

Central Bank Digital Currency and Banking: Macroeconomic Benefits of a Cash-like Design*

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Abstract

Should a central bank digital currency (CBDC) be issued? Should its design be cash- or deposit-like? To answer these questions, we theoretically and quantitatively assess the effects of a CBDC on consumption, banking and welfare. Our model introduces new general equilibrium linkages across different types of retail transactions as well as a novel feedback effect from transactions to deposits creation. The general equilibrium effects of a CBDC are decomposed into three channels: payment efficiency, price effects and bank funding costs. We show that a cash-like CBDC is more effective than a deposit-like CBDC in promoting consumption and welfare. Interestingly, a cash-like CBDC can also crowd in banking, even in the absence of bank market power. In a calibrated model, at the maximum, a cash-like CBDC can increase bank intermediation by 5.8% and capture up to 25% of the payment market. In contrast, a deposit-like CBDC can crowd out banking by up to 2.6% thereby grabbing a market share of about 16.7%.

Keywords: Central bank digital currency, banking, general equilibrium, payments.

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1 Introduction

Several central banks are considering issuing a central bank digital currency (CBDC) for retail payments.¹ It is commonly believed that the introduction of a CBDC can have profound implications for the efficiency and stability of the macroeconomy. In particular, one frequently raised policy concern is that a CBDC, by competing with banks for deposit funding, could crowd out banking and reduce output—a point discussed in recent reports by the International Monetary Fund (Mancini-Griffoli et al., 2018) and the report by the Committee on Payments and Market Infrastructures of the Bank for International Settlements (2018). To inform this policy discussion, this paper provides a theoretical and quantitative assessment. We show that the concern that a CBDC could crowd out banking is not warranted when an appropriate design is adopted.

We develop a model of payments and banking to evaluate the general equilibrium effects of introducing a CBDC on bank intermediation and retail transactions. In our model, banks finance investment by issuing deposits to households. Goods produced from the investment are sold to households in a frictional retail market. In the absence of perfect credit, households bring a portfolio of cash and deposits to finance these trades. As in the real world, the model features different types of transactions. In type-1 transactions, cash is the only viable payment option (e.g., offline transactions). In type-2 transactions, cash is not viable and only deposits and credit are used (e.g., online transactions). In type-3 transactions, all payment instruments can be used (e.g., in most physical retail stores). As deposits are used in transactions, the implied liquidity premium lowers banks' funding costs. An important feature of our model is that bank deposits are used to finance investments that then produce goods traded in different types of retail transactions.² This new feature introduces two general equilibrium linkages that are overlooked in the existing literature: (i) an inter-market price linkage across different types of retail transactions, and (ii) a feedback effect

¹The Bank for International Settlements surveyed 65 central banks in 2020, covering 72% of the world population and 91% of world output. Of these central banks, 86% are engaged in work regarding a CBDC, 60% have started experiments or proofs-of-concept for a CBDC, and 14% have moved forward to development and pilot arrangements for a CBDC (see Boar and Wehrli 2021).

²Many firms in the retail and manufacturing industries rely on loans from deposit-taking financial institutions. According to the Survey on Financing and Growth of Small and Medium Enterprises in Canada, among those enterprises that requested debt financing in 2017, 93.4% in the manufacturing industry and 93% in the retail trade industry obtained their loans from domestic chartered banks or credit unions; i.e., deposit-taking institutions. (Source: <https://bit.ly/3l21yuv>).

from retail transactions to deposits creation. In addition, the inclusion of type-3 transactions in the model allows cash and deposits to compete directly, leading to the novel implications discussed below.³

We investigate the effects of introducing different types of CBDCs: the cash-like, deposit-like and universal CBDC. The cash-like CBDC can be used in type-1 and type-3 transactions, while a deposit-like CBDC can be used in type-2 and type-3 transactions. The universal CBDC can be used in all transactions. We then show that the general equilibrium effects of introducing a CBDC can be decomposed into the following three channels. First, an interest-bearing CBDC lowers the opportunity costs of holding payment balances and increases payment efficiency, promoting aggregate demand for consumption and investment. We call this the *payment efficiency channel*. Second, the increase in aggregate demand raises the price level, thereby reducing the quantity of trades in all types of transactions, including those where the CBDC is not used. This is the inter-market price linkage that is overlooked by the existing literature. We call this channel the *price effect*. Third, the introduction of a CBDC may induce banks to raise the interest paid on deposits, increasing banks' funding costs and lowering consumption and investment. This is the *bank funding cost channel*.

Applying this decomposition, we demonstrate analytically that an interest-bearing CBDC can *crowd in* bank intermediation even when banks do not have any market power—a novel result in this literature. Furthermore, the effects depend crucially on the design of the CBDC. In particular, the introduction of a cash-like CBDC will promote consumption, banking and welfare. To understand this, note that an interest-bearing, cash-like CBDC lowers the opportunity costs of holding payment balances. The direct effect is that households will buy more goods in transactions where a CBDC is used. An additional, indirect effect is that if banks are forced to raise the interest rate on deposits, then households will also buy more goods in transactions where deposits are used. In other words, a cash-like CBDC generates an interest-rate *spillover effect* from cash to non-cash transactions.⁴ Through the payment efficiency channel, the higher consumption demand will induce

³For simplicity, many existing models of CBDC and banking (e.g., Keister and Sanches (2019), Williamson (2020a)) only consider type-1 and type-2 trades. Given that over 75% of trades in the US accept both cash and cards, one may argue that our setup is more realistic. More importantly, this feature significantly changes the positive and normative implications of the CBDC.

⁴This highlights the importance of incorporating type-3 transactions because this spillover effect is overlooked in those existing models where type-3 transactions are not captured.

banks to create more deposits to finance production in order to clear the goods market. This is the feedback effect from transactions to deposits creation—the second linkage mentioned above—that is neglected by the literature. We show that, when cash is used in type-3 transactions, the positive effect through the payment efficiency channel can outweigh the other two channels and lead to higher consumption, intermediation and welfare.

In contrast, a deposit-like CBDC may not promote consumption and banking. First, this type of CBDC cannot be used to buy type-1 goods, so it cannot improve payment efficiency in these transactions. Second, the introduction of a deposit-like CBDC drives up the price level and lowers type-1 consumption, further worsening the allocative efficiency. Finally, the payment efficiency channel is weaker because, unlike deposits, cash is not interest bearing. Hence the introduction of a deposit-like CBDC cannot induce an endogenous reduction in the opportunity cost of holding cash. As a result, there are no interest-rate spillover effects from non-cash transactions to cash transactions. This explains why a deposit-like CBDC can crowd out banking and lower output.

In our quantitative exercise, we calibrate the model with US data and assess the effects of different types of CBDCs on different transactions that take place through different channels. As stated above, for a cash-like CBDC, the payment efficiency gains dominate the price and funding cost channels, leading to more bank intermediation. With a nominal interest rate of 5%, the CBDC demand is maximized, bank intermediation increases by 5.8%, retail consumption goes up by 3.5%, and the CBDC captures 25% of the payment market. Again, the positive effect is due to the payment efficiency channel, with a positive spillover to other transactions through banks' endogenous responses. In contrast, a deposit-like CBDC leads to weaker payment efficiency gains. At the maximum, this type of CBDC can obtain 16.7% of the payment market and reduce bank intermediation by 2.6% in equilibrium.

Overall, our study provides two design recommendations in order to realize the potential benefits of a CBDC. First, it should aim to serve the current market segments where cash is the only option (e.g., offline payments, anonymous transactions) and where cash directly competes with deposits (e.g., physical retail stores). Second, the CBDC should bear interest (or other perks) that reduce the opportunity costs of holding payment balances.

Our work is related to recent papers that study the implications of a CBDC on banking and the macroeconomy. Keister and Sanches (2019) develop a model with perfectly competitive banks that face a pledgeability constraint. They find that, while the CBDC always crowds out bank

intermediation, social welfare can still increase when the exchange efficiency significantly improves, especially when financial frictions are not severe. In models with imperfect competition among banks in the deposit market, Andolfatto (2020) and Chiu et al. (2019) show that introducing an interest-bearing CBDC does not necessarily lead to disintermediation. Using an overlapping generations model, Andolfatto (2020) shows that a CBDC could compel a central bank to increase its deposit rate, leading to an increase in bank deposits and financial inclusion. In Chiu et al. (2019), the impact of a CBDC is non-monotonic in its interest rate. It expands bank intermediation if its interest rate lies within an intermediate range and it causes disintermediation if its interest rate is set too high.⁵ In our model, banks do not have market power and are not subject to a pledgeability constraint. Instead, the model incorporates the above-mentioned general equilibrium linkages that are important for understanding the effects of different types of CBDCs on different types of transactions. In addition, in our calibrated model, we disentangle and quantify the contribution of different channels to the overall effect of the CBDC.

A growing number of papers have been studying the effects of introducing a CBDC. Some discuss the role of the CBDC as a tool for monetary policy: Barrdear and Kumhof (2021) study the macroeconomic effects of a CBDC in a rich DSGE model. Davoodalhosseini (2021) studies the co-existence of cash and a CBDC in a model where the CBDC allows for balance-contingent transfers; Brunnermeier and Niepelt (2019) and Niepelt (2020) show that, under certain conditions, a CBDC has no effects on macroeconomic outcomes. Jiang and Zhu (2021) study the interactions between a CBDC and reserves that are used as separate tools for monetary policy. Williamson (2020a) examines the role of a CBDC when monetary policy is endogenous and the central bank faces a financial constraint. Others discuss the implications of a CBDC for financial stability and bank runs (See Chiu et.al 2020; Fernandez-Villaverde et al. 2020; Schilling et al. 2020; Keister and Monnet 2020; Monnet et. al 2020; and Williamson 2020b). Our paper, instead, focuses on the general equilibrium effects on consumption, intermediation, and welfare in the steady state as a result of introducing a CBDC. Other papers study motivations for issuing a CBDC as well as various CBDC design options and principles. For example, Berentsen and Schar (2018) argue for central banks issuing electronic money to let agents save money outside of the private financial sector. For a

⁵In models closely related to Chiu et al. (2019), Lagos and Zhang (2019; 2020) show that the monetary policy can be effective because it can set the value of an outside option for agents as long as the use of money is above zero. One may conclude from their paper that if cash is no longer used, the introduction of a CBDC may keep monetary policy effective. Similarly, Rocheteau et al. (2018) show that money holdings can limit the market power of banks.

non-exhaustive list of papers, see Agur et al. (2020), Davoodalhosseini and Rivadenyra (2020), Mancini-Griffoli et al. (2018), Fung and Halaburda (2016), and Kumhof and Noone (2018).⁶

This paper is organized as follows. In Section 2, we introduce a model with cash, deposits and credit. In Section 3, we introduce a cash-like, interest-bearing CBDC. We discuss the three channels through which a CBDC affects the economy. We also show that the cash-like CBDC can improve consumption, intermediation, and welfare. In Section 4, we study a deposit-like CBDC. Section 5 discusses policy-relevant design issues. In Section 6, we calibrate the model and evaluate the quantitative implications. Section 7 concludes. The omitted proofs and some details are relegated to the appendix.

2 Model

Time is discrete and continues forever: $t = 0, 1, 2, \dots$. Each period consists of two sub-periods with two markets that open sequentially. In the first sub-period, a frictional decentralized market (DM) opens. In the second sub-period, a Walrasian centralized market (CM) opens.

In this economy, there is a measure one of infinitely lived households. In addition, in each CM, a measure one of new competitive bankers enters the economy and exits in the following CM. There are four goods: three consumption goods (c_1 , c_2 and c_3) produced by bankers and consumed by households in the DM, and numeraire good y produced by households and consumed by both bankers and households in the CM. In the model, one should interpret a banker as a banker-producer pair that is engaged in both banking and production activities. One can easily introduce producers explicitly and separate these two activities.⁷

In the CM, households work and produce the numeraire good that is subject to a constant marginal disutility of one. Households can consume the numeraire good in the CM. Alternatively, they can transfer these goods to young bankers who possess investment technology that converts k units

⁶There is also related literature on private cryptocurrencies. See, for example, Biais et al. (2019), Chiu and Koepl (2019), Cong et al. (2021), and the references herein. Other papers study currency competition with a focus on cryptocurrencies. See Fernández-Villaverde and Sanches (2019) and Zhu and Hendry (2019).

⁷We do not separate the banker and the producer because this will not enrich our analysis. In particular, the current setup is equivalent to one where bankers and producers trade in a competitive loan market (e.g., see Chiu et al. (2019)).

of the numeraire goods into $F(k)$ units of the consumption goods in the next DM. The DM is frictional because perfect credit is not available. Different payment options are used to facilitate the exchange of consumption goods. Households can use credit, subject to a credit limit, B , which is discussed below. In addition, fiat money and deposits serve as means of payment.

Fiat money is supplied by the central bank. In our model, money represents both traditional banknotes (denoted as “cash” below) and a central bank digital currency (denoted as “CBDC” below). We will start with a model with cash and then introduce the CBDC later. One difference between cash and a CBDC is that, for technological reasons, a central bank can pay interest only on CBDC balances. The money stock at the beginning of period t is denoted as M_t . The central bank maintains a constant money growth rate $\mu = M_{t+1}/M_t$ by making lump-sum transfers and paying interest to the CBDC holders in the CM.

A young banker in the CM issues deposits to finance the purchase of the numeraire good for investment. Bankers redeem deposits in the next CM where their promises are backed by consumption sales in the DM. Deposits can be used by households as a medium of exchange in the DM.

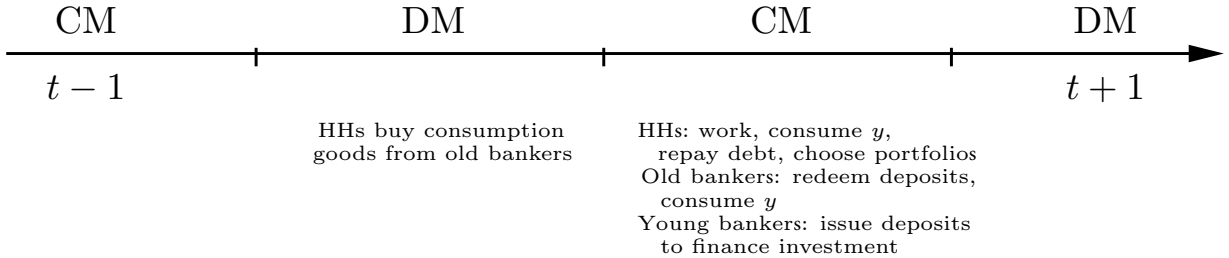


Figure 1: Time-line

The presence of trading frictions in the DM implies that households may not be able to trade. In particular, in the DM, households have an opportunity to consume with probability $\sigma > 0$. In the case where a household gets to consume, it buys the three consumption goods in three segmented markets. Consumption good c_1 , or simply good 1, can be bought only with cash (e.g., offline transactions where electronic payments are not available). Consumption good c_2 , or simply good 2, cannot be purchased with cash but can be purchased only with deposits or credit (e.g., online transactions). Consumption good c_3 , or simply good 3, can be purchased using cash, deposits or

credit (e.g., most physical retail stores). Anticipating their DM transactions, households acquire a portfolio of money and deposits in the CM. Figure 1 illustrates the time-line of the model.

2.1 Bankers

We first consider the decision problem of bankers who derive utility from the numeraire good in the CM and only when they are old. Their utility function is linear. Each of these bankers possesses an investment technology. By investing k units of the numeraire good in the CM, the banker produces $F(k)$ units of consumption goods in the next DM, where F is an increasing and strictly concave function. We also assume that $kF'(k)$ is increasing. The real price of consumption goods (i.e., in terms of the numeraire good) is p . The banker finances the investment by issuing deposits k_d that bear a gross interest rate R_d . Each banker takes (p, R_d) as given and maximizes their profit (i.e., CM consumption when old)

$$\pi = \max_{k_d} pF(k_d) - R_d k_d.$$

The first-order-condition (FOC) is then given by

$$pF'(k_d) = R_d. \tag{1}$$

2.2 Households

We now consider the households' decisions in the two markets. We use y and h to denote respectively the consumption and production of the numeraire good in the CM, and we use $\mathbf{c} = (c_1, c_2, c_3)$ to denote the consumption bundle in the DM. Households' period utility is given by

$$U(y) - h + \sum_{i=1}^3 u_i(c_i),$$

where the utility from the numeraire good is $U(y)$, and the utility from the consumption good i is $u_i(c_i)$. We assume $u_i(0) = 0$, $U'(0) = u'_i(0) = \infty$, $U'(y) > 0$, $u'_i(c_i) > 0$, $U''(y) < 0$, and $u''_i(c_i) < 0$.

In the CM, we use $W(Z, D)$ to denote the value function of a household that is carrying real money balance Z and real deposit balance D . The household chooses numeraire good consumption y , production h and continuation portfolio (\hat{Z}, \hat{D}) to solve

$$\begin{aligned} W(Z, D) &= \max_{y, h, \hat{Z}, \hat{D}} \left\{ U(y) - h + \beta V(\hat{Z}, \hat{D}) \right\} \\ \text{st. } h + Z + D + T &\geq y + \psi_z \hat{Z} + \psi_d \hat{D}, \end{aligned}$$

where V is the value function in the following DM. On the LHS of the budget constraint, a household's real income consists of production income h , portfolio market value $Z + D$, and lump-sum transfers T . The expenditure on the RHS consists of numeraire consumption y and the costs of purchasing new portfolio $\psi_z \hat{Z} + \psi_d \hat{D}$ for the next DM. In the CM, when a household deposits one unit of the numeraire good with the banker, the banker promises to repay R_d units of the numeraire good tomorrow. The price of each unit of real deposit balances is thus $\psi_d = R_d^{-1}$. Similarly, due to inflation and interest payments, money balances carry a real return rate of R_z . For example, if the inflation rate is μ_z and the nominal interest rate on the money paid by the central bank is i , then we have $R_z = (1 + i)/\mu_z$. Again, $\psi_z = R_z^{-1}$ is the price of a unit of the real money balance in the CM. Since we first consider a world without a CBDC, the interest on the balance of the money (i.e., cash) is $i = 0$ and $T = Z(\mu_z - 1)$.⁸

Using the above budget constraint, we obtain:

$$\begin{aligned} W(Z, D) = & Z + D + T + \max_y [U(y) - y] \\ & + \max_{\hat{D}, \hat{Z}} \left\{ -\frac{\mu_z}{1+i} \hat{Z} - \frac{\hat{D}}{R_d} + \beta V(\hat{Z}, \hat{D}) \right\}. \end{aligned}$$

Due to the linearity of the value function, we can rewrite W as a function of one variable:

$$W(Z + D) \equiv W(Z, D) = Z + D + W(0).$$

The FOCs are given by

$$\begin{aligned} y & : U'(y) = 1, \\ \hat{Z} & : \frac{\mu_z}{1+i} \geq \beta V_1(\hat{Z}, \hat{D}), \text{ equality if } \hat{Z} > 0, \\ \hat{D} & : \frac{1}{R_d} \geq \beta V_2(\hat{Z}, \hat{D}), \text{ equality if } \hat{D} > 0, \end{aligned}$$

where the subscript j denotes the derivative of V with respect to argument j .

In the DM, a household takes the real price of consumption, p , as given,⁹ and this makes the consumption choice subject to the following payment constraints:

$$(c_1 + f c_3)p \leq Z, \tag{2}$$

$$[c_2 + (1 - f)c_3]p \leq D + B. \tag{3}$$

⁸While the interest rate is zero, we keep the term i in the problem to facilitate the comparison with the setup that includes an interest-bearing CBDC.

⁹Since bankers can use the same technology to transform the numeraire good into any one of the three consumption goods, the prices of the consumption goods are equalized in equilibrium: $p_i = p$.

Here, (2) denotes the constraint on the total cash expenditure on good 1 (i.e., c_1p) and good 3 (i.e., fc_3p), where f is the fraction of good 3 paid in cash. We call this the *cash-payment constraint* (CC). Similarly, (3) denotes the constraint on the total non-cash expenditure spent on good 2 (i.e., c_2p) and good 3 (i.e., $(1-f)c_3p$), where B is the real credit limit. We call this the *deposit-payment constraint* (DC). The household's DM problem is given by

$$\begin{aligned} V(Z, D) &= \max_{c_1, c_2, c_3, f} \sigma \left[\sum_{i=1}^3 u_i(c_i) + W(w) \right] + (1 - \sigma)W(Z + D) \\ \text{st. } w &= Z - (c_1 + fc_3)p + D - (c_2 + (1 - f)c_3)p, \\ &\quad (\text{CC}), (\text{DC}), \end{aligned}$$

where w is the unspent wealth after consumption. The CM solution implies that the DM problem can be rewritten as

$$\begin{aligned} V(Z, D) &= \max_{c_1, c_2, c_3, f} \sigma \sum_{i=1}^3 u_i(c_i) + Z + D + W(0) \\ &\quad - \sigma(c_1 + c_2 + c_3)p \\ &\quad + \sigma\lambda_z [Z - (c_1 + fc_3)p] \\ &\quad + \sigma\lambda_d [D + B - (c_2 + (1 - f)c_3)p], \end{aligned}$$

where $\sigma\lambda_z$ and $\sigma\lambda_d$ are the Lagrangian multipliers that are associated with CC and DC. The envelope conditions are given by

$$\begin{aligned} V_1(Z, D) &= \sigma\lambda_z + 1, \\ V_2(Z, D) &= \sigma\lambda_d + 1. \end{aligned}$$

Following the literature, we call $\lambda_z \geq 0$ and $\lambda_d \geq 0$ the liquidity premiums that are associated with cash and deposits. We obtain the following equations that characterize the optimal DM choices:

$$c_1 : u'_1(c_1) = (1 + \lambda_z)p \quad (4)$$

$$c_2 : u'_2(c_2) = (1 + \lambda_d)p \quad (5)$$

$$c_3 : u'_3(c_3) = (1 + f\lambda_z + (1 - f)\lambda_d)p \quad (6)$$

$$f : \lambda_d - \lambda_z \begin{cases} \leq 0, & \text{if } f = 0, \\ = 0, & \text{if } f \in (0, 1), \\ \geq 0, & \text{if } f = 1, \end{cases} \quad (7)$$

The first two equations equalize the marginal utility of consuming a good to the marginal cost of tightening the respective payment constraint that is associated with that good. The third equation has a similar interpretation. The only difference is that consuming good 3 tightens both constraints, so the marginal cost is equal to the weighted average of the Lagrangian multipliers of the two constraints. The last equation states that the payment choice for good 3 is related to liquidity premiums λ_d and λ_z . An instrument is used exclusively if and only if its liquidity premium is smaller than the other instrument's liquidity premium. Both instruments are used when the premiums are equalized.

Combining the FOCs in the CM and DM, we obtain the optimal portfolio choice in the CM, which is characterized by the following Euler equations:

$$\frac{\mu_z}{\beta(1+i)} \geq \sigma\lambda_z + 1, \text{ equality if } Z > 0, \quad (8)$$

$$\frac{1}{\beta R_d} \geq \sigma\lambda_d + 1, \text{ equality if } D > 0. \quad (9)$$

The LHS of (8) captures the opportunity costs of holding cash (i.e., rates of inflation, discounting net of the interest return). The RHS captures the resale value of one plus the liquidity premium. Equation (9) has a similar interpretation. The liquidity premiums are positive only if the following payment constraints are binding:

$$Z \geq (c_1 + fc_3)p, \text{ “=” if } \lambda_z > 0, \quad (10)$$

$$D + B \geq (c_2 + (1-f)c_3)p, \text{ “=” if } \lambda_d > 0, \quad (11)$$

where the quantity of deposits in equilibrium is given by

$$D = R_d k_d. \quad (12)$$

Finally, the market clearing condition for the DM is given by

$$\sigma(c_1 + c_2 + c_3) = F(k_d). \quad (13)$$

Let us highlight two important distinct features of this model, relative to the existing literature that is based on the Lagos and Wright (2005) framework. First, since the investments that are used to produce consumption goods are financed by deposits issued by banks, there is a feedback linkage from consumption demand to deposits creation. In particular, a higher consumption in the DM can lead to higher deposits because, other things being equal, k_d is increasing in p in (1). This linkage is

missed in models of CBDC and banking that are based on Lagos and Wright (2005), because they typically assume that investment goods are not traded in the DM. Second, since all consumption goods are produced by a common concave production function, our setup incorporates an inter-market linkage that endogenously links cash and non-cash transactions in different markets (i.e., (4)-(6) are linked by p). This linkage is missing in existing models where DM trades are bilateral and/or are subject to linear production costs.

2.3 Efficiency and Equilibrium without a CBDC

Given monetary policy (i, μ_z) , a steady-state monetary equilibrium consists of $\mathbf{c}, f, k_d, p, Z, D, R_d, \lambda_z, \lambda_d$ that satisfy (1)-(13). Before analyzing the equilibrium outcome, it is useful to characterize the first-best allocation, which is given by

$$\max_{c_1, c_2, c_3, k_d} \sigma \beta [u_1(c_1) + u_2(c_2) + u_3(c_3)] - k_d$$

subject to

$$\sigma (c_1 + c_2 + c_3) = F(k_d).$$

That is, we choose consumption and investment to maximize the sum of the period utilities of households and bankers, subject to the production technology. We denote the Lagrangian multiplier associated with the constraint as βp^* . The FOCs are given by

$$\begin{aligned} k_d &: \beta p^* F'(k_d) = 1, \\ c_i &: u'_1(c_1) = u'_2(c_2) = u'_3(c_3) = p^*. \end{aligned}$$

The solution, k_d^* and c_i^* , gives us the first-best allocation. In a frictionless economy (without payment constraints), p^* is the price that supports (k_d^*, c_i^*) as an equilibrium allocation. In the presence of the payment constraint, the following lemma states the conditions under which the equilibrium allocation is efficient.

Lemma 1. *The equilibrium allocation is efficient when $R_z = \beta^{-1}$ and $B \geq \max\{0, B^*\}$, where $B^* \equiv c_2^* p^* - \frac{k_d^*}{\beta}$.*

Intuitively, efficiency is achieved when the central bank follows the Friedman rule and when the credit limit is not binding. The former condition is required to ensure that the CC is not binding. The second condition is required to ensure that the DC is not binding. Notice that for efficient

consumption, credit is needed only to make the consumption of good 2 efficient because good 3 can be consumed using cash. Moreover, when $B^* < 0$, credit is not needed in this economy because the real value of deposits is sufficiently high so that all type-2 trades can be financed.

When the monetary policy (i, μ_z) is away from the Friedman rule; i.e., $R_z = (1 + i)/\mu_z < \beta^{-1}$, the CC is binding and hence the liquidity premium $\lambda_z = (1 - \beta R_z)/\sigma \beta R_z$ is positive. The following proposition then characterizes the effects of credit limit B in this situation.

Proposition 2. *Away from the Friedman rule, there exists a credit threshold, $\bar{B}(\mu_z)$, such that*

$$(i) \text{ if } B \geq \bar{B}(\mu_z), \text{ then } \lambda_d = 0, f = 0, R_d = \frac{1}{\beta}, k_d \leq k^*,$$

$$(ii) \text{ if } B < \bar{B}(\mu_z), \text{ then } \lambda_d > 0 \text{ and } \begin{cases} \text{if } \lambda_d \leq \lambda_z, & f = 0, \\ \text{if } \lambda_d = \lambda_z, & f \in (0, 1), \\ \text{if } \lambda_d \geq \lambda_z, & f = 1. \end{cases}$$

This proposition states that when credit is abundant, the DC is slack. Cash is not used to finance good-3 transactions because the CC is still binding. Under-consumption of good 1 implies that aggregate investment and output are below their efficient levels. If credit is scarce, then both the CC and DC are binding. Depending on the relative tightness of the constraints, cash may or may not be used in type-3 transactions.

What are the effects of changing credit limit B ? When cash and deposits are both used in type-3 transactions, relaxing the credit limit has no effects on the real economy except for changing the composition of the means of payment. This is because the return on deposits is pinned down by monetary policy. When cash is not used in type-3 transactions, relaxing the credit limit reduces the consumption of good 1 as well as aggregate consumption. This result is similar to that of Chiu et al. (2018), who show that, due to price externality, improving credit arrangements can have a negative impact on money users' consumption. Details of these and other comparative static exercises with respect to B are reported in the appendix.

3 Cash-like CBDC

Different ways to design a CBDC have been proposed; e.g., cash-like, deposit-like, and a universal CBDC. A cash-like CBDC can be used in type-1 and type-3 transactions (e.g., a stored-value card that cannot support online transactions). A deposit-like CBDC can be used in type-2 and type-3

transactions (e.g., a payment account that is not accessible in an offline setting). A universal CBDC can be used in all transactions (See Table 1). In this section, we study the effects of a cash-like CBDC.

	Cash	Deposits	Credit	Cash-like CBDC	Deposit-like CBDC	Universal CBDC
Type 1	✓			✓		✓
Type 2		✓	✓		✓	✓
Type 3	✓	✓	✓	✓	✓	✓

Table 1: Acceptability of Payment Instruments in Different Markets

This section focuses on a cash-like CBDC that has growth rate μ_m and bears interest rate $i_m > 0$, implying a real rate of $R_m = (1 + i_m)/\mu_m$. Since cash and cash-like CBDCs are perfect substitutes in payments, a household compares R_m and R_z and holds the CBDC that offers the higher real rate. Equation (8) implies that the CBDC will crowd out cash whenever $R_m > R_z$.¹⁰ Otherwise, a CBDC is not used and the equilibrium is not changed.

In the following analysis, we focus on the case where $R_m > R_z$, so that cash is replaced by the cash-like CBDC. All of the equilibrium conditions remain unaffected, with the exception that the subscript “ z ” is replaced by “ m ”. Below, we study the comparative statics with respect to the real rate, R_m , on the CBDC. The effects will depend on the value of f . Here, we focus on the case of the interior $f \in (0, 1)$ and we consider the boundary cases in Appendix B. The interior case is the relevant one in reality as the data suggests that the share of the type-3 consumption good is more than 70% of all consumption goods. More details are given in the numerical section. In addition, we focus on the interesting case where credit is limited so that $\lambda_d > 0$. Otherwise, deposits do not serve as a means of payments.

When a CBDC replaces cash and has an interior share $f \in (0, 1)$, households are indifferent between using the CBDC or deposits in good-3 transactions. Equation (7) implies that their liquidity premiums are equalized; i.e., $\lambda_d = \lambda_m$. The real interest rate on the CBDC, R_m , set by the central

¹⁰This can happen either with a sufficiently high i_m or a sufficiently low CBDC growth rate, μ_m . In practice, a central bank will likely set the same inflation rates for the CBDC and cash. Hence, the nominal CBDC rate will become the key factor—a case we consider in the quantitative analysis.

bank will determine the real interest rate on deposits:

$$R_d = \frac{1}{\beta(\sigma\lambda_d + 1)} = \frac{1}{\beta(\sigma\lambda_m + 1)} = R_m.$$

We can then obtain the following equilibrium conditions:

$$k_d = F'^{-1} \left(\frac{1}{\beta p(1 + \sigma\lambda_m)} \right), \quad (14)$$

$$F(k_d) = \sigma u'^{-1}((1 + \lambda_m)p), \quad (15)$$

which then imply that

$$F \left(F'^{-1} \left(\frac{1}{\beta p(1 + \sigma\lambda_m)} \right) \right) = \sigma u'^{-1}((1 + \lambda_m)p), \quad (16)$$

where $u'^{-1}(\cdot) \equiv u_1'^{-1}(\cdot) + u_2'^{-1}(\cdot) + u_3'^{-1}(\cdot)$. Equation (14) determines a banker's level of investment, given the liquidity premium and the price level. Equation (15) is the market clearing condition in the DM and determines the total output in equilibrium. Equation (16) combines the above two equations to determine the equilibrium price, p . We can then solve for consumption, investment, and the bank interest rate.

Finally, to ensure that f is interior, we need B to not be too low (so that $f < 1$) or too high (so that $f > 0$):¹¹

$$[c_2 - F'(k_d)k_d] p \leq B \leq [c_2 + c_3 - F'(k_d)k_d] p.$$

A higher real rate, R_m , on the CBDC affects the equilibrium allocation through three channels, which we will explain one by one. These channels are shown in Figure 2, which uses traditional demand and supply curves plotted in the (c, p) space.

The first channel is related to *payment efficiency*. As the real rate on the CBDC rises, payments in the DM become more efficient as the opportunity costs of holding payment balances decline. This is mathematically evident from (4). While the CBDC is only used for good-1 and good-3 purchases, there is also an interest rate *spillover effect* on good-2 transactions. As the real interest rate paid

¹¹The LHS inequality requires B to be high enough. Otherwise, the sum of the deposits and credit is insufficient to finance even type-2 transactions. In that case, the interior f cannot be a solution and the corner case with $f = 1$ should be considered. If B is greater than the threshold on the RHS, then credit is so abundant that the DC is not binding. The corner case with $f = 0$ should then be considered. Note that when $\lambda_m = 0$, the RHS inequality is redundant because, in that case, even when B is large, agents are indifferent between cash and deposits, so the interior f can be a solution.

on the CBDC rises, bankers are forced to raise the interest rate on deposits in order to retain households. This implies that payments in all DM transactions become more efficient. As shown in Figure 2, an increase in the CBDC rate shifts up the demand curves in all markets (market 3 is not shown in the diagram). For a fixed price level, consumption in both markets increases from c_i to c'_i .

The second channel is the *price channel*. As payments become more efficient, aggregate demand for consumption rises, pushing up the price level and the marginal cost of production in the DM. This leads to an endogenous reduction in consumption. Because of the price linkage, there are responses in the quantities of all types of purchases. In Figure 2, a higher marginal cost of production lowers consumption from c'_i to c''_i . Note that the magnitude of this channel depends on the shape of the production cost function. This channel vanishes when the cost function is linear, as is assumed in many previous models.

The third channel is related to *bank funding costs*. In response to an increase in the real rate on the CBDC, bankers need to raise their deposit rates to retain clients, and this will lead to higher funding costs for investment. In Figure 2, a higher funding cost shifts the aggregate supply curve to the left, raising the price level and decreasing consumption from c''_i to c'''_i . This mechanism is mathematically evident from (14) and (15), where an increase in R_d decreases k_d and consequently raises the price level.

The above discussion implies that, while a rise in R_m must drive up the price level, the effects on the quantities of goods consumed are ambiguous, depending on the relative strength of the supply- and demand-side effects. To determine the sign of the equilibrium effects, we examine the demand and supply conditions separately. Equation (15) gives us the demand side of consumption,

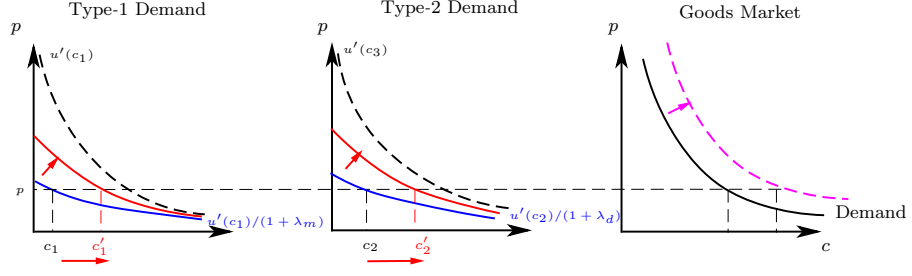
$$c = F(k_d) = \sigma u'^{-1}((1 + \lambda_d)p), \quad (17)$$

which is determined by the marginal valuation of the liquidity balances in a trade, captured by $\Delta = (1 + \lambda_d)p$. Increasing R_m will have two opposite effects on the demand side: a positive effect on c by lowering λ_d as well as a negative effect by raising p . If the decline in λ_d dominates, then Δ decreases and the demand for goods increases. The idea is that households increase their consumption demand when the liquidity constraint is relaxed.

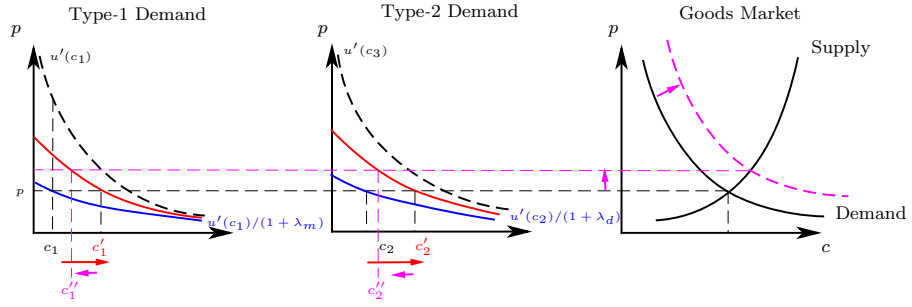
Similarly, (14) describes the supply side of consumption, which depends on Δ and p :

$$F'(k_d) = \frac{R_d}{p} = \frac{1}{\beta(\sigma\Delta + (1 - \sigma)p)}. \quad (18)$$

Payment efficiency



Price channel



Bank funding costs

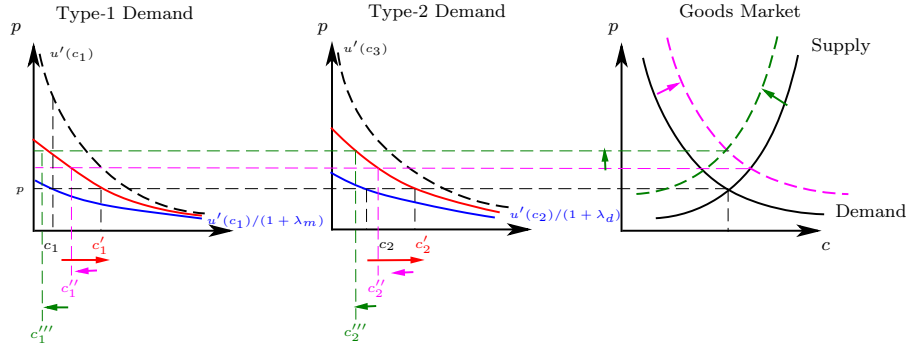


Figure 2: Effects of a Cash-like CBDC ($f \in (0, 1)$)

The denominator on the RHS captures the household's returns from holding deposits: with probability σ , the balance is spent with its marginal valuation given by Δ ; with probability $1 - \sigma$, the balance is held to maturity with its real return rate increasing in price p . A decline in Δ or p will incentivize households to hold smaller deposits, increasing the funding cost of investment and leading to a lower supply of goods.

As we can see, a decline in Δ generates two opposite effects on k_d through the demand and supply sides. In the special case where $\sigma = 1$, these two effects cancel each other out because the term p

Cash-like CBDC ($f \in (0, 1)$)	c_1	c_2	c_3	c
- Payment efficiency	+	+	+	+
- Price effects	-	-	-	-
- Bank funding effects	-	-	-	-
Total effects	+	+	+	+

Table 2: Disentangling the Effects of a Cash-like CBDC

disappears from (18) as the balances are always spent. In this case, changing R_m has no effect on k_d .¹²

In the general case where $\sigma < 1$, the term p re-emerges in (18). This implies an additional channel on the supply side through which a price increase can induce households to increase their deposit balances, mitigating the effect of a decline in Δ . This is why when this additional channel operates, the supply side will be weakened and dominated by the demand side, resulting in higher aggregate consumption and higher deposits in equilibrium.¹³ Hence, we have the following proposition.

Proposition 3. *Suppose $f \in (0, 1)$ and $\sigma < 1$. A higher real rate, R_m , paid on the cash-like CBDC leads to higher p , c_i , c , k_d and welfare.*

This result implies that, as long as households are indifferent between a CBDC and deposits in type-3 transactions, a higher R_m will lead to higher consumption and intermediation. Notice that a key channel in the above argument is the effect of p on deposits creation through the supply side—the missing linkage in the previous literature. Table 2 summarizes the equilibrium effects.

Obviously, the case with $R_m = R_z$ is equivalent to a world without a CBDC. Hence, if both cash and deposits are initially used in type-3 transactions ($f \in (0, 1)$), then the effects of introducing a cash-like CBDC with $R_m > R_z$ are captured by Proposition 3, as long as $f \in (0, 1)$ remains valid. Hence we have the following corollary, which applies to cases where $R_m - R_z$ is not too big to ensure $f \in (0, 1)$.

¹²Specifically, when $\sigma = 1$, the two conditions imply that $F(F'^{-1}(\frac{1}{\beta\Delta})) = u'^{-1}(\Delta)$, which uniquely determines Δ . As a result, consumption and investment are invariant to a change in R_m .

¹³To show this, we can first combine (17) and (18) to obtain $\frac{1}{\sigma}F\left(F'^{-1}\left(\frac{1}{\beta(\sigma\Delta+(1-\sigma)p)}\right)\right) = u'^{-1}(\Delta)$, which is a downward-sloping curve in the (Δ, p) space. Second, we can plot $p = \Delta/(1 + \lambda_d)$ as an upward-sloping curve in this space. As λ_d decreases, the second curve becomes steeper, implying a higher p and a lower Δ in equilibrium. Equation (17) then implies that k_d must go up. See the details in the appendix.

Corollary 4. *If both cash and deposits are initially used in type-3 transactions, then introducing a cash-like CBDC with $R_m \in (R_z, \bar{R}_m)$ increases bank intermediation and welfare for some $\bar{R}_m > R_z$.*

In other words, as long as cash and deposits are directly competing as payment options in some trades, introducing a cash-like CBDC with an appropriate rate can increase welfare and *crowd in* banking. Again, the mechanism is that the CBDC improves payment efficiency in all transactions in equilibrium. Welfare goes up as consumption gets closer to the efficient level for all goods. Despite the higher interest rate, the rise in aggregate consumption induces a higher derived demand for bank deposits, leading to more bank intermediation. Notice that this result is quite general as it is independent of the forms of the utility and production functions. This also provides a novel economic insight to the literature that ignores the general equilibrium feedback effect from transactions to deposits creation.

4 Deposit-like CBDC

In this section, we focus on the deposit-like CBDC. We first derive a new set of equilibrium conditions and then conduct the comparative statics exercise with respect to R_m . In general, cash, the CBDC and deposits can coexist. The following table describes their equilibrium market shares:

	Cash Z	CBDC M	Deposits D
Good 1 (c_1)	1	0	0
Good 2 (c_2)	0	v	$1 - v$
Good 3 (c_3)	f	n	s

Here, v and n denote the fractions of type-2 and type-3 trades, respectively, that are financed by a CBDC; s denotes the fraction of type-3 trades that are financed by deposits. The real value of the

CBDC chosen by households is denoted by M . The households' DM problem becomes

$$\begin{aligned}
V(Z, D, M) &= \max_{\mathbf{c}, f, v, n, s} \sigma \left[\sum_{i=1}^3 u_i(c_i) + W(w) \right] + (1 - \sigma)W(Z + D + M) \\
\text{st. } w &= Z + M + D - (c_1 + c_2 + c_3), \\
Z &\geq (c_1 + fc_3)p, \\
D + B &\geq ((1 - v)c_2 + sc_3)p, \\
M &\geq (vc_2 + nc_3)p, \\
f + n + s &= 1.
\end{aligned}$$

Here, the objective function and the constraints are slightly modified for the introduction of the CBDC. In particular, the fourth constraint denotes the new *CBDC payment constraint* (MC).

Again we focus on the interesting case where $R_m > R_z$ (otherwise, the CBDC is not used and the equilibrium is not affected). In this case, cash is dominated by CBDC in terms of the interest rate, hence, cash is not used to purchase good 3. In addition, bankers need to match the deposit rate to the CBDC rate in equilibrium. The equilibrium effects of increasing R_m —either by raising i_m or lowering μ_m —are given in the following proposition.

Proposition 5. *A higher real rate, R_m , paid on the deposit-like CBDC leads to*

- (i) *a higher p, c_2 and c_3 , a lower c_1 , and*
- (ii) *a lower c and k_d for σ that are not too low.*

We can again understand the equilibrium impacts that were obtained using the three channels discussed earlier: Regarding the payment efficiency channel, as R_m rises, the payments for goods 2 and 3 become more efficient, but the payments for good 1 are not affected as cash is not used in good-3 transactions. Regarding the price channel, the increase in demand for goods 2 and 3 pushes up the aggregate price level, which in turn reduces the consumption for all goods. Finally, regarding the bank funding cost channel, since banks respond to changes in the interest rate on deposit-like CBDCs, bank funding costs go up, further reducing consumption.

The effects of an increase in the interest rate on deposit-like CBDCs on all three types of consumption goods are summarized in Table 3 and illustrated in Figure 4.¹⁴ As stated in Proposition 5(ii), the total effect on aggregate consumption is negative if the payments frictions are small (i.e., σ is

¹⁴Note that the effects of the deposit-like CBDC are qualitatively identical whether $f = 0$ or $f > 0$.

Deposit-like CBDC	c_1	c_2	c_3	c
- Payment efficiency	0	+	+	+
- Price effects	—	—	—	—
- Bank funding effects	—	—	—	—
Total effects	—	+	+	+/-

Table 3: Disentangling the Effects of a Deposit-like CBDC

large). Note that, since different types of consumption go in opposite directions, the overall effect on welfare cannot be signed without specifying the exact functional form of the utility functions.

Unlike in the case of a cash-like CBDC, increasing R_m will reduce type-1 consumption, implying an ambiguous effect on aggregate consumption. The next section studies the differences between these two designs.

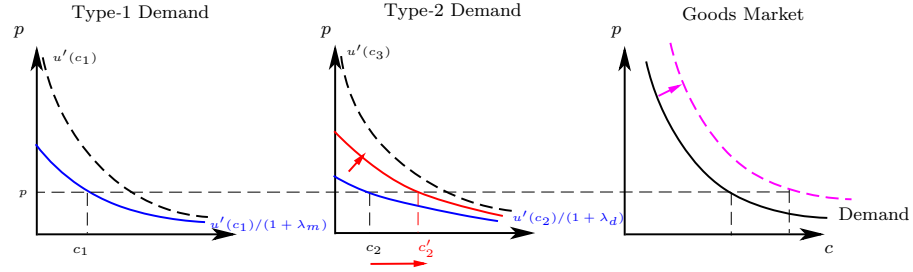
5 Guidance for Policy Makers

This section discusses policy-related issues. We first compare the macroeconomic effects of cash- and deposit-like CBDCs. Then we explore the effects of a universal CBDC that can be used in all transactions. Finally, we summarize some policy lessons learned so far.

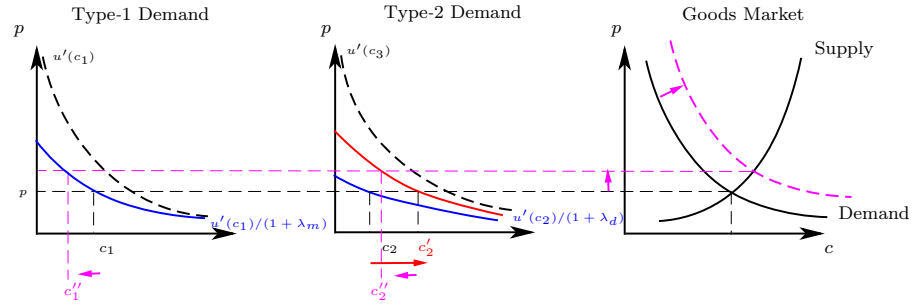
5.1 Comparing Cash- and Deposit-like CBDCs

Our analysis suggests that the equilibrium effects of a CBDC depend crucially on its design. In particular, the introduction of a cash-like CBDC can promote consumption and crowd in banking. To understand this, note that an interest-bearing, cash-like CBDC lowers the opportunity costs of holding payment balances. The direct effect is that households will buy more goods in transactions where a CBDC is used. An additional, indirect effect is that if banks are forced to raise the interest rate on deposits, then households will also buy more goods in transactions where deposits are used. In other words, a cash-like CBDC generates a positive *spillover effect* from cash to non-cash transactions. Through the payment efficiency channel, the higher consumption demand will induce banks to create more deposits to finance production in order to clear the goods market. This is the positive feedback effect from transactions to deposits creation that is overlooked in the literature.

Payment efficiency



Price channel



Bank funding costs

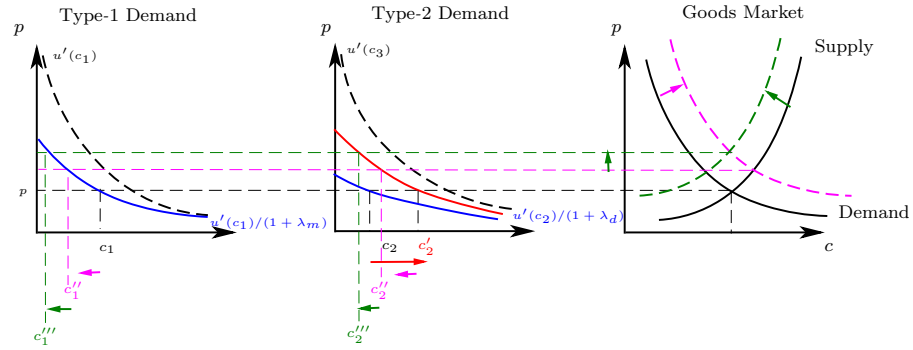


Figure 3: Effects of a Deposit-like CBDC

We show that when cash is used in type-3 transactions, the positive effect through the payment efficiency channel can outweigh the other two channels and lead to higher intermediation.

In contrast, a deposit-like CBDC may not promote consumption and banking. First, a deposit-like CBDC cannot be used to purchase type-1 goods, so it cannot improve payment efficiency in these transactions. Second, the introduction of a CBDC drives up the price level and lowers type-1 consumption, further worsening the payment efficiency in type-1 transactions. Finally, the payment efficiency channel is weaker because, unlike deposits, cash is not interest bearing. Hence the introduction of a deposit-like CBDC cannot induce an endogenous reduction in the opportunity costs of holding cash. As a result, there are no positive spillover effects from non-cash transactions to cash transactions. This explains why a deposit-like CBDC can crowd out banking and lower output.

We now provide a formal comparison between the two designs. We start with an economy with an interior f when there is no CBDC. We then compare two policies: one with a cash-like CBDC that bears interest rate $R_m > R_z$, and another with a deposit-like CBDC that pays the same interest rate. The equilibrium effects of these two policies are given by the following proposition.

Proposition 6. *If f is interior under both policies, then*

- (i) a cash-like CBDC leads to higher c_1 , c , k_d , π ,*
- (ii) a deposit-like CBDC leads to higher c_2 , c_3 , and*
- (iii) overall, the cash-like CBDC leads to higher welfare.*

The proposition suggests that for a given interest rate, introducing a cash-like CBDC is more effective in raising aggregate consumption and investment. It also implies that a cash-like CBDC is a better choice if the policy maker aims to minimize the negative impacts on banks in terms of both the volume of intermediation and bank profits. The intuition is that in this economy, the main source of inefficiency is the high opportunity cost of carrying cash, which leads to a low interest rate being paid by deposits in equilibrium. Hence, offering a cash-like CBDC helps directly tackle this problem. Through the channels discussed above, a cash-like CBDC induces higher demand and output. As banks endogenously raise the interest rate on deposits, consumption demand rises not only in type-1 transactions but also in the other two types of transactions. As a result, the price level increases more under the cash-like CBDC than under the deposit-like one. This implies that the consumption of type-2 and type-3 goods is lower in the cash-like case compared with the deposit-like CBDC that carries the same interest rate. Since aggregate output is higher, type-1

consumption must be higher under a cash-like CBDC. Under a deposit-like CBDC, banks will also raise the deposit rates, but the opportunity cost of carrying cash is not affected.

5.2 Universal CBDC

Another potential policy option is to design a universal CBDC that can be used in all types of transactions. A natural expectation is that this would be a more powerful design than a cash- or deposit-like CBDC. We will show that this design is not essential in the sense that it does not support more-desirable allocations relative to a cash-like CBDC.

Note that as long as $R_m > R_z$, the universal CBDC replaces cash in type-1 transactions while competing with deposits in type-2 and type-3 transactions. This is similar to introducing a cash-like CBDC to an economy without type-2 transactions: the cash-like CBDC replaces cash in type-1 transactions while competing with deposits in type-3 transactions. Therefore, as long as $f > 0$, introducing a universal CBDC is equivalent to introducing a cash-like CBDC to an economy where there are no type-2 goods (i.e., $\bar{u}_2(\cdot) \equiv 0$), and the utility of type-3 goods is given by $\bar{u}_3(\cdot) \equiv u_2(\cdot) + u_3(\cdot)$. Hence, we have the following result.

Proposition 7. *As long as both the CBDC and the deposits are used in type-3 transactions, then the effect of a universal CBDC is quantitatively the same as that of a cash-like CBDC.*

It is true that a universal CBDC can support type-2 transactions while a cash-like CBDC cannot. With an interior f , however, this is not really an advantage because by exerting competitive pressure on deposits in type-3 transactions, a cash-like CBDC can also indirectly lower the opportunity cost of trading in type-2 transactions.

5.3 Policy Lessons

To conclude this section, we summarize the policy lessons regarding the CBDC design. As long as a CBDC and deposits are both used in type-3 transactions, we have the following:

- A cash-like CBDC can promote consumption, investment and welfare.
- A cash-like CBDC can also crowd in bank intermediation.

- A cash-like CBDC is more effective than a deposit-like CBDC, and it also performs as well as a universal CBDC.
- Being interest bearing is an essential feature for generating the above benefits.

Overall, our findings suggest two key policy recommendations. To realize the potential benefits of issuing a CBDC, this type of currency needs to be designed so that we have the following:

- The CBDC can serve the current market segments where cash is the only option (e.g., offline payments, anonymous transactions) and where cash competes directly with deposits (e.g., physical retail stores);
- The CBDC can bear interest (or other perks) to reduce the opportunity costs of holding payment balances.

In particular, our analysis demonstrates that if the above recommendations are followed, then policy makers no longer need to worry about the potential disintermediation concerns of issuing a CBDC.¹⁵

6 Quantitative Analysis

We have analytically investigated the effects of different types of CBDCs and identified three channels through which a CBDC can affect the economy. In this section, we use a calibrated model to evaluate the effect of introducing various types of CBDCs and quantify the contributions of different channels to the aggregate effect.

6.1 Calibration

We first assume that the utility functions in the CM and DM are given by

$$\begin{aligned} U(y) &= A \log y, \\ u_i(c_i) &= \frac{a_i}{1 - \eta_i} c_i^{1 - \eta_i}, \end{aligned}$$

¹⁵Our study does not consider banks' market power. Chiu et. al (2019) show that a CBDC can also promote banking when the deposit market is non-competitive.

where we normalize a_3 to 1. To reduce the number of parameters, we assume that $\eta_2 = \eta_3$ for the two goods that are accepting deposit payments. The production function takes the form

$$F(k) = \frac{k^{1-\gamma}}{1-\gamma}.$$

To calibrate the model, we first follow the literature to set $\beta = 0.97$ and $\mu_z = 2\%$ to reflect the long-term inflation rate in the US. We also set $\sigma = 0.6$.¹⁶ Next, we construct some calibration targets by using payments and monetary statistics from three data sets: the Survey of the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; the new M1 series obtained from Lucas and Nicolini (2015); and a FRED dataset. We now discuss our calibration strategy. First, as in Chiu et al. (2019), we base this on the DCPC and FRED to compute the targets for aggregate payment behaviour according to Table 4.¹⁷ Second, we derive an empirical relationship between the money stock and nominal interest rates (i.e., money demand function). This is constructed by using the new M1 series from Lucas and Nicolini (2015) for the period 1982-2008.¹⁸

Cash share of transactions	$\frac{Z}{Z+D+B}$	21%
Debit share of transactions	$\frac{D}{Z+D+B}$	34%
Type-1 share of transactions	$\frac{c_1}{c}$	14%
Type-2 share of transactions	$\frac{c_2}{c}$	10%

Table 4: Calibration Targets

¹⁶Lagos and Wright (2005) set $\sigma = 0.5$ for most of their quantitative analyses because σ and θ (bargaining power) cannot be “precisely” identified in their model; similarly, σ and γ together cannot be precisely identified in our analysis. (See Appendix B.) The fit and welfare implications do not change significantly as long as σ is not very close to 1.

¹⁷Cash and debit shares are obtained directly from the DCPC 2017. From FRED, we obtain the share of e-commerce retail sales as a percentage of total sales, which is 8.2% at the end of 2016. According to the DCPC 2016, debit and credit cards are not accepted in 15% of transactions and cash is not accepted in 2% of transactions. Then we calculate type-1 transactions as all non-online transactions that do not accept debit and credit cards. This implies that type-1 trades account for $(1 - 8.2\%)15\% = 13.77\%$. Similarly, we calculate type-2 transactions as all online transactions plus those non-online transactions that do not accept cash. This implies that type-2 trades account for $8.2\% + (1 - 8.2\%)2\% = 10.04\%$. Type-3 transactions account for the remaining share. For the DCPC, see Greene and Stavins (2017, 2018). For the FRED data, see the entry [34] in the reference list.

¹⁸We exclude the post-crises period because the demand for M1 increased significantly after the crises, perhaps due to agents’ store-of-value motives or foreign demand, which are not related to the transactional demand that we study in this paper.

Parameters	Notation	Value	Notes
Calibrated externally			
Discount factor	β	0.97	Standard in literature
Coefficient of type 3	a_3	1	Normalization
Curvature of utility for type 3	η_3	η_2	Restriction
Money growth rate	μ	1.02	2% inflation
Prob. of consumption shock	σ	0.6	Fixed
Calibrated internally			
Coefficient of type 1	a_1	0.720	Payment data
Coefficient of type 2	a_2	0.156	Payment data
Coeff. of CM consumption	A	2.023	Money demand curve
Credit	B	0.517	Payment data
Curvature of utility for type 1	η_1	0.050	Money demand curve
Curvature of utility for type 2	η_2	0.798	Money demand curve
Curvature of production	γ	0.397	Payment data

Table 5: Calibration Results

Finally, we set parameter values for the utility functions (a_1, a_2, A, η_i) , the production function (γ) and the credit limit (B) to match the payment and monetary targets. We follow a two-step procedure: (i) Given (a_1, a_2, B, γ) , we parameterize A, η_1 , and η_2 to fit the empirical money demand; and (ii) the values of (a_1, a_2, B, γ) are then chosen to minimize the distance between the payment targets and the model-implied values at 2% inflation. Table 5 summarizes the parameter values.¹⁹ Figure 4 plots the money demand curve predicted by the model against the data for the period 1982 to 2008.

6.2 Effects of a Cash-like CBDC

We now use the calibrated model to quantify the effects of introducing a cash-like CBDC. Since the denomination of cash and the CBDC will likely be identical, we assume that the CBDC has

¹⁹If we use the money demand data for the period 1987 to 2009, then the effects of CBDCs are quantitatively similar to the benchmark estimation.

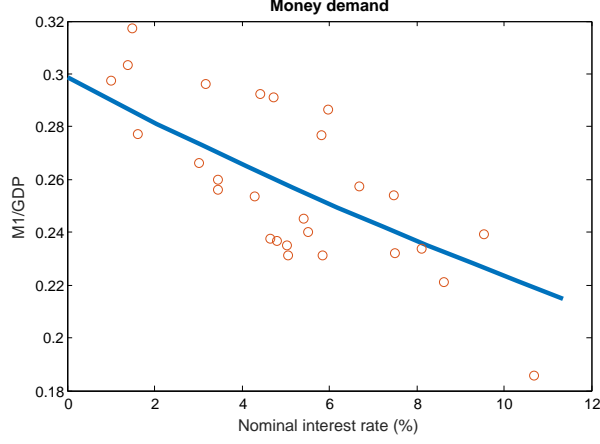


Figure 4: Money Demand Curve; Model vs. Data

the same growth rate of 2% but carries an interest rate of i_{CL} . Section 3 suggests that the equilibrium effects of introducing a CBDC depend on whether cash is used in type-3 transactions. Our calibration exercise indicates that f is about 10% in the status quo with 2% inflation. Figure 5 reports the responses of consumption and investment as i_{CL} increases from 0% to 5%. The values along the vertical axis are normalized to 1 when $i_{CL} = 0$ (i.e., the status quo). In each plot, the effects are further decomposed into the three channels discussed earlier, with the solid line combining all of them. The illustrated effects confirm our analytical result that paying a higher i_{CL} improves payment efficiency for type-1 transactions, with spillovers to type 2 and type 3 through the endogenous responses of banks. The increases in consumption and investment are mitigated by the negative general equilibrium price effect and higher bank funding costs. Consistent with findings in Proposition 3, the overall effects on consumption and investment are positive. The model predicts that a cash-like CBDC crowds in banking by 5.8% at the maximum. Table 6 reports the decomposition when $i_{CL} = 5\%$ (i.e., at the Friedman rule). Since this is the maximum feasible rate, we conclude that introducing a cash-like CBDC can induce a rise in retail transactions by 3.5% at the maximum, including a significant increase in cash-only type-1 transactions (up to 20%). Regarding the uptake of the new CBDC relative to other payment instruments, the model predicts that the market share of a cash-like CBDC is from about 21 to 25%, depending on i_{CL} . Cash will be completely driven out of the market and replaced by the CBDC. The welfare gain of introducing a cash-like CBDC is 5 bps at the maximum.

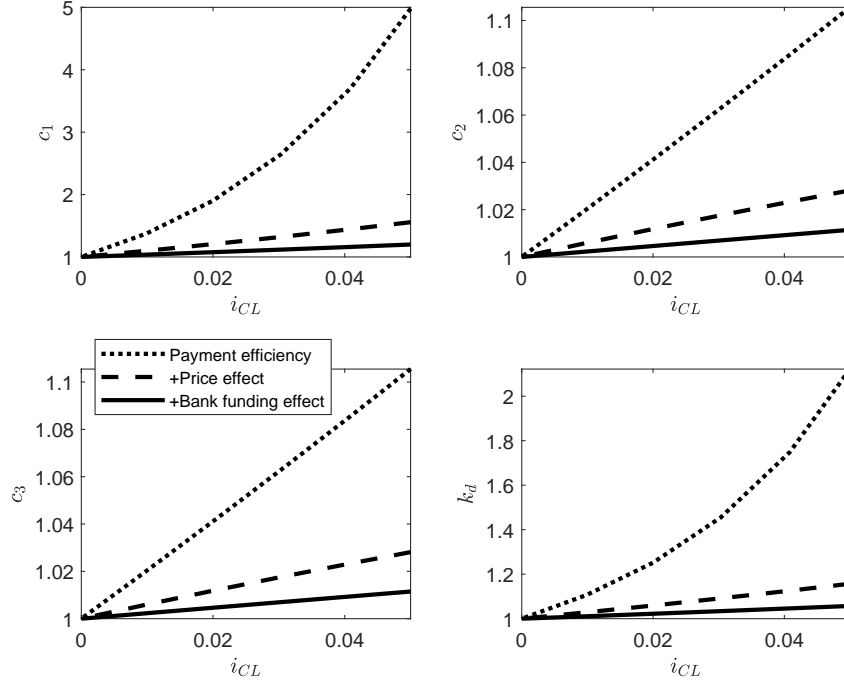


Figure 5: Effects of Interest on a Cash-like CBDC

Percentage change	c_1	c_2	c_3	c
- Payment efficiency	419.43	10.89	10.89	60.56
- Price effects	-361.84	-8.00	-8.00	-51.02
- Bank funding effects	-36.99	-1.71	-1.71	-6.00
Total effects	20.61	1.18	1.18	3.54

Table 6: Decomposition of Effects for a Cash-like CBDC with $i_{CL} = 5\%$

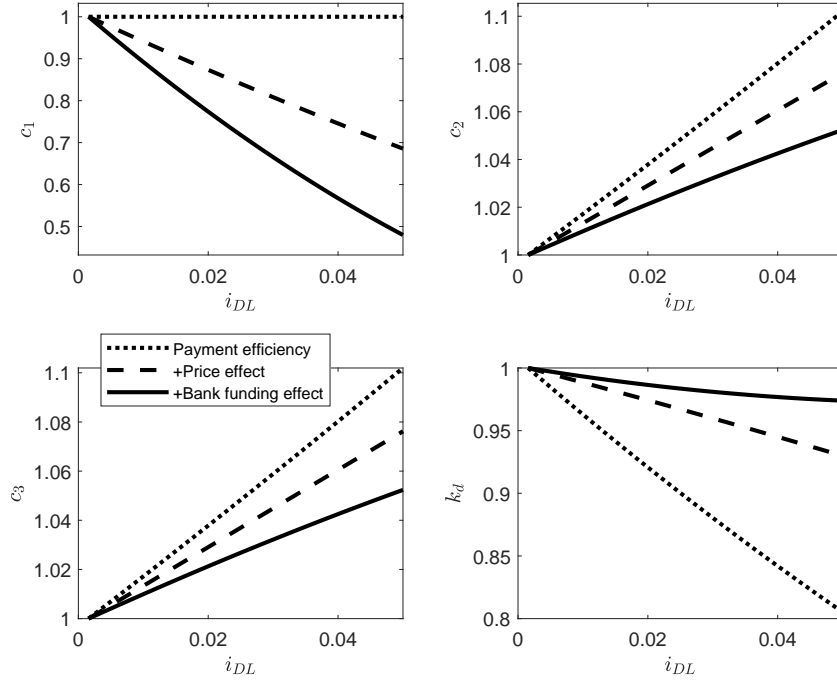


Figure 6: Effects of Interest on a Deposit-like CBDC

6.3 Effects of a Deposit-like CBDC

Figure 6 quantifies the effects of introducing a deposit-like CBDC with its rate, i_{DL} , ranging from 0% to 5%. Paying interest on the CBDC improves payment efficiency for type-2 and type-3 transactions, while there are no spillover effects to type-1 transactions since cash is dominated in the rate of return. The effects due to a higher price and higher bank funding costs are again negative. Overall, type-1 transactions decrease, while the other transactions increase. The calibrated model predicts that the effect on type-1 transactions dominates, leading to lower aggregate consumption and investment. At the maximum, type-1 consumption drops by 50%, while the other types of consumption go up by about 5%, as reported in Table 7. Overall, a deposit-like CBDC crowds out banking by 2.6% at the maximum. Regarding the uptake of a deposit-like CBDC, the model predicts that its market share is from about 9.5% to 16.7%, depending on i_{DL} . Since the CBDC is not strictly dominating, cash can still maintain at least 5% of the payment market. The welfare cost of introducing a deposit-like CBDC is 9 bps at the maximum.

Percentage change	c_1	c_2	c_3	c
- payment efficiency	0	10.54	10.54	9.28
- price effects	-31.69	-2.61	-2.61	-6.08
- bank funding effects	-21.60	-2.54	-2.54	-3.20
Total effects	-53.30	5.39	5.39	-1.61

Table 7: Decomposition of Effects for a Deposit-like CBDC with $i_{DL} = 5\%$

7 Conclusion

Our study extends a standard monetary model to assess different CBDC designs by incorporating two realistic features. First, cash competes with deposits as payment instruments in some transactions. Second, deposit-taking banks help fund the production of consumption. We found some policy-relevant results that are both interesting and novel. Specifically, a cash-like CBDC can promote consumption and welfare, thereby out-performing a deposit-like CBDC. In addition, a cash-like CBDC can crowd in banking, which suggests that the worry about disintermediation is not warranted. More importantly, a CBDC generates these benefits only when it bears interest—a result that casts doubt on the optimality of not paying interest on CBDC balances. Overall, our results show that ignoring the general equilibrium effects results in misleading qualitative and quantitative predictions.

In this paper, we abstract from other frictions in the banking sector such as bank market power and pledgeability constraints, both of which have already been explored in the literature. While it would be interesting to introduce these features into our model, this extension is left for future research.

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Appendix

A. Proofs and Derivations

Proof of Lemma 1

Proof. From the first-best problem, we obtain c_i^* and k_d^* , with the price p^* solving

$$\frac{1}{\sigma} F \left(F'^{-1} \left(\frac{1}{\beta p^*} \right) \right) = u_1'^{-1}(p^*) + u_2'^{-1}(p^*) + u_3'^{-1}(p^*).$$

To ensure that the CC is not binding, we must have

$$R_z = \frac{1}{\beta}. \quad (19)$$

To ensure that the DC is not binding, we must have

$$R_d k_d + B \geq (c_2 + (1 - f)c_3)p.$$

This gives a threshold for B such that the equilibrium is efficient when

$$B > \max\{0, B^*\}, \quad (20)$$

where $B^* \equiv c_2^* p^* - \frac{k_d^*}{\beta}$.

□

Proof of Proposition 2

Proof. Assume that cash is not used for the purchase of good c_3 and that the DC is slack (i.e., $f = 0$ and $\lambda_d = 0$). Using equilibrium conditions together with the market clearing condition, we obtain

$$\frac{1}{\sigma} F \left(F'^{-1} \left(\frac{1}{\beta p} \right) \right) = u_1'^{-1}((1 + \lambda_z)p) + u_2'^{-1}(p) + u_3'^{-1}(p). \quad (21)$$

This equation implicitly defines $p = \bar{p}(\lambda_z)$ as a decreasing function of λ_z .

Notice that at the first best, the price is given by $\bar{p}(0)$; i.e., $p^* = \bar{p}(0)$. In this case, we need to verify that the DC is not binding; i.e., $R_d k_d + B \geq (c_2 + c_3)p$, which is equivalent to

$$B \geq \bar{B}(\lambda_z) \equiv p \left[u_2'^{-1}(p) + u_3'^{-1}(p) - \frac{1}{\beta p} F'^{-1} \left(\frac{1}{\beta p} \right) \right]. \quad (22)$$

Note that $p = \bar{p}(\lambda_z)$, but we have simplified the exposition. In this case, cash is not used in good-3 transactions because it is more costly than deposits, so $f = 0$ is verified. Moreover, from $k_d = F'^{-1}\left(\frac{1}{\beta p}\right)$, we know that k_d is also decreasing in λ_z . At $\lambda_z = 0$, $k_d = k^*$, so $k_d \leq k^*$ for $\lambda_z \geq 0$.

For a given cash inflation rate, if credit is scarce; i.e., $B < \bar{B}(\lambda_z)$, then the DC is binding, the liquidity premium on deposits is positive, $\lambda_d > 0$, and cash may or may not be used for good 3, depending on whether λ_d is greater than λ_z .

□

Proof of Proposition 3

Proof. We separate the proof into two parts.

[1] The effects of R_m on price and quantity

Suppose $\sigma < 1$. First, we want to know how p changes with R_m . Equivalently, we study how p changes with λ_m , as we know that λ_m and R_m move in opposite directions because $R_m = 1/(\beta(1 + \sigma\lambda_m))$. We use this trick in most proofs.

As R_m increases, or equivalently as λ_m decreases, the RHS and LHS of (16) (demand and supply) both shift upwards, so p increases.

Now we show that $(1 + \lambda_m)p$ must decrease. Suppose by way of contradiction that $(1 + \lambda_m)p$ increases, then the RHS of (15) decreases, so the LHS should decrease as well. Then, (14) implies that $\sigma(1 + \lambda_m)p + (1 - \sigma)p$ should decrease. But p increases, so $(1 + \lambda_m)p$ should decrease. This is a contradiction.

Hence, $(1 + \lambda_m)p$ must decrease. As a result, all c_i s and consequently c and k_d increase.²⁰

[2] Effects of R_m on welfare

To calculate welfare, we first need to calculate the derivative of p with respect to R_m . Equivalently, we calculate the derivative of p with respect to λ_m , as we know that λ_m and R_m move in opposite

²⁰For completeness, suppose $\sigma = 1$. Note that the RHS of (16) is a strictly decreasing function in $(1 + \lambda_m)p$ and the LHS is strictly increasing in $(1 + \lambda_m)p$. Therefore, if a solution exists, there exists a unique one. Hence, $(1 + \lambda_m)p$ is constant and the values of the LHS and the RHS, which are each equal to σc , is kept constant as λ_m increases. Consequently, k_d is kept constant too.

directions because $R_m = 1/(\beta(1 + \sigma\lambda_m))$. Denote $p' \equiv \frac{\partial p}{\partial \lambda_m}$. We take the derivative of (16) with respect to λ_m :

$$\frac{1}{\sigma} F' \frac{-p'}{F'' p^2 \beta(1 + \sigma\lambda_m)} + \frac{1}{\sigma} F' \frac{-p\sigma}{F'' p^2 \beta(1 + \sigma\lambda_m)^2} = \frac{(1 + \lambda_m)p' + p}{u_1''} + \frac{(1 + \lambda_m)p' + p}{u_2''} + \frac{(1 + \lambda_m)p' + p}{u_3''}.$$

Note that we drop the arguments of the functions (e.g., $F' \equiv F'(k_d)$ and $u_1'' \equiv u_1''(c_1)$). Therefore,

$$p' = \frac{-\left(\frac{F'}{F'' p^2 \beta(1 + \sigma\lambda_m)^2} + \frac{1}{u_1''} + \frac{1}{u_2''} + \frac{1}{u_3''}\right)p}{\frac{F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} + \frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''}}.$$

Welfare is given by

$$W = U(Y^*) - Y^* + \beta\sigma(u_1(c_1) + u_2(c_2) + u_3(c_3)) - k_d$$

The change in welfare is given by

$$\begin{aligned} \frac{1}{\beta\sigma} \frac{dW}{d\lambda_m} &= u_1' dc_1 + u_2' dc_2 + u_3' dc_3 - \frac{dk_d}{\beta\sigma} \\ &= u_1' \frac{(1 + \lambda_m)p' + p}{u_1''} + u_2' \frac{(1 + \lambda_m)p' + p}{u_2''} + u_3' \frac{(1 + \lambda_m)p' + p}{u_3''} \\ &\quad + \frac{p'}{\beta\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} + \frac{p}{\beta F'' p^2 \beta(1 + \sigma\lambda_m)^2} \end{aligned}$$

We have $u' \equiv u_1' = u_2' = u_3' = (1 + \lambda_m)p$ and $F' = \frac{1}{p\beta(1 + \sigma\lambda_m)}$, therefore,

$$\begin{aligned} \frac{1}{\beta\sigma} \frac{dW}{d\lambda_m} &= \left[\frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''} + \frac{F'}{[u' F'] \beta\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} \right] u' p' \\ &\quad + u' p \left(\frac{1}{u_1''} + \frac{1}{u_2''} + \frac{1}{u_3''} \right) + \frac{u' F' p}{[u' F'] \beta F'' p^2 \beta(1 + \sigma\lambda_m)^2}, \end{aligned}$$

so

$$\frac{dW}{d\lambda_m} = \left[\frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''} + \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \frac{F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} \right] p' + p \left(\frac{1}{u_1''} + \frac{1}{u_2''} + \frac{1}{u_3''} + \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \frac{F'}{F'' p^2 \beta(1 + \sigma\lambda_m)^2} \right).$$

Hence,

$$\begin{aligned} \frac{1}{\beta\sigma u'} \frac{dW}{d\lambda_m} &= \left[\frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''} + \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \frac{F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} \right] p' + p \left(\frac{1}{u_1''} + \frac{1}{u_2''} + \frac{1}{u_3''} + \frac{F'}{F'' p^2 \beta(1 + \sigma\lambda_m)^2} \right) \\ &\quad - \left(1 - \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \right) \frac{F'}{F'' p \beta(1 + \sigma\lambda_m)^2} \\ &= \left(1 - \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \right) \frac{-F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} p' + \frac{\sigma p}{1 + \sigma\lambda_m} \left(1 - \frac{1 + \sigma\lambda_m}{1 + \lambda_m} \right) \frac{-F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)} \\ &= \underbrace{\frac{-F'}{\sigma F'' p^2 \beta(1 + \sigma\lambda_m)}}_{>0} \underbrace{\frac{(1 - \sigma)\lambda_m}{(1 + \lambda_m)(1 + \sigma\lambda_m)} (p' (1 + \sigma\lambda_m) + \sigma p)}_{= \frac{d[p(1 + \sigma\lambda_m)]}{d\lambda_m} \leq 0}. \end{aligned}$$

Note that $\frac{d[p(1 + \sigma\lambda_m)]}{d\lambda_m}$ is negative because k_d decreases with λ_m , as shown above. Therefore, $dW/d\lambda_m$ is strictly negative for $\sigma < 1$. \square

Solving the Deposit-like CBDC Equilibrium

We first collect the equations for the deposit-like CBDC and then prove the result.

$$\begin{aligned}
 V(Z, D, M) = & \max_{\mathbf{c}, f, v, n, s} \sigma \sum_{i=1}^3 u_i(c_i) + Z + D + M + W(0) \\
 & - \sigma(c_1 + c_2 + c_3)p \\
 & + \sigma\lambda_z [Z - (c_1 + fc_3)p] \\
 & + \sigma\lambda_m [M - (vc_2 + nc_3)p] \\
 & + \sigma\lambda_d [D + B - ((1-v)c_2 + sc_3)p] \\
 & + \sigma\lambda_0 [-1 + f + n + s],
 \end{aligned}$$

where $\sigma\lambda_m$ denotes the Lagrangian multiplier for the MC. Also, λ_0 denotes the Lagrangian multiplier for the $f + n + s = 1$ constraint.

The equilibrium conditions are given by

$$k_d : pF'(k_d) = R_d$$

$$c_1 : u'_1(c_1) = (1 + \lambda_z)(p + \tau)$$

$$c_2 : u'_2(c_2) = m(1 + \lambda_m)p + (1 - m)(1 + \lambda_d)p$$

$$c_3 : u'_3(c_3) = [f(1 + \lambda_z) + n(1 + \lambda_m) + s(1 + \lambda_d)]p$$

$$\text{DM good market} : \sigma(c_1 + c_2 + c_3) = F(k_d)$$

$$v : -\lambda_m + \lambda_d \begin{cases} \leq 0, & \text{if } v = 0, \\ = 0, & \text{if } v \in (0, 1), \\ \geq 0, & \text{if } v = 1, \end{cases}$$

$$f : -(1 + \lambda_z)pc_3 + \lambda_0 \begin{cases} \leq 0, & \text{if } f = 0, \\ = 0, & \text{if } f \in (0, 1), \\ \geq 0, & \text{if } f = 1, \end{cases}$$

$$n : -(1 + \lambda_m)pc_3 + \lambda_0 \begin{cases} \leq 0, & \text{if } n = 0, \\ = 0, & \text{if } n \in (0, 1), \\ \geq 0, & \text{if } n = 1, \end{cases}$$

$$\begin{aligned}
s &: -(1 + \lambda_d)pc_3 + \lambda_0 \begin{cases} \leq 0, & \text{if } s = 0, \\ = 0, & \text{if } s \in (0, 1), \\ \geq 0, & \text{if } s = 1, \end{cases} \\
\lambda_0 &: f + n + s = 1,
\end{aligned}$$

$$\begin{aligned}
Z &: \frac{\mu_z}{\beta(1+i)} \geq \sigma\lambda_z + 1, \text{ equality if } Z > 0, \\
D &: \frac{1}{\beta R_d} \geq \sigma\lambda_d + 1, \text{ equality if } D > 0, \\
M &: \frac{1}{\beta R_m} \geq \sigma\lambda_m + 1, \text{ equality if } M > 0, \\
ZZ &: Z \geq (c_1 + fc_3)p, \text{ " = " if } \lambda_z > 0 \\
DC &: R_d k_d + B \geq [(1 - m)c_2 + (1 - f - n)c_3]p, \text{ " = " if } \lambda_d > 0 \\
MM &: M \geq (mc_2 + nc_3)p, \text{ " = " if } \lambda_m > 0
\end{aligned}$$

In addition, bankers need to match the deposit rate to the CBDC rate in equilibrium.

$$\lambda \equiv \lambda_d = \lambda_m = \frac{1}{\sigma} \left[\frac{1}{\beta R_m} - 1 \right].$$

The market clearing condition then can be written as

$$F \left(F'^{-1} \left(\frac{1}{p\beta} \frac{1}{\sigma(1+\lambda) + 1 - \sigma} \right) \right) = \sigma u_1'^{-1} ((1 + \lambda_z)p) + \sigma u_{23}'^{-1} ((1 + \lambda)p), \quad (23)$$

where $u_{23}'^{-1} \equiv u_2'^{-1} + u_3'^{-1}$.

The quantity of the CBDC and cash can be calculated from the following equations:

$$\begin{aligned}
DC &: R_d k_d + B = (1 - n)c_3 p \Rightarrow n = 1 - \frac{k_d F'(k_d) + B}{F(k_d)} \sigma > 0 \\
MM &: M = nc_3 p, \\
ZZ &: Z = c_1 p
\end{aligned}$$

Proof of Proposition 5

Proof. Given $\lambda_m < \lambda_z$, we have $\lambda_d = \lambda_m = \frac{1}{\sigma} \left[\frac{1}{\beta R_m} - 1 \right]$ as discussed in the text.

As λ decreases, the LHS of (23) shifts downward and the RHS shifts upward, so p increases.

Suppose by way of contradiction that $(1 + \lambda_m)p$ increases, then the RHS of (23) decreases, so the LHS should decrease as well. As a result, $\sigma(1 + \lambda_m)p + (1 - \sigma)p$ should decrease. But $(1 - \sigma)p$ weakly increases (it is constant when $\sigma = 1$), implying that $(1 + \lambda_m)p$ should decrease. This is a contradiction!

Therefore, $(1 + \lambda_m)p$ should decrease. As a result, c_1 decreases, and c_2 and c_3 increase.

We know that $\beta F'(k_d) = \frac{1}{\sigma(p(1+\lambda_m))+(1-\sigma)p}$. We take the derivative with respect to p :

$$\beta F''(k_d) \frac{\partial k_d}{\partial p} = -\beta^2 F'^2(k_d) \left[\sigma \frac{\partial(p(1 + \lambda_m))}{\partial p} + 1 - \sigma \right]$$

We showed above that $p(1 + \lambda_m)$ is strictly decreasing in p . Therefore, $\frac{\partial k_d}{\partial p}$ is strictly negative at $\sigma = 1$.²¹ As a result, if σ is sufficiently close to 1, then $\frac{\partial k_d}{\partial p}$ continues to be strictly negative. Therefore, k_d and consequently c should decrease with an increase in R_m for σ sufficiently close to 1.

□

Proof of Proposition 6

Proof. The following equations characterize the equilibrium price:

$$\text{Cash-like CBDC: } F \left(F'^{-1} \left(\frac{1}{p\beta} \frac{1}{\sigma(1 + \lambda_m) + 1 - \sigma} \right) \right) = \sigma u_1'^{-1}((1 + \lambda_m)p) + \sigma u_{23}'^{-1}((1 + \lambda_m)p), \quad (24)$$

$$\text{Deposit-like CBDC: } F \left(F'^{-1} \left(\frac{1}{p\beta} \frac{1}{\sigma(1 + \lambda_m) + 1 - \sigma} \right) \right) = \sigma u_1'^{-1}((1 + \lambda_z)p) + \sigma u_{23}'^{-1}((1 + \lambda_m)p). \quad (25)$$

Consider the RHS and LHS in the (p, y) space for both equations. The curve on the LHS is the same for both cash- and deposit-like CBDCs. The curve on the RHS; i.e., aggregate demand, is higher for the cash-like CBDC and, as a result, the price level is higher for that. Also, aggregate consumption and output are higher. A higher price level for the cash-like CBDC implies lower consumption of goods 2 and 3. Since aggregate consumption is higher for the cash-like CBDC, the consumption of good 1 should also be higher for this type of CBDC.

Note that, given R_d , the profit function is increasing in p by the envelope theorem. Hence, the equilibrium profit is higher under the cash-like CBDC.

²¹When $\sigma = 1$, (23) gives p and $F'(k_d) = 1/(\beta p(1 + \lambda_m))$ gives k_d . Both p and k_d are in $(0, \infty)$ and are well defined. As a result, $\frac{\partial k_d}{\partial p}$ is also well defined and is in $(-\infty, 0)$.

Now we compare the welfare level, for which we need to calculate the derivative of p with respect to λ_z from (25):

$$\frac{1}{\sigma} F' \frac{-p'}{F'' p^2 \beta (1 + \sigma \lambda_m)} = \frac{(1 + \lambda_z) p' + p}{u_1''} + \frac{(1 + \lambda_m) p'}{u_2''} + \frac{(1 + \lambda_m) p'}{u_3''},$$

where $p' \equiv \frac{\partial p}{\partial \lambda_z}$. We also drop the arguments of the functions (e.g., $F' \equiv F'(k_d)$ and $u_1'' \equiv u_1''(c_1)$) to simplify the exposition. Therefore,

$$p' = \frac{-\frac{p}{u_1''}}{\frac{F'}{\sigma F'' p^2 \beta (1 + \sigma \lambda_m)} + \frac{1 + \lambda_z}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''}}. \quad (26)$$

Welfare is given by

$$W = U(Y^*) - Y^* + \beta \sigma (u_1(c_1) + u_2(c_2) + u_3(c_3)) - k_d$$

Now we calculate the change in welfare:

$$\begin{aligned} \frac{1}{\beta \sigma} \frac{dW}{d\lambda_z} &= u_1' dc_1 + u_2' dc_2 + u_3' dc_3 - \frac{dk_d}{\beta \sigma} \\ &= u_1' \frac{(1 + \lambda_z) p' + p}{u_1''} + u_2' \frac{(1 + \lambda_m) p'}{u_2''} + u_3' \frac{(1 + \lambda_m) p'}{u_3''} + \frac{p'}{\beta \sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)} \end{aligned}$$

At $\lambda_z = \lambda_m$, we have $u' \equiv u_1' = u_2' = u_3' = (1 + \lambda_m)p$ and $F' = \frac{1}{p\beta(1+\sigma\lambda_m)}$, therefore

$$\begin{aligned} \frac{1}{\beta \sigma} \frac{dW}{d\lambda_z} &= \left[\frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''} + \frac{F'}{[u' F'] \beta \sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)} \right] u' p' + u_1' \frac{p}{u_1''}, \\ \Rightarrow \frac{1}{\beta \sigma u'} \frac{dW}{d\lambda_z} &= \left[\frac{1 + \lambda_m}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''} + \frac{(1 + \sigma \lambda_m)}{1 + \lambda_m} \frac{F'}{\sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)} \right] p' + \frac{p}{u_1''} \\ &= \frac{\left(1 - \frac{1 + \sigma \lambda_m}{1 + \lambda_m} \right) \frac{F'}{\sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)}}{\frac{F'(k_d)}{\sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)} + \frac{1 + \lambda_z}{u_1''} + \frac{1 + \lambda_m}{u_2''} + \frac{1 + \lambda_m}{u_3''}} \frac{p}{u_1''} \\ &= \underbrace{\left(1 - \frac{1 + \sigma \lambda_m}{1 + \lambda_m} \right)}_{>0} \underbrace{\frac{-F'}{\sigma F''(k_d) p^2 \beta (1 + \sigma \lambda_m)}}_{>0} \underbrace{p'}_{<0} < 0, \end{aligned}$$

where p' is obtained from (26), and $p' < 0$ is obtained from the first part of the proposition. Therefore, $dW/d\lambda_z < 0$.

As we change $\lambda_{DL} = \lambda_z$ to $\lambda_{CL} = \lambda_m$ where $\lambda_m < \lambda_z$, welfare increases as long as λ_m is close to λ_z . In other words, when we change the type of CBDC from the deposit-like to the cash-like CBDC, welfare increases.

□

B. Boundary Cases of $f = 0$ and $f = 1$

We focus on two special cases. We assume that the DC constraint is binding (i.e., $\lambda_d > 0$) for both.

Case 2: A CBDC is not used in good-3 transactions ($f = 0$)

The following proposition characterizes the equilibrium effects.

Proposition 8. *Suppose $f = 0$ and $\sigma = 1$. With a higher real rate, R_m , paid on the cash-like CBDC,*

(i) p , k_d , c_1 and c increase, and

(ii) c_2 and c_3 decrease.

Proof of Proposition 8. A combination of $\lambda_d = \frac{1}{\sigma} \left[\frac{1}{\beta R_d} - 1 \right] = \frac{1}{\sigma} \left[\frac{1}{\beta p F'(k_d)} - 1 \right]$, $c_2 = u_2'^{-1}[(1 + \lambda_d)p]$, $c_3 = u_3'^{-1}[(1 + \lambda_d)p]$, and (11) gives

$$F'(k_d)k_d + \frac{B}{p} = u_{23}'^{-1} \left(\frac{1}{\sigma \beta F'(k_d)} - \frac{1 - \sigma}{\sigma} p \right). \quad (27)$$

From (27), we know that the RHS is decreasing and the LHS is increasing in k_d , given that $kF'(k)$ is increasing. Therefore, for a given p , there exists at most one k_d . Also, the RHS is increasing in p and the LHS is decreasing in p . Therefore, k_d is increasing in p .

We show that, as R_m increases (or equivalently as λ_m decreases), p increases. By way of contradiction, assume p decreases, then c_1 increases. Also, k_d is increasing in p , so k_d falls. Given that σ is equal to 1, and c_2 and c_3 rise. But the fact that c_1 , c_2 and c_3 all increase and k_d decreases is inconsistent with the market clearing condition:²²

$$\frac{F(k_d)}{\sigma} = c_1 + c_2 = u_1'^{-1}((1 + \lambda_m)p) + u_{23}'^{-1} \left(\frac{1}{\sigma \beta F'(k_d)} - \frac{1 - \sigma}{\sigma} p \right). \quad (28)$$

Therefore, p increases with R_m . As a result, c_2 and c_3 decrease as λ_m decreases. Moreover, the increase in k_d implies that total consumption, c , increases. Given that c_2 and c_3 have decreased, c_1 must increase, as does $(1 + \lambda_m)p$.

□

²²Note that (27) and (28) together give us k_d and p .

Case 3: Deposits are not used in good-3 transactions ($f = 1$)

Proposition 9. *Suppose $f = 1$ and $\sigma = 1$. With a higher real rate R_m paid on the cash-like CBDC,*

(i) p , k_d , c_1 , c_3 and c increase, and

(ii) c_2 decreases.

Proof of Proposition 9. Again, note that $\sigma\lambda_d + 1 = \frac{1}{\beta R_d} = \frac{1}{\beta p F'(k_d)}$. This, together with (11), can be written as

$$pF'(k_d)k_d + B = u_2'^{-1} \left(\frac{1}{\beta F'(k_d)} \right),$$

given that $\sigma = 1$. This equation implies that k_d is increasing in p because the LHS is increasing in k_d , the RHS is decreasing in k_d , and the LHS shifts downwards with an increase in p . Since k_d goes up, $\frac{1}{\beta F'(k_d)}$ also goes up, and so does $(1 + \lambda_d)p$. Therefore, c_2 goes down.

As R_m goes up, λ_m goes down. Here, we argue that p must also go up. By way of contradiction, suppose p goes down. Then c_1 and c_3 go up. Also, as shown above, c_2 is decreasing in p , so c_2 also goes up. As a result, aggregate consumption should go up and, consequently, k_d should go up. However, as shown above, k_d is increasing in p , so a lower p implies a lower k_d . This is a contradiction!

Now we have shown that p should go up as λ_z decreases. A higher p implies a higher k_d . Therefore, c goes up. Since c_2 goes down, c_1 and c_3 should go up too. \square

C. Comparative Static Exercises with Respect to the Credit Limit

B

Here, we conduct comparative statics with respect to B . We focus on two sub-cases: one with $f \in (0, 1)$ and one with $f = 0$. The case with $f = 1$ does not give us new insights.

Case 1: Cash and deposits are used in good-3 transactions ($f \in (0, 1)$)

Proposition 10. *As long as $0 < f < 1$, as B increases, p , R_d , k_d , c_1 , c_2 and c_3 are all constant but f increases and Z decreases.*

Proof of Proposition 10. Because $0 < f < 1$, λ_d and consequently R_d are fixed by monetary policy because $\lambda_d = \lambda_z$. Then, (15) implies that p cannot change. Given $\lambda_d = \lambda_z$ and the fact that λ_d , λ_z and p are all constant, c_i should also be constant. Given that R_d and p are fixed, k_d should be constant too. As B increases, only f changes to satisfy the DC. Consequently, Z changes to have the CC satisfied. \square

This result simply states that as long as agents are indifferent between using cash or deposits, extending credit conditions has no effect on the real economy. This is because the return on deposits is pinned down by monetary policy and extending credit conditions only changes the composition of the means of payment. Credit crowds out cash, but the quantity of deposits remains unchanged.

Case 2: Cash is not used for good 3, $f = 0$

Characterization in general is not easy, so we focus on the case in which $\sigma = 1$.

Proposition 11. *Assume $f = 0$ and $\sigma = 1$. As B increases, p , c_2 and c_3 increase and c_1 , c , and k_d decrease.*

Proof of Proposition 11. We solved for $f = 0$ in Appendix B. The following two equations characterize the equilibrium:

$$F'(k_d)k_d + \frac{B}{p} = u'_{23} \left(\frac{1}{\sigma\beta F'(k_d)} - \frac{1-\sigma}{\sigma} p \right), \quad (29)$$

$$F(k) = \sigma u'^{-1}_1((1 + \lambda_z)p) + \sigma u'^{-1}_{23} \left(\frac{1}{\sigma\beta F'(k_d)} - \frac{1-\sigma}{\sigma} p \right). \quad (30)$$

Consider the (k_d, p) space. The first equation illustrates an increasing function of p in terms of k_d . If $\sigma = 1$, the second equation illustrates a decreasing function of p in terms of k_d . Therefore, there exists at most one solution. With an increase in B , the first curve shifts upwards, so p increases and k_d decreases. These together imply that c_2 and c_3 increase. A higher p implies a lower c_1 . A lower k_d implies lower aggregate consumption. \square

According to this result, when cash is only used in good-1 transactions and credit is scarce, improving credit conditions reduces the consumption of non-credit goods as well as aggregate consumption.

There are spillovers in this environment. As B increases, the DC becomes more relaxed, increasing aggregate demand and consequently the price level. This, in turn, tightens the CC, so c_1 decreases. To determine the overall effect on investment, note that the increase in B means that agents would need smaller deposits to make their payments. This decreases the demand for deposits, thus increasing R_d . The DC tells us that as σ gets close to 1, the second effect is dominant. That is, the overall effect is that credit crowds out bank intermediation and leads to less production in the special case where $\sigma = 1$. Yet, the comparative statics may change when σ is small.

Discussion of comparative statics

We learn that the effect of credit on bank intermediation and real variables will be different across the two above cases. If agents are indifferent between cash and deposits, extending credit conditions does not have real effects on the economy but crowds out cash. Yet, monetary policy is quite effective as it determines the interest rate on deposits and the cost of funding for banks. If agents prefer deposits to cash, then extending credit conditions crowds out bank intermediation and has negative spillovers on cash transactions.

D. On the Calibration Exercise

In this appendix, we discuss some points about the calibration exercise to understand the details better.

First, the share of deposits out of the total means of payment turns out to be almost fixed in our quantitative exercise. This is not a coincidence and is an implication of the fact that in our model the elasticity of production function is constant. To see this, denote by $\eta_F = \frac{k_d F'(k_d)}{F(k_d)}$ the elasticity of production function with respect to the investment level in the general case, without imposing the functional form in Section 6. Denote by y the total production in the economy, then we have

$$\sigma(Z + D + B) = py = pF(k_d) = \frac{pF(k_d)}{R_d} R_d = \frac{F(k_d)}{F'(k_d)} R_d = \frac{F(k_d)}{k_d F'(k_d)} R_d k_d = \frac{D}{\eta_F},$$

where $\sigma c = y$ in equilibrium. Given the functional form of F in this section, $F(k) = k^{1-\gamma}/(1-\gamma)$, we have

$$\frac{D}{py} = \frac{D}{\sigma(Z + D + B)} = 1 - \gamma. \quad (31)$$

Therefore, the share of deposits, $\frac{D}{Z+D+B}$, is fixed and equal to σ times the elasticity of production function. As a cross-check, (31) implies that $\frac{D}{Z+D+B}$ must be around $0.6 \cdot 0.603 = 0.36$; this is close to 0.34, which is taken from the payments data.

Second, to get a sense of the model-implied values for the α_i 's,

$$\begin{aligned} \alpha_1 &= \frac{Z}{Z + D + B} = \frac{Z}{pc} = 21\% \text{ (estimated)} \approx 21\% \text{ (target)} \\ \alpha_2 &= \frac{D}{Z + D + B} = \frac{k_d R_d}{pc} = 36.5\% \text{ (estimated)} \approx 34\% \text{ (target)} \\ \alpha_3 &= \frac{B}{Z + D + B} = \frac{B}{pc} = 42\% \text{ (estimated)} \approx 44\% \text{ (target)}^{23} \\ 1 - \gamma &= \frac{D}{py} = \frac{1}{\sigma} \frac{D}{pc} = \frac{1}{0.6} * 34\% = 56.67\% \Rightarrow \gamma = 43.33\% \end{aligned}$$

So, if we fix γ to 43.33%, then we precisely match the target for α_2 but we do worse on other targets. We include γ in the minimization problem, so it turns out that $\gamma = 39.7\%$ minimizes the sum of squared error of all targets.

Moreover, note that γ in Chiu et al. (2019) is set at 0.34, which is close to 0.397 in this paper, although they take a different approach to calculate γ ; they use the elasticity of loans with respect to the real prime rate.

Third, the value of B derived in the model is 0.52. This is consistent with $pc = \frac{B}{pc} = 0.525/0.421 = 1.248$, which we derive from the model. Using this, we can calculate $\frac{M1}{GDP}$:

$$\begin{aligned}\frac{M1}{GDP} &= \frac{Z + D}{pc + X} = \frac{Z + D}{Z + D + B} \frac{Z + D + B}{pc + X} \\ &= (21\% + 36.5\%) \frac{pc}{pc + 2.04} = \mathbf{0.217},\end{aligned}$$

which is not far from the data in which the average of the ratio of $\frac{M1}{GDP}$ in our data is 0.257 and the median is 0.254.

Finally, as a cross-check to verify the consistency of different data sources, let's compare the share of $\frac{B}{Z+D+B}$ from our estimation against other data sources that do not use payments data. The share of deposits to M1 in our model can be calculated as follows:

$$\frac{D}{Z + D} = \frac{\frac{D}{Z+D+B}}{1 - \frac{B}{Z+D+B}} = 0.34/(1 - 0.44) = 0.61.$$

The historical average of the ratio of total checkable deposits to M1 (from FRED) for the period 1975 to 2020 is 0.6155.