# Delegated Costly Screening

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**Abstract**

A policymaker relies on regulators or bureaucrats to screen agents along a costly dimension. How can she maintain some control over the design of the screening process? She solves a two-layer mechanism design problem: she restricts the set of allowable allocations, after which a screener picks a menu that maps an agent's costly evidence to this restricted set. In general, the policymaker can set a floor in a way that dominates full delegation no matter how the screener's objectives are misaligned. When this misalignment is only over the relative importance of reducing allocation errors or agent's screening costs, the effectiveness of this restriction hinges sharply on the direction of the screener's bias. In the min-max optimal mechanism, if the screener is more concerned with reducing errors, setting this floor is robustly optimal for the policymaker. But if the screener is more concerned with keeping costs down, not only does this particular floor have no effect: any restriction that strictly improves over full delegation is complex and sensitive to the details of the screener's preferences. I consider the implications for regulatory governance.

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# 1 Introduction

Policymakers rely on intermediaries to screen agents on their behalf: Lawmakers rely on local government officials to screen workfare applicants by their willingness to engage in menial labor. Managers of a public healthcare program rely on hospitals to screen patients by their willingness to wait for procedures. Governments let consumer protection regulators and patent offices screen innovators by their willingness to undergo costly applications.

These intermediaries might not be better informed than policymakers ex-ante about which applicant is relatively well off or poor or which innovations are good or bad. But they may be better placed to observe the costly evidence or effort that applicants or innovators produce, and to therefore screen these agents on that basis. But this advantage may also allow intermediaries to seize control over mechanism design. How can a policymaker influence an intermediary's design? When is this easy to do? How much does she need to know about an intermediary's biases to do it?

To answer these questions, I study a model of delegated costly screening. There are three players: a policymaker (she), a screener (he), and an agent. First, the policymaker limits the screener to a set of allowable allocations. Next, the screener designs a menu that offers the agent larger allowable allocations for more costly evidence. Finally, the agent takes stock of his private information and decides how much costly evidence to generate. For example, a policymaker can require that workfare applicants either receive nothing, between \$50 and \$100 per week, or between \$200 and \$400 per week. A local government official can respond by offering any applicant \$70 if they meet basic eligibility requirements or \$300 if they also complete some menial labor. An eligible agent considers his cost of labor and utility for money to decide between the two options.

A good screening program has high targeting efficiency so that relatively worse (well) off applicants are not too under (over) allocated. At the same time, the program should not be too onerous for applicants. The policymaker and screener minimize a weighted sum of their (agent-type dependent) disutility from allocation errors and agent screening costs but disagree about the relative importance of these objectives, e.g., one is less concerned with careful targeting and wants to move closer to a welfare rather than workfare system.

The benchmark setting considers a tractable model where the policymaker and screener evaluate menu choices by the worst-case realization over the agent's type. One interpretation is that both may be ambiguity averse and seek good guarantees in settings like innovation approval. Alternatively, they may worry about worst-case outcomes even if they can express priors. For example, managers of a workfare or publicly funded health care program may try to defend against political opponents who would cherry-pick cases to attack those programs, e.g., by highlighting patients who undergo significant hurdles only to receive poor healthcare coverage or beneficiaries who have little need and are still awarded substantial welfare.

| Screener Bias: | Reducing Allocation Errors | Reducing Screening Costs |
|---|---|---|
| **Payoffs and Form:** | optimal delegation is *simple* and *perfect* | delegation is either *complex or futile* |
| **Information:** | optimal delegation is *robustly optimal* | no delegation set is *robustly improving* |

Figure 1: This table summarizes the main results for the benchmark setting.

To paraphrase the main result, the policymaker can easily align the screener's behavior if the latter cares more about keeping down allocation errors than she does. But aligning the screener's behavior is hard if he worries more about keeping down an applicant's screening costs (e.g., would prefer moving closer to a welfare rather than workfare scheme).

More precisely, Theorem 1 (summarized in Figure 1) shows that when the screener places more weight on reducing allocation errors than the policymaker, optimal delegation rules are *simple*: they take the form of intervals. An optimal rule for the policymaker is to set a floor for the screener at the lowest allocation level she would award, had she been the one designing the menu. This delegation set is *perfect*: the constrained screener can do no better than to choose the policymaker's favorite menu. Finally, this delegation set is *robust*: the policymaker can identify her optimal rule knowing only the direction of the screener's bias.

Theorem 2 shows that these results all fail when the screener places more weight on reducing screening costs. Unless the players' preferred menus coincide, delegation is either *futile*, i.e, no restriction to the agent's action space strictly improves over full delegation, or delegation is *complex*, i.e., strict improvements exist but none of them are simple. Moreover, a strict improvement over full delegation, even if it exists, can never be *robust*: there always exists a screener, biased in the same direction but to a different extent, for whom this restriction produces a worse outcome for the policymaker than full delegation would.

A rough intuition is that a bias for keeping down allocation errors versus screening costs corresponds to the screener wanting to offer a steeper or flatter menu than the policymaker. In the former case, the policymaker can compress the allowable allocation space from the bottom to force the screener to pick a more favorable menu. In the latter case, the policymaker has to stretch the screener's menu by perforating the allocation space. This can backfire if the allowable allocations are not precisely chosen.

The results imply that floors are min-max optimal and undominated when even the direction of the screener's bias is unknown. A practical implication is that the policymaker should always pressure the screener to worry more about allocation errors and set a floor, even if she cares about screening costs herself. If a screener still cares more about costs despite this, the floor would not bind and at least the gap in preferences shrinks. Otherwise, the floor ensures that the screener uses the policymaker's preferred menu all the same.

The next set of results generalize beyond the benchmark setting. Policymakers and screeners may disagree on many dimensions, such as the relative importance of type I or type II errors or what even constitutes an allocation error. Proposition 1 shows that even when both have distinct loss functions, setting a floor precisely as in Theorem 1 dominates full delegation for the policymaker. Proposition 2 shows that if allocation error losses are strictly convex and screening cost losses are linear, the same result holds in a model where the players care about expected losses. Setting floors is therefore a sound policy even in contexts where optimal mechanisms are hard to find.

## 1.1 Related Literature

Describing the role of dissipative costs for screening users of publicly funded healthcare services, Zeckhauser (2019) writes:

> Ordeals currently play a prominent and critical role in directing resources to high-value users... Unlike pricing, the primary instrument of resource allocation in developed societies, ordeals are scarcely studied, little understood, and often accepted without thought... conscious attention to their design and operation could greatly enhance that value.

One difference from pricing is that it is harder for a policymaker to know whether intermediaries are offering her intended menu when they screen through ordeals. So to give "conscious attention to their design", a policymaker needs to consider how an intermediary's ideal mechanism differs from her own. This paper studies how policymakers can mitigate this friction.

The results may speak to various applications where costly screens are used, like in innovation approval. Masur (2010) and De Rassenfosse and Jaffe (2018) consider how applications costs and high attorney fees can induce selection of worthwhile patents. Lemley and Shapiro (2005) even propose a system that "thinks of the process of issuing patents in terms of designing a mechanism", for example by "[letting] patent applicants select either the normal, brief examination process, which would lead to a Standard Patent if the application were approved, or a more rigorous application process, which would lead to Super Patent if the application were approved". Another application is targeting welfare, where researchers explored the efficacy of different ordeals (Alatas et al., 2016; Deshpande and Li, 2019) and compared workfare to welfare (Besley and Coate, 1992; Ravallion, 2009).

This paper belongs to the literature on delegation, which, starting with Holmstrom (1977, 1980), focuses on aligning the behavior of experts who are better informed (e.g., Frankel (2014)) or can more cheaply acquire information (e.g., Szalay (2005); Chade and Kovrijnykh (2016)). Here I focus on how to align the behavior of intermediaries whose advantage lies solely in being able to design richer mechanisms to elicit applicants' private information than the policymaker can design alone.[1]

This paper contributes in particular to the literature on delegated contracting. Hiriart and Martimort (2012) study a problem of delegating the choice of punishment to a biased regulatory agency to deter a firm from causing accidents. They focus on a moral hazard problem, whereas I study adverse selection.[2] Papers that consider adverse selection include Amador and Bagwell (2016), who study the classic Baron and Myerson (1982) monopolist regulation problem in a setting without transfers; Guo and Shmaya (2019), who study this problem when the regulator minimizes worst-case regret; and Kundu and Nilssen (2020) and Martimort et al. (2020) who study problems of regulating a regulator of a firm to mitigate bureaucratic drift (and so are closest to the motivation here). In these papers, the intermediary (e.g., monopolist) is given discretion because he is *ex-ante* better informed (e.g., about costs). I explore a different reason to delegate which arises when only an (equally uninformed) intermediary can contract on the evidence that an agent generates.

In the aforementioned monopoly regulation papers, the intermediary's choice is a single price-quantity pair; for example, Amador and Bagwell (2016) restrict the monopolist from using schemes like two-part tariffs. In Hiriart and Martimort (2012) too, a mechanism is a single punishment level in the event of an accident. Kundu and Nilssen (2020) and Martimort et al. (2020), on the other hand, restrict their principals to a simple class of delegation mechanisms. A contribution here is to study optimal delegated mechanism design without imposing simplifying restrictions on the delegation or contracting space.

This paper draws on the literature on mechanism design and collusion, where one agent (like the screener in my model) has all the bargaining power to offer side contracts; seminal papers include Laffont and Martimort (1998); Mookherjee and Tsumagari (2004); Faure-Grimaud et al. (2003); Celik (2009). In models with transfers, binary types, or perfectly informed intermediaries, these papers address whether delegation to a subcontractor can be as effective as directly contracting with all parties. I find that delegation is indeed optimal in my setting and focus on characterizing optimal rules.

To make progress on a delegated mechanism design problem, this paper takes a max-min approach, as in Hurwicz and Shapiro (1978), Chassang (2013), Frankel (2014), and Carroll (2015, 2017)[3]. It is especially related to those papers with min-max regret criteria like Bergemann and Schlag (2008) and Guo and Shmaya (2019), who respectively study robust monopoly pricing and monopoly regulation. Minimizing worst-case regret can be seen as a special case of the model where allocation error losses are linear.

[1] See Appendix C for a comparison between delegated screening and delegated information acquisition.

[2] Iossa and Martimort (2016) consider adverse selection and moral hazard. There, the principal delegates the choice between complete and incomplete contracts but designs both contracts herself.

[3] See Carroll (2019) for an overview of the robust mechanism design literature.

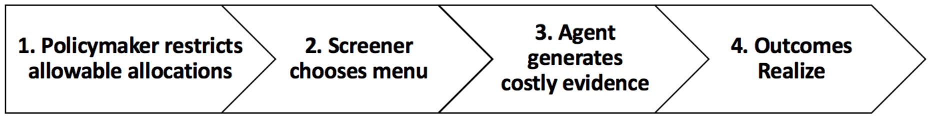


Figure 2: The game proceeds in four periods, with the policymaker committing to a delegation set first and the screener committing to a menu thereafter.

# 2 The Model

This section proposes the delegated costly screening game. Section 2.1 discusses how the model can be viewed from the lens of optimal mechanism design.

**Players, Actions and Timing** There is a policymaker (she), a screener (he), and an agent.

In the first period, the policymaker chooses a set of *allowable allocations*, which is *any* nonempty closed subset $Y$ of the $[0, 1]$ interval.

In the second period, the screener chooses a *menu* that maps costly evidence generated by the agent to an allocation in $Y$. That is, the screener is free to choose any right-continuous menu $y : \mathbb{R}_+ \to Y$.

In the third period, the agent generates costly evidence $n \in \mathbb{R}$, given its private information and the menu selected by the screener.

In the last period, payoffs of all agents are realized. The timing is summarized in Figure 2.

**Other Primitives and Information** The agent has a type $\theta \in \Theta$. The agent's type determines its marginal cost $c(\theta) > 0$ of generating costly evidence, and its marginal benefit $b(\theta) \in \mathbb{R}_+$ of receiving an allocation. The agent's payoff is normalized to 0 if it is allocated nothing. The agent's type also determines the type-specific losses that the policymaker and screener face from misallocating to the agent, $f_\theta : [0, 1] \to \mathbb{R}_+$. Finally, the agent's type determines the type-specific losses that the policymaker and screener face from the agent having to generate costly effort, $h_\theta : \mathbb{R}_+ \to \mathbb{R}_+$.

The type $\theta$ is the agent's private information, while $\Theta, c, b, f_\theta, h_\theta$ are common knowledge. The policymaker and screener evaluate their choices by considering their payoffs under their worst-case realization of the agent's type; it is immaterial for the results whether one interprets this as ambiguity aversion or just a statement about the players' objectives despite having full support priors over $\Theta$ (see Section 5.2.1 for more on the two interpretations).

I assume that the allocative loss functions $f_\theta$ are continuous and strictly quasiconvex for all $\theta \in \Theta$, so deviations from the unique optimal allocation level for each type lead to losses.

For example, policymakers and screeners may worry about disbursing too little to a relatively poor workfare applicant or too much to one who is relatively better off (alternatively, approving a bad innovation or rejecting a good one). Next, I assume that the screening cost loss functions $h_\theta$ are continuous and strictly increasing and that $h_\theta(0) = 0$ for all $\theta \in \Theta$. An interpretation is that the policymaker and screener partly internalize the strain of a workfare applicant engaged in low-productivity labor or the costs of a firm generating supporting evidence for a patent application. Finally, I assume that $\Theta$ is finite to simplify exposition and sidestep issues like the existence of optimal menus.[4]

**Agent's Objective** The agent produces an amount of costly evidence $n$ to solve:

$$\max_{n \in \mathbb{R}_+} b(\theta)y(n) - c(\theta)n.$$

Note that $b$ and $c$ need not be co-monotone. Let $N_y(\theta)$ denote the set of optimal evidence levels for a type $\theta$ agent facing menu $y$.

**Screener's Objective** The screener minimizes the worst-case weighted sum of losses due to allocation errors and screening costs, where the worst case is taken over the realization of the agent's type. In particular, the screener chooses a menu $y$ to solve:

$$\min_{y} \max_{\theta \in \Theta} \min_{n \in N_y(\theta)} \alpha f_\theta(y(n)) + (1 - \alpha) h_\theta(n),$$

where $\alpha \in (0, 1)$ is the screener's *weight on allocation errors*. The solution concept assumes that the agent breaks ties in favor of the screener when indifferent among multiple menu choices. Let $\mathcal{X}(Y)$ denote the set of screener optimal menus when she is restricted to menus with co-domain $Y$.[5] Among evidence levels in $N_y(\theta)$, let $n_y(\theta)$ denote the screener's favored choice for a type $\theta$ agent.

**Policymaker's Objective** The policymaker is also motivated by a desire to minimize worst-case weighted sum of losses. She differs from the screener only by the relative weights she places on the losses from allocation errors versus screening costs. She chooses a delegation set $Y \subset [0, 1]$ to solve:

$$\inf_{Y} \min_{y \in \mathcal{X}(Y)} \max_{\theta \in \Theta} \alpha_P f_\theta(y(n_y(\theta))) + (1 - \alpha_P) h_\theta(n_y(\theta))$$

[4] Appendix A extends the model to the case with infinitely many types.

[5] The screener's preference for reducing evidence production costs for the agent endogenously puts a cap on the largest evidence level that any agent will be required to show in equilibrium. This coupled with the facts that $Y$ is closed and the loss functions are continuous ensures that $\mathcal{X}(Y)$ is nonempty.

The policymaker's choice of delegation set, $Y$, changes the set of screener optimal menus, $\mathcal{X}(Y)$. Of these menus, the screener chooses the one that minimizes the policymaker's worst-case loss. Note that when $\alpha_P \neq \alpha$, the worst-case type realization for the policymaker need not coincide with the worst-case type realization for the screener.

The preference misalignment can go in either direction. For example, policymakers and local officials may stand on different sides of the workfare versus welfare debate, depending on whether they prioritize better targeting or keeping down applicant screening costs. Regulators in the Food and Drug Administration (FDA) may worry primarily about approval errors, while lawmakers might also be concerned with trial costs that are partly passed onto consumers. On the other hand, while regulators in the Federal Aviation Administration (FAA) may be concerned with imposing too costly and intrusive a screening process for Boeing, lawmakers may primarily care about the safety of approved flights.

## 2.1 Discussion

**Optimality of Delegated Screening** The delegated screening game can be seen as an implementation of the policymaker's optimal mechanism (without transfers) in a more general contracting environment. Suppose (1) the policymaker can design any centralized communication protocol mapping the screener's and agent's messages to an allocation for the agent, (2) the screener can subsequently offer a side contract mapping the evidence the agent produces to a message that the screener tells the policymaker, (3) the agent observes the communication protocol and side contract before choosing an evidence level and message. Then an analogue of the taxation principle implies that it is without loss of generality for the policymaker to restrict to delegated screening mechanisms in her search for optimal mechanisms; see Appendix B.

**Frictions** This mechanism design perspective clarifies the two main frictions. First, the type of the agent is hidden to everyone but the agent, which is the usual friction in screening problems. Second, the screener can observe and contract on the evidence produced by the agent, while the policymaker cannot. In light of these two frictions, the apparent absence of communication between the policymaker and agent in delegated screening mechanisms is not an additional handicap to the policymaker.

**No Transfers** The model assumes no transfers between the policymaker and screener for a few reasons.

First, it may be difficult for the policymaker to pay the screener based on the outcomes of the screening process. Policymakers in a central government may not get a clear picture of how bureaucrats implement a national workfare program in different locations. Similarly, incentivizing patent officers or consumer protection regulators can be difficult when the efficacy of approved innovations (and more so, rejected ones) can take years to understand.

Next, even if outcomes can be observed, a policymaker may wish to avoid giving bureaucrats high-powered incentives on that basis when their jobs involve more than just designing the screening process (Holmstrom and Milgrom, 1991).

Finally, one need not interpret the inability of the principal to offer transfers in the model literally: it may be possible and beneficial for the policymaker to monitor outcomes and award contingent transfers. But these are costly measures. Studying the delegation model reveals when the bureaucrat's behavior is costly to align and when it is not.

# 3 The Screener's Problem

This section presents a characterization of screener optimal menus that is key for solving the policymaker's delegation problem in Section 4.

## 3.1 Implemenetability and Reformulating the Screener's Problem

We start by reducing our search for optimal menus to a more manageable set. The analysis is standard after accounting for the fact that the agent has a two-dimensional type but is screened along a single dimension.

**Effective Types** Since agent types vary in both costs of producing evidence and benefits of receiving allocations, different types can have the same preferences over the screener's menu. Define $\tau(\theta) \equiv \frac{b(\theta)}{c(\theta)}$ to be the *effective type* of a type $\theta$ agent. Let $\Theta_\tau \subset \Theta$ be the set of all types $\theta$ with effective type $\tau$, and let $\mathcal{T}$ denote the set of all effective types. Let $\underline{\tau}$ and $\overline{\tau}$ denote the lowest and highest effective types, respectively. All types in $\Theta_\tau$ for a given $\tau \in \mathcal{T}$ have identical preferences over the screener's menu (see the left panel in Figure 3).[6]

It is convenient to assume the following tie-breaking rule: for every $\tau$, all types in $\Theta_\tau$ break indifferences among choices in the screener's menu in the same way[7]

**Revelation Principle** We can now apply the *revelation principle* to the one-dimensional space of effective types, $\mathcal{T}$. Abusing notation, the screener effectively chooses an *allocation*

[6]Consider two menu items $(n, y(n))$ and $(n', y(n'))$, and two types of agents $\theta$ and $\theta'$ such that $\frac{b(\theta)}{c(\theta)} = \frac{b(\theta')}{c(\theta')}$. Note that, $b(\theta)y(n) - (\theta)n \geq b(\theta)y(n') - c(\theta) \iff \frac{b(\theta)}{c(\theta)}y(n) - n \geq \frac{b(\theta)}{c(\theta)}y(n') - n' \iff \frac{b(\theta')}{c(\theta')}y(n) - n \geq \frac{b(\theta')}{c(\theta')}y(n') - n' \iff b(\theta')y(n) - c(\theta')n \geq b(\theta')y(n') - c(\theta')n'$.

[7]The results hold for any tie-breaking convention, but this assumption ensures that the mapping from effective types to allocations is a function rather than a correspondence, which simplifies exposition.

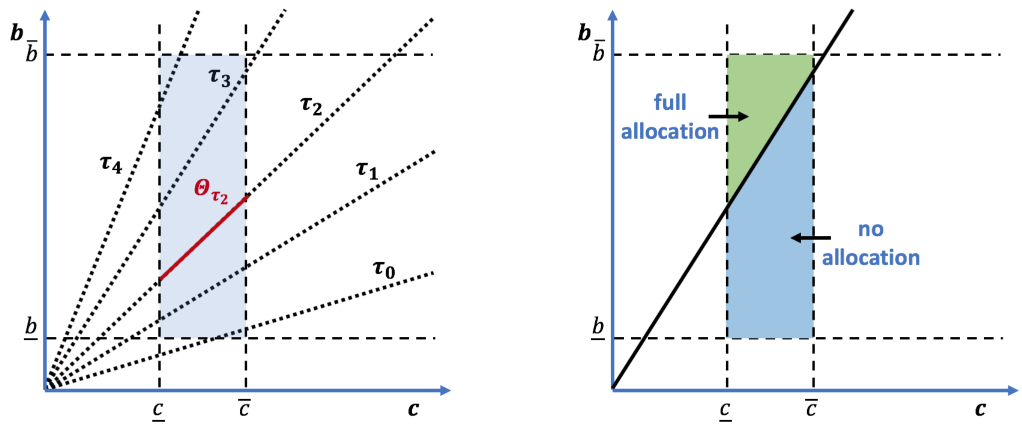


Figure 3: In this example, the agent's costs lie between $\underline{c}$ and $\bar{c}$ and benefits between $\underline{b}$ and $\bar{b}$. The left figure shows iso effective type curves. The highlighted line segment in red consists of all types with effective type $\tau_2$. The right figure depicts participating (green) and nonparticipating (blue) types under a cutoff allocation rules which awards full allocation if the agent produces a certain amount of evidence and no allocation otherwise. Types with sufficiently low benefits or sufficiently high costs choose not to produce evidence.

*rule* and *standard of proof*, $(y, n) : \mathcal{T} \to [0,1] \times \mathbb{R}_+$, that satisfy incentive compatibility (IC): for all $\tau, \tau' \in \mathcal{T}$,

$$\tau y(\tau) - n(\tau) \geq \tau y(\tau') - n(\tau') \tag{IC}$$

An allocation rule $y$ is *implementable* if there exists some standard of proof $n$ such that $(y, n)$ is incentive compatible. We have the familiar characterization of implementability:

LEMMA **1** (Myerson's Lemma)**.** *An allocation rule is implementable if and only if it is non-decreasing.*

Let $\mathcal{I}$ denote the set of implementable allocation rules, and let $\mathcal{I}_Y$ denote the implementable allocation rules with co-domain $Y \subset [0,1]$. Finally, let $\mathcal{X}(Y)$ now refer to the set of screener optimal *allocation rules* in $\mathcal{I}_Y$.

**Optimal Standard of Proof** For an allocation rule $y \in \mathcal{I}_Y$, let $y_0 < \ldots < y_k$ denote the set of allocation levels in its range. For $i < k$, let $\tau_i$ denote the highest effective type $\tau \in \mathcal{T}$ for which $y(\tau) = y_i$. Define a standard of proof $n$ inductively: $n(\tau_0) = 0$, and $n(\tau_{i+1})$ is set so that upward IC constraints bind, i.e., all agents in $\Theta_{\tau_i}$ are indifferent between $(y_i, n_i)$ and $(y_{i+1}, n_{i+1})$. Among all standards of proof that implement $y$ and for any allocation level in its range, $n$ requires the least evidence. The screener's choice of allocation rule $y$ therefore pins down $n$ in this way.

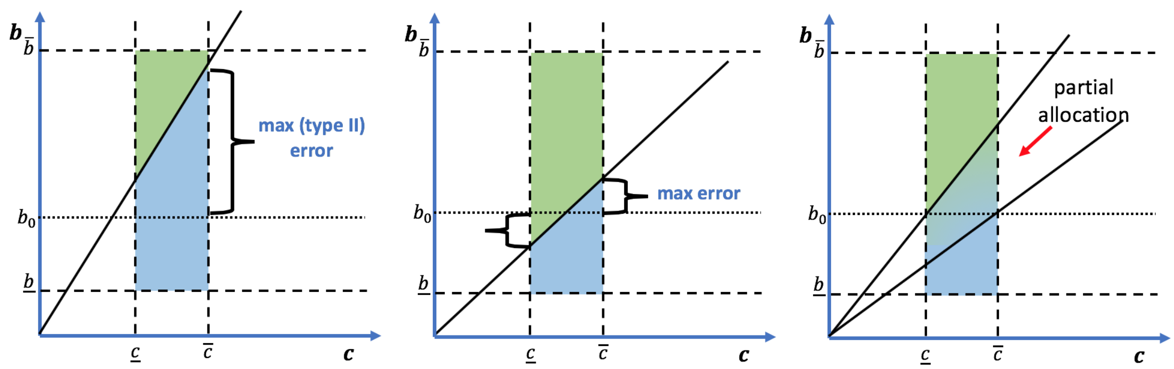


Figure 4: The first two panels depict cutoff allocation rules: full allocation if the agent shows sufficient evidence and no allocation otherwise. The types in green, with low costs or high benefits, choose to submit the requisite evidence; the types in blue do not. With lower evidence requirements (center panel), the screener reduces the worst-case error from failing to allocate to types with benefits above $b_0$. But he introduces the possibility of allocating to a type with benefits below $b_0$. The screener can reduce worst-case errors further by partially allocating to intermediate effective types, as depicted in the third panel.

**Screener's Problem and Policymaker's Problem** We can now re-express the screener's and objective in a more manageable form. The screener chooses $y \in \mathcal{I}_Y$ to minimize:

$$R(y) \equiv \max_{\tau \in \mathcal{T}} R_\tau(y) \equiv \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha f_\theta(y(\tau(\theta))) + (1-\alpha) h_\theta(n(\tau(\theta))) \tag{1}$$

A solution to this minimization problem is a *constrained optimal allocation rule for the screener.*

**Example** Figure 4 shows a simple example where under complete information, the screener would fully allocate to types who have a benefit above $b_0$ and allocate nothing otherwise. However, types are private information and agents also differ in their costs of submitting evidence. Those with lower (higher) costs obtain larger (smaller) allocations, whether the screener wants this or not. Losses due to misallocations are greater when the agent's benefits are further from $b_0$. In particular, the screener chooses an allocation rule $y$ to solve

$$\min_{y \in \mathcal{I}} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha \max\{(1-y(\tau))(b(\theta)-b_0), y(\tau)(b_0 - b(\theta))\} + (1-\alpha) h_\theta(n(\tau)).$$

Here, $f_\theta(y) \equiv \max\{(1-y)(b(\theta)-b_0), y(b_0 - b(\theta))\}$. The screener stands to gain by using partial allocations since they minimize the size of the largest error he can make.

## 3.2 Screener Optimal Menus

The last panel of Figure 4 suggests that, as in the expected utility case, min-max optimal menus need not take on a “simple” form. They can be nonlinear even with interval spaces and linear loss functions (e.g., where there is no need for ironing). Still, we can identify certain useful properties that they must always satisfy.

**Example** Consider an example with two types of agents, $\tau = l$ or $h$, with $0 < l < h$. Suppose that if types were common knowledge, the screener would allocate each type $y_l^*$ or $y_h^*$ respectively, where $0 \leq y_l^* < y_h^*$. Finally, suppose that the screener's losses due to allocation errors are quadratic, and his losses due to agent screening costs are linear, i.e., the screener solves:

$$\min_{y \in \mathcal{I}} \max_{\tau \in \{l,h\}} \alpha(y_\tau - y_\tau^*)^2 + (1-\alpha)n_\tau.$$

Whatever $y$ is chosen, the screener sets $n_l = 0$ and $n_h = l(y_h - y_l)$, so that no evidence is required for the lower allocation and the requisite evidence for the higher allocation leaves the low type indifferent between the two menu items.

This immediately implies that $y_l > y_l^*$, i.e., that the screener *over-allocates* to the low type agent. Were this not the case, the screener would bear no losses on the low type, from either allocation errors (since $n_l = 0$) or screening costs (since $y_l = y_l^*$). Facing a type $h$ agent would be his worst-case scenario, since for any choice of $y_h$, he would either face allocation errors ($y_h \neq y_h^*$) or screening costs ($y_h > y_l = y_l^*$). But then his loss due to screening costs for the high type could be brought down by increasing $y_l$ (since $n_h = l(y_h - y_l)$).[8]

The optimal menu would require distorting the allocation to $l$ upward until the losses for facing either type were equalized. A screener who cares less about keeping down allocation errors and more about keeping down screening costs would rely on this distortion more heavily. That is, $y_l$ is *decreasing* in $\alpha$.

This logic example extends more generally.

**Terminology** Let $y \in \mathcal{I}$, and let $\mathcal{T}_0 = \{\tau \in \mathcal{T} : y(\tau) = y_0\}$. The allocation rule $y$ *attains its worst-case payoff on* $y_0$ if $\sup_{\tau \in \mathcal{T}_0} R_\tau(y) = R(y)$.

The lowest allocation level, in $y$ (i.e., $y(\underline{\tau})$) is the *free option*, as $n(\underline{\tau}) = 0$ in the optimal standard of proof.[9]

Finally, we say *screening costs matter* if dropping the losses from screening costs strictly reduces the screener's loss:

$$\min_{y \in \mathcal{I}_Y} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha f_\theta(y(\tau(\theta))) + (1-\alpha)h_\theta(n(\tau(\theta))) > \min_{y \in \mathcal{I}_Y} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha f_\theta(y(\tau(\theta))).$$

[8] Or if $y_h = y_l$, so the implementability constraint is binding, allocation errors losses on the high type can be reduced by increasing both $y_l$ and $y_h$ in tandem.

[9] There may be $\tau > \underline{\tau}$ which are also allocated the free option.

Screening costs would not matter, for example, if the optimal allocation rule is constant for any $\alpha$.

We can now state necessary conditions that min-max optimal menus must satisfy when the screener is restricted to an interval of allowable allocations.

Lemma **2.** *Suppose the set of allowable allocations $Y$ is connected. Let $y$ be a constrained optimal allocation rule for the screener, and let $y_0$ denote the free option in $y$.*

1. *The screener attains his worst-case payoff on $y_0$.*
2. *If screening costs matter, then $y_0 > 0$.*
3. *The size of the free option is equal across all screener-optimal allocation rules,*
4. *The size of the free option is weakly decreasing in the screener's weight on allocation errors.*[10]

**Interpretation** Note that Lemma 2, part 1 leaves open the possibility that the screener may also realize his worst-case payoff with some types who would choose larger allocations from his menu. However, Lemma 2, part 1 and part 3 together suggest that any constraint on the allocation space that precludes him from offering his ideal free option necessarily increases his worst-case loss.

Lemma 2, part 2 has the clearest implication for the design of costly screens when the agent's costs matter to the screener: even if introducing certain costs induce advantageous selection, their value is always enhanced by giving away some part of the allocation for free. The positive effect of reducing the evidence procurement burden on types who receive larger allocations initially outweighs the allocation errors that may arise, if any.

An example would be a welfare program applicant whose readily observable traits neither outright qualify or disqualify him from receiving benefits. Suppose the applicant's willingness to participate in a costly appeals process is typically an indication of greater need. One design the screener may adopt is to ask all borderline candidates to either participate in the appeals process or give up on their application. Lemma 2 suggests that the program should instead have options for such an applicant to either (1) walk away with minimal benefits (alternatively, with lower quality in-kind transfers), or (2) participate in a less stringent appeals process to receive more benefits.[11]

[10] All proofs are in Appendix E.

[11] In the context of innovation approval processes, Lemma 2 suggests including a regulatory sandbox, where ex-ante qualified firms can freely test or market their products in a restricted setting or show costly evidence to obtain greater approval from regulators.

# 4 Main Results: Simplicity, Effectiveness and Robustness of Delegation

This section uses the characterization of the unconstrained screener's optimal allocation rules from Section 3 to solve the policymaker's problem. Section 4.1 considers the case where the screener is biased towards minimizing allocation errors, while Section 4.2 considers the case where the screener is biased towards keeping down screening costs. The results aim to answer the following questions: When is the optimal delegation set just an interval, and when is it a more complex set? Can these restrictions be designed in such a way that the screener decides to offer the policymaker's favorite menu? How much does the policymaker need to know about the screener's preferences to design these restrictions?

**Policymaker's Problem** The policymaker chooses a closed delegation set $Y \subset [0,1]$ to minimize:

$$\min_{y \in \mathcal{X}(Y)} R^P(y) \equiv \min_{y \in \mathcal{X}(Y)} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha_P f_\theta(y(\tau(\theta))) + (1-\alpha_P) h_\theta(n(\tau(\theta))) \tag{2}$$

**Terminology** Recall that $\alpha$ and $\alpha_P$ are the weights that the screener and policymaker place on allocation errors, respectively.

The screener is *biased towards reducing allocation errors* if $\alpha > \alpha_P$. And the screener is *biased towards reducing screening costs* if $1-\alpha > 1-\alpha_p$.

If the set of screener optimal allocation rules in $\mathcal{I}_Y$ and policymaker optimal allocation rules in $\mathcal{I}$ intersect for some $Y$, then delegation set $Y$ is *perfect*.

*Delegation is futile* if full delegation is optimal for the policymaker but not perfect. In other words, there are no restrictions that the policymaker can set in place to improve the screener's behavior.

A delegation set $Y$ is *simple* if it is connected. A particular simple delegation set is *full delegation*: $Y = [0,1]$. The policymaker chooses *no delegation* if $Y$ is a singleton.

*Delegation is complex* if full delegation is not optimal for the policymaker, but no simple delegation set strictly improves (i.e., reduces the policymaker's loss as given by eq. (2)) over full delegation.

For a fixed $\alpha_P$, a delegation set $Y$ is *robustly optimal* if it solves the policymaker's problem either for any $\alpha > \alpha_P$ or all $\alpha < \alpha_P$. This captures the notion that all the policymaker needs to know about the screener's preferences is contained in just the direction of his bias.

For a fixed $\alpha_P$, a delegation set $Y$ is *robustly improving* if it is a strict improvement over full delegation for some $\alpha > \alpha_P$ ($\alpha < \alpha_P$) and a weak improvement over full delegation for all $\alpha > \alpha_P$ ($\alpha < \alpha_P$). Full delegation is dominated for a policymaker who knows the direction of the screener's bias and has robustly improving delegation sets at his disposal.

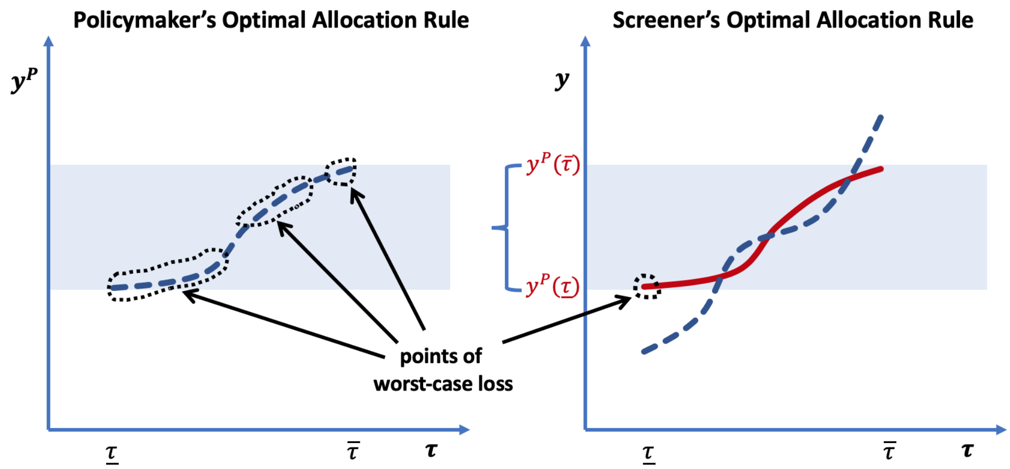


Figure 5: The blue curve on the left panel is a policymaker optimal allocation rule, $y_P$. The blue curve on the right panel is an optimal allocation rule for the unconstrained screener. Restricted to allocation rules with co-domain $[y_P(\underline{\tau}), y_P(\overline{\tau})]$, the shaded blue region, the screener finds $y_P$ to be optimal (red curve).

## 4.1 Screener is Biased Towards Reducing Allocation Errors

Theorem 1 characterizes optimal delegation when the screener is biased towards reducing allocation errors.

Theorem 1. *Suppose the screener is biased towards reducing allocation errors. Let $y_P$ be an optimal allocation rule for the policymaker, and let $Y \equiv [y_P(\underline{\tau}), 1]$. $Y$ is a simple, perfect and robustly optimal delegation set for the policymaker.*

When $\alpha > \alpha_P$, Theorem 1 shows that optimal delegation has a very simple form. The policymaker simply restricts the screener from awarding smaller allocations than she would ever award, had she directly been in charge of screening. That is, if $y_P$ is optimal for the policymaker, she sets a floor at $y_P(\underline{\tau})$. With this restriction, $y_P$ is also optimal for the screener in the set of allowable allocation rules, and this result holds regardless of how much larger $\alpha$ might be than $\alpha_P$.

The intuition for this result is easiest to see in the case where the policymaker's optimal allocation rule is fully separating in effective types, as shown in Figure 5. The screener's unconstrained optimal allocation rules typically have a smaller free option (less upward distortion on low effective types) than the policymaker's optimal allocation rule (Lemma 2, part 4). Once the screener is forced to use an allocation rule with a larger free option, the screener has no incentive to increase this upward distortion even further.

Next, when adopting the policymaker's favorite screening rule, the screener only faces worst-case losses at the lowest effective type. Elsewhere, up to a re-scaling of his objective function, the screener faces smaller costs than the policymaker. So any other approval rule with a free option of the same size can only have a weakly larger worst-case loss for the screener, as he is already weighed down by the loss faced on the lowest effective type.[12]

This last argument does not work when the screener optimal rule pools multiple effective types at the lowest allocation level. The proof extends the argument to cover this case as well.

## 4.2 Screener is Biased Towards Reducing Screening Costs

The next theorem shows that the results on the simplicity, effectiveness and robustness of delegation flip when $\alpha < \alpha_P$. To begin with, the optimal delegation set is typically a non-interval set and fails to be robustly optimal.

But the problem runs deeper. In fact, *any* restriction that improves over full delegation is a non-interval set. And moreover, *any* improvement over full delegation for some $\alpha < \alpha_P$ is guaranteed to worsen the policymaker's outcome if the screener's weight on allocation errors was instead some other $\alpha' < \alpha_P$. There are therefore no easy restrictions that the policymaker can impose to improve her outcome over the status-quo option of fully delegating to the screener.

Theorem **2.** *Suppose the screener is biased towards reducing screening costs.*

1. *If the unconstrained screener's optimal allocation rules do not coincide with the Policymaker's, delegation is either complex or futile.*

2. *No delegation set is robustly improving.*

For intuition, first note that the policymaker's ideal menu now has a smaller free option than the screener's ideal menu (Lemma 2, part 4). Binding floors clearly do not help the policymaker, since they only increase the size of the screener's free option.

A binding cap that still allows the screener to have the same free option that he would have in his ideal menu cannot help. Such a cap would (weakly) increase the screener's worst-case loss. The screener would then try to at least alleviate agent screening costs by picking a menu with a larger free option, a counterproductive move from the policymaker's perspective.[13]

[12] Note that the policymaker's ideal screening rule is one constrained optimal rule for the screener, though there may be others. See Appendix D for a discussion of selection criteria among the set of screener constrained optimal allocation rules.

[13] This still leaves open the possibility that the policymaker benefits by setting a cap so stringent that none of the allocations in the screener's unconstrained optimal menu are allowable. The proof in the appendix argues why such a cap only makes matters worse.

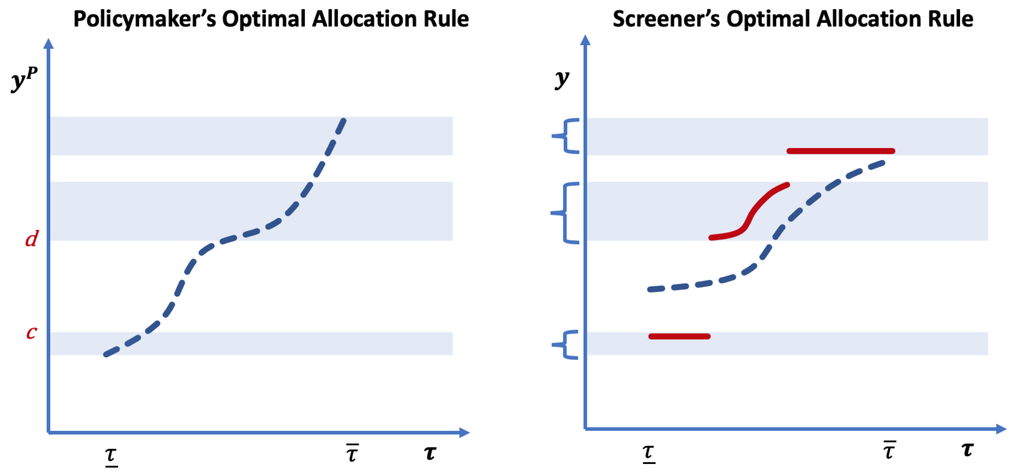


Figure 6: The blue curve on the left panel is a policymaker optimal allocation rule, $y_P$. The blue curve on the right panel is an optimal allocation rule for the unconstrained screener, $y$. When restricted to allocation rules with co-domain in the delegation set shown, the screener may find an allocation rule with a smaller free option, such as the red curve, to be optimal.

This reasoning suggests a necessary condition that all strict improvements over full delegation share. The only way that the policymaker might improve over full delegation is by using a delegation set which excludes some open interval $(c, d)$ about $y(\underline{\tau})$, as shown in Figure 6. If the delegation set is chosen correctly, the screener may choose a menu with a discontinuously *smaller* free option (than in his unconstrained optimal menu) over a menu with a discontinuously larger free option.

But crafting such a delegation set is finicky business. We next argue that any delegation set which might improve over full delegation for some $\alpha > \alpha_P$ can make matters worse for some other $\alpha' > \alpha_P$.

There are two ways that removing an interval $(c, d)$ can backfire. Consider the optimal allocation rule for a screener who places weight $\alpha' \approx 0$ on allocation errors. Such a screener is primarily concerned about allocation costs, and therefore uses a (nearly) flat allocation rule at some level $y_0$. The two cases to consider are $y_0 \in (c, d)$ and $y_0 \geq d$, as pictured in Figure 7.[14]

If $y_0 \in (c, d)$, then removing this interval forces the type $\alpha'$ screener to pick the flat allocation rule at $c$ or $d$ instead (see Figure 7). But among the constant allocation rule, the type $\alpha'$ screener chooses the one that minimizes allocation errors. Therefore, removing the interval $(c, d)$ makes the policymaker worse off when facing a type $\alpha'$ screener.

[14]Lemma 2, part 4 rules out the case where $y_0 < c$: since the optimal allocation rule of a type $\alpha$ screener has a free option in $(c, d)$, the free option for the type $\alpha'$ screener must be weakly larger.

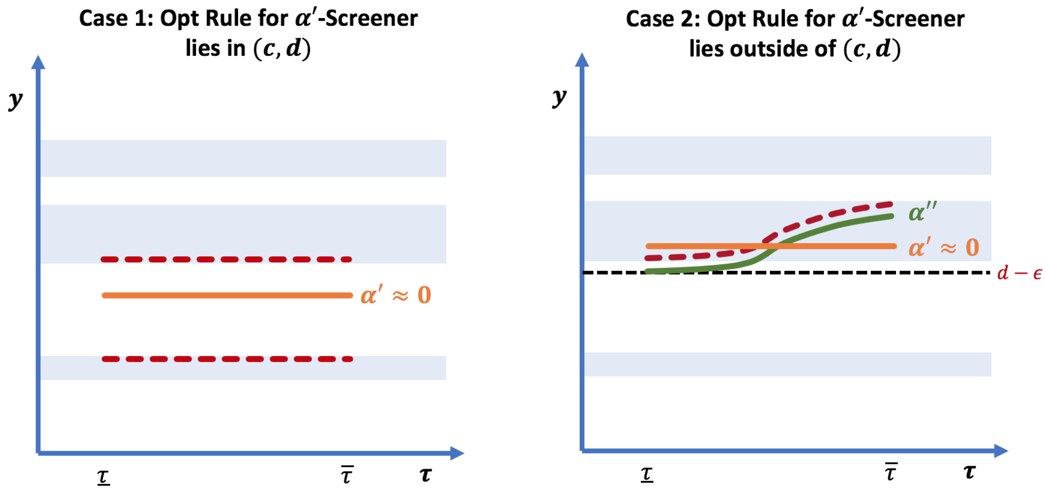


Figure 7: In the first case, an $\alpha'$ type screener moves chooses one of the two dashed red lines after the policymaker's restriction is imposed. In the second case, an $\alpha''$ type screener moves to the dashed red line after the restriction. The free option is larger in this new allocation rule, making the policymaker worse off.

If $y_0 \geq d$, this allocation level may or may not belong to the policymaker's delegation set. If it does belong, then the type $\alpha'$ screener no longer delivers the desired counterexample. Instead, note that there is one screener type who would choose a free option in $(c, d)$ and another type who would choose a free option in $[d, 1]$. Therefore, there is a screener type who would choose a free option for every level in between.[15] So an unconstrained screener who would choose a free option of size $d - \epsilon$ would react to the policymaker's delegation rule by choosing an allocation rule with a free option of size $d$ or greater rather than drop down discontinuously to $c$, as shown in Figure 7. But increasing the size of the free option makes the policymaker worse off; she is better off not constraining such a screener.

Theorem 2, part 1 says that even if the policymaker is certain about $\alpha$, simple interval restrictions would not improve outcomes from her perspective. Theorem 2, part 2 implies that knowing only the direction of the screener's bias is grossly insufficient for the policymaker to be able to enact positive change.

[15]The mapping from screener types to the size of the free option under optimal allocation rules is continuous. This follows from the continuity of the screener's value function in $\alpha$.

## 4.3 Screener Private Information

The results so far discuss properties of min-max optimal delegation sets when the policymaker knows the relative weight, $\alpha$, that the screener places on allocation errors. In some cases, the policymaker might not know $\alpha$ or even the direction of the screener's bias. Here we solve for the policymaker's min-max optimal delegation set, taking worst case over both the screener's type and the agent's type.

Let $\mathcal{X}(Y,\alpha)$ denote the set of screener optimal menus when her private type is $\alpha$ and she is restricted to menus with co-domain $Y$. The policy maker chooses a delegation set $Y$ to solve:

$$\inf_{Y} \min_{y\in\mathcal{X}(Y,\alpha)} \max_{\alpha\in[0,1]} \max_{\theta\in\Theta} \alpha_P f_\theta(y(n_y(\theta))) + (1-\alpha_P)h_\theta(n_y(\theta))$$

A set $Y$ which solves this objective is *min-max optimal with respect to the screener's type.*

A delegation set $Y$ is *undominated* if there is no delegation set $Y'$ that gives the policymaker weakly lower loss for some $\alpha$ and $\theta$ and strictly lower loss for some $\alpha'$ and $\theta'$.

Corollary **1.** *Let $y_P$ be an optimal allocation rule for the policymaker, and let $Y = [y_P(\underline{\tau},1)]$. $Y$ is undominated and min-max optimal with respect to the screener's type for the policymaker.*

That this floor is undominated follows from Theorem 1 (i.e., the fact that it is optimal when the screener is biased towards reducing allocation errors) and Theorem 2, part 2 (i.e., the fact that no other restriction strictly dominates full delegation when the screener is biased towards reducing screening costs; and since the floor is not binding in this case, it is effectively full delegation). Min-max optimality is a weaker condition and follows from one additional observation: for any delegation set the policymaker chooses, her worst-case scenario happens when the agent cares only about keeping down screening costs (i.e., $\alpha = 0$). A delegation set that does not alter such a screener's choice is min-max optimal, and the floor is one such delegation set.

## 4.4 Implications for Regulatory Governance

Theorem 1 and Theorem 2 present a stark dichotomy in the difficulty of delegation between the cases where the screener is more biased towards minimizing approval errors versus screening costs. Depending on the context in which they find themselves, policymakers may or may not be able to effectively delegate the task of costly screening.

However, policymakers often have many coarse tools for altering the incentives of regulators, even if they abstain from offering high-powered contracts out of concern for adverse reactions.

For example, Congress delegates authority to regulatory agencies and partially shapes its objectives by determining the agency's mandate. They can make the agency's stated objective to ensure the safety of innovations or they may declare that the agency should consider both consumer protection and costs of regulated business.[16]

We can think of a coarse tool as the policymaker having the ability to increase or decrease $\alpha$ by some $x\%$. How should the policymaker use this additional lever in conjunction with her capacity to restrict the space of allocation rules?

**Result.** *A robust policy for the policymaker is to increase $\alpha$ by $x\%$ and set a floor at the minimum of her optimal allocation rule, $y_P(\underline{\tau})$.*

If $(1 - \frac{x}{100})\alpha$ is greater than $\alpha_P$, then this policy allows the screener to achieve her first best allocation rule.

But even if $(1 - \frac{x}{100})\alpha < \alpha_P$, the screener is better off. The floor is nonbinding by Lemma 2 part 4, so effectively, the policymaker maintained full delegation while decreasing the extent of preference divergence between herself and the screener. Again, she ends up better off. This result is driven by the fact that preference divergence is inconsequential in one direction but not the other.

An implication for innovation approval processes is that regulators should not be incentivized to care about screening costs for regulated firms, even if policymakers themselves care. Emphasizing consumer protection in a regulatory agency's mandate while limiting its capacity to avoid approval errors is a better form of governance than holding it accountable for approval costs directly. In the latter situation, policymakers are forced to either suffer the costs of full delegation or make restrictions that have the potential of backfiring if the agency's preferences are misestimated.

# 5 Extensions: The Case for Setting Floors

This section extends the model in different directions. Section 5.1 considers fully general differences in objectives between the policymaker and the screener. Section 5.2 considers the model of delegated screening where the players are Bayesian and minimize expected rather than worst-case loss. In both extensions, setting floors is a simple and robust policy that always dominates full delegation.

[16]To give another example, policymakers can automate certain processes to alter the incentives of bureaucrats to patiently review multiple appeals decisions by the same applicant.

## 5.1 General Divergence in Objectives

Section 2 and Section 4 model the disagreement between the policymaker and the screener as arising solely from placing different weights on allocation errors and screening costs. This may be a salient friction in many contexts where costly screening is delegated, as discussed in the introduction.

But other disagreements are also possible. Consumer protection regulators may be more worried about type I errors than type II errors, where policymakers might have a more balanced weight on both. Alternatively, bureaucrats may be biased toward extending welfare benefits to applicants, while policymakers are more discerning.[17]

We can extend the model to capture fully general preference divergence, imposing only that both the policymaker and screener are still averse to screening costs.

Let $\{f_\theta\}$ and $\{h_\theta\}$ denote the allocation loss functions for the screener as before. But now let $\{f_\theta^P\}$ and $\{h_\theta^P\}$ denote the corresponding loss functions for the policymaker, where $f_\theta$ is strictly quasiconvex and $h_\theta$ is strictly increasing for all $\theta \in \Theta$.

As before, the screener chooses $y \in \mathcal{I}_Y$ to minimize:

$$R(y) \equiv \max_{\tau \in \mathcal{T}} R_\tau(y) \equiv \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha f_\theta(y(\tau(\theta))) + (1-\alpha) h_\theta(n(\tau(\theta))) \tag{3}$$

The policymaker chooses a closed delegation set $Y \subset [0,1]$ to minimize:

$$\min_{y \in \mathcal{X}(Y)} R^P(y) \equiv \min_{y \in \mathcal{X}(Y)} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha_P f_\theta^P(y(\tau(\theta))) + (1-\alpha_P) h_\theta^P(n(\tau(\theta))) \tag{4}$$

We refer to this as *the general preference divergence formulation of the model.*

Even at this generality, the policymaker has a policy at her disposal that can only make her weakly better off and does not require knowing anything about the screener's preferences. She simply considers her optimal allocation rule and sets a floor at the minimum of the range for that rule. If the floor is not binding, there was no harm in applying it. If it is binding, she is better off for it.

PROPOSITION 1. *Consider the general preference divergence formulation of the model. Let $y_P$ be an optimal allocation rule for the policymaker. Setting a floor at $y_P(\underline{\tau})$ is a weak improvement over full delegation for the policymaker.*

## 5.2 Delegated Costly Screening in the Expected Loss Case

Consider the expected loss analog of the problem, where both the screener and the policymaker with a common prior, letting $p_\theta$ be the probability that the agent is of type $\theta$. Let $u, u^P : \mathbb{R}_+ \to \mathbb{R}_+$ be increasing, concave functions.

[17] See Prendergast (2007) for a discussion on various forms of biases that bureaucrats may have relative to policymakers.

The screener chooses $y \in \mathcal{I}_Y$ to minimize:

$$B(y) \equiv \sum_{\tau \in \mathcal{T}} \sum_{\theta \in \Theta_\tau} p_\theta u(\alpha f_\theta(y(\tau(\theta))) + (1-\alpha) h_\theta(n(\tau(\theta)))) \tag{5}$$

The policymaker chooses a closed delegation set $Y \subset [0,1]$ to minimize:

$$\min_{y \in \mathcal{X}(Y)} \sum_{\tau \in \mathcal{T}} \sum_{\theta \in \Theta_\tau} p_\theta u^P(\alpha_P f_\theta^P(y(\tau(\theta))) + (1-\alpha_P) h_\theta^P(n(\tau(\theta)))) \tag{6}$$

Notice that we allow the preference divergence between the policymaker and the screener to be fully general, as in Section 5.1. We call this *the expected loss formulation of the model*.

### 5.2.1 Relationship to the Max-Min Problem

There are two ways of going from the expected loss to the max-min formulation of the problem, and each has different interpretations.

One way is to take the curvature of $u$ to infinity so that the players are infinitely risk-averse in the max-min case. While they have a prior over the state space, the worst-case loss is particularly salient in determining their choices. For example, regulators and politicians may worry about the poor optics of very deserving candidates being given insufficient care under a public healthcare program or undeserving candidates getting too many benefits. Media sources and political opponents to the program may have incentives to cherry-pick evidence about the program's efficacy and report the most egregious errors.

Another way of reaching the model of in Section 2 is to replace the double summation and priors with a max over the realization of agent types. This can be interpreted as a model of an agency making decisions in the face of large uncertainties, as is the case when regulators are charged with approving innovations.

### 5.2.2 Floors are (Weakly) Robustly Improving in the Linear Costs Case

Optimal delegation need not be simple in the expected loss case, even when $\alpha > \alpha_P$. The easiest way to see this is to consider a three type case. Even if the cap and floor are set optimally, the screener's optimal allocation to the middle type generally will not coincide with the policymaker's choice.

Still, when the loss functions satisfy some additional conditions, setting floors in the same way as Proposition 1 is at least robustly improving.

Proposition 2. *Consider the expected loss formulation of the model. Suppose the allocation loss functions $\{f_\theta\}_{\theta \in \Theta}$ and $\{f_\theta^P\}_{\theta \in \Theta}$ are strictly convex*[18]*, and $\{h_\theta\}_{\theta \in \Theta}$ and $\{h_\theta^P\}_{\theta \in \Theta}$ are*

[18] Strict convexity rather than strict quasiconvexity is required to retain the notion of a 'right' allocation

*linear*[19]*, and both the policymaker and screener are risk-neutral (i.e., $u$ and $u^P$ are linear). Let $y_P$ be an optimal allocation rule for the policymaker. Setting a floor at $y_P(\underline{\tau})$ is a weak improvement over full delegation for the policymaker.*

While the benchmark model focuses on examples where max-min analysis is applicable, other situations of delegated costly screening involve neither ambiguity nor worst-case payoffs. Proposition 2 shows that the same simple and robust policy can be used to improve outcomes in this case as well.

# 6 Conclusion

When policymakers create a new welfare program or an innovation approval process, various intermediaries invariably take control of certain aspects of the design. Screening downstream agents is one of those aspects. This paper introduces a model of delegated costly screening to study when and how a policymaker can influence a screener's choices.

Two themes emerge from the analysis. First, aligning the behavior of a screener who cares more about reducing errors is easy (i.e., simple restrictions are optimal and robust) while dealing with screeners who care more about agent costs is not. Second, the policymaker can always set floors in a way that does not require knowing anything about the screener's preferences and weakly dominates setting no restrictions. Together, these results suggest that a policymaker should take any measures available to reduce a screener's concern for an agent's costs and increase his concern for errors. Even if such a measure runs the risk of pushing the screener's objective further away from the policymaker's, divergence in this direction is easier to fix through simple restrictions than divergence in the other.

While this paper studies the question of delegated screening, there are many other settings where policymakers rely on intermediaries who have a richer contract space than they do. For example, lawmakers delegate sentencing to judges who commit to their own preferred mappings from evidence to sentences. How should lawmakers set allowable ranges of punishments if they disagree with judges on the importance of fairness versus deterrence? Other examples include firms delegating the design of incentive schemes and promotion schedules to managers who can observe worker output; or a state government delegating the design of a procurement process to a local government that can verify product quality.

level for any given effective type. Quasiconvexity suffices in the max-min model since the max of quasiconvex loss functions of types in some $\Theta_\tau$ is quasiconvex. In the expected loss case, the required condition is that convex functions are closed under addition, so that the probability-weighted sum of errors is convex.

[19]The linearity in screening costs ensures that changes to the allocation for some effective type have constant marginal benefits in terms of lower costs for higher type. This implies that the level of allocation awarded to lower types does not affect the choice of allocation to award to higher types.

The min-max approach taken in this paper was useful in characterizing the properties of optimal mechanisms. Moreover, we showed that delegation rules that were “optimal” in the min-max case were “good” in the expected loss case. This may be a fruitful approach for tackling other delegated mechanism design problems.

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# A Infinite Type Spaces

This sections gives the auxiliary assumptions needed to extend the results to the case where $\Theta$ is infinite. I give a proof of the analogs of Lemma 2 parts 1 and 2, which is the crucial step in translating the other theorems.

In addition to the assumptions in Section 2, assume:

1. $\mathcal{T}$ is compact.

2. The family of allocative loss functions $\{f_\theta\}_{\theta\in\Theta}$ is uniformly bounded and equicontinuous.

3. The correspondence $\tau \rightrightarrows \{f_\theta : \theta \in \Theta_\tau\}$ is upper-hemicontinuous, under the topology induced by the sup-norm metric.

4. The family of loss functions for costly evidence provision, $\{h_\theta\}_{\theta\in\Theta}$, is uniformly bounded.

LEMMA **3.** *Let $y$ be a screener-optimal allocation rule, and let $y_0$ denote the free option in $y$.*

1. *The screener attains his worst-case payoff on $y_0$.*

2. *If screening costs matter, then $y_0 > 0$.*

*Proof.* Suppose $y$ is an optimal allocation rule and let $\mathcal{T}_0$ be the set of effective types which take the free option (i.e., the allocation requiring no costly evidence), $y(\underline{\tau})$. Suppose for contradiction that $y$ does not attain its worst-case payoff for $\tau \in \mathcal{T}_0$, i.e., $R(y) - sup_{\tau\in\underline{T}} R_\tau(y) > \epsilon > 0$. There are two cases to consider.

*Case 1:* Suppose $y(\underline{\tau})$ is an isolated point in Range$(y)$, so that a $\delta' > 0$ radius ball around $y(\underline{\tau})$ does not intersect with Range$(y)$.

By equicontinuity, there exists a $\delta < \delta'$ sufficiently small such that $|f_\theta(y(\underline{\tau})) - f_\theta(y(\underline{\tau}) + \delta)| < \frac{\epsilon}{2}$ for all $\theta$ with $\tau(\theta) \in \mathcal{T}_0$. Consider the new approval rule $y_1(\tau) = \max\{y(\tau), y(\underline{\tau}) + \delta\}$ for all $\tau \in \mathcal{T}$. This reduces the evidence every effective type in $\mathcal{T} - \mathcal{T}_0$ has to show by $\epsilon\underline{T}$. Therefore $R(y_1) < R(y)$, a contradiction.

*Case 2:* Suppose $y(\underline{\tau})$ is not an isolated point in Range$(y)$. Then $\mathcal{T}_0$ is a closed set; let $\tau' = \max \mathcal{T}_0$.

Claim: there is an open interval $O \subset \mathbb{R}$ such that $O \cap \underline{T}$ contains $\tau'$, and such that $R_\tau(y(\underline{\tau})) < R(y) - \epsilon$. Suppose not and consider the correspondence $F : \tau \rightrightarrows \{f_\theta : \theta \in \Theta_\tau\}$. Then there exists a sequence of effective types $\tau_n \in \tau$, such that $\lim_{n\to\infty} \tau_n = \tau'$, and $f_{\theta_n}(\underline{\tau}) \geq R(y) - \epsilon$ for some $\theta_n$ such that $\tau(\theta_n) = \tau_n$. Then by equicontinuity and uniform boundedness, there exists a subsequence of $\{f_{\theta_n}\}_{n\in\mathbb{N}}$ that converges uniformly to some limit

$f$. By upper-hemicontinuity of $F$, $f \in F(\tau')$, so $f(y(\underline{\tau})) \geq R(y) - \epsilon$, i.e., there is some $\theta$ with $\tau(\theta) = \tau'$ such that $f_\theta(y(\underline{\tau})) \geq R(y) - \epsilon$, a contradiction.

Therefore, there exists a closed ball centered at $\tau'$ with radius $\delta'' > 0$ such that the claim above holds. Let $\epsilon' \equiv y(\underline{\tau} + \delta'') - y(\underline{\tau})$ ($\epsilon' > 0$ by assumption of Case 2). Appealing to equicontinuity again, let $0 < \delta < \epsilon'$ be such that $|f_\theta(y(\underline{\tau})) - f_\theta(y(\underline{\tau}) + \delta)| < \frac{\epsilon}{2}$ for all $\theta \in \Theta$. The rule $y_2(\tau) = \max\{y(\tau), y(\underline{\tau}) + \delta\}$ for all $\tau \in \mathcal{T}$ has $R(y_2) < R(y)$, contradicting the optimality of $y$. □

# B Optimality of Delegated Screening

Section 2 describes a game where the policymaker chooses sets of allocations and the screener chooses menus satisfying this co-domain restriction. This section shows that if we instead suppose that both the policymaker and screener can write complete contracts, the policymaker's optimal deterministic grand contract can be implemented using an indirect mechanism of the form in Section 2.

To establish this result, we appeal to an analogue of the taxation principle that applies when one of the participants in the mechanism (i.e., the screener) has commitment power, and there are both hidden actions and hidden types (i.e., the agent's evidence choice and type).

The timing of contracting is as follows.

1. The policymaker chooses a communication protocol: $M_s$ and $M_a$ are the sets of all message strategies for the screener and agent, and $q : M_s \times M_a \to [0, 1]$ maps message strategies to an allocation for the agent.[20]

2. Before either player participates in the communication protocol, the screener can offer a side contract to the agent. The screener chooses a direct mechanism $m_s : \Theta \times \mathbb{R}_+ \to M_s$ that maps the agent's reported type and costly evidence to a messaging strategy that the screener will subsequently follow in the policymaker's protocol.

3. The agent chooses what type $\theta'$ to report and how much costly evidence $n$ to generate for the screener.

4. The agent then reports a message $m_a \in M_a$ to the policymaker, while the screener reports $m_s(\theta', n)$ to the policymaker.

[20] As usual, communication may be dynamic and among the players or between the players and the policymaker at various stages. Here, we treat any messages sent by the policymaker as part of the communication protocol and assume $q$ is measurable with respect to what she knows. We restrict attention to deterministic mechanisms.

Note the following features of this contracting environment.

We allow the screener to offer a contract after the policymaker offers hers. This timing seems natural for many instances and only serves to increase the agency problems faced by the policymaker.[21]

Next, the outcomes of the policymaker's communication protocol depend only on the messages sent by the screener and agent, since the agent's costly action is hidden from her. On the other hand, the screener observes and can contract on the evidence generated by the agent. There is no loss of generality in restricting attention to direct mechanisms for the screener, as the usual revelation principle applies at this stage.

To analyze this game, note that the screener's messaging strategy $m_s \in M_s$ is already determined before the players join the communication protocol. The agent chooses the messaging strategy $\arg\max_{m_a \in M_a} q(m_s, m_a)$, as its payoff is strictly increasing in allocation. Therefore, $Y = \{\max_{m_a \in M_a} q(m_s, m_a) | m_s \in M_s\}$ is the set of possible allocations any agent may receive in equilibrium through the policymaker's communication protocol.

The policymaker's communication protocol composed with the screener's direct mechanism induces a mapping from the agent's type and costly evidence generation to $Y$. Indeed, the screener may choose any incentive compatible and individually rational mechanism with co-domain $Y$. Therefore, restricting attention to the game of delegated screening in 2 is without loss of generality.

# C Paying for versus Eliciting Information

Setting floors is a sound prescription for policymakers: it is a robust policy with fully general preference divergence, in max-min and expected loss settings, and without regularity assumptions on the agent's type space. These results are driven by the assumption that the intermediary has to elicit information from agents in an incentive compatible way.

This sets the theory of delegated screening apart from delegated information acquisition, where the intermediary can acquire signals about the agent at a cost to her and the policymaker (but the costs are not contractible). There, the assumptions on the signal structure drive the form of delegation, precluding more unified results.

When the intermediary pays to learn exogenous information about the agent's private type, setting floor as in Proposition 1 or Proposition 2 can leave the policymaker strictly worse off. This highlights an important difference between this and the model of delegated screening.

[21] Mookherjee and Tsumagari (2004) model collusion similarly in a setting where a principal contracts with two agents, one of which can offer a side contract with the other.

This section gives a simple example to illustrate this difference (compare to Proposition 2). In place of eliciting information about the agent's type, suppose the intermediary can purchase costly signals, or tests, to learn this information.

There are four agent types, $\theta_1, \theta_2, \theta_3, \theta_4$. If an agent of type $\theta_i$ is allocated $y$ units, the policymaker and screener face an allocation error loss of $f_{\theta_i}(y) = |y - \frac{i}{5}|$. Types $\theta_1$ and $\theta_2$ occur with equal probability and with probability 0.99.

There are two tests, $t_1$ and $t_2$. Test $t_1$ reveals that the agent's type is a member of $\{\theta_1, \theta_2, \{\theta_3, \theta_4\}\}$. Test $t_2$ reveals that the agents type is a member of $\{\{\theta_1, \theta_2\}, \theta_3, \theta_4\}$.

The loss of each player when the intermediary awards allocation $y$ to a type $\theta_i$ agent is $f_{\theta_i}(y)$ plus that player's costs for the tests the intermediary used. Test $t_1$ costs 1 to the policymaker, whereas test $t_2$ costs 2. Both tests cost $c > 0$ to the intermediary: assume that $c$ is small enough that the unconstrained intermediary would strictly prefer to use either test over making a decision with no test at all.

The policymaker would not use either test if she was in charge of picking allocations. She would instead pick an allocation just slightly to the right of $\frac{1}{2}(\frac{1}{5} + \frac{2}{5}) = \frac{3}{10}$, say $\frac{3}{10} + \delta$.

An optimal policy for the intermediary, however, would be to first use test $t_1$, and stop there 99% of the time (it is immaterial to the example whether or not $c$ is small enough such that he would continue on to use $t_2$ in the 1% this case arises).

Suppose that the policymaker places a floor on the allocations that the intermediary can award at the lowest level she would ever award, $\frac{3}{10} + \delta$. Since this ties the intermediary's decision in the case where the state is $\theta_1$ or $\theta_2$, $t_2$ is now a strictly more valuable test than $t_1$. Moreover, its value is not much diminished from before (assuming without loss of generality that $\delta$ is sufficiently close to 0), so he would still prefer using $t_2$ over no test at all.

Note that after imposing the floor, the policymaker's loss from tests increased from at most $0.99 + 0.01 \times 2$ to 2. The change in the accuracy of the intermediary's decision is not enough to compensate for this. Therefore, by setting a floor in this way, the policymaker is worse off than under full delegation.

# D Additional Constraints on Implementation

One feature of the max-min analysis is that the screener may have many optimal allocation rules. The model assumes that these ties are broken in favor of the policymaker. While retaining this assumption, this section considers two additional considerations in the screener's choice, aimed at thinning out the size of the indifference sets.

One 'natural' way for the screener to choose a constrained optimal allocation rule is to pick the one closest to the allocation rule he was using under full delegation. That is, if the screener uses allocation rule $y$ under full delegation, he picks a rule in $\arg\min_{y' \in \mathcal{X}(Y)} d_\infty(y, y')$

when delegation set $Y$ is imposed. We refer to this as the *least-change criterion*.

Another consideration for the screener may be to select among the optimal allocation rules only those which are not pointwise dominated in terms of worst-case loss. That is, the screener will not pick $y \in \mathcal{X}(Y)$ if there exists a $y' \in \mathcal{X}(Y)$ such that $R_\tau(y') \leq R_\tau(y)$ for all $\tau \in \mathcal{T}$ and $R_{\tau'}(y') \leq R_{\tau'}(y)$ for some $\tau' \in \mathcal{T}$. This is the *undominated criterion*.

We can modify Proposition 1 to allow for these stronger solution concepts.

**Result.** *Consider the general preference divergence formulation of the model. Let $y_P$ be an optimal allocation rule for the policymaker. Setting a floor at $y_P(\underline{\tau})$ is a weak improvement over full delegation for the policymaker even when the screener applies the least-change or undominated criteria.*

The proof of this result follows directly from the proof of Proposition 1, which shows that if $y$ is optimal for the screener under full delegation, $y' \equiv \{y_P(\underline{\tau}), y\}$ is optimal for the screener in $\mathcal{I}_{[y_P(\underline{\tau}),1]}$. Notice $y'$ is the closest allocation rule to $y$, so the least-change criterion does not affect the result. Next, it is straightforward to check that if $y$ is undominated in the sense of pointwise worst-case loss, $y'$ is undominated as well. This implies that the policymaker's preferred undominated allocation rule in $\mathcal{X}([y_P(\underline{\tau}), 1])$, is a (weak) improvement over $y$ as well.

Consider again the model of Section 2 where the policymaker and screener diverge only on the weights they place on allocation errors and screening costs. When the screener is biased towards minimizing screening costs, introducing new conditions on the screener's choice does not change the results on the complexity or cost of delegation or the non-existence of robustly improving policies. But when the screener is biased towards minimizing allocation errors, the policymaker may no longer be able to induce the screener to utilize her favorite allocation rule.

However, Lemma 2, part 4 combined with Appendix D guarantee that simple and robust improvements continue exist in the case where $\alpha > \alpha_P$. Unless the policymaker's and unconstrained screener's optimal allocation rules already overlap, these improvements are strict.

# E Omitted Proofs

*Proof of Lemma 2.* Let $\mathcal{T}_0^y$ be the set of effective types which take the free option. Let $y_1$ be the next lowest allocation level after the free option, if $y$ is not constant, and let $y_1 = y_0$ otherwise. We show the statements in turn.

*Part 1:* Suppose for contradiction that $y$ does not attain its worst-case payoff at $y_0$, i.e., $R(y) - \max_{\tau \in \mathcal{T}_0^y} R_\tau(y) > \epsilon > 0$. By this assumption, $y_1 > y_0$.

There exists a $0 < \delta < y_1 - y_0$ such that $y_0 + \delta \in Y$ (since $Y$ is connected) and $|f_\theta(y_0) - f_\theta(y_0 + \delta)| < \frac{\epsilon}{2}$ for all $\theta$ with $\tau(\theta) \in \mathcal{T}_0^y$. Consider the new allocation rule $y'$, where $y'(\tau) \equiv \max\{y(\tau), y_0 + \delta\}$ for all $\tau \in \mathcal{T}$. This reduces the evidence every effective type in $\mathcal{T} \setminus \mathcal{T}_0^y$ has to show by at least $\delta\underline{\tau}$, under the optimal standard of proof that implements $y'$.[22] Them $R_\tau(y') < R_\tau(y)$ for $\tau \in \mathcal{T} \setminus \mathcal{T}_0^y$, since $h_\theta$ is strictly increasing for all $\theta$. Therefore, $R(y') < R(y)$, a contradiction.

*Part 2:* If $0 \notin Y$, then this is automatically true, so suppose $0 \in Y$.

Suppose that screening costs matter and for contradiction that $y_0 = 0$. Let $L \equiv \min_{y' \in \mathcal{I}} \max_{\tau \in \mathcal{T}} \max_{\theta \in \Theta_\tau} \alpha f_\theta(y'(\tau(\theta)))$, and let $\mathcal{Y} \subset \mathcal{I}$ be the corresponding set of optimizers. Let $\mathcal{T}_0 = \{\tau \in \mathcal{T} : y'(\tau) = 0 \forall y' \in \mathcal{Y}\}$.

First, we show that $\mathcal{T}_0$ must be empty.

Suppose first $\mathcal{T}_0$ is non-empty and $\mathcal{T}_0^y \subset \mathcal{T}_0$. Then the worst-case loss that the screener faces among agents that take the free option is no more than $L$, which by the assumption that costs matter is smaller than $R(y)$. This contradicts the first part of this theorem.

Suppose next that $\mathcal{T}_0$ is non-empty and $\mathcal{T}_0 \subset \mathcal{T}_0^y$. By the first part of the theorem, combined with the assumption that costs matter, the screener faces worst-case loss on $\mathcal{T}_0^y \setminus \mathcal{T}_0$ and a loss of no more than $L$ on $\mathcal{T}_0$. Note that $f \equiv \max_{\tau \in \mathcal{T}_0^y \setminus \mathcal{T}_0} \max_{\theta \in \Theta_\tau} f_\theta$ is strictly quasiconvex, being the max of strictly quasiconvex functions. Combined with the fact that $y'(\tau) > 0$ for all $y' \in \mathcal{Y}$, this implies that there exists a region $[0, \epsilon]$ for some $0 < \epsilon < y_1$ where $f$ is strictly decreasing. Finally, let $0 < \delta < \epsilon$ be such that $y + \delta \in Y$ and $|f_\theta(y_0) - f_\theta(y_0 + \delta)| < R(y) - L$. Then the allocation rule $y'$ where $y'(\tau) \equiv \{y(\tau), y_0 + \delta\}$ for all $\tau$ has a lower worst-case loss loss than $y$ by the same arguments as before, a contradiction to the optimality of $y$.

This proves that $\mathcal{T}_0$ is empty, which means there exists a $y' \in \mathcal{Y}$ such that $y'(\underline{\tau}) > 0$. It can be shown, by similar arguments to those above, that $y''$, where $y''(\tau) \equiv \max\{y(\tau), y'(\underline{\tau})\}$ has strictly lower worst-case loss for the screener. This implies that $y_0 > 0$.

*Part 3:* Next, the fact that the size of the free option is equal across all screener optimal allocation rules follows as a simple consequence of part 4 of the theorem: if $y'$ is an optimal allocation rule for a screener with weight $\alpha'$ on allocation errors and $\alpha \geq \alpha'$, then the free option is weakly larger under $y'$ than under $y$. Applying this statement to $\alpha' = \alpha$ proves part 3. Therefore, we only need to prove part 4.

*Part 4:* Suppose for contradiction that the free option is larger under $y$ than under $y'$. Let $0 <$

[22] Let $y_k$ be the next largest allocation awarded under $y'$ after the free option and the $k$th smallest allocation offered in $y$. Then evidence is reduced for every effective type in $\mathcal{T} \setminus \mathcal{T}_0^y$ by exactly $\delta\tau_{k-1} + n_{k-1}$, where $\tau_k$ is the effective type just indifferent between receiving the free option and paying the cost of receiving $y_k$ under $y'$.

$\epsilon < y(\underline{\tau}) - y'(\underline{\tau})$. Let $\tilde{y} = \max\{y(\underline{\tau}) - \epsilon, y'\}$ and let $\tilde{n}$ be the corresponding optimal standard of proof. Let $\tilde{\mathcal{T}}_0 \subset \mathcal{T}$ be the set of effective types who are allocated the free option under $\tilde{y}$. Let $R^s \equiv R(y)\frac{\alpha'}{\alpha}$. Let $R' \equiv \max_{\tau\in\mathcal{T}} \max_{\theta\in\Theta_\tau} \alpha' f_\theta(y'(\tau(\theta))) + (1-\alpha')h_\theta(n'(\tau(\theta)))$ be the type-$\alpha'$ screener's worst-case loss under $y'$ ($n'$ is the optimal standard of proof corresponding to $y'$).

The optimality of $y$ over $y'$ for the type-$\alpha$ screener and the fact that $\frac{\alpha'}{\alpha}(1-\alpha) < 1-\alpha < 1-\alpha'$ implies that $R^s \leq R'$.

Suppose $\theta$ is such that $\tau(\theta) \in \tilde{\mathcal{T}}_0$. Then,

$$\begin{aligned}\alpha' f_\theta(\tilde{y}(\tau(\theta))) &< \alpha' \max\{f_\theta(y(\tau(\theta))), f_\theta(y'(\tau(\theta)))\} \\ &\leq \max\{R^s, R'\} \\ &= R',\end{aligned}$$

where the first inequality follows from the strict quasi-convexity of $f_\theta$. This implies $R' > \max_{\tau\in\mathcal{T}} \max_{\theta\in\Theta_\tau} \alpha' f_\theta(\tilde{y}(\tau(\theta))) + (1-\alpha')h_\theta(\tilde{n}(\tau(\theta)))$, contradicting the optimality of $y'$ for the type-$\alpha'$ screener. □

*Proof of Theorem 1.* Suppose the policymaker uses delegation set $Y$. Let $y_0 \equiv \min Y$. If $y_P$ is constant, then $Y$ is a singleton, and the theorem follows trivially. Suppose $y_P$ is not constant, and let $y_1$ be the second lowest allocation under $y_P$.

The screener is worse off by choosing an allocation rule $y$ with a larger free option than $y_0$ than she would be by choosing the best allocation rule with a free option of this size.[23]

To complete the proof, we have to show that $y_P$ is optimal for the screener among all allocation rules $y$ with $y(\underline{\tau}) = y_0$.

Let $L$ denote the policymaker's worst-case loss when the allocation rule $y_P$ is used. Let $\tau \in \mathcal{T}$.

$$\begin{aligned}\frac{\alpha_P}{\alpha} R_\tau(y_P) &= \max_{\theta\in\Theta_\tau} \alpha_P f_\theta(y_P(\tau(\theta))) + \frac{\alpha_P}{\alpha}(1-\alpha)h_\theta(n_P(\tau(\theta))) \\ &\leq L.\end{aligned}$$

The inequality is strict for any $\tau$ for which $y_P(\tau) > y_0$, since $\frac{\alpha_P}{\alpha}(1-\alpha) < 1-\alpha_P$ and $n_P(\tau) > 0$. Next, the inequality is an equality for any $\tau$ such that $y_P(\tau) = y_0$, since $n_P(\tau) = 0$ and $h_\theta(0) = 0$. Therefore $R(y_P) = \frac{\alpha}{\alpha_P}L$, but the worst-case loss for the screener only occurs at the free option.

[23] This follows from entirely symmetric arguments to those in the proof of Lemma 2, part 4: were this not true, the policymaker would be strictly better off using some other allocation rule with a larger free option as well, contradicting the optimality of $y_P$.

Letting $\mathcal{T}_0$ be the set of effective types for which $y_P = 0$, the above shows that for any allocation rule $y$ with $y(\tau) = 0$ on $\mathcal{T}_0$, $R(y) \leq R(y_P)$.[24]

The only remaining case to consider address is whether or not an allocation rule $y$ with $y(\underline{\tau}) = y_P(\underline{\tau})$ but $y(\tau) > y_P(\tau)$ for some $\tau \in \mathcal{T}_0$ can have strictly lower worst-case loss than $y_P$ for the screener.

Suppose for contradiction that this is indeed the case. Then let $\tau_1 \in \mathcal{T}_0$ denote the smallest effective type for which $y(\tau) > y(\underline{\tau})$.

By assumption, the worst-case loss on $\tau < \tau_1$ is strictly less than $\frac{\alpha}{\alpha_P}L$ for the screener, under $y$. And since $y$ and $y_P$ coincide on this region, the worst-case loss for the policymaker is also strictly under $L$ on this region.

Then let $0 < \epsilon < \min y(\tau_1), y_1 - y_0$ be such that $\alpha_P f_\theta(y_0 + \epsilon) < L$ for all $\theta$ with $\tau(\theta) < \tau_1$. For $\theta$ with $\tau(\theta) \in [\tau_1, \overline{\tau}] \cap \mathcal{T}_0$,

$$\begin{aligned}
\alpha_P f_\theta(y_0 + \epsilon) &< \max\{\alpha_P f_\theta(y_0), \alpha_P f_\theta(y(\tau_1))\} \\
&\leq \max\{\alpha_P f_\theta(y_0), \alpha_P f_\theta(y(\tau(\theta)))\} \\
&\leq \max\{L, \frac{\alpha_P}{\alpha}\alpha f_\theta(y(\tau(\theta)))\} \\
&\leq \max\{L, \frac{\alpha_P}{\alpha}R(y)\} \\
&\leq \max\{L, \frac{\alpha_P}{\alpha}R(y_P)\} \\
&= L
\end{aligned}$$

This raises the contradiction that $y'_P = \max\{y_0 + \epsilon, y_P\}$ has lower worst-case loss for the policymaker than $y_P$.

Therefore $y_P$ is optimal for the screener. □

*Proof of Theorem 2, part 1.* Let $Y$ be any delegation set. Let $y$ be the screener's unconstrained optimal allocation rule.

If $Y$ were an interval, then either $Y \subset [0, y(\underline{\tau}))$, $Y \subset (y(\underline{\tau}), 1]$ or $y(\underline{\tau}) \in Y$.

Claim: in each of these cases, the policymaker is weakly better off under full delegation rather than using delegation set $Y$.

First, suppose that $Y \subset [0, y(\underline{\tau}))$. Let $y'$ be the constant allocation rule with $y'(\tau) = \max Y$ for all $\tau \in \mathcal{T}$. Let $y'' \in \mathcal{I}_Y$ be any other rule; note $y''$ must have a smaller free option, since it is weakly increasing. At any point where $y''(\tau) = y'(\tau)$, $R_\tau(y') < R_\tau(y'')$, since the

[24] Therefore, if $y_P$ was fully separating on $\mathcal{T}$, for example, this is enough to conclude that it is an optimal choice for the screener in $\mathcal{I}_Y$.

screening costs under $y'$ are 0. If $y''(\tau(\theta)) < y'(\tau(\theta))$, then:

$$\begin{aligned}\alpha f_\theta(y'(\tau(\theta))) &< \max\{\alpha f_\theta(y(\tau(\theta))), \alpha f_\theta(y''(\tau(\theta)))\}\\ &\leq \max\{R(y), R(y'')\}\\ &= R(y'')\end{aligned}$$

This shows that $R(y') < R(y'')$. Therefore, the screener's unique optimal allocation rule in $\mathcal{I}_Y$ is $y'$, a constant delegation rule. But full delegation is better than no delegation for the policymaker:

$$\max_\tau \max_{\theta\in\Theta_\tau} \alpha_P f_\theta(y(\tau)) + (1-\alpha_P)h_\theta(n(\tau)) < \frac{\alpha_p}{\alpha}R(y) < \frac{\alpha_p}{\alpha}R(y') = \max_{\theta\in\Theta_\tau} \alpha_P f_\theta(y'(\tau)).$$

Next suppose $Y \subset (y(\underline{\tau}), 1]$. Take any allocation rule $y'$ in $\mathcal{I}_Y$. The policymaker prefers the screener's chosen rule, $y$, to $y'$.[25] Therefore, full delegation is better than $Y$ for the policymaker.

Finally, suppose that $Y$ is such that $y(\underline{\tau}) \in Y$. Then, for any $y'$ with $y'(\underline{\tau}) < y(\underline{\tau})$, the allocation rule $y''$ with $y''(\tau) \equiv \max\{y(\underline{\tau}), y'(\tau)\}$ is a strict improvement for the screener. This means the screener will only choose those allocation rules with free option weakly larger than $y(\underline{\tau})$, leaving the policymaker weakly worse off.

This proves the claim that full delegation is weakly better than any interval restrictions. Now if the unconstrained screener's optimal allocation rules do not coincide with the policymaker's, the screener implements a sub-optimal allocation rule for the policymaker under full delegation. Therefore, delegation is either futile (e.g., if full delegation is optimal) or complex (if full delegation is not optimal). □

*Proof of Theorem 2, part 2.* Suppose that for some $\alpha_p > \alpha$, there exists a delegation set which is a strict improvement over full delegation. Let $y$ be the screener's unconstrained optimal allocation rule. Following the proof of Theorem 2, part 1, this rule must exclude an interval $(c, d)$ containing $y(\underline{\tau})$. There are two cases to consider. Let $y_0$ denote the optimal constant allocation rule (by strict quasiconvexity of every $f_\theta$, $f(a) \equiv \sup_{\theta\in\Theta} f_\theta(a)$ is also strictly quasiconvex and therefore has a unique minimum).

Case 1: $y_0 \in (c, d)$. This implies that a screener with a weight $\alpha'$ close enough to 0 would have chosen an allocation rule in $(c, d)$ if unconstrained, but is forced to choose between the allocation rule that is constant at $c$ or an allocation rule with free option of size $d$ instead. Full delegation is superior to both outcomes for such a screener.

[25] This follows by the same arguments as in the proof of Theorem 1, but with the role of the policymaker and the screener reversed: if the policymaker strictly preferred some allocation rule with a larger free option, then the screener prefers some other rule with a larger free option as well, contradicting the optimality of $y$ (by Lemma 2, part 3).

Case 2: $y_0 \geq d$. By Lemma 2 part 3, the correspondence from the screener's weight on allocation errors to the sizes of free options in optimal menus is singleton valued.

Claim: this mapping is continuous. To see this, let $\alpha' < \alpha$ be two weights the screener may place on allocative errors, and let $y$ and $y'$ be corresponding optimal allocation rules. By Lemma 2 part 4, $y(\underline{\tau}) \leq y'(\underline{\tau})$. Let $\epsilon > 0$. By Lemma 2, there is an $\epsilon' > 0$ such that if the free option under an allocation rule $y''$ is greater than or equal to $y(\underline{\tau})+\epsilon$, then the worst-case loss to a type-$\alpha$ screener increases by at least $\epsilon'$. By continuity of the value function in $\alpha$, there is a $\delta > 0$ such that if $\alpha - \alpha' < \delta$, then $\frac{\alpha}{\alpha'}$ times the worst-case loss of the $\alpha'$-type screener under $y'$ minus the worst-case loss of the $\alpha$-type screener under $y$ is less than $\epsilon'$. This difference is an upper-bound on difference in worst-case loss for the type $\alpha$ screener under $y$ versus $y'$. This means if $\alpha - \alpha' < \delta$, $y'(\underline{\tau}) - y(\underline{\tau}) < \epsilon$, proving the claim

Since the mapping is continuous, by the intermediate value theorem, then there exists a sequence $\alpha'_n > \alpha$ for $n$ large enough, who would choose a free option of size $d - \frac{1}{n}$ if unconstrained. For large enough $n$, the type-$\alpha'_n$ screener would choose a free option of size $d$ or larger under the policymaker's restriction. Since the policymaker increases the size of the free option for such a screener, she increases her worst-case error on this type.

By Lemma 2, part 4, the case where $y_0 \leq c$ is an impossibility.

Therefore case 1 and case 2 collectively show that robust improvement is not possible. □

*Proof of Corollary 1.* Theorem 1 and Theorem 2 immediately imply that $Y$ is undominated: $Y$ is optimal for any $\alpha > \alpha_P$, and any $Y'$ that gives the policymaker a lower loss than $Y$ does for some $\alpha < \alpha_P$ must be strictly worse than $Y$ for some other $\alpha' < \alpha_P$.

To see that this strategy is min-max optimal with respect to the screener's type, note first that for any delegation set $Y'$, the optimal choice of a screener for whom $\alpha = 0$ is some constant allocation rule $y$. Consider another screener $\alpha' \in (0, \alpha_P)$ for whom the optimal allocation rule is $y'$ when the delegation set is $Y'$. The optimality of $y'$ for such a screener implies that there is some $\theta' \in \Theta$ such that for all $\theta \in \Theta$,

$$\alpha' f_{\theta'}(y(n_y(\theta'))) + (1-\alpha')h_{\theta'}(n_y(\theta')) \geq \alpha' f_{\theta'}(y'(n_{y'}(\theta))) + (1-\alpha')h_\theta(n_{y'}(\theta)).$$

Since $y$ is flat, $n_y = 0$, so this inequality becomes,

$$\alpha'(f_{\theta'}(y(n_y(\theta'))) - f_{\theta'}(y'(n_{y'}(\theta)))) \geq (1-\alpha')h_\theta(n_{y'}(\theta)),$$

which implies the same inequality holds when replacing $\alpha'$ with $\alpha_P > \alpha'$. In other words, the policymaker prefers $y'$ to $y$. Therefore the worst-case loss for the policymaker for any delegation set occurs when the choice of allocation rule is being made by a screener with weight zero on allocation errors. Then the policymaker's min-max optimal delegation set is one which allows such a screener to choose the best constant allocation rule. Since a floor at $y_P(\underline{\tau})$ does not interfere with such a screener's choice, it is min-max optimal with respect to the screener's type for the policymaker. □

Lemma **4.** *Let $y$ be an optimal allocation rule for the screener, and suppose that a floor at $y_0 \in [0,1]$ is set. Then the allocation rule $y'$ where $y'(\tau) \equiv \max\{y_0, y(\tau)\}$ is optimal for the screener.*

*Proof of Lemma 4.* The statement is true if the floor is non-binding; suppose the floor binds.

Note first that $R(y) < R(y')$ by Lemma 2, part 3. Therefore, the worst-case loss on the set of effective types $\mathcal{T}_0$ for which $y' = y$ is larger under $y'$, since the worst-case loss on $\mathcal{T} \setminus \mathcal{T}_0$ is smaller under $y'$. To show an allocation rule $y'' \geq y_0$ cannot have a lower worst-case loss than $y'$, it suffices to show it cannot have a lower worst-case loss on $\mathcal{T}_0$.

If the worst-case loss is greater under $y'$ than under $y$ at some $\tau \in \mathcal{T}_0$, by strict quasi-convexity of $\max_{\theta \in \Theta_\tau} f_\theta$, increasing allocation to this effective type further only increases worst-case loss. If worst-case loss at some $\tau \in \mathcal{T}_0$ is smaller under $y'$ than under $y$, then $R_\tau(y') < R_\tau(y) < R(y)$.

This implies that $y''$ cannot have lower worst-case loss than $y'$: if $\tau \in \mathcal{T}_0$ is such that $R_\tau(y'') < R_\tau(y')$, then $R_\tau(y'') < R(y) < R(y'')$, so this does not affect the worst-case loss at $y''$. The worst-case loss for $y''$ happens at an effective type $\tau$ where $R_\tau(y') \leq R_\tau(y'')$. □

*Proof of Proposition 1.* By Lemma 4 (and using the same notation as in the proof there), $y'$ (with the floor set at $y_P(\underline{\tau})$) is an optimal allocation rule for the screener. In the case where the policymaker's floor is binding, the policymaker faces lower worst-case loss on every effective type for whom the allocation stayed the same between $y'$ and $y$. Among those effective types for whom the floor was binding, the policymaker's loss due to screening costs is reduced to 0 under $y'$. Finally, consider those effective types $\tau$ for whom worst-case losses are larger under $y'$ than under $y$. The allocation error losses must be weakly larger still under $y_P$, since $y(\tau) < y'(\tau) \leq y_P(\tau)$, and the screening costs are weakly larger at $\tau$ under $y_P$ as well. But the worst-case loss here is less than or equal to the worst-case loss of the policymaker under $y_P$. By the optimality of $y_P$ for the policymaker, the policymaker does not attain her worst-case loss under $y'$ or $y$ at $\tau$.

Since $y'$ is therefore a weak improvement over $y$ for the policymaker, the screener's choice of allocation rule (which is the one the policymaker prefers among those optimal for the screener) after imposing the floor is a weak improvement over $y$. □

Lemma **5.** *Consider the expected loss formulation of the delegated screening problem and suppose the assumptions in Proposition 2 hold. Let $y$ be an optimal allocation rule for the screener, and suppose that a floor at $y_0 \in [0,1]$ is set. Then the allocation rule $y'$ where $y'(\tau) \equiv \max\{y_0, y(\tau)\}$ is optimal for the screener.*

*Proof of Lemma 5.* We first make some definitions to save on notation. Let $k(\theta) = p_\theta \alpha$. Since $h_\theta(n(\theta))$ is linear, let $l_\theta n(\theta) \equiv p_\theta(1-\alpha)h_\theta(n(\theta))$.

If the floor is not binding, the statement holds.

Suppose the floor is binding. Let $\mathcal{T}_0$ be the set of effective types for whom $y(\tau) < y'(\tau)$ . Let $y'' \in \mathcal{I}_{[y_0,1]}$ be any implementable allocation rule such that $y'(\tau) \geq y_0$ for all $\tau$. There are two cases to consider.

Case 1: $y''(\tau) = y_0$ for all $\tau \in \mathcal{T}_0$. Define the allocation rule $y'''$ to be equal to $y$ on $\mathcal{T}_0$ and $y''$ on $\mathcal{T} \setminus \mathcal{T}_0$.

In this case,

$$\begin{aligned} B(y') - B(y'') &= \sum_{\tau \in \mathcal{T} \setminus \mathcal{T}_0} \sum_{\theta \in \Theta_\tau} k_\theta (f_\theta(y'(\tau(\theta))) - f_\theta(y''(\tau(\theta)))) + l_\theta(n'(\tau(\theta)) - n''(\tau(\theta))) \\ &= \sum_{\tau \in \mathcal{T} \setminus \mathcal{T}_0} \sum_{\theta \in \Theta_\tau} k_\theta (f_\theta(y(\tau(\theta))) - f_\theta(y'''(\tau(\theta)))) + l_\theta(n(\tau(\theta)) - n'''(\tau(\theta))) \\ &= \sum_{\tau \in \mathcal{T}} \sum_{\theta \in \Theta_\tau} k_\theta (f_\theta(y(\tau(\theta))) - f_\theta(y'''(\tau(\theta)))) + l_\theta(n(\tau(\theta)) - n'''(\tau(\theta))) \\ &= B(y) - B(y''') \\ &\geq 0 \end{aligned}$$

The second equality follows from the fact that $n$ differs from $n'$ by a constant on $\mathcal{T} \setminus \mathcal{T}_0$, and $n''$ differs from $n'''$ by the same constant. The third equality follows from the facts that $y'''$ and $y$ agree on $\mathcal{T}_0$. The last inequality follows from the optimality of $y$ for the screener.

The screener therefore weakly prefers $y'$ to $y''$.

Case 2: $y''(\tau) > y_0$ for some $\tau \in \mathcal{T}_0$. Let $\tau_0$ denote the smallest effective type for which this holds. Let $\tau_1$ be the largest effective type for which $y(\tau_0) = y(\tau_1)$. Note that by definition, $\tau_0$ and $\tau_1$ are in $\mathcal{T}_0$. Finally, let $\tau_2$ be the smallest effective type for which $y''(\tau_2) = y''(\tau_1)$.

To recap: $\tau_0 \leq \tau_2 \leq \tau_1$, $y$ is constant on $[\tau_0, \tau_1]$, and $y''$ is constant on $[\tau_2, \tau_1]$. Therefore, both $y$ and $y''$ are constant on $[\tau_2, \tau_1]$. Next, $y''(\tau) > y_0$ on this region, while $y(\tau) < y_0$ for $\tau \in [\tau_2, \tau_1]$. Denote the constant values these allocation rules take on this interval by $y''_c$ and $y_c$ . Finally, compared to the values they take on $[\tau_2, \tau_1]$, $y''(\tau)$ is strictly smaller for $\tau \in [0, \tau_2)$ while $y$ is strictly larger for $\tau \in (\tau_1, \overline{\tau}]$.

Let $\Theta_{[\tau_2,\tau_1]}$ denote the set of all types $\theta$ with $\tau(\theta) \in [\tau_2, \tau_1]$. Define $\mu(x) \equiv \sum_{\theta \in \Theta_{[\tau_2,\tau_1]}} k_\theta f_\theta(x)$ for all $x \in [y_c, 1]$. Note $\mu$ is strictly convex, since it is a sum of strictly convex functions. Define $\eta(x) \equiv \sum_{\theta \in \Theta_{[\tau_2,\tau_1]}} l_\theta \tau_3 x$ for all $x \in [y_c, 1]$ Next, let $\Theta_{(\tau_1,\overline{\tau}]}$ denote the set of all types $\theta$ with $\tau(\theta) > \tau_1$. Let $\tau_3$ denote the next lowest effective type to $\tau_3$. Define $\phi(x) \equiv \sum_{\theta \in \Theta_{(\tau_1,\overline{\tau}]}} l_\theta(\tau_3 - \tau_2)x$ for $x \in [y_c, 1]$. Notice that both $\eta$ and $\phi$ are linear in $x$.

Consider the allocation rule $y_\epsilon$, which is equal to $y$ on $[\underline{\tau}, \overline{\tau}] \setminus [\tau_2, \tau_1]$ and equal to $y_c + \epsilon$ on $[\tau_2, \tau_1]$, where $\epsilon$ is sufficiently small so that $y_\epsilon$ is still non-decreasing. Then it follows from the definitions that

$$B(y_\epsilon) - B(y) = (\eta(y_c + \epsilon) - \eta(y_c)) + (\phi(y_c + \epsilon) - \phi(y_c)) + (\mu(y_c + \epsilon) - \mu(y_c)) \geq 0$$

. by the optimality of $y$.

But now consider the allocation rule $y''_\epsilon$, which is equal to $y''$ on $[\underline{\tau}, \overline{\tau}] \setminus [\tau_2, \tau_1]$ and equal to $y_c - \epsilon$ on $[\tau_2, \tau_1]$, where $\epsilon$ is the same as before and we assume without loss of generality that it was chosen to be sufficiently small so that $y''_\epsilon$ is still non-decreasing. Then

$$\begin{aligned} B(y'') - B(y''_\epsilon) &= (\eta(y''_c) - \eta(y''_c - \epsilon)) + (\phi(y''_c) - \phi(y''_c - \epsilon)) + (\mu(y''_c) - \mu(y''_c - \epsilon)) \\ &= (\eta(y_c + \epsilon) - \eta(y_c)) + (\phi(y_c + \epsilon) - \phi(y_c)) + (\mu(y''_c) - \mu(y''_c - \epsilon)), \end{aligned}$$

where the equality follows from the linearity of $\phi$ and $\eta$. Note, however, that by strict convexity, $(\mu(y''_c) - \mu(y''_c - \epsilon)) > (\mu(y_c + \epsilon) - \mu(y_c))$. This implies $B(y'') - B(y''_\epsilon) > 0$, so $y''$ is not an optimal allocation rule among those that have a floor at $y_0$.

Case 1 and case 2 together imply (since optimal allocation rules exist) that $y'$ is an optimal allocation rule for the screener among those that have a floor at $y_0$. □

*Proof of Proposition 2.* Let $y_P$ be a policymaker optimal allocation rule, and let $y_0 \equiv y_P(\underline{\tau})$. We also carrying over all the notation from Lemma 5.

$y'$ is optimal for the screener by the same Lemma 5. We want to show that the policymaker weakly prefers $y'$ to $y$.

We prove this statement under the assumption that $y_P$ and $y$ are fully separating in effective types, as the case where there is some pooling is handled precisely as in case 2 of the proof of Lemma 5.

Let $\tau \in \mathcal{T}_0$. Let $y_\epsilon$ be a perturbation which is equal to $y$ everywhere but $\tau$, where it equals $y(\tau) + \epsilon$. Let $y_{P,\epsilon}$ be a perturbation which is equal to $y_P$ everywhere but $\tau$, where it equals $y_P - \epsilon$. Assume that $\epsilon > 0$ is sufficiently small so that $y_\epsilon$ and $y_{P,\epsilon}$ are non-decreasing (such an $\epsilon$ exists by the assumption of full separation). The reduction in loss due to corresponding evidence costs on types with effective types in $(\tau, \overline{\tau}]$ is the same when moving from $y$ to $y_\epsilon$, as when moving from $y_{P,\epsilon}$ to $y_P$. The same is true for the increase in loss due to evidence costs on types with effective type $\tau$. However, the increase in loss due to total allocation errors is larger moving from $y_{P,\epsilon}$ to $y_P$ than moving from $y$ to $y_\epsilon$, by strict convexity of the allocation loss functions. The optimality of $y_P$ implies that the policymaker prefers $y_P$ to $y_{P,\epsilon}$. Then the policymaker should prefer $y_\epsilon$ to $y$.

By applying this argument to each $\tau$ in $\mathcal{T}_0$, starting by perturbing the allocation function upward to $y_0$ on the largest effective type in this set, we see that each of these moves is beneficial to the policymaker. Therefore, the policymaker prefers $y'$ to $y$. □