Modelling yield curve in less liquid markets
An integrated approach

Marcin Dec
FAME—GRAPE, Poland and Kozminski University, Poland

Main Objectives

In this article we develop a class of weighting schemes which improve fit relative to conventionally used methods. We operationalise fit as mean absolute error and smoothness of estimated forward curves. We propose to combine the outstanding amounts and market liquidity data with market level information on prices of government bonds in the form of weighting schemes. Prior literature for LLMs and for developed markets offered approaches that lack a reference to liquidity measures such as: equal weights of yield differences or weights derived from modified duration on price differences. We postulate that inferring from the liquidity in particular maturities raises the information content of the estimation and, unlike earlier approaches, is model-consistent.

We posit here that Pure Expectations Hypothesis (PEH) does not hold universally in LLMs. Empirical research demonstrated that in liquid markets (US) Pure Expectations Hypothesis, claiming that future interest rates are the expected future values of interest rates. We have found heuristically a class of weighting systems which are model-consistent.

Stylised facts on the Polish yield curve

1. Monthly secondary market’s turnover is of the same magnitude as monthly sum of NBP bills auctioned at reference rate.
2. The volume traded on BondSpot platform and on the market as a whole, as well as daily number of transactions follow common pattern during the lifetime of a certain bond type.
3. The bid ask spread rises approximately two- or threefold above bond’s lifetime in the last year.
4. Zero trading days presentable mirror reflection of the ones observed for volume and number of trades.
5. Historically, the lowest bid ask spread were observed in the segment of (6, 12] and the highest in the short-end of the curve [0, 1.5].

Filtering rules and weight system framework

- Principle 1. Minimise the share of arbitrary decisions.
- Principle 2. Do not exclude bonds from the sample entirely, but diminish their weight accordingly, unless the pricing is systematically distorted.
- Principle 3. Include as much reliable and useful information (static and dynamic data) as possible to reflect importance of a certain bond in the yield curve formation.
- Principle 4. Include only risk free rates.

To that end we propose the following filtering rules:

- Rule 1. From the broad set of Polish government bonds we exclude CPI-linkers, foreign, foreign denominated and retail bonds. This is a usual choice in yield curve estimation.
- Rule 2. We take all pricing information of fixed and zero coupon bonds which are subject to fixing on BondSpot, with an exception in the next bullet.
- Rule 3. We exclude bonds with less than 0.5% or 1.2 years to maturity (till mid 2017 and after that period) because their prices and therefore yyle are distorted significantly by the switch operations of Polish Mint (as it was characteristic).

We estimate and experiment with a parametric yield curve environment, namely, in the popular Nelson-Siegel-Svensson form and its extension - henceforth: NSS). We propose to overcome all the typical numerical challenges of the NSS approach:

1. each market may have its own heuristics with regard to possible shapes on the yield curve - we have acknowledged it by carefully examining Polish market’s data and have run multiple tests and visual data inspection in order to create a list of different starting vectors of parameters β implying various shapes of yield curves encountered.
2. the constrained optimisation methods may become particularly slow - we have tested and used Mathlab’s routine of internal-point whereas key step.
3. Pricing efficiency is the preparation of bonds cash flow matrix and time to maturity vectors prior to the iteration pre se. The details will be given in this subsection.
4. different combinations of starting parameters may produce an equally good fit to observed data - we have not experimented the pre-arranged Polish market’s data and have run multiple tests and visual data inspection in order to create a list of different starting vectors of parameters β implying various shapes of yield curves encountered.
5. The optimisation problem we face may be defined:

\[ \min_{\mathbf{\beta}} \left\{ \sum \frac{W_i}{2} (p_i - R_i)^2 + W_{oa} \right\} \quad \text{s.t. } \mathbf{C} \mathbf{\beta} = 0 \]

where \( \mathbf{R} \) is a vector of NSS parameters, \( p_i \) are observed prices, \( \mathbf{F}(\mathbf{\beta}) \) are theoretical prices given \( \mathbf{\beta} \), \( \mathbf{C} \mathbf{\beta} = 0 \) stands for estimated short term interest rate given \( \mathbf{\beta} \), \( W_{oa} \) is a combined weight of modified duration \( \mathbf{W}_{md} \) and either \( \mathbf{W}_{oa} \) or \( \mathbf{W}_{oa} + \mathbf{W}_{md} \) are their sum.

We have analysed the results of Polish government yield curve estimations in 28 different weight/filtering systems in the ensuing groups of measures:

1. statistics of estimated parameters \( \mathbf{\beta} \), i.e. mean, median, standard deviation, interquartile range, max-min range
2. characteristics of goodness-of-fit MAE, MWAPE, maximum absolute difference, hit ratio, cheap/rich ratio
3. auxiliary characteristics of estimated interest rates: smoothness, short rates fit, volatility and level of symmetric interest rates in segments
4. optimisation algorithm - related: exit flags, number of iterations, number of calls to objective function, execution time (in seconds)

Results

General observations:
- conventional weighting (all equal weights in the yield space) results in the roughest and not well fitted yield curves
- systems based on combined outstanding amounts and turnover give less rough curves than conventional with only slightly worse error
- systems based on solely on outstanding amounts produce even smoother curves but with a trade-off with MAE
- systems based on solely on turnover give the smoothest curves and yet again worse MAE

Conclusions

We have collected stylised facts on Polish government bonds market, which were consequently used in filtering and weight system design. These facts revealed importance of liquidity in bonds lifespan, heterogeneity of liquidity and distributions and duration to estimate for the US and compare the results between the less liquid (Poland) markets and liquid market (US).

We have found heuristically a class of weighting system for Polish government bonds yield curve which significantly improves fit and smoothness as compared to the traditional approach of all equal weight.

This class has three core characteristics:
- at least the same weight for the short end of the curve as a sum for all other tenors of bonds
- exclusion of eligible-for-switch bonds from the estimation
- bonds’ weights based on at least outstanding amounts (in the best systems we had either \( W_{oa} \) or \( W_{oa} + W_{md} \))

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