Shaping the future: Policy shocks and the GDP growth distribution

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The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Canada.

Introduction

We incorporate quantile regressions into a structural VAR model to empirically assess how monetary and fiscal policy influence risks around future GDP growth.

A Hybrid Model

In a traditional VAR, we write a bivariate process \(\{Y_t, Z_t\}\) with one lag as

\[
\begin{pmatrix}
Y_{t+1} \\
Z_{t+1}
\end{pmatrix} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \begin{pmatrix} Y_{t} \\
Z_{t}
\end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\
\epsilon_{2,t}
\end{pmatrix},
\]

where the error term \(\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t})\) is such that \(\mathbb{E}(\epsilon_t) = 0\) and \(\text{Cov}(\epsilon_t, \epsilon_{t-h}) = \Sigma_{t-h}\), where \(\Sigma\) is PSD.

In the QR-VAR hybrid, we generalize the linear model of one variable, \(Y_t\), to a linear quantile model. For \(N\) evenly spaced quantiles \(\{p_1, \ldots, p_N\}\), the linear conditional quantiles are written as

\[
\begin{pmatrix}
Q_{p_1}(Y_{t\mid y_{1:t-1}}) \\
Q_{p_2}(Y_{t\mid y_{1:t-1}})
\end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} Y_{1:t-1} \\
Y_{2:t-1}
\end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\
\eta_{2,t}
\end{pmatrix},
\]

and independently estimated with the linear quantile regression method of Koenker and Bassett (1978). Given the representation of \(Y_{1:t}\) in Eq (1), we propose a QR-VAR hybrid model

\[
\begin{pmatrix}
y_{1,t} \\
y_{2,t}
\end{pmatrix} = \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\
y_{2,t-1}
\end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\
\eta_{2,t}
\end{pmatrix}. \tag{3}
\]

For sufficiently large \(N\), we can approximate the conditional mean of \(Y_t\) as \(\frac{1}{N} \sum_{n=1}^{N} Q_{p_n}(Y_{1:t\mid y_{1:t-1}})\), from which we estimate residuals

\[
\epsilon_{1,t} = y_{1,t} - \frac{1}{N} \sum_{n=1}^{N} Q_{p_n}(Y_{1:t\mid y_{1:t-1}}), \quad \epsilon_{2,t} = y_{2,t} - \frac{1}{N} \sum_{n=1}^{N} \eta_{n,t}. \tag{4}
\]

The residuals \((\epsilon_{1,t}, \epsilon_{2,t})\) are decomposed structurally as in the standard VAR.

Questions

- How can we assess risks around GDP growth?
  - We simulate the entire distribution of GDP growth with a new quantile-vector autoregression hybrid model (QR-VAR).
- How can monetary and fiscal policy tools influence these risks?
  - Both policies change the location of the distribution of future GDP growth, but fiscal shocks also impact its shape.
  - When the ZLB binds, fiscal policy scales the distribution up, esp. for lower quantiles.

Literature

- Larger spending multipliers in recessions / ZLB
  Auerbach and Gorodnichenko (2010); Christiano, Eichenbaum and Rebello (2011); Romey and Zalacay (2015)
- Effectiveness of monetary policy tools
  Sime and Wu (2019); Ulate (2019)
- Asymmetric dynamics output / financial stress
  Adrian, Brunnermeier and Giannone (2019); Charlebois and Manganelli (2019)

Benefits of the QR-VAR

When \(Y_t\) is governed by non-gaussian, conditionally heteroskedastic, and skewed dynamics, estimating the QR-VAR compared to the VAR:

- Does not worsen conditional mean predictions;
- Slightly over-estimates conditional variance;
- Accurately estimates conditional skewness.

Simulation Approach

- Fit conditional quantiles. For all \(p \in \{p_1, \ldots, p_N\}\) we estimate
  \[
  \hat{Q}_p(Y_{1:t\mid y_{1:t-1}}) = \hat{\beta}_p Y_{1:t-1} + \hat{\eta}_p \]
  where \(\hat{\beta}_p(u) = u \cdot (p - i_{[p<0]}).
  
  We obtain \(N = 99\) fitted quantiles \(\{\hat{Q}_{0.01}(Y_{1:t\mid y_{1:t-1}}) \ldots \hat{Q}_{0.99}(Y_{1:t\mid y_{1:t-1}})\}.

- Approximate conditional density \(\widehat{f}(y_{1:t\mid y_{1:t-1}})\) with Kernel Density Approximation (KDA)
  \[
  \hat{K}(y_{1:t} - \hat{Q}_p(Y_{1:t\mid y_{1:t-1}})),
  \]
  \[
  \frac{1}{N} \sum_{n=1}^{N} \frac{1}{\hat{H} \lambda_p} K((y_{1:t} - \hat{Q}_{p_n}(Y_{1:t\mid y_{1:t-1}}))),
  \]
  where \(K(\cdot)\) is a Gaussian kernel, \(\hat{H} > 0\) is a bandwidth, and \(\lambda_p\) is a local bandwidth.

Simulate: re-sampling from \(\hat{f}(\cdot\mid y_{1:t-1})\)

Application to Monetary and Spending Shocks

Panel QR-VAR (US, UK, Japan, Canada, Australia, Finland; 1964Q1–2019Q4) with 5 variables:

- Annual real GDP growth per cap.
- Annual real government spending growth per cap.
- Change in the short-term rate
- Annual CPI inflation
- Log of the financial stress index (FSI)

Structural shock identification:

<table>
<thead>
<tr>
<th>Shock</th>
<th>Spending</th>
<th>Supply</th>
<th>Monetary</th>
<th>Financial</th>
<th>Demand</th>
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<tbody>
<tr>
<td>M2 spending</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Inflation</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Rate</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Imp. GDP</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Spending shocks identified with zero restrictions

Results

Monetary shocks move all quantiles of GDP growth evenly while spending shocks increase the odds of faster GDP growth.

Impact response of quantiles 10 (dotted), 50 (plain), 90 (dashed) and mean (stared) of GDP growth, after 50 bps monetary easing (left) or 5ppt spending increase (right).

Spending shocks increase upside risks during ZLB events.

Quantile impulse (Panels 1 to 5) or mean impulse (Panel 6) Normal times (blue) or ZLB (red) with 90% confidence interval

Conclusion

- We build a hybrid of quantile regressions and structural VAR to relax assumptions of linearity, symmetry, and uni-modality for one variable
- QR-VAR better for skewed variable
- Monetary policy does not change the shape of the distribution of future GDP growth
- Fiscal policy changes the shape of the distribution
- More so when the ZLB is binding
- Increases lower quantiles: speeds up the recovery