Measures of Model Risk for Continuous-time Finance Models

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Highlights

- We propose an expected shortfall based measurement to measure parameter estimation risk (***PER***) and model specification risk (***MSR***) of continuous-time finance models.
- We apply the model risk measure to **affine jump-diffusion models** and **Lévy jump models** and investigate the impact of PER and MSR on the models' ability to capture the **joint dynamics of stock and option prices**.
- We **estimate** the parameters using Markov chain Monte Carlo techniques, under the risk-neutral probability measure and the real-world probability measure **jointly**.
- We find strong evidence supporting modeling of price jumps.

Motivation

- **Identify and measure model risk is essential.** (Basel Committee on Banking Supervision, 2009; Federal Reserve Board of Governors, 2011; European Banking Authority, 2012).
- Previous studies are typically based on **point-wise** estimation methods, thus **ignoring PER**; few studies measure PER and MSR separately $\Longrightarrow$ A general method to measure PER and MSR separately (Schilling et al., 2020).
  - Bayesian approach (Jacquier and Jarrow, 2000; Jacquier et al., 2002; Chung et al., 2013), the estimated posterior distribution reflects the uncertainty of parameters.
  - Expected shortfall (***ES***); jump models.
- Model risk is asymmetric $\Longrightarrow$ Measure the model risk for long and short positions separately.

Model Risk

Definition of Model Risk:

For option $\mathcal{H}$ and model $\mathcal{M}$ with the vector of parameters Θ.

PER: The **parameter estimation risk** refers to the uncertainty in the values of parameters Θ obtained via the estimation process $\mathcal{K}$ given dataset $\mathcal{D}$.

MSR: The **model specification risk** of $\mathcal{M}$ refers to the risk that, based on dataset $\mathcal{D}$ and methodologies $\mathcal{K}$, the model is unable to produce the features of $\mathcal{H}$.

TMR: The **total model risk** (***TMR***) is defined as the sum of PER and MSR.

Model Risk Measures:

PER: Bayesian MCMC estimation $\Rightarrow$ posterior distribution of parameters $\Rightarrow$ estimated price distribution: uncertainty of model prices $\Rightarrow$ potential loss due to parameter estimation. The model risk of the long/short position is measured with the left/right tail.

MSR: A model is misspecified if the pricing error cannot be completely explained by PER.

TMR: TMR = PER + MSR.

Models

The joint dynamics of the daily spot and option prices upon discretization:

$$Y_{t+1} = Y_t + \left(r_t - \frac{1}{2}V_t + \psi_J^{\mathbb{Q}}(-i) + \eta_s V_t\right)\delta^* + \sqrt{V_t\delta^*}\epsilon_{t+1}^Y + J_{t+1}^Y,$$
$$V_{t+1} = V_t + \kappa(\theta - V_t)\delta^* + \sigma_V\sqrt{V_t\delta^*}\epsilon_{t+1}^V + J_{t+1}^V,$$
$$PE_{t+1,\Delta_n}(\mathcal{M}(\Theta), Y, V) = a_{\Delta_n} + \rho_c PE_{t,\Delta_n}(\mathcal{M}(\Theta), Y, V) + \sigma_c\epsilon_{t+1,\Delta_n}^c,$$
$$a_{\Delta_n} \sim \mathbb{N}(a_m, a_s^2),$$

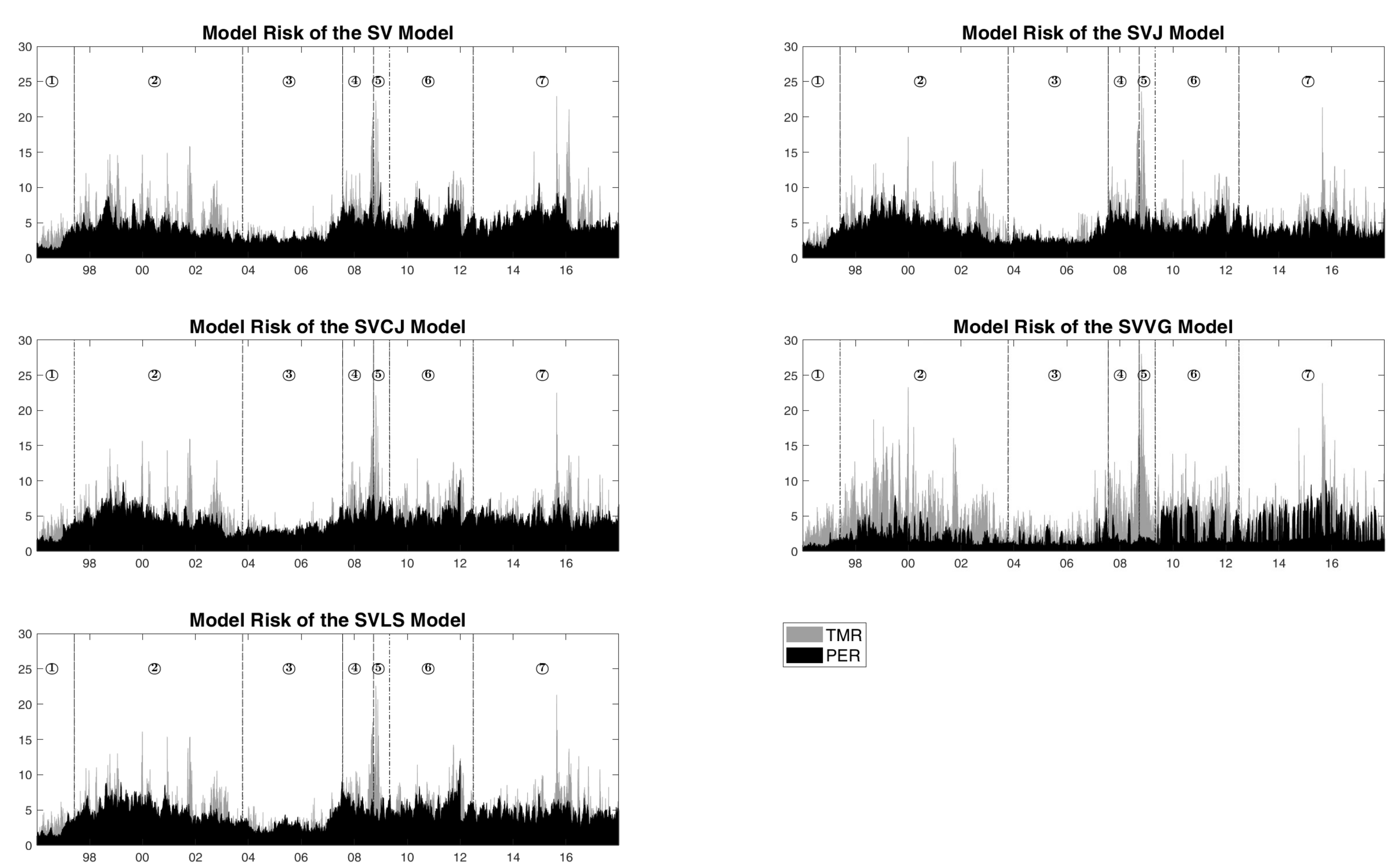
The description of all parameters can be found in the paper. $PE_{t+1,\Delta_n}(\mathcal{M}(\Theta), Y, V) = OP_{t+1,\Delta_n} - F_{t+1,\Delta_n}(\mathcal{M}(\Theta), Y, V)$ is the option pricing error. Building on the the autoregressive specification used in Eraker (2004) and Yu et al. (2011), **we further introduce the drift term** $a_{\Delta_n} \sim \mathbb{N}(a_m, a_s^2)$, which provides random effects to the autoregressive pricing error process; a_m is the average size of a_{Δ_n} while a_s modulates the varying effects of the drift term across options with different strike prices as determined by the Delta values.

We consider five models with different jump specifications:

Model	**Return Jump** $J_t^Y(\mathbb{P})$	**Variance Jump** $J_t^V(\mathbb{P})$
SV	0	0
SVJ	$\xi^Y N_t^Y$	0
SVVG	$X_t^{VG}(\sigma,\gamma,\nu) = \gamma G_t^\nu + \sigma W_{G_t^\nu}$	0
SVLS	$X_t^{LS}(\alpha,\sigma)$	0
SVCJ	$\xi^Y N_t^Y$	$\xi^V N_t^V$

$N_t^Y = N_t^V$ denotes the Poisson process with rate λ; ξ^Y is normally distributed with mean μ_J and volatility σ_J; $\{X^{VG}\}$ is the arithmetic Brownian motion with drift γ and volatility σ; $X_t^{LS}(\alpha,\sigma) - X_s^{LS}(\alpha,\sigma) \sim S_\alpha(\beta, \sigma(t-s)^{\frac{1}{\alpha}}, \gamma),\ t > s$; ξ^V is exponentially distributed with mean μ_V.

Model Risk Estimates



Model Risk from 1996 to 2017. The size of the grey line is the MSR.

Further Results

Period	①	②	③	④	⑤	⑥	⑦
SV	-0.4985**	-0.5898**	-0.2329**	-0.6771**	-0.1266	-0.7017**	-1.1023**
SVJ	-0.3924**	-0.5441**	-0.1178**	-0.3356**	-0.9025**	-0.4824**	-0.5175**
SVCJ	-0.3994**	-0.5876**	-0.1531**	-0.4612**	**0.4611****	-0.4878**	-0.4174**
SVVG	-0.0630**	-0.1375**	**0.0138***	-0.1310**	**0.0169****	-0.2865**	-0.5011**
SVLS	-0.4746**	-0.4879**	-0.1976**	-0.4598**	-0.3405**	-0.8413**	-0.7408**

The mean values of the differences between the PER of long and short positions.
The short position tends to bear higher model risk.

	Pricing Error				MSR				TMR			
	SVLS	SVVG	SVCJ	SVJ	SVLS	SVVG	SVCJ	SVJ	SVLS	SVVG	SVCJ	SVJ
SVVG	-4.90***				-7.98***				1.92**			
SVCJ	-2.53***	3.71***			-1.02	7.02***			-0.13	-1.91**		
SVJ	-1.05	4.68***	1.11		-1.57*	8.04***	-0.25		2.16**	-0.83	2.13**	
SV	-2.79***	2.53***	-1.34*	-2.04**	-1.59*	7.24***	-1.11	-0.72	**-2.82*****	**-3.50*****	**-2.21****	**-2.43*****

This table reports Diebold and Mariano statistics for squared pricing errors, MSR, and TMR.

Explaining Pricing Error with Model Risk

Is that necessary to measure PER and MSR separately?
Let $\epsilon_t(\mathcal{H}; \mathcal{M}(\Theta), \mathcal{D}, \mathcal{K})$ represent the absolute pricing error of option $\mathcal{H}$.

$$\epsilon_t(\mathcal{H}) = \beta_0 + \alpha\rho_{\eta,t}^{PER}(\mathcal{H}) + \beta_2\rho_{\eta,t}^{TMR}(\mathcal{H}) + \varepsilon_t.$$

Test whether $\alpha = 0$.

	β_0	α	β_2	Adj. R^2
SV	0.43**	**-1.05****	1.47**	72.96%
SVJ	0.27**	**-0.97****	1.45**	73.54%
SVCJ	0.13*	**-0.98****	1.48**	72.57%
SVVG	0.36**	**-0.63****	1.10**	89.95%
SVLS	0.44**	**-1.06****	1.48**	70.34%

Summary and Further Research

SVLS has the smallest **MSR**, while **SVVG** has the lowest **PER and TMR**.
All jump models have significantly **smaller TMR** compared with SV.
A **short position** bears a **greater model risk**.
Further: Investigate the model risk of high-dimensional models.

References

Basel Committee on Banking Supervision, 2009. Revisions to the Basel II market risk framework. Bank for International Settlements.
Chung, T.-K., Hui, C.-H., Li, K.-F., 2013. Explaining share price disparity with parameter uncertainty: Evidence from Chinese A-and H-shares. Journal of Banking and Finance 37, 1073–1083.
Eraker, B., 2004. Do stock prices and volatility jump? Reconciling evidence from spot and option prices. Journal of Finance 59, 1367–1403.
European Banking Authority, 2012. Discussion paper on draft regulatory technical standards on prudent valuation, under Article 100 of the draft Capital Requirements Regulation (CRR).
Federal Reserve Board of Governors, 2011. Supervisory guidance on model risk management. Federal Reserve SR Letter 11-7 Attachment.
Jacquier, E., Jarrow, R., 2000. Bayesian analysis of contingent claim model error. Journal of Econometrics 94, 145–180.
Jacquier, E., Polson, N. G., Rossi, P. E., 2002. Bayesian analysis of stochastic volatility models. Journal of Business and Economic Statistics 20, 69–87.
Schilling K., Bauer D., Christiansen MC., Kling A., 2020 Decomposing dynamic risks into risk components. Management Science 66, 5738–5756.
Yu, C. L., Li, H., Wells, M. T., 2011. MCMC estimation of L´evy jump models using stock and option prices. Mathematical Finance 21, 383–422.