Liquidity Allocation and Endogenous Uncertainty

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For most of advanced economies, during the slow economic recovery from the Great Recession,

- **Weak Investment** but **Soar of Stock Market**
- **Saving Glut of Non-Financial Corporation**: net funds inflow from real economy to financial system
- high level of **Uncertainty**
Motivation: Phenomenon
Growth Rate of Equity Price and Investment Ratio
Fund Flows and Uncertainty

Shaded areas indicate U.S. recessions

Sources: BIS, Baker, Scott R., CBOE

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Empirical Evidences

- Evidences for Slow Economic Recovery:
  - Becker, Davis and Murphy (2010)
  - Fernald, Hall, Stock, and Watson (2017)
  - Ball (2014)
  - Hall (2015)

- Evidences for Saving Glut of non-Financial Corporate and Weak Investment:
  - Chen, Karabarbounis and Neiman (2017)
  - Gruber and Kamin (2016)
What I Do

- A Tractable Theory to Rationalize Above Phenomenon
  - An **Endogenous Liquidity Allocation Mechanism** between Real Economy and Financial System
  - The Interaction between Endogenous Liquidity Allocation and Endogenous Aggregate Uncertainty

- Numerical Analysis
Main Intuitions

- Assumptions:
  - Physical capital has less liquidity than its corresponding equity
  - Entrepreneurs have to take partial risk of their own investment

- Holding of risky physical capital depends on the capital structure of corporation, so does the investment:
  - costly adjustment of physical capital position
  - incomplete risk-sharing
  - Net worth works as risk buffer
Main Intuitions

Endogenous Liquidity Allocation Mechanism:

- **Recovering:**
  - Entrepreneurs with low net worth level, prefer high liquidity financial assets, and even disinvest.
  - More funds from real economy flow into financial system, push up equity price and amplify financial risks.
  - Higher financial risks retard investment and leads to an adverse liquidity loop.

- **Booming:**
  - High net worth level of entrepreneurs stimulates high investment demand and high equity price.
Related Literature

- **DSGE of Endogenous Risks**
  - Di Tella (2017)

- **Dynamic Corporate Investment**
  - Isohatala, Milne, and Robertson (2014)
  - He and Kondor (2016)

- **Liquidity Difference between Assets**
  - Kiyotaky and Moore (2012)
Economic Environment

- Infinite Identical Risk-Averse Entrepreneurs Whose Total Mass is 1
- Infinite Identical Risk-Neutral Financial Investors Whose Total Mass is 1
- Only Entrepreneurs Can Run Physical Capital
- Entrepreneurs Can Raise Funds from Financial Markets by Issuing Equity and Bonds
- Financial Investors ONLY Participate in Transactions in Financial Market
Key Assumptions

1. Slower Adjustment of Macroeconomy than Financial Market
   - Physical Capital Has Less Liquidity than Its Corresponding Equity
   - Transaction Cost for Purchasing Capital: \( \Psi(\kappa) < \kappa \)  
     - Capital and Equity are Imperfect Substitutive

2. Financial Frictions: “Skin in the Game”
   - Entrepreneurs Have to Take Partial Risk of Their Own Investment
   - Equity Issuance Constraint: \( \chi \geq \chi \)
Other Assumptions

- The Evolving Process of Physical Capital:

\[ dK = \left[ \Phi(\nu) + \Psi(\kappa) - \delta \right] K dt + \sigma K dZ \]

- \(dZ\): Aggregate Productivity Shock

- Guessed Process of Equity Price:

\[ dq = \mu^q q dt + \sigma^q q dZ \]

- Return Rate of Equity:

\[ dR = \left( \frac{A - \nu - p\kappa}{q} \right) dt + \left[ \Phi(\nu) + \Psi(\kappa) - \delta + \mu^q + \sigma \sigma^q \right] dt + (\sigma + \sigma^q) dZ \]
Dynamic Optimization Question of Entrepreneurs

$$\max_{\{C, \nu, \iota, \kappa, \chi\}} E_0 \left[ \int_0^\infty e^{-\rho t} \frac{C^{1-\gamma}}{1-\gamma} dt \right]$$

s.t.

$$0 \leq h \leq 1,$$

$$(1 - h)\nu W = \chi qK \geq \bar{\chi} qK,$$

$$dW = (1 - h)\nu WdR + h\nu Wd\tilde{R} + (1 - \nu) Wrdt - Cdtd,$$

$$\frac{d(qK)}{qK} = \left[ \Phi(\iota) + \psi(\kappa) - \delta + \mu^q + \sigma \sigma^q \right] d\tau + \left( \frac{\sigma + \sigma^q}{\sigma^V} \right) dZ$$
Optimal Choices of Entrepreneurs

- Optimal Investment Ratio

\[
\frac{(1 - \gamma)\varphi(w) - w\varphi'(w)}{(1 - h)\nu w\varphi'(w)} + 1 = \frac{1}{q}
\]

Relative Price of Capital Denominated by Equity

- \( w \equiv \frac{W}{qK} \): the capital ratio
- \( \varphi(w) \equiv J(W, qK)/(qK)^{1-\gamma} \)

- Relative Price of Capital

\[
\frac{(1 - \gamma)\varphi(w) - w\varphi'(w)}{(1 - h)\nu w\varphi'(w)} + 1 = \frac{qKJ'_qK + (1 - h)\nu WJ'_W}{(1 - h)\nu WJ'_W} < 1
\]

because \( J'_qK < 0 \)
Optimal Choices of Entrepreneurs

- Trade-Off between Producing Capital and Purchasing Capital
  \[
  \Phi'(\iota)K = \Psi'(\kappa)K/p
  \]
  Capital Formation by Producing = Capital Formation by Purchasing

- Comparison between Capital Price and Equity Price: At Equilibrium, \( p < q \)
  \[
  \left[ \frac{(1 - \gamma)\varphi(w) - w\varphi'(w)}{(1 - h)w\varphi'(w)} + 1 \right]\Psi'(\kappa) = \frac{p}{q}
  \]

- Different from BS (2014, 2017) Who Assume Physical Capital ⇐⇒ Equity:
  \[
  \Phi'(\iota)K = K/q
  \]
Optimal Choices of Entrepreneurs

- **Asset Pricing of Equity**

\[
(1 - h)E(d\hat{R})/dt + h\mu\tilde{R} + \left(\lambda(1 - h)\right)
\]

\[
\left\{\frac{(qK)^{\gamma}\varphi'(w)}{(1 - h)(\sigma + \sigma^q) + h\sigma\tilde{R}}\right\}
\]

\[
\quad = r + \pi^e[(1 - h)(\sigma + \sigma^q) + h\sigma\tilde{R}]
\]

- **\(\lambda\)**: the Lagrangian Multiplier of Equity Issuance Constraint

\[
\chi \geq \chi_L
\]

- **Liquidity Premium Comes from “Skin of Game”**

- **Risk Pricing by Entrepreneurs:**

\[
\pi^e \equiv \gamma(\sigma + \sigma^q) - \frac{\nu[(1 - h)(\sigma + \sigma^q) + h\sigma\tilde{R}] - (\sigma + \sigma^q)w\varphi''(w)}{\varphi'(w)}
\]
Optimal Choices of Investors

- Dynamic Optimization Question

\[
\max_{\{C, \nu\}} E_0 \left[ \int e^{-rt} C \, dt \right]
\]

s.t.

\[
dW = \nu W \, dR + (1 - \nu) W \, r \, dt - C \, dt
\]

- Asset Pricing of (Outside) Equity

\[
\frac{A - \iota - p\kappa}{q} + \Phi(\iota) + \Psi(\kappa) - \delta + \mu^q + \sigma^q = r
\]

\[
E(dR)/dt
\]
Markov Equilibrium

- The Markov Equilibrium Has A Single State Variable
  \[ \eta \equiv \frac{\int_0^1 W(i)di}{\int_0^1 qK(i)di} \]

- No One Purchases Physical Capital
  \[ \kappa = 0 \Rightarrow \Psi(\kappa) = 0 \]

- The Net Liquidity Flow to Financial System
  \[ \int_0^1 dF(i)di \equiv \int_0^1 d[(1 - \nu(i))W(i)]di - \int_0^1 d[(1 - \chi(i))qK(i)]di \]

  \( \text{inflow} \)

  \( \text{outflow} \)

- The Ratio of Net Liquidity Flow:
  \[ \frac{\int_0^1 dF(i)di}{\int_0^1 qK(i)di} \]
Parameterization

- Similar Parameter Values as BS (2014)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>time discount rate of entrepreneurs</td>
<td>6%</td>
</tr>
<tr>
<td>$r$</td>
<td>time discount rate of investors</td>
<td>5%</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>risk aversion of entrepreneurs</td>
<td>2</td>
</tr>
<tr>
<td>$A$</td>
<td>productivity</td>
<td>12%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>depreciation rate</td>
<td>3%</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>capital quality shock</td>
<td>2%</td>
</tr>
<tr>
<td>$\chi$</td>
<td>equity issuance constraint</td>
<td>70%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>capital formation function by investment</td>
<td>10</td>
</tr>
<tr>
<td>$\psi$</td>
<td>capital formation function by purchasing</td>
<td>10</td>
</tr>
</tbody>
</table>
Global Dynamics
Investment Ratio, Equity Price Growth and Equity Market Risk
Global Dynamics
Equity Price Growth and Net Liquidity to Financial System

\[ q/\mu^q \]

\[ \text{Liquidity Ratio} \]
Global Dynamics with Different Extents of Equity Issuance Constraint
Global Dynamics with Different Extents of Exogenous Risk
Conclusion

- A Tractable DSGE Model of **Endogenous Liquidity Allocation Mechanism** between Real Economy and Financial System
- The **Interactions between Endogenous Risks and Liquidity Allocation** Help Us to Understand the Inconsistency between Business Cycle and Financial Cycles.

**Policy Implications:**
- QE policy is not perfect
- QE policy has a potential to lead to the weak investment and saving glut of non-financial corporation during the economic recovery by allocating more liquidity into financial markets rather than into the real economy
Ext1: Capital Requirement for Financial Investors

- Financial investors face capital requirement constraint: \( \nu \leq \bar{m} \)
- Financial investors can be risk-averse

\[
\rho = \nu [E (dR/dt) - r] + r \geq r
\]

\[
E (dR/dt) - r = \frac{\zeta}{W_{RPI}}
\]

\[
\frac{\zeta}{W_{RPI}} + \frac{\lambda_2 (qK)^{-\gamma \varphi'}}{(\varphi'_{LPe})} = \left[ \gamma + (1 - \nu) \frac{\eta \varphi''}{\varphi'} \right] (\sigma + \sigma^q)^2 > 0
\]
Proposition: When $0 \leq \eta < 1 - \frac{1-\chi}{m}$, entrepreneurs face a binding equity issuance constraint and hold no outside equity, and financial investors’ capital requirement constraint are not binding, i.e., $\nu = \frac{\chi}{\eta}, \nu < \overline{m}$, and $qH = 0$; When $1 - \frac{1-\chi}{m} < \eta \leq 1$, entrepreneurs’ equity issuance constraint is not binding, and financial investors face a binding capital requirement constraint, i.e., $\nu > \frac{\chi}{\eta}, \chi > \overline{\chi}$, and $\nu = \overline{m}$; When $\eta = 1 - \frac{1-\chi}{m}$, both entrepreneurs’ equity issuance constraint and investors’s capital requirement constraint are binding, entrepreneurs hold no outside equity, i.e., $\nu = \frac{\chi}{\eta}, \nu = \overline{m}$, and $qH = 0.$
Ext2: Capital Misallocations

- Entrepreneurs can rent out $1 - \psi$ ($0 \leq \psi \leq 1$) fraction of physical capital to investors whose productivity is $A \ll A$ to hedge against partial labor productivity shock $dZ$:

$$dR = \left[ \frac{\psi A + (1 - \psi)A - \iota - p\kappa}{q} + \Phi (\iota) + \Psi (\kappa) - \delta + \mu^q + \psi\sigma q}{E(dR/dt)} \right] + (\psi\sigma + \sigma^q)dZ$$

$$LPe = RPe = \left[ \gamma + (1 - \nu)\frac{w\varphi''(w)}{\varphi'(w)} \right] (\psi\sigma + \sigma^q)^2 = \pi^e (\psi\sigma + \sigma^q)$$

- Or, Entrepreneurs can sell out physical capital to investors and pay some transaction cost
Ext3: Heterogenous Productivity Agents

Agent $i$'s choice will affect his return rate of physical capital, $dr^K(i)$

$$
\max \{C^i, \chi^i, h^i, \nu^i, \iota^i, \kappa^i\} \quad E_0 \int_0^\infty e^{-\rho t} \frac{(C^i)^{1-\gamma}}{1 - \gamma} dt
$$

s.t.

$$
\chi^i \geq \bar{\chi},
$$

$$
(1 - h^i) \nu^i W^i = \chi^i qK^i,
$$

$$
1 \geq h^i \geq 0,
$$

$$
dW^i = (1 - h^i) \nu^i W^i dr^K(i) + h^i \nu^i W^i dR + (1 - \nu^i) W^i rdt - C^i dt,
$$

$$
\frac{d(qK^i)}{qK^i} = \left[ \Phi \left( \nu^i \right) + \Psi \left( \kappa^i \right) - \delta + \mu^q + \sigma \sigma^q \right] dt + (\sigma + \sigma^q) dZ.
$$