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A Keynesian Approach to Modeling the Long-Term Interest Rate

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ABSTRACT

There are several widely used benchmark models of the long-term interest rate in quantitative finance. However, these models have yet to incorporate Keynes's valuable insights about interest rate dynamics. The Keynesian approach to interest rate dynamics can be readily incorporated in the benchmark models of the long-term interest rate. This paper modifies several benchmark interest rate models. In these modified models the long-term interest rate is related to the short-term interest rate and a Wiener process. The Keynesian approach to interest rate dynamics can be useful in addressing theoretical and policy issues.

KEYWORDS: Long-Term Interest Rate; Bond Yields; Monetary Policy; Short-Term Interest Rate; John Maynard Keynes

JEL CLASSIFICATIONS: E12; E43; E50; E58; E60; G10; G12; G41

1. INTRODUCTION

There are a number of widely used benchmark models of the long-term interest rate in quantitative finance. Both researchers and practitioners use these models for theoretical, empirical, and policy purposes. However, these models have yet to incorporate John Maynard Keynes's insights about interest rate dynamics. Keynes's insights on interest rates are based on his analysis of uncertainty, liquidity preference, financial markets and institutions, investors' expectations, and animal spirits.

Keynes conjectured that the central bank has a decisive influence on the long-term interest rate on a country's risk-free government bonds, mainly through its policy rate and its various monetary policy actions. He argued that the central bank's decisions on the policy rate set the short-term interest rate. In turn the short-term interest rate has a crucial effect on the long-term interest rate of risk-free government bonds. Thus, Keynes believed the central bank's actions influence government bond yields and the shape of the government bond yield curve.

Keynes's approach to interest rate dynamics is stands in contradistinction to the loanable funds theory, as articulated in classical economists such as Fisher (1930) and Wicksell ([1936] 1962). Nonetheless Keynes's approach to interest rate dynamics is based on both theoretic arguments and stylized empirical facts (Kregel 2011). The theoretical basis of his approach originates from his analysis of the central bank's policy rate, open market operations, and balance sheet policies (Keynes 1930: I, 185–220; 1930: II, 362–64, 369–73), his theory of interest rates (1930: II, 352–61; [1936] 2007 165–85, 222–44; 1937), uncertainty and expectations ([1936] 2007, 148–51, 166–72), animal spirits ([1936] 2007, 161–63), and liquidity preference ([1936] 2007 , 194–209). The empirical basis for Keynes's views originates from Riefler's (1930) pioneering statistical analysis of money markets and bond markets in the United States in the 1920s and Keynes's (1930: II, 355–56) own observations and analysis of money markets and bond markets in the United Kingdom during the same years.

The Keynesian approach to modeling long-term government bond yields has been revamped in recent years in several empirical studies, such as Akram and Das (2015, 2017, 2019, 2020), Akram and Li (2020a, 2020b, 2020c, 2020d, 2020e), Akram and Uddin (2020, 2021), Chakraborty (2016), Das and Akram (2020), Gabrisch (2021), Levrero and Deleidi (2020), Rahimi (2014), Rahimi, Chu, and Lavoie (2017), Simoski (2019), and Vinod, Chakraborty, and Karun (2014). There are also a few theoretical studies, such as Akram (2021a, 2021b) and Wray (1992), that have advanced the Keynesian approach to interest rate modeling, building on Keynes's approach to interest rate dynamics, as reflected in Keynes (1937) and Robinson (1951), and money and credit as articulated in Keynes (1930, [1936]2007) and the works of Keynesians, such as Lavoie (1984). This approach has contested the conventional view based on the loanable funds theory of interest rates. The conventional approach is based on the view that the interest rate depends on the demand for and supply of funds. In the conventional approach to interest rate modeling, increased fiscal deficit and higher government debt ratios exert upward pressures on government bond yields, and higher deficit and debt ratios increase the likelihood of sovereign debt defaults. Simoski (2019, 8–21) provides a critical review of both the conventional and the Keynesian approaches to interest rate modeling.

The Keynesian approach to interest rate modeling, however, has yet to be generally deployed in the substantial theoretical work on interest rate modeling that exists in the quantitative finance literature, though Akram (2021b) is a recent attempt to do so. The quantitative finance literature would benefit from tapping Keynes's remarkable and prescient insights on the dynamics of the long-term interest rate. Keynes's conjecture on interest rate dynamics (Akram and Li 2020c) is founded on his unique and original analysis of uncertainty, liquidity preference, financial institutions and financial markets, investors' expectations, herding, and animal spirits.

Relating the long-term interest rate as a function of the short-term interest rate and a Wiener process comprise a Keynesian approach to interest rate dynamics. This paper modifies several benchmark interest rate models in order to advance a Keynesian approach to interest rate dynamics in quantitative finance. This paper bridges an important gap in interest rate modeling by applying a Keynesian approach to the modeling of the long-term interest rate based on a Wiener process using stochastic differential equations.

The paper is structured as follows. Section 2 briefly recounts Keynes's views on interest rates. Section 3 presents several models of the long-term interest based on the short-term interest rate and a Wiener process. The models presented here are modifications of benchmark models widely used in quantitative finance to illustrate Keynes's approach to interest rate dynamics. The modified models demonstrate the central bank's decisive influence over the long-term interest rate based on the short-term interest rate and a Wiener process. Section 4 discusses some policy implications of these models. Section 5 concludes.

2. KEYNES'S VIEWS ON INTEREST RATES

Keynes was acutely aware of the institutional features of capitalist production processes and financial systems, as pointed out in Kregel (1980). For Keynes ([1936] 2007, 148), economic activity and financial decisions occur amid fundamental uncertainty. Hence, he argues that the investors' "usual practice" is to "take the existing situation and project it into the future, modified only to the extent that we have more or less definite reasons for expecting changes." He emphasizes "the extreme precariousness of the basis of knowledge on which our estimates of prospective yields have to be made" ([1936] 2007, 149). As a result, investors often "fall back on what is, in truth, a *convention*" [italics in the original] ([1936] 2007, 152). Keynes notes that "human decisions affecting the future […] cannot depend on strict mathematical expectation, since the basis for making such calculation does not exist" ([1936] 2007, 162–63).

For Keynes, interest rates have a psychological and sociological basis in a world characterized by ontological uncertainty. Economic agents in the real world have liquidity preferences, whereas in the conventional view interest rates depend on the demand for and supply of funds in financial markets. Moreover, in the conventional view an increase (decrease) in government debt and deficit ratios leads to higher (lower) government bond yields. Keynes ([1936] 2007, 175–85, 239–44) firmly rejects this loanable funds theory of interest rates, as advocated in Fisher (1930), Wicksell ([1936] 1962), and others.

Keynes's discussion of the relationship between the short-term interest rate and the long-term interest rate occurs in volume II of his *Treatise on Money*. Keynes (1930: II, 352) notes that "[t]he main direct influence of the Banking System is over the short-term rate of interest" and "the influence of the short-term rate of interest on the long-term rate is much greater than anyone […] would have expected" (1930: II, 362).

Keynes holds that the central bank drives the long-term interest rate through the short-term interest rate. After analyzing the data on interest rate dynamics in the United States and the United Kingdom, he concludes that there is a tight relationship between the short-term and long-term interest rates. Keynes (1930: II, 354–55) approvingly quotes W. W. Riefler (1930, 123) as stating: "[T]he surprising fact is not that [long-term] bond yields are relatively stable in comparison to short-term [interest] rates, but they have reflected fluctuations in short-term [interest] rates so strikingly and to a such a considerable extent." Keynes provides theoretical justification for these stylized facts based on institutional features of financial markets and organizations, profit maximization and the lack of arbitrage conditions, portfolio managers' incentives, and excessive focus on the near term (Keynes 1930: II, 352–62). Under such circumstances, Keynes (1930: II, 362) firmly maintains that "there is no reason to doubt the ability of a Central Bank to make its short-term rate of interest effective in the market."

Keynes ([1936] 2007, 205) asserts that the central bank's actions—through its control of the short-term interest rate via the policy rate, as well as its open market operations or balance sheet policies—can determine "a rate of interest or, more strictly, a determinate complex rate of interest for debt of different maturities." Specifically, if the central bank is willing to buy and sell securities of various maturity tenors, it can exert effective and unequivocal control throughout the yield curve on government securities, as well as over the slope of the yield curve. Kregel (2011), Akram and Li (2020c), and Akram (2021a) have undertaken extended discussions of Keynes's view regarding the influence of central bank policy on the long-term interest rate on government bonds via the short-term interest rate.

3. KEYNESIAN MODELS OF LONG-TERM INTEREST RATE DYNAMICS

Notation

The long-term interest rate is r_{LT}, while the short-term interest rate is r_{ST}. The volatility of the long-term interest rate is $V \geq 0$, and the Wiener process is dW. Here $\mu, \sqrt{V}, \kappa,$ and ρ, are constants, whereas $\mu(t), \sqrt{V(t)}, \alpha(t),$ and $\beta(t)$ are time-dependent functions. A Wiener process, $W(t)$, is a continuous-time stochastic process. For $t \geq 0$ with $W(0) = 0$ and such that the increment $W(t) - W(s)$ is Gaussian with mean 0 and variance $(t - s)$ for any $0 \leq s < t$, and increments for nonoverlapping time intervals are independent.

Concepts and Background

Papoulis (1984) and Karatsas and Shreve (1997) provide background on the mathematical concepts that are used as the building blocks for modeling interest rates and asset prices in continuous time. Rebonato (1996, 2004) describes interest rate models that are widely used in quantitative finances. There are many interest rate models in quantitative finance. However, the models associated with Merton (1973, 1974), Vasicek (1977), Cox, Ingersoll, and Ross (1985), Ho and Lee (1986), Black, Derman, and Toy (1990), Black and Karasinski (1991), Hull and White (1990a, 1990b), Heath, Jarrow, and Morton (1992), Heston (1993), Jamshidian (1995), and Brace, Gatarek, and Museila (1997) are regarded as benchmark interest models in quantitative finance.

Interest Rate Models Modified to Represent a Keynesian Approach

The following equations present various interest rate models, which have been modified to represent a Keynesian approach to interest rate dynamics.

$$dr_{LT} = \mu dt + \sqrt{V} dW \quad [1]$$

Equation [1] follows Metron's (1973) model. It states that the long-term interest rate follows a Wiener process, dW, in which μ is the drift and V is the volatility of the long-term interest rate. The model has two constants: the drift term, μ, and the volatility, V.

$$dr_{LT} = \mu r_{ST} dt + \sqrt{V} r_{ST} dW \quad [2]$$

Equation [2] is based on Vaisek (1977). It states that the long-term interest rate is a geometric function, but it also depends on the short-term interest rate, adjusted by a drift term, and the volatility of the long-term interest rate. The model has two constants, the drift term, μ, and the volatility, V.

$$dr_{LT} = \sqrt{V} r_{ST} dW \quad [3]$$

Equation [3] states that the long-term interest rate follows the short-term interest rate, adjusted by the volatility, along a Wiener process. The model has only one constant, the volatility V. This model is based on Dothan (1978).

$$dr_{LT} = \rho(\kappa - r_{ST})dt + \sqrt{V r_{ST}} dW \quad [4]$$

Equation [4] expresses that the long-term interest rate follows a mean reverting process of the short-term interest rate and a Wiener process adjusted by its volatility and the short-term interest rate. This model is based on Cox, Ingersoll, and Ross (1985). Here $\rho, \kappa,$ and V are constants. In this model for $\rho, \kappa > 0$, the long-term interest rate moves toward κ, while the speed adjustment is ρ. In a recent paper, Akram (2021b) interprets the Keynesian approach to interest rate dynamics as represented in this model.

$$dr_{LT} = \mu(t) r_{ST} dt + \sqrt{V} dW \quad [5]$$

Equation [5] conveys that the long-term interest rate is a function the short-term interest rate and a Wiener process adjusted by its volatility. This model is based on Ho and Lee (1986). It has no mean reversion. Here the drift term, $\mu(t)$, is a function, while V is a constant. The effect of the short-term interest rate on the change in the long-term rate varies over time, since $\mu(t)$ is a function of time. The effect of the Wiener process is adjusted by volatility. There is no mean reversion.

$$dr_{LT} = \left\{\alpha(t) + \left[\frac{\sqrt{V\prime(t)}}{\sqrt{V(t)}}\right] ln(r_{ST})\right\} r_{ST}dt + \sqrt{V(t)}r_{ST}dW \quad [6]$$

Equation [6] formulates that the long-term interest rate is a function of the logarithm of the short-term interest rate and the Wiener process adjusted by volatility and the short-term interest rate. This is inspired by Dothan (1978), Ho and Lee (1986), and Black, Derman, and Toy (1990). This model also has no mean reversion. Here $\alpha(t)$, $V(t)$, and $V'(t)$ are functions. The model emphasizes the influence of the interest rate's volatility on the adjustment of the interest rate, reflecting the stylized fact that the long-term interest rate in advanced capitalist economies tends to exhibit less volatility than the short-term interest rate.

$$dr_{LT} = [\alpha(t) - \beta(t)r_{ST}]dt + \sqrt{V(t)}dW \quad [7]$$

Equation [7] implies that the long-term interest rate is a function of the short-term interest rate that adjusts at a time-dependent pace and a Wiener process adjusted by volatility and the short-term interest rate. This is based on Black, Derman, and Toy (1990). Here $\alpha(t)$, $\beta(t)$, and $V(t)$ are functions. This model incorporates a mean reverting trend with an implied drift trend.

$$dr_{LT} = [\alpha(t) - \beta(t)ln(r_{ST})]r_{ST}dt + \sqrt{V(t)}r_{ST}dW \quad [8]$$

Equation [8] implies that the long-term interest rate is a function of the logarithm of the short-term interest rate and a Wiener process adjusted both by the volatility and the short-term interest rate. This model is derived from Hull and White (1990a, 1990b) and Black and Karasinski (1991). Here $\alpha(t)$, $\beta(t)$, and $V(t)$ are functions.

$$dln[r_{LT}] = \mu(t)ln[r_{ST}]dt + \sqrt{V(t)}r_{ST}dW \quad [9]$$

Equation [9] implies that the logarithm of the long-term interest rate is a function of the log of the short-term interest rate and the Wiener process. Here both the drift term and the volatility are functions of time. This model is derived from Kalotay, Williams, and Fabozzi (1993), but it is also similar to Ho and Lee (1986).

$$dln[r_{LT}] = \rho(\kappa - ln[r_{ST}])dt + \sqrt{Vr_{ST}}dW \qquad [10]$$

Equation [10] conveys that the logarithm of the long-term interest rate is a function of a mean reverting process of the logarithm of the short-term interest rate and a Wiener process adjusted by volatility and the short-term interest rate. This model is based on a modification of Cox, Ingersoll, and Ross (1985). Here $\rho, \kappa,$ and V are constants. Here, for $\rho, \kappa > 0$, the logarithm of the long-term interest rate moves toward κ, where the speed adjustment is ρ.

The modified versions of the above-mentioned benchmark models can be useful in translating Keynes's insights to specific functional forms to understand interest rate dynamics and to evaluate which specifications best capture the underlying processes. Questions such as which of these models best fits the data, under what circumstances, and in which time periods are of paramount interest to researchers, practitioners, and policymakers. However, these are empirical issues that can be only addressed through detailed, careful, and rigorous examination of the data, along with proper institutional and historic analysis of economic and financial conditions.

4. POLICY RELEVANCE

The Keynesian approach to interest rate dynamics is germane not merely for macroeconomic theory and quantitative finance but also for macroeconomic and monetary policy. It can be valuable for understanding and assessing the central bank's policy actions and monetary transmission mechanism.

In the modified models presented in this paper, a higher (lower) short-term interest rate leads to a higher (lower) long-term interest rate on government bonds. The central bank influences government bond yields primarily through its policy rates (such as its target interbank rate, interest rate on reserves, discount rate, and repo and reverse repo rates), which in turn affect the short-term interest rate. These models emphasize the role of the short-term interest rate as the determinant of the Treasury yield curve and its shape. The central bank has various other tools, such as policy pronouncements, forward guidance, large-scale asset purchases, yield curve control, and administrative measures, for influencing government bond yields, even though the policy rates and short-term interest rate are the most important drivers of the long-term interest rate on government bonds.

If the short-term interest rate is the primary driver of the long-term interest rate, then there are consequential implications for macroeconomics, financial markets, and monetary policy. If the data is in concordance with Keynesian models of the long-term interest rate, then it would support the conjecture that the central bank of a country with monetary sovereignty controls government bond yields, the Treasury yield curve, and the slope of the yield curve mainly through the influence of the policy rates on the short-term interest rate. Other aspects of monetary policy, such as calendar-based and information-conditional forward guidance, policy pronouncements, large-scale asset purchases, and yield curve control, should be viewed as secondary. Moreover, the influence of the fiscal deficit ratio or fiscal government debt ratio on government bond yields of a country with monetary sovereignty is likely to pale in comparison to the short-term interest rate. The Keynesian approach to interest rate dynamics can be fruitful for analyzing the central bank's operations and their effect on the economy and financial markets (Fullwiler 2017 [2008]).

5. CONCLUSION

Keynes's approach to interest rate dynamics is based on his conception of ontological uncertainty, liquidity preference, investors' expectations and animal spirits, financial institutions, financial markets, and institutional practices. He maintained that the central bank's policy rate drives the short-term interest rate, which in turn influences the long-term interest rate.

This paper modifies benchmark interest rate models that are widely used in quantitative finance. These modified models show the long-term interest rate as a function of the short-term interest rate and a Wiener process, as represented in stochastic differential equations, to model Keynes's approach to interest rate dynamics. The Keynesian approach to interest rate dynamics has been revived in numerous empirical papers in recent years. The Keynesian approach to interest rate dynamics, as specified in the modified models, can be valuable in addressing issues in macroeconomics, finance, and monetary policy.

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