Modelling yield curve in less liquid markets

An integrated approach

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Main Objectives

In this article we **develop a class of weighting schemes which improves fit relative to conventionally used methods. We operationalise fit as mean absolute error and smoothness of estimated forward curves**. We propose to combine the outstanding amounts and market liquidity data with market level information on prices of government bonds in the form of weighting schemes. Prior literature for LLMs and for developed markets offered approaches that lack a reference to liquidity measures such as: equal weights of yield differences or weights derived from modified duration on price differences. We postulate that inferring from the liquidity in particular maturities raises the information content of the estimation and, unlike earlier approaches, is model-consistent.

We posit here that Pure Expectations Hypothesis (PEH) does not hold universally in LLMs. Empirical research demonstrated that in liquid markets (US) Pure Expectations Hypothesis, claiming that that future rate is an unbiased predictor of the future spot rate, holds for short horizons and durations, but not universally Fama (1987), Campbell(1991), Cochrane(2005). We will obtain verification of PEH across horizons and durations analogous to the literature for the US and compare the results between the less liquid (Poland) markets and liquid market (US).

Stylised facts on the Polish yield curve

1. Monthly secondary market's turnover is of the same magnitude as monthly sum of NBP bills auctioned at reference rate.
2. The volume traded (both on BondSpot platform and on the market as a whole), as well as daily number of transactions follow common pattern during the lifetime of a certain bond type.
3. The bid-ask spread rises approximately two- or threefold above bond's lifetime mean in the last year.
4. Zero trading days patterns resemble mirror reflection of the ones observed for volume and number of trades.
5. Historically the lowest bid-ask spreads were observed in the segment of (6, 12] and the highest in the short-end of the curve [0, 1.5].
6. Ultra long end of Polish yield curve (12, 30] is very erratically inhabited with only one or two series quoted on fixing, and no representation since 2018 till now.
7. Switch auctions influence prices by increasing BAS due to very limited motivation on both sides: potential buyers' side who have alternative strategy of rolling NBP bills and potential sellers' who maybe better off using these bonds to buy longer and more liquid ones.
8. Historically the least liquid segments were [0, 1.5] and (12,30].
9. Switch operations make short bonds (eligible to switch from) richer than the interpolated interest rate between NBP bills and $[1.0, 1.5]$ segment of bonds.
10. All segment-wise average yield time series are *trend stationary* when corrected for long term variance a mode de Newley-West for lags of at least 18-months.
11. Share of BondSpot in total secondary market turnover is erratic and the list of bonds traded is periodically shallow.

Filtering rules and weight system framework

- **Principle 1.** Minimise the share of arbitrary decisions.
- **Principle 2.** Do not exclude bonds from the sample entirely, but diminish their weight accordingly, unless the pricing is systemically distorted.
- **Principle 3.** Include as much reliable and useful information (static and dynamic data) as possible to reflect importance of a certain bond in the yield curve formation.
- **Principle 4.** Include only *risk free* rates.

To that end we propose the following *filtering rules*:

- **Rule 1.** from the broad set of Polish government bonds we exclude CPI-linkers, floaters, foreign denominated and retail bonds. This is a usual choice in yield curve estimation literature.
- **Rule 2.** we take all pricing information of fixed and zero coupon bonds which are subject to fixing on BondSpot, with an exception in the next bullet:
- **Rule 3.** we exclude bonds with less than 0.85 or 1.20 years to maturity (till mid 2017 and after that period) because their prices and therefore *ytm* are distorted significantly by the switch operations of Polish MinFin (as it was clearly shown).

We estimate and experiment with a parsimonious yield curve environment, namely, in the popular **Nelson-Siegel-Svensson** form and its extension - henceforth: NSS).

We propose to overcome almost all the typical numerical challenges of the NSS approach:

1. *each market may have its own heuristics with regard to possible shapes on the yield curve* - we have acknowledged it by carefully examining Polish market's data and have ran multiple tests and visual data inspection in order to create a list of different starting vectors of parameters Θ implying various shapes of yield curves encountered.
2. *the constrained optimisation methods may become particularly slow* - we have tasted and used Matlab's routine of *internal-point* whereas key step in achieving efficiency is the preparation of bonds cash flow matrix and time to maturity vectors prior to the iteration *pre se*. The details will be given in this subsection.
3. *different combinations of starting parameters may produce an equally good fit to observed data* - we have not experienced the precise same *goodness-of-fit* measures for different parameters, but we accept that such situation is probable, and therefore we have developed more composite measure of *goodness* including wider scope of statistics. Rules of such a ranking will be presented in subsequent subsections.
4. *there is usually a set of starting values of* Θ *needed* - such a set was prepared with more than 30 starting vectors, based on historical contexts.
5. *overly smooth specification of the yield curve fitting may disguise some important issue-characteristic or term structure related economic information in government bonds' prices (i.e. tax effects, supply effects)* - we have already proposed in filtering rules a treatment of such specific bonds in Poland either by excluding them (*switch bonds*) or limiting their informational value in line with the importance for the market, which is measured by the outstanding amounts and turnover on the secondary market.

The optimisation problem we face may be defined:

$$\min_{\Theta}\left\{\sum_{i=1}^{N} W_i(P_i(\Theta)-p_i)^2 + W^{nbp}(R(\Theta)-r^{nbp})^2\right\} \quad \text{s.t.} \quad \mathcal{C}(\Theta) \tag{1}$$

where $\Theta = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ is a vector of NSS parameters, p_i are observed prices, $P_i(\Theta)$ are theoretical prices given Θ, $R(\Theta)$ stands for estimated short term interest rate given Θ, W_i is a combined weight of modified duration W_i^{md} and either W_i^{oa} or W_i^{vol} or their sum:

$$W_i = \begin{cases} W_i^{md} W_i^{oa}, & \text{outstanding amounts weights only} \\ W_i^{md} W_i^{vol}, & \text{turnover weights only} \\ W_i^{md}\left(W_i^{oa} + W_i^{vol}\right), & \text{combined weights} \end{cases} \tag{2}$$

and the weight for the *ultra-short* end of the curve is defined as follows (with γ being a scalar multiplier (1 or 2, in our simulations)):

$$W^{nbp} = \begin{cases} \gamma \sum_{i=1}^{N} W_i^{oa}, & \text{outstanding amounts weights only} \\ \gamma \sum_{i=1}^{N} W_i^{vol}, & \text{turnover weights only} \\ \gamma \sum_{i=1}^{N}\left(W_i^{oa} + W_i^{vol}\right), & \text{combined weights} \end{cases} \tag{3}$$

We have analysed the results of Polish government yield curve estimations in 28 different weight/filtering systems in the ensuing groups of measures:

1. **statistics of estimated parameters** Θ_t i.e. mean, median, standard deviation, interquartile range, max-min range
2. **statistics of *goodness-of-fit*** MAE, WMAE, maximum absolute difference, hit ratio, cheap/rich ratios,
3. **auxiliary characteristics of estimated interest rates**: smoothness, short rates fit , volatility and level of synthetic interest rates in segments
4. **optimisation algorithm - related**: exit flags, number of iterations, number of calls to objective function, execution time (in seconds)

Results

General observations:

- conventional weighting (all equal weights in the yield space) results in the roughest and not well fitted yield curves
- systems based on combined outstanding amounts and turnover give less rough curves then conventional with only slightly worse error
- systems based on solely on outstanding amounts produce even smoother curves but with a trade-off with MAE
- systems based on solely on turnover give the smoothest curves and yet again worse MAE
- excluding *eligible-for-switch* bonds improves smoothness (lowers the roughness) and error (lowers MAE) significantly (roughness by the factor of 5 and MAE by approx. 30%).

We have found heuristically a class of weighting system for Polish government bonds yield curve which significantly improves the fit and smoothness as compared to the traditional approach of all equal weight. This **class** has three core characteristics:

- at least the same weight for the short end of the curve as a sum for all other tenors of bonds
- exclusion of *eligible-for-switch* bonds from the estimation
- bonds' weights based on at least outstanding amounts (in the best systems we had either W^{oa} or $W^{oa} + W^{vol}$)

Conclusions

We have collected stylised facts on Polish government bonds market, which were consequently used in filtering and weight system design. These facts revealed importance of liquidity in bonds lifespan, heterogeneity of liquidity in different yield curve segments, bid-ask spreads behaviour as well as price distortion in the very short end of the curve due to *switch auctions*.

Moreover, we argued that as for the ultra short end ($1/52$ years) rate we should use NBP bill rate. We developed efficient algorithm which allowed for relatively quick estimations of 28 systems with approximately 4 thousand days each. We have tested 28 different weight systems and rank them in the space of *goodness-of-fit* and *smoothness* and confirmed that there is a class of weights that systematically gives better results than the classic approach of all equal weights. The highest ranked systems have at least the same weight for the short end of the curve as a sum for all other tenors of bonds, *eligible-for-switch* bonds were excluded from the estimation and weights were be based on at least outstanding amounts.

The evidence presented suggests that indeed PEH does not hold universally for Polish government bonds yield curve. Contrary to the US data and research where PEH is almost always rejected, we have found that for Poland there is a limited domain where PEH cannot be ruled out. The scope where *pure expectations hypothesis* probably holds in Poland is bounded by (1) the investment horizon of approximately 12 months and (joined condition) and (2) by maturity of the bond of circa 36 months. It is still unclear (as in the bewildering variety of research) what causes the rejection of PEH for all other combinations of horizon length and maturity: existence of some kind of risk premia or unexpected excess yield (we have only a mixture of these two contained in β coefficient estimators).

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