T\right)^2$$

from which we obtain that

$$A \quad = \quad \frac{a^T+c^T}{2\tau_\eta\left(y^T\right)^2}\left[(\beta+\lambda)c^T-(\beta t_p+\delta)a^T\right]-\frac{\delta\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2}.$$

Next, use Condition (19) to express c^T as a function of $\hat{c}^T$ and rewrite A as follows

$$A=-\frac{\left(a^T\right)^2}{2\tau_\eta\left(y^T\right)^2}\left[\frac{\beta(\beta+\lambda)\hat{c}^T}{\left(1+\beta\hat{c}^T\right)^2}+\frac{\beta t_p+\delta}{1+\beta\hat{c}^T}\right]-\frac{\delta\left(a^T\right)^2}{2\tau_e\left(y^T\right)^2}.$$

Finally, one can verify numerically that the function $\hat{V}(y^T, y_i)$ is globally quasi-concave in y_i. We thus conclude that the proposed policy implements the efficient acquisition and usage of information. Q.E.D.

6.2 Extension: Cournot Case (traders submitting market orders)

In this subsection we show that, in a Cournot equilibrium, there is no inefficiency in either the collection or usage of information. The environment is the same as in the baseline model except for the fact that traders are restricted to submitting market orders instead of a collection of limit orders (equivalently, a demand schedule).

6.2.1 Efficiency in Usage

Suppose that $y_i = y$ for all i. In any symmetric equilibrium in which the price is affine in (θ, u, η), each trader's market order is an affine function of her private signal. That is,

$$x_i = as_i + b$$

for some scalars (a, b) that depend on the exogenous parameters of the model. Aggregate demand is then equal to

$$\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b.$$

Combining the above expression with the inverse aggregate supply function

$$p = \alpha - u + \beta\tilde{x}$$

from the representative investor, we then have that the equilibrium price must satisfy

$$p = \alpha - u + \beta b + \beta a(\theta + f(y)\eta). \tag{41}$$

For each s_i, the equilibrium market order $x_i = as_i + b$ must maximize trader i's expected profits

$$\Pi_i \quad = \quad \mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|s_i\right] - \mathcal{C}(y_i),$$

where $x_i = a_i s_i + b$.

Following steps similar to those in the baseline model, we have that, for any s_i, the derivative of Π_i with respect to x_i, evaluated at $x_i = a_i s_i + b$, must be equal to zero, which yields[18]

$$\mathbb{E}\left[\theta|s_i\right] - \alpha - \beta b - \beta a\mathbb{E}\left[\theta + f(y)\eta|s_i\right] \quad = \quad \lambda(as_i + b).$$

We conclude that the equilibrium value of b, which we denote by b^*, is equal to $b^* = -\frac{\alpha}{\beta+\lambda}$. To obtain the equilibrium value of a, which we denote by a^*, we replace $\mathbb{E}\left[\theta|s_i\right] = \frac{\tau_\epsilon}{\tau_\epsilon+\tau_\theta}s_i$ and $\mathbb{E}\left[\eta|s_i\right] \frac{f(y)\frac{1}{\tau_\eta}\tau_\theta\tau_\epsilon}{\tau_\epsilon+\tau_\theta}s_i$ into the above FOC from which we obtain that

$$a^* \quad = \quad \frac{\tau_\epsilon}{\lambda\left(\tau_\epsilon + \tau_\theta\right) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}.$$

Next, we can derive the expression for the welfare losses when agents do not condition on the price. When the market orders are affine with coefficients a and b,

$$x_i - \tilde{x} \quad = \quad a(s_i - \theta - f(y)\eta)$$

from which we obtain that

$$\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = \frac{a^2}{y\tau_e}.$$

as in the baseline model. Recall that the first-best action is $x^o = \frac{\theta-\alpha+u}{\beta+\lambda}$. One can then show that, for any (a, b), the welfare losses are equal to

$$WL = \tfrac{(\beta+\lambda)\mathbb{E}[(\tilde{x}-x^o)^2]+\lambda\mathbb{E}[(x_i-\tilde{x})^2]}{2} =$$

$$\tfrac{1}{2(\beta+\lambda)^2}\left(\tfrac{(\beta a+\lambda a-1)^2}{\tau_\theta} + \tfrac{(\beta+\lambda)^2 a^2}{y\tau_\eta} + \tfrac{1}{\tau_u} + b^2(\beta+\lambda)^2 + \alpha^2 + 2\alpha b(\beta+\lambda)\right).$$

[18]Note that $\mathbb{E}\left[u|s_i\right] = 0$.

For any a, the value of b that minimizes the welfare losses is thus given by the FOC

$$\frac{\partial WL}{\partial b} = b + \frac{\alpha}{\beta + \lambda} = 0.$$

We conclude that the optimal value of b is the equilibrium one: $b^T = b^* = -\frac{\alpha}{\beta+\lambda}$. Replacing the above into the expression for the welfare losses, we have that the latter can be expressed as a function of a as follows

$$WL(a; y) = \frac{1}{2}\left(\frac{(\beta a + \lambda a - 1)^2}{(\beta + \lambda)\tau_\theta} + \frac{(\beta + \lambda)a^2}{y\tau_\eta} + \frac{1}{(\beta + \lambda)\tau_u} + \frac{\lambda a^2}{y\tau_e}\right).$$

Differentiating $WL(a; y)$ with respect to a and setting the derivative equal to zero gives us the socially-optimal value of a, which we denote by a^T:

$$\frac{\partial WL(a^T; y)}{\partial a} = \frac{(\beta a^T + \lambda a^A - 1)}{\tau_\theta} + \frac{(\beta + \lambda)a^T}{y\tau_\eta} + \frac{\lambda a^T}{y\tau_e} = 0$$

from which we obtain that

$$a^T = \frac{\tau_\epsilon}{\lambda\tau_\epsilon + \beta\tau_\epsilon + \lambda\tau_\theta + \frac{\beta\tau_\epsilon\tau_\theta}{y\tau_\eta}} = a^*.$$

We thus conclude that there is no inefficiency in the usage of information in the Cournot game.

6.2.2 Efficiency in Acquisition

We first characterize the equilibrium acquisition of private information. When each trader $j \neq i$ chooses $y_j = y$ and then submits the equilibrium affine market order $x_j = as_j + b$ for quality of information y, and trader i instead acquires information of quality y_i and then, after observing s_i, submits the market order x_i, his expected payoff is equal to

$$\Pi_i = \mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|s_i, y_i\right] - \mathcal{C}(y_i)$$

where $p = \alpha - u + \beta\tilde{x}$, with $\tilde{x} = a(y)(\theta + f(y)\eta) + b$, with

$$a = a(y) = \frac{\tau_\epsilon}{\lambda\,(\tau_\epsilon + \tau_\theta) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}$$

and $b = -\frac{\alpha}{\beta+\lambda}$, as shown above. For any (s_i, y_i), the optimal market order for trader i is given by the FOC with respect to x_i which yields $x_i = a_i s_i + b$ with

$$a_i = a_i(y, y_i) = \frac{y_i\tau_e\tau_\eta(1 - \beta a(y)) - \beta a(y)\frac{\sqrt{y_i}}{\sqrt{y}}\tau_\theta\tau_e}{\lambda\,(y_i\tau_e\tau_\eta + \tau_\theta(\tau_e + \tau_\eta))}$$

and $b = -\frac{\alpha}{\beta+\lambda}$. That is, for any (y, y_i), trader i's expected profits when all other traders acquire information of quality y and then submit the equilibrium market orders for quality of information y, and trader i instead acquires information of quality y_i and then submits the market order that maximizes his payoff (the one described above) is given by

$$\begin{aligned}\Pi_i(y, y_i) &= \mathbb{E}\left[(\theta - \alpha + u - \beta\,(a\theta + af(y)\eta + b))\,(a_i s_i + b) - \lambda\frac{(a_i s_i + b)^2}{2}; y, y_i\right] - \mathcal{C}(y_i) \\ &= \frac{a_i - \beta a a_i}{\tau_\theta} - \frac{\beta a a_i}{\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda a_i^2}{2}\left(\frac{1}{\tau_\theta} + \frac{1}{y_i\tau_\eta} + \frac{1}{y_i\tau_e}\right) - \mathcal{C}(y_i) - \alpha b + (1-\beta)b^2\end{aligned}$$

where we used the shortcuts $a = a(y)$ and $a_i = a_i(y, y_i)$ and the fact that $s_i = \theta + f(y_i)(\eta + e_i)$.

Replacing a_i with $a_i(y, y_i)$ and a with $a(y)$, and using the Envelope Theorem, we then have that

$$\frac{\partial \Pi_i(y, y_i)}{\partial y_i} = \frac{1}{2}\frac{\beta a(y) a_i(y, y_i)}{y_i\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda\,(a_i(y, y_i))^2}{2}\left(-\frac{1}{y_i^2\tau_\eta} - \frac{1}{y_i^2\tau_e}\right) - \mathcal{C}'(y_i).$$

When y is equal to the equilibrium level, which we denote by y^*, it must be that

$$\frac{\partial \Pi_i(y^*, y^*)}{\partial y_i} = 0$$

which, using the fact $a_i(y^*, y^*) = a(y^*)$ yields

$$\mathcal{C}'(y^*) = \frac{1}{2}\left(\frac{(\beta+\lambda)\,(a(y^*))^2}{(y^*)^2\tau_\eta} + \frac{\lambda\,(a(y^*))^2}{(y^*)^2\tau_e}\right).$$

Next, we characterize the socially-optimal value of y. Because for any y, the socially-optimal usage of information coincides with the equilibrium, as shown above, using the Envelope Theorem, we have that the optimal value of y, which we denote by y^T is given by the condition

$$\frac{\partial WL(a(y^T); y^T)}{\partial y} = \frac{1}{2}\left(-\frac{(\beta+\lambda)\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_\eta} - \frac{\lambda\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_e}\right) + \mathcal{C}'(y^T) = 0.$$

We conclude that the the optimal value of y, which we denote by y^T, is given by the solution to the following condition

$$\mathcal{C}'(y^T) = \frac{1}{2}\left(\frac{(\beta+\lambda)\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_\eta} + \frac{\lambda\left(a(y^T)\right)^2}{\left(y^T\right)^2\tau_e}\right).$$

It is immediate to see that $y^T = y^*$, implying that the equilibrium acquisition of information is also efficient. Q.E.D.