Bankruptcy Exemption of Repo Markets: Too Much Today for Too Little Tomorrow?

Viral V. Acharya, V. Ravi Anshuman and S. Viswanathan*

ABSTRACT

We examine the desirability of granting "safe harbor" provisions to the sale and repurchase (repo) markets, i.e., granting repo contracts exemption from bankruptcy and automatic stay. Such exemption can enable financial firms to raise greater liquidity and operate at higher leverage in normal times. This liquidity creation occurs, however, at the cost of ex-post inefficiency when there are adverse aggregate shocks to the fundamental quality of collateral underlying the contracts. When exempt from bankruptcy, creditors of highly leveraged financial firms respond to such shocks by engaging in collateral liquidations, which lead to fire sales. Financial arbitrage by less leveraged financial firms equilibrates returns from acquiring financial assets at fire sale prices and those from real-sector lending, inducing a rise in lending rates, a deterioration in endogenous asset quality, a liquidity crunch, and in the extremis, a complete credit crunch in the real sector. Given this inefficiency, an automatic stay on repo contracts in bankruptcy *can be* not only ex-post optimal, but also ex-ante optimal, especially for illiquid collateral with high exposure to aggregate risk, and for economies with a large real sector.

Keywords: Automatic stay, safe harbor provisions, sale and repurchase contracts, fire sales, credit crunch, financial crises, systemic risk

JEL Classification: G01, G21, G28, G33, D62, K11, K12

*Viral Acharya is from New York University (Stern School of Business), V. Ravi Anshuman is from IIM Bangalore, and S. Viswanathan is from Duke University. We acknowledge research assistance of Samidh Pratap Singh, Simrat Singh and especially, Srijith Mohanan. We are grateful to participants at the Federal Reserve Bank of New York workshop on Safe Harbors (June 2012). The usual disclaimer applies. Contact author: V. Ravi Anshuman, C/I 104, IIM Bangalore India 560076, Tel: +91-80-26993062, e-mail: anshuman@iimb.ac.in

1. Introduction

A repurchase agreement – also known as a "sale and repurchase agreement" or more popularly as a "repo" – is a short-term transaction between two parties in which one party borrows cash from the other by pledging a financial security as collateral. One important feature of the repo market in the United States is that all transactions falling under the umbrella of repos are exempt from the automatic stay in bankruptcy of the counter-parties and, therefore, all repo transactions can be settled with immediacy, for example, underlying collateral can be liquidated following a bankruptcy filing. This exemption from bankruptcy, sometimes also called as a "safe harbor" provision, has been extended gradually to different repo markets, starting with Treasuries and Agency (Fannie Mae and Freddie Mac) securities, and most recently in 2005, to repos for non-Agency mortgage-backed assets.[1] The failures of financial firms exposed to mortgages or mortgage-backed securities, such as Countrywide, Bear Stearns and Lehman Brothers, all involved in some part a "repo run," that is, an inability of the borrower to roll over the repo contracts with the financiers. Indeed, since the global financial crisis, there has been stress in the form of Treasury fire sales and "repo rate spikes" even in the Treasury repo market, notably as observed during September 2019 and March 2020.[2]

We build a theory to understand the desirability of granting repo contracts the exemption from bankruptcy. We start with a model (based on Acharya and Viswanathan [2011]) in which financial intermediaries (such as, bank-holding companies) borrow funds from financiers (such as, money-market funds) to originate assets. Since the backdrop we have in mind is one of trading-based financial institutions, which are typically highly levered and are primary borrowers in repo markets, we focus on the agency problem of asset substitution or risk-shifting by borrowers as in Jensen and Meckling [1976]: a borrower, after raising debt, has incentives to transfer wealth away from lenders by switching to riskier assets unless the expected profits from safer assets are sufficiently high.[3]

Given the agency problem and taking a purely partial equilibrium view of the bilateral contract, the ex-ante liquidity of borrowers is greater if they grant liquidation rights on underlying assets to the financiers. The intuition is that if the borrower instead has the right to renegotiate the contracts, then financiers can be driven down to their reservation payoff under the inefficient asset choice. Financiers will anticipate this ex ante and provide lower liquidity to borrowers. The implication

[1] See Acharya and Öncü [2014] for a chronology of these exemptions.

[2] See, in particular, Copeland et al. [2021] and d'Avernas and Vandeweyer [2020].

[3] Related to the work of Stiglitz and Weiss [1981] and Diamond [1989, 1991], this risk-shifting problem rations potential borrowers in that it limits the maximum amount of financing they can raise from lenders.

is that absent considerations other than the agency problem between borrowers and financiers, bankruptcy exemption of collateralized borrowing, as presently accorded to repo contracts, enables financial firms to raise greater liquidity, operate at a higher leverage, and, originate more assets.

This liquidity creation via extension of bankruptcy exemption occurs, however, at potentially significant costs when a general equilibrium view is considered. In particular, when financial firms can also originate assets in the future, say in the form of loans to the real asset sector, the partial equilibrium result can get overturned. We show that there is an inherent conflict between current asset origination based on leverage and future asset origination; bankruptcy exemption amplifies this wedge in inter-temporal asset origination and can lead to too much origination today for too little asset origination tomorrow.

We consider a three date model in which an aggregate economic shock at the intermediate date affects the funding liquidity of financial firms. Upon arrival of adverse news about underlying asset quality, highly-leveraged borrowers face greater funding or rollover stress. Their ability to raise new financing to pay off earlier creditors is diminished, prompting them to sell some legacy financial assets to relax their financing constraint. For an adverse enough shock, partial asset sales do not suffice to roll over existing debt and all assets of such borrowers are liquidated by creditors when given exemption from bankruptcy. Less-leveraged borrowers, in contrast, have surplus capacity to raise financing and acquire the assets being liquidated by highly-leveraged firms. Hence, our general equilibrium model considers asset sales by borrowers (or liquidations by financiers) in an industry equilibrium, and the market-clearing price of legacy financial assets reflects fire-sale effects [Shleifer and Vishny, 1992, Gale and Allen, 1994, Allen and Gale, 1998].

Absent the consideration of new asset origination, such a market-based transfer of assets from highly-leveraged borrowers to less-leveraged borrowers, who can own and manage the assets as efficiently as the original asset owners, does not affect ex-post efficiency. However, if less-leveraged or liquidity-surplus financial firms can also originate new assets in future, then this result is substantially overturned. Let us elaborate. Bankruptcy exemption facilitates a greater degree of ex-ante leverage, which, in turn, causes greater consequent liquidations in the event of an adverse economic shock. The secondary market for the liquidated assets clears at a fire-sale discount because of a moral hazard problem of risk shifting. The extent of the risk shifting problem depends on the state of the economy. Thus, while bankruptcy exemption facilitates greater ex-ante asset creation, it also causes greater consequent liquidations at fire sale discounts, which provide excess returns to capital.

In pursuit of higher returns when liquidated assets are available "too cheap," less-leveraged firms engage in financial arbitrage between the secondary market for liquidated assets (financial sector secondary market) and the new loan origination market (real sector primary market). This cross-market arbitraging activity implies that the expected return from originating new loans must match the expected return from investing in the secondary market for legacy financial assets; in the new loan market, interest rates rise in tandem with the the extent of liquidation, i.e., less-leveraged firms engage in price discrimination in the new loan market. A moral hazard problem arises in the new loan market because household borrowers in the new loan market invest less effort when faced with higher interest rates, resulting in (an endogenously determined) lower loan quality. The drop in loan quality adversely affects lender's (i.e., less-leveraged firm's) expected profits. Thus, there is an upper bound on the interest rate that lenders would charge; the marginal benefit of increasing the interest rate beyond this level is more than offset by the marginal reduction in new loan quality.

When bankruptcy exemption causes too much ex-post liquidation, the returns from investing in the financial asset market and the new loan market (both returns being equal) hit the upper bound. Less-leveraged firms' are no longer interested in deploying additional capital in the new loan market. Instead, they withdraw capital from potential real sector loan origination (causing a real sector liquidity crunch) and deploy the same in the secondary market for liquidation of financial assets. In short, less-leveraged firms engage in price discrimination in the new loan market to the extent possible; however, when this rent extraction hits its limit, they engage in quantity discrimination (reduction in supply). In the extreme, if the state of the economy is such that bankruptcy exemption triggers excessive ex-post liquidation, the market for new loans shuts down. Less-leveraged firms divert all their surplus-liquidity to the legacy financial asset market. This happens when the return from investing in the secondary market for legacy financial assets exceeds the maximum possible return from investing in the new loan market.

To summarize, our model captures the interaction between the moral hazard problem in the financial sector (risk shifting) and the moral hazard problem in the real sector (effort aversion). Cross-market arbitraging activity causes these moral hazard problems to move in sync with each other. Bankruptcy exemption causes too much ex-ante leverage creation, which, in turn, causes greater moral hazard problems and consequent ex-post liquidations, resulting in fire-sale effects, either in the form of "price" effects or "quantity" effects.

We show that bankruptcy exemption *can be* sub-optimal in our model, i.e., the negative externality of bankruptcy exemption (in the form of fire-sale effects in future periods) *can* overwhelm the positive effect (in the form of greater financial intermediation in the current period). The intuition for the result in the context of our model is as follows. While bankruptcy exemption induces ex-ante (Date 0) asset creation, the *incremental* beneficiaries are borrowers with larger investment requirements; these are borrowers who lenders would not have financed if there was no safe harbor. Two key implications arise for borrowers in this situation. First, their investments are necessarily low NPV because of the high level of investment. Second, the financing of a larger investment using debt implies that these borrowers will be the most leveraged ones. They are more likely to be forced to liquidate their assets in the event of an economic shock. The attendant adverse consequences of fire-sale effects due to liquidation of financial assets reduces social welfare in future periods. Put differently, bankruptcy exemption creates highly leveraged financial intermediation (associated with low value addition) in the present; however, in the event of an economic shock, the high leverage in the economy causes a significant reduction in both the quality and quantity of financial intermediation in the future.

The main contribution of our model then can be summarized as follows. First, we show that bankruptcy exemption can cause inefficient financial intermediation. It facilitates the entry of highly-leveraged financial intermediaries that lock up the future surplus liquidity of less leveraged firms for acquiring assets at fire-sale prices; this intermediation capacity could be used otherwise to finance real investment activity. Thus, the model sheds lights on the debate among policy makers about the role of bankruptcy exemption - whether it reduces or exacerbates systemic risk.[4] We show that the former view (that bankruptcy exemption reduces systemic risk) is based on a myopic perspective of preventing ex-post liquidations that can have follow-on effects on lending markets. Once we take an ex-ante perspective and endogenize the implications of ex-post liquidation on ex-ante lending behavior, we can show that bankruptcy exemption never decreases systemic risk.

Second, our model helps derive conditions under which bankruptcy exemption is optimal, and also the conditions under which an automatic stay (the polar opposite policy of bankruptcy exemption) is optimal. An automatic stay on repo contracts in bankruptcy may be especially useful when fire-sale effects in underlying collateral are likely, for instance, in case of less liquid collateral, such as mortgages, that lose value when aggregate risk materializes. An automatic stay is also beneficial when the real sector is very large (as in a developed country) and fire-sale effects can severely reduce economic surplus. On the other hand, bankruptcy exemption of repo contracts can be ex-ante optimal *only* when there are no fire-sale effects; such a situation arises when the magnitude of the economic shock is mild, the collateral is of good quality, and the real sector is small.

[4] Federal Reserve Report [2011], written in the aftermath of the global financial crisis.

In the general case, there is an interior optimum level of bankruptcy exemption, suggesting that a partial form of bankruptcy exemption may be optimal. We show that, ceteris paribus, the optimal extent of bankruptcy exemption is lower when the economic shock is more adverse, collateral quality is weak, and when the real sector is large in magnitude. We also map the different solutions for the optimal level of bankruptcy exemption in the space where all three parameters, the extent of the economic shock, the quality of collateral, and the size of the real sector, vary together.

Section 2 relates our work to theoretical and empirical literature. Section 3 sets up the basic features of the model. Section 4 analyzes the model and presents the ex-post equilibrium outcomes, taking ex-ante leverage as given. Section 5 augments the benchmark model to study the ex-ante debt capacity of firms. Section 6 derives results on ex-ante welfare analysis. Section 7 sheds light on the debate about the role of bankruptcy exemption in affecting systemic risk. Section 8 discusses the determinants of an optimal level of bankruptcy exemption. Section 9 concludes. Proofs are in the Appendix.

2. Related Literature

Our paper is motivated by the empirical literature on the role played by runs in the repo market in exacerbating the financial crisis (Copeland et al. [2010, 2014], Gorton et al. [2010], Gorton and Metrick [2010, 2012], Gorton et al. [2020a] and Gorton et al. [2020b]).[5] In addition, **?** point out that the over-dependence of systematically important financial institutions (SIFI) on repo financing exposed the financial system to systemic risk, which eventually led to an economic contraction. The institutional arrangements of the repo market model can play a critical role in determining how systemic risk propagates in the economy. Our paper addresses a key design feature of repo markets, namely, bankruptcy exemption of repo collateral, in exacerbating crisis-like situations.

The model presented in this paper is closely related to three strands of literature: (i) the role of financial frictions in creating inefficient fire sales; (ii) the welfare implications of leverage-induced fire sales; and (iii) the role of bankruptcy exemptions of repo collateral.

The first strand deals with the role of financial frictions in exacerbating the impact of macroeconomic shocks. These frictions limit the ability of a highly leveraged firm from continuing as a going-concern during an economic shock unless it liquidates some of its assets at fire-sale prices. In addition to the seminal papers referred in the Introduction, this literature is now rather vast. Our model is most closely related to the work of Acharya and Viswanathan [2011] and Lorenzoni [2008].

[5]In a related paper on financial crisis, Gorton et al. [2020a] argue that counter party risk may cause (implicit) debt runs due to fear of solvency. However, our study does not model credit risk.

In Lorenzoni [2008], fire sales are generated by financial frictions that arise due to the limitation of agents to commit credibly to future loan repayments. In Acharya and Viswanathan [2011], funding liquidity is constrained by financial frictions that arise due to a risk-shifting problem; our model extends their framework and considers the interaction of fire sales generated by moral hazard problems in the financial sector (in the form of risk-shifting incentives) with the moral hazard problem in the real sector (in the form of lower endogenous asset quality). This interaction is also a feature of Diamond and Rajan [2011] model of credit freeze in anticipation of future fire sales.

The second strand of literature deals with the welfare implications of leverage-induced fire sale effects. Such liquidation have been argued to cause inefficiencies in the economy (Bordo and Jeanne [2002], Lorenzoni [2008], Stein [2012]). The central feature of these studies is that aggregate leverage and fire-sale effects are endogenously related. Bordo and Jeanne [2002] analyze the ex-post consequences of a sharp decline in asset prices (following an asset price boom) on real economic activity. They advocate that under certain conditions, ex-ante adoption of a tight monetary policy (during the boom period), over and beyond what mere inflation targeting would imply, is optimal when based on aggregate welfare considerations. Lorenzoni [2008] points out there is excess ex-ante borrowing that fails to internalize the ex-post inefficiency due to fire sales and a central planner can improve social welfare by limiting the amount of ex-ante aggregate leverage in the economy. Finally, Stein [2012] examines the financial stability implications of short-term private money creation and how monetary policy and complementary tools such as open-market operations can be deployed to limit the negative externalities arising from fire sales on loan originations.

We build on these two strands of literature in the context of bankruptcy exemption of repo contracts. In particular, our model shows that bankruptcy exemption affects the trade-off between ex-ante credit creation and inefficient ex-post fire sales that limit future credit creation. More importantly, we show that an automatic stay that precludes bankruptcy exemption effectively can be a policy tool that a central planner can employ in reducing ex-ante aggregate leverage in the economy to the welfare-maximizing level. Two recent studies have also explicitly modeled the bankruptcy exemption provision; both use fundamentally different assumptions from our work. First, Antinolfi et al. [2015] show that fire-sale externalities arise due to bankruptcy exemption. However, as they themselves point out, this externality disappears in their model if the exchange of fire-sale assets arises in a a competitive equilibrium. In contrast, fire-sale effects in our model are endogenously determined in a competitive equilibrium and the resulting welfare implication are analyzed. Second, Ma [2017] considers a structural model of the bankruptcy exemption provision to evaluate how it affects the coordination problem of creditors in a repo run and the strategic declaration of bankruptcy by a firm; the model, however, does not consider the spillovers effects on the real sector, which is the focus of our analysis.

The third strand of related literature argues that bankruptcy exemption of repo collateral increases creditor rights and thereby exacerbates ex-post fire-sale liquidations (Tuckman [2010], Acharya and Öncü [2014]). Duffie and Skeel [2012] recognize the role of bankruptcy exemption in increasing systemic risks and propose limiting the bankruptcy exemption to repos and (centrally cleared) derivative contracts that are backed with highly liquid collateral. Tuckman [2010], too, advocates restricting the safe harbor provision to only those derivatives that are centrally cleared to reduce the risk of fire sales in the event of an adverse shock and to also reduce the incentives of market participants to take up large position in complex, illiquid derivatives wherein the underlying assets are most susceptible to crashes. Acharya and Öncü [2014] recommend withdrawing the safe harbor exemption from all repo transactions other than those having government backed claims as collateral. We confirm the intuition of this literature that stronger creditor rights accorded as safe-harbor provisions to repo contracts facilitate ex-ante credit creation, but cause ex-post fire sales in the event of an adverse aggregate shock to the economy. Our theoretical setup allows for a welfare analysis factoring in both of these effects. More recently, Zhong and Zhou (2021) endogenize ex-post bankruptcy payoffs to evaluate the ex-ante decision to stay invested in a firm. Thus, they are able to establish a timing consistent approach to ex-post and ex-ante credit runs.

The issue of bankruptcy exemption has also caught the attention of the legal profession. Several articles in law journals have discussed the costs and benefits of the safe harbor provision which allows lenders to liquidate collateral in the event of bankruptcy. These articles also discuss collateral runs as an important factor in evaluating bankruptcy exemption (e.g., Edwards and Morrison [2005], Jackson [2009], Skeel and Jackson [2011], Federal Reserve Report [2011], Duffie and Skeel [2012], Mooney Jr [2014] and Morrison et al. [2014]).

3. Model Setup

We build a model of financial intermediation to understand the role of bankruptcy exemption. Our objective is to determine the optimal extent of bankruptcy exemption. We partition our analysis into two sections: first, we examine the role of bankruptcy exemption on ex-post liquidation effects under an exogneous assumption about the ex-ante leverage in the economy; in the second section, we endogenize the leverage decisions from an ex-ante perspective and derive the ex-ante optimal level of bankruptcy exemption.

Our model follows the setup in Acharya and Viswanathan [2011]. Financial intermediary firms make investment decisions in a two-period, three-date world – a start date (Date 0), an intermediate date (Date 1), and a terminal date (Date 2). Figure 1 shows the payoffs on the assets in the economy and the time line of the model. We discuss below the role of financial intermediaries, the available assets in the economy and their Date 2 payoffs, followed by a summary of the sequence of key events in the model.

3.1. Financial Intermediaries

The economy consists of a continuum of financial intermediaries. They start out with differing levels of financial infrastructure and/or human capital, both of which are required for participating in the financial intermediation sector. Depending on the accumulation of infrastructure assets, these financial intermediaries require differing amounts of capital (investment shortfall, s) to start a business by acquiring a financial asset. Similar to the approach followed by Anderson and Sundaresan [1996] in analyzing debt contract design, we assume that the investment shortfall is financed in the short-term debt market; more specifically, in the short-term repo market.[6] Thus, at Date 0, financial firms vary in terms of the degree of leverage in their balance sheets.

3.2. Assets in the Economy

There are two sectors in the economy, the financial sector (consisting of financial assets) and the real sector (consisting of real assets/loans). The financial asset is originated at Date 0, but the real asset is originated at Date 1.

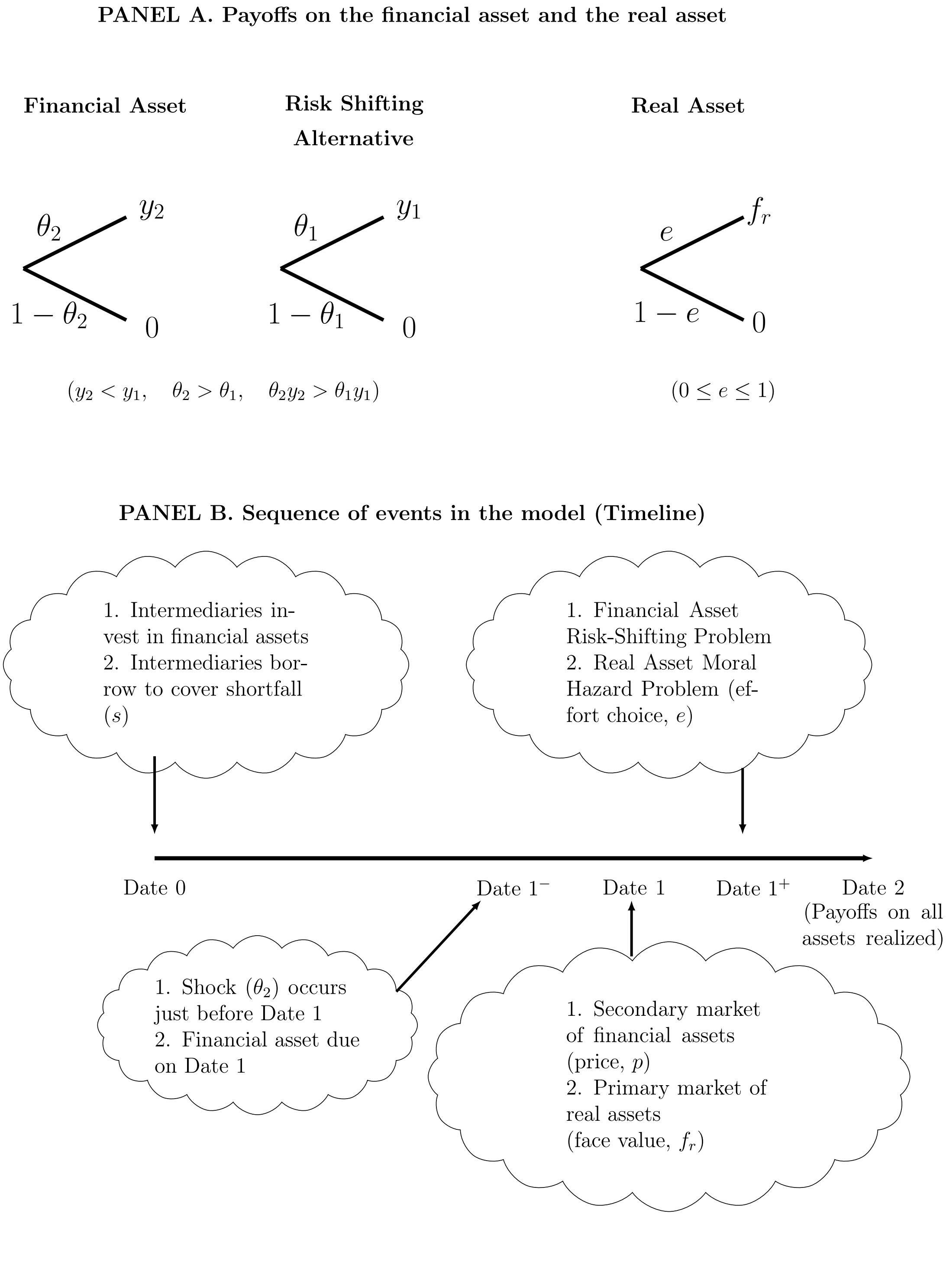
The financial asset could be a legacy loan or a pool of loans, which produces uncertain cash flows at Date 2, and against which intermediaries can raise leverage at Date 0 in the form of repo contracts maturing at Date 1. We model the financial asset as a commoditized product, which allows for an exchange between financial intermediaries without loss of value.

There is an additional asset in the financial sector, which is a risk-shifting alternative to the financial asset. The risk-shifting alternative is never taken up in equilibrium but it affects the decisions of agents in the economy, as will be discussed further down.

The real sector is characterized by asset specificity (because of a moral hazard problem that

[6] In earlier studies, Aghion and Bolton [1992] and Hart and Moore [1994] have used this approach in the context of security design.

Figure 1: **Description of the Model.** Panel A shows the Date 2 payoffs on the financial asset, the real asset, and the risk-shifting alternatives. Panel B show the sequence of events in the model.



is borrower-specific, as will be elaborated further down). The real asset can be thought of as relatively illiquid loans, e.g., a mortgage or small-business loan to households that are originated at the intermediate date, Date 1, and mature at Date 2. The cash flows from the real asset are not pledgeable by intermediaries (at Date 0 or at Date 1) to raise finances.

The payoffs on the financial asset, the risk-shifting alternative, and the real asset are shown in Figure **1** (Panel A).

3.2.1. Financial Asset Payoffs

The risk-shifting asset (denoted 1) and the safer asset (denoted 2) have the following payoffs at Date 2: the safer asset has a payoff of y_2 with a probability of θ_2 and a payoff of 0 with a probability of $(1-\theta_2)$; the risk-shifting alternative has a payoff of y_1 with a probability of θ_1 and a payoff of 0 with a probability of $(1-\theta_1)$. Further, $\theta_1 < \theta_2$, $y_1 > y_2$ and $\theta_1 y_1 \leq \theta_2 y_2$. Thus, while the first asset has a higher payoff in the non-default state, it experiences a higher likelihood of the default state. More importantly, the first asset is riskier in that it has a lower expected payoffs as compared to the second asset (i.e., $\theta_1 y_1 \leq \theta_2 y_2$) and has a higher variance per unit expected payoff compared to second asset (i.e. $(1-\theta_1)y_1 > (1-\theta_2)y_2$).[7]

3.2.2. Real Asset Payoffs

The real sector (of size $\mathcal{B}$) consists of assets such as new mortgage or small business loans taken up by households at Date 1. The payoffs on the real asset are shown in Figure 1 (Panel A). There is an outflow of 1 unit at Date 1 and there is an uncertain binary payoff at Date 2: with a probability e, the payoff is a face value (f_r); otherwise, it is 0. The probability e reflects the household effort choice based on a moral hazard problem. Both e and f_r are endogenously determined, as will be discussed further down. Given that the loan amount is normalized to unity, the face value (f_r) effectively determines the interest rate of household loans.

3.3. Summary of Sequence of Events/Decisions

The time-line of the model is shown in Figure **1** (Panel B). At Date 0, financial firms invest in a financial asset after borrowing the required financing (to cover the investment shortfall, s) in the short-term debt market. At Date 1^-, the economy experiences a shock, which renders firms as

[7] As in Acharya and Viswanathan [2011], we assume that assets are financial sector specific and cannot be redeployed by financiers in the event of the closure of the firm. Furthermore, following Acharya and Viswanathan [2011], we also assume that risk-shifting is costless to implement.

either surplus-liquidity firms (less leveraged firms) that are looking for investment opportunities or credit-constrained firms (highly leveraged firms) that are unable to roll over their short-term debt claims to the next period. At Date 1, firms with surplus-liquidity face two investment opportunities. On the one hand, they could invest in the (financial) asset re-sale market where they can acquire the financial assets of credit-constrained firms at a price p (that is endogenously determined as an equilibrium outcome). On the other hand, surplus-liquidity firms can also consider investing their surplus in the real sector by investing 1 unit in each real asset. At Date 1^+, the firm can exercise the risk-shifting alternative, if desired, and the household makes the effort choice on the real asset. At Date 2, all asset payoffs are realized. Note that the distinction in teh sequence of events at Date 1^-, Date 1, and Date 1^+ is a precise representation; for convenience, we refer to the set of events as Date 1 events, e.g., a Date 1 economic shock.

Intermediation decisions are thus made at Date 0 (raising repo financing to enter the financial sector) and Date 1 (repaying repo contracts and extending illiquid loans to the real sector). We refer to intermediary decisions/outcomes at Date 1 as coming from the *ex-post* model and decisions/outcomes at Date 0 as coming from the *ex-ante* model. The ex-ante model must take into account the optimal decision strategies and outcomes of the ex-post Date 1 equilibrium; at the same time, the ex-post equilibrium strategies and outcomes are affected by the strategies of ex-ante optimization, a key feature of the model, as in Acharya and Viswanathan [2011].

3.4. Salient Features of the Model

Our model differs from the Acharya and Viswanathan [2011] setup in three significant ways. First, we recognize that not all firms can be forced by lenders to liquidate their assets. In practice, we often observe strategic write-downs as a result of renegotiation between the firm and its lenders. We define a parameter (q) that reflects the probability of a credit-constrained firm being unable to roll over its debt and consequently forced to take recourse to the asset sale market to meet its liabilities at Date 1. Conversely, $(1 - q)$ is the probability that such a credit-constrained firm is able to renegotiate with the lender and write-down its obligations. One could view q in the context of the bankruptcy code. If $q = 1$, the asset is exempt from an automatic stay and the lender enjoys exclusive rights over the asset in the event of bankruptcy, a feature that allows the lender to liquidate the asset in the secondary market. We, therefore, refer to q as the bankruptcy exemption parameter; it describes the likelihood of the lender retaining control of the asset in the event of a borrower default.

The second major point of departure from the Acharya and Viswanathan [2011] model is that we allow for the existence of a new loan market at Date 1. After the Date 1 shock has been realized, firms that are not credit-constrained, i.e., surplus-liquidity firms, can also invest in a new loan market. These new loans can be thought of as new mortgage or small business loans. Thus, firms with surplus-liquidity at Date 1 can either invest in the secondary market for financial assets or in the primary (origination) market of real assets. This characterization allows us to analyze the interplay between the financial asset sale market and the real economy.

The third major point of departure is that we take into account moral hazard in the real economy. For instance, in the case of mortgage loans, households being residual claimants on levered assets would have lower incentives to invest in maintenance of the asset if the borrowing rate is too high (as we will show to be the case when an adverse shock occurs in the economy). Fixed claims, such as debt, exacerbate moral hazard problems in the real sector when loan rates are too high and our model captures this insight.

4. Optimizing Behavior of Agents

In this section, we discuss the ex-post equilibrium at Date 1. After solving the ex-post model at Date 1, we will then solve the ex-ante model (at Date 0) in Section 5.

In the ex-post setup, we will be working with an exogenously given distribution of leverage choices made by individual financial intermediaries at Date 0. We analyze the optimizing behavior of the agents in the economy, i.e., the decisions of lenders, households, credit-constrained firms, and surplus-liquidity firms at Date 1, for a given economic shock (θ_2).

In the ex-ante model, leverage choices made by financial intermediaries at Date 0 are endogenized. More specifically, the leverage decision is determined after taking into account the liquidation price of the financial asset at Date 1, as determined in the ex-post model. Because the liquidation price (at Date 1) is itself a function of the distribution of leverage in the economy (determined at Date 0), there is an endogenous relation between leverage distribution in the economy at Date 0 and the financial asset's liquidation price. The ex-ante model accounts for this endogeneity. Keeping this objective in the background, we begin by analyzing the optimizing behavior of agents in the economy at Date 1 (ex-post model).

4.1. Lender's Decision to Roll Over Short-term Debt

At the intermediate date, Date 1, the economy suffers an observable, but unverifiable shock (θ_2). Depending on the shock, lenders demand repayments at Date 1 or agree to roll over debt to Date 2. A financial asset sale market exists where firms can liquidate their claims on the asset in order to service outstanding debt. The counter-parties in this asset sale market are firms with surplus-liquidity. After the realization of the Date 1 shock, the asset sale market is cleared and (some) debts rolled over, a moral hazard situation occurs. Credit-constrained firms that have successfully raised capital can explore the possibility of investing in an alternative riskier financial asset (the risk-shifting asset) which may become attractive once rollover debt has been tied up after the Date 1 shock. Thus, Date 1 financing must account for this risk-shifting possibility. We present a lemma on the funding liquidity of the financial asset at Date 1 after accounting for the risk-shifting problem.

LEMMA 1: *The funding liquidity at Date 1 per unit of the safer asset is $\rho^* = \theta_2 \frac{\theta_2 y_2 - \theta_1 y_1}{\theta_2 - \theta_1}$. The reduction in funding liquidity is given by k_1, such that $k_1 = \theta_2 y_2 - \rho^* = \frac{\theta_2 \theta_1 (y_1 - y_2)}{\theta_2 - \theta_1}$. k_1 is decreasing in y_2 and θ_2.*

The funding liquidity of an asset at Date 1 is the amount of rollover debt that can be raised by pledging the asset. Since the risk-shifting asset is a negative value investment, lenders would want to set the face value (f) in such a way that the borrower has no incentives to risk shift. This requires $\theta_2(y_2 - f) > \theta_1(y_1 - f)$, which implies that $f < \frac{\theta_2 y_2 - \theta_1 y_1}{\theta_2 - \theta_1}$. The funding liquidity of the financial asset is given by the loan amount that lenders would be able to finance, given the face value (f). In this case, funding liquidity is equal to $\theta_2 f$. The funding liquidity (ρ^*) is represented as $\theta_2 y_2 - k_1$. One can think of k_1 as the non-pledgable portion of expected cash flows ($\theta_2 y_2$) or the funding illiquidity of the asset due to the risk-shifting agency problem. It can be easily seen that this funding illiquidity (k_1) reduces as the payoff of the asset (y_2) or the economic outlook for the asset (θ_2) improves.

The key implication of the above lemma is that funding liquidity of the financial asset depends on the economic shock to asset quality (θ_2). Because firms differ in the amount of debt assumed at Date 0, the economic shock has differing implications for different firms. The deterioration in asset quality may cause highly leveraged firms to become credit-constrained at Date 1, and they may be forced to liquidate some or all their positions in the financial asset to meet the liabilities due at Date 1. In contrast, less leveraged firms would enjoy excess liquidity and would be looking for opportunities to invest their capital. Such surplus-liquidity firms can either participate in the (financial) asset sale market created to meet the liquidation requirements of credit-constrained firms, or in a new loan (real asset) market, which represents demands from the real sector at Date 1.

While the financial asset is subject to risk-shifting concerns which affect its funding liquidity, we assume that the real asset cannot be pledged to raise capital, i.e, it has no funding liquidity.

4.2. Household's Moral Hazard Problem

Surplus-liquidity firms that invest in the real asset provide one unit of capital at Date 1 to households in return for a promised payment of f_r at Date 2. Households use this capital to finance investment in a physical asset that provides a rental income of R at Date 2. Thus, households view their leveraged investment as paying a cash flow of $R - f_r$ in the high state (which occurs with a probability of e) and a cash flow of 0 in the low state (which occurs with a probability of $1 - e$). The probability e, which is endogenously determined, reflects the effort choice of the household, and, thus, the asset quality.

The expected benefit from renting is $e(R - f_r)$. The household must expend effort to maintain the physical asset (i.e., ensure asset quality) but is effort averse; this conflict results in a moral hazard problem. To capture the dis-utility associated with expending effort, we consider its pecuniary equivalent to estimate the net expected payoffs from investing in the physical asset. For tractability, we assume that the pecuniary equivalent of expending effort is quadratic in the level of effort; more specifically, the cost is equal to $\frac{1}{2}\gamma e^2$, where γ captures the intensity of effort aversion. A risk neutral household chooses an effort level e that trades off the benefits of asset quality with effort aversion, and maximizes net expected payoffs of $e(R - f_r) - \frac{1}{2}\gamma e^2$. Given the bounds on the effort choice ($0 \leq e \leq 1$), the optimal solution is given by,

$$e^* = \min\left[\max[0, \frac{1}{\gamma}(R - f_r), 1]\right]. \tag{1}$$

To ensure that the second order condition holds for maximization, we need $\gamma > 0$. Then, Lemma 2 follows; it implies that the moral hazard problem worsens when interest rate (or face value) increases.

LEMMA 2: *The optimal effort level of the representative household (e^*), and, thus, the asset quality, is negatively related to the face value (f_r) of the real asset loan.*

4.3. *Liquidation Decisions of Credit-Constrained Firms*

The continuum of intermediary firms differ from each other in terms of the investment shortfalls (s) required to enter the financial intermediation sector; thus, these intermediaries differ in terms of their outstanding liabilities (ρ) due at Date 1. Suppose the distribution of ρ is given by $\rho \sim G(\rho)$ over $[\rho_{\min}, \rho_{\max}]$, where $\theta_1 y_1 \leq \rho_{min} < \theta_2 y_2 \leq \rho_{\max}$ and $\rho^* \in [\rho_{\min}, \rho_{\max}]$, where ρ^* is the funding liquidity of the financial asset. At Date 1^-, when the economy-wide shock (θ_2) is realized, firms will either be credit-constrained ($\rho \geq \rho^*$) or will enjoy surplus-liquidity ($\rho < \rho^*$). Thus, a market for the financial asset is created in which credit-constrained firms supply the financial asset and surplus-liquidity firms demand the financial asset. The market for financial assets clears at a price p, which will be derived keeping in mind that surplus-liquidity firms can also divert their surplus liquidity into the household loan market (i.e, real asset market), which opens at Date 1.

To raise ρ units to roll over debt, a firm must choose a liquidation policy $\delta \geq 0$ such that $[\delta p + (1-\delta)\rho^*] \geq \rho$. It follows that $\delta(p,\rho) \geq \frac{(\rho-\rho^*)}{(p-\rho^*)}$. Note that $\delta(p,\rho) > 0$ if and only if $\rho > \rho^*$, i.e., only credit-constrained firms will liquidate some of their assets. In contrast, surplus-liquidity firms ($\rho < \rho^*$) will take long positions in the financial asset. Furthermore, notice that some firms with extremely high values of ρ (i.e. $\rho > p$) would be forced to liquidate their entire holdings ($\delta = 1$).

The aggregate supply of financial asset depends on the bankruptcy exemption parameter (q) as follows. Only a fraction q of the credit-constrained firms actually go into liquidation. The remaining fraction $(1-q)$ attempt a strategic write-down by entering into negotiations with the lenders.[8] We assume that the liability can be renegotiated downward to its capacity to raise funds, or its funding liquidity, ρ^*. Thus, given an adverse shock θ_2 at Date 1, a fraction q of the credit-constrained firms will be forced to liquidate some or part of their assets (depending on the liabilities they face).

If $g(\rho)$ denotes the p.d.f. of ρ, the aggregate supply of financial assets in the market is given by

$$S(p,\rho^*) = \int_{\rho^*}^{\rho_{\max}} q \min\left[\frac{(\rho-\rho^*)}{(p-\rho^*)}, 1\right] \; g(\rho) d\rho. \tag{2}$$

4.4. *Ex-post Equilibrium*

Consider a firm that has surplus liquidity, i.e., $\rho < \rho^*$. It can use this excess liquidity to acquire new assets. Suppose that this firm acquires α units of the financial asset in the asset sale market

[8] The bankruptcy exemption parameter (q) can be thought of as an average value that captures the average "style" of heterogeneous judges who interpret the bankruptcy code in their individual style. From a cross-sectional perspective, q can be thought of as capturing judge fixed effects.

and lends β units in the new loan market at Date 1. Such surplus-liquidity firms would optimally choose α and β, for a given p and f_r and a conjectured household effort choice (e^*), as stated in Equation (1).

For a given realization of the economic shock at Date 1, the competitive behavior of agents in the economy results in a competitive equilibrium, which is defined formally below:

An ex-post competitive equilibrium, as characterized by $(\bar{\alpha}, \bar{\beta}, p, f_r, e^*)$, is determined as follows:

(i) Households maximize their effort given the face value (f_r) of the real asset loan, as given by Equation (1), which is stated below:

$$e^* = \min \left[\max[0, \frac{1}{\gamma}(R - f_r), 1] \right]. \tag{3}$$

(ii) Surplus-liquidity firms maximize the incremental benefits from acquiring financial assets (α)of credit-constrained firms in the secondary market of legacy financial assets and providing unit amount loans (β) to households in the primary market of real asset loans; they assume a given p and f_r, and conjecture on e^*.

$$\max_{\alpha \geq 0, \beta \geq 0} \quad (1 + \alpha)(\theta_2 y_2 - \rho^*) + \beta e f_r, \tag{4}$$

subject to the budget constraint on liquidity.

$$\alpha(p - \rho^*) + \beta \leq \rho^* - \rho, \tag{5}$$

(iii) Let the optimal choice for α and β be $\alpha^*(\rho)$ and $\beta^*(\rho)$, respectively. The aggregate demand for the financial asset is given by

$$\bar{\alpha} = \int_{\rho_{min}}^{\rho_*} \alpha^*(\rho) g(\rho) d\rho, \tag{6}$$

and the aggregate demand for the real asset is given by

$$\bar{\beta} = \int_{\rho_{min}}^{\rho^*} \beta^*(\rho) g(\rho) d\rho \leq \mathcal{B}, \tag{7}$$

where $\mathcal{B}$ denotes the size of the real sector.

The objective function in (4) captures the incremental benefits associated with acquiring financial and real assets. Acquiring one unit of the financial asset yields an expected payoff of $\theta_2 y_2$, which implies that the incremental benefit over and above the funding liquidity of the financial asset is

$(\theta_2 y_2 - \rho^*)$. Since the real asset cash flows cannot be pledged, the incremental benefit of acquiring one unit of the real asset is the same as its expected payoff, i.e., ef_r.

The constraint in (5) is the liquidity budget constraint of a surplus liquidity firm. The right hand side reflects the available surplus liquidity in the firm. The left hand side represents the allocation of liquidity by surplus-liquidity firms toward acquiring α financial assets and lending on β household loans in the real asset market. The other two constraints (not explicitly stated) are that there is a non-negative demand for the financial asset and the real asset.

4.4.1. *Equilibrium Restrictions on face value (f_r) and price p*

Some basic restrictions on the face value (f_r and the finacial asset price (p) must be satisfied in equilibrium.

(i) For non-trivial effort choice, we require $e^* > 0$, i.e., $\frac{1}{\gamma}(R - f_r) > 0$, i.e, $f_r < R$.

(ii) We require $f_r \leq f_r^m = \frac{R}{2}$,[9] where f_r^m denotes the surplus-liquidity firm's profit-maximizing face value (since profits are concave in f_r, lenders have no incentive to post a higher face value).

(iii) $ef_r \geq 1$, otherwise there is no investment in real sector, i.e., $ef_r = \frac{1}{\gamma}(R - f_r)f_r \geq 1$.[10]

(iv) Combining all the above constraints, we get: $\frac{R}{2} - \sqrt{R^2 - 4\gamma} \leq f_r \leq \frac{R}{2}$.

(v) The financial asset price (p) must lie in the interval $(\rho^*, \theta_2 y_2)$.

The last restriction on the price of the financial asset (p) follows because (i) it cannot exceed the expected payoffs on the asset ($\theta_2 y_2$) and (ii) it must be strictly higher than the funding liquidity (ρ^*), otherwise the demand for the asset would be infinite.

[9] Note that f_r^m can be solved as $\text{argmax}_{f_r} ef_r$ s.t. $e = \frac{1}{\gamma}(R - f_r)$; it follows that $f_r^m = \frac{R}{2}$.

[10] We also require additional restrictions on γ, as discussed in what follows. Under fair pricing of household loans (i.e., when $ef_r = 1$), the face value f_r is equal to $\frac{R}{2} - \frac{\sqrt{(R^2 - 4\gamma)}}{2}$, which is the lower root of the quadratic equation in f_r. To ensure that $e \leq 1$, we require $f_r \geq 1$, i.e., we require $(R-2)^2 \geq (R^2 - 4\gamma)$ which implies $\gamma \geq R - 1$. Furthermore, we also require $\gamma \leq \frac{R^2}{4}$; a greater value of γ would result in an imaginary solution for f_r. Combining these restrictions, we require $R - 1 \leq \gamma \leq \frac{R^2}{4}$.

4.5. *Cross-Market Equilibrium Returns*

The optimization exercise of surplus-liquidity firms yields an equilibrium relation between the incremental expected return from investing in the financial asset market ($= \frac{k_1}{p-\rho^*}$)[11] and the real asset markets ($= ef_r$),[12] as stated in the lemma below.

LEMMA 3: *(i) when both the financial asset market and the real asset market are open:*

$$\bar{\beta} > 0 \implies \frac{k_1}{p-\rho^*} = ef_r. \tag{8}$$

(ii) when only the financial asset market is open:

$$\bar{\beta} = 0 \implies \frac{k_1}{p-\rho^*} > ef_r. \tag{9}$$

Equation (8) states that the incremental expected return from investing in two asset markets must be strictly equal. If they are unequal, all surplus capital will flow to the market offering higher return, thereby causing a shutdown of the other market. Thus, when both markets are open, it must be the case the returns are equal across the two markets.[13] Equation (9) states that when only the financial asset market is open, the return from investing in the financial asset must necessarily be strictly greater than the return from investing in the real asset. Note that the financial market must necessarily clear (i.e., α is strictly greater than 0) because it is a secondary market of legacy assets. In contrast, the real asset market is a primary market and is constrained by supply and therefore it can be closed in equilibrium.

4.6. *Financial Asset Market Clearing Price (p)*

For a firm that has surplus-liquidity., i.e., $\rho < \rho^*$, let the optimal choice for α and β be $\alpha^*(\rho)$ and $\beta^*(\rho)$, respectively. Then, the aggregate demand for the financial asset is given by $\bar{\alpha} =$

[11] The numerator and denominator of the expression $\frac{k_1}{p-\rho^*}$ represent the marginal benefit (expected benefits net of funding liquidity) and marginal cost (market price net of funding liquidity) of acquiring the financial asset.

[12] The return per dollar of investment in the real asset market is given by the ratio of the marginal benefit ($ef_r - 0$) and the marginal cost is given by $(1 - 0)$, where 0 indicates the funding liquidity of the real asset and 1 indicates the loan amount of 1 unit.

[13] This feature of the model is an important insight that resonates with the views of some economists on the import of fire sales during a crisis (Diamond and Rajan [2011], Hanson et al. [2011], and Stein [2012]). We produce below a specific example used in Hanson et al. [2011] to bring this point home effectively: "*If a toxic mortgage security falls in price to the point where it offers a (risk-adjusted) 20 percent rate of return to a prospective buyer, this will tend to drive the rate on new loans up towards 20 percent as well - since from the perspective of an intermediary that can choose to either make new loans or buy distressed securities, the expected rate of return on the two must be equalized. In other words, in market equilibrium, the real costs of fire sales manifest themselves in the further deepening of credit crunches.*"

$\int_{\rho_{min}}^{\rho_*} \alpha^*(\rho)g(\rho)d\rho$, and the aggregate demand for the real asset is given by $\bar{\beta} = \int_{\rho_{min}}^{\rho^*} \beta^*(\rho)g(\rho)d\rho$. The aggregate demand for financial and real assets should be equal to the aggregate liquidity of the surplus-liquidity firms. We thus obtain the following aggregate budget constraint.

$$\bar{\alpha}(p - \rho^*) + \bar{\beta} = \int_{\rho_{\min}}^{\rho^*} (\rho^* - \rho)g(\rho)d\rho. \tag{10}$$

Since financial asset markets must necessarily clear, the aggregate demand for financial assets ($\bar{\alpha}$) should be equal to the aggregate supply of financial assets, as given by Equation (2). This implies that the aggregate budget constraint becomes

$$\int_{\rho^*}^{\rho_{\max}} q \min\left(\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right) \ g(\rho)d\rho + \bar{\beta}\frac{1}{p - \rho^*} = \int_{\rho_{\min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*} \ g(\rho)d\rho. \tag{11}$$

Equation (11) can be solved to determine the market clearing price of the financial asset (p):

Lemma 4: *The financial asset market clears at an equilibrium price ($p(\bar{\beta};\theta_2)$) given by*

$$p = \rho^* + \frac{1}{q}\int_{\rho_{min}}^{p} G(\rho)d\rho - \frac{1-q}{q}\int_{\rho^*}^{p} G(\rho)d\rho - \frac{\bar{\beta}}{q}. \tag{12}$$

The first term on the right hand side of Equation (12) represents the funding liquidity of the asset. The second and third term capture the excess liquidity in the financial market (i.e., the net of the liquidation demand of credit-constrained firms and the liquidity surplus of surplus-liquidity firms). The last term represent the liquidity flowing into the real sector in the form of real asset loans. The combination of these three terms reflects the spare liquidity in the economy.

If the spare liquidity in the economy is sufficiently high and exceeds the funding illuquidity of the asset (k_1), the financial asset will trade at its fair value of $\theta_2 y_2$. This situation would arise when the economic shock is too mild. When the spare liquidity in the economy is lower than k_1, fire sales arise and the financial asset trades at a discount to its fair value. In short, whether the financial asset price trades at a discount or a fair value depends on the spare liquidity in the economy. For a given economic shock (θ_2), the spare liquidity depends on the bankruptcy exemption parameter (q). A high value of q triggers greater liquidation by credit-constrained firms. Thus, as q increases, the spare liquidity in the economy diminishes, fire sales arise, and the financial asset trades at a discount to the fair value.[14]

[14] In the ex-post equilibrium, we take the economic shock (θ_2) as given on Date 1, but in general, the combination of (θ_2, q) determines the aggregate liquidation of financial assets by credit-constrained firms, as described in Equation (2), which in turn, causes the market price to trade at or below the fair value.

Proposition 1: *Conditional on the economic shock (θ_2), the economy lies in either one of two mutually exclusive regions: the Fair Pricing Equilibrium Region, where both the financial asset and the real asset are fairly priced, and the Fire Sale Equilibrium Region, where both the financial asset and the real asset are priced a discount to the fair value. In the Fair Pricing Equilibrium Region, the equilibrium characteristics are given by*

$$p = \theta_2 y_2, \tag{13}$$

$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma} < \frac{R}{2}, \tag{14}$$

$$\bar{\beta} = \mathcal{B}. \tag{15}$$

The critical factor driving the type of equilibrium region is the amount of spare liquidity in the economy. For a given economic shock (θ_2), the spare liquidity depends on the bankruptcy exemption parameter (q). At lower values of q, bankruptcy exemption is rarely applicable and most credit-constrained firms are able to renegotiate their debt to a lower face value and roll over their obligations. There is minimal liquidation in such an economy and the spare liquidity of surplus-liquidity firms is sufficiently high to cause the equilibrium market clearing price to hit the corner, i.e., the financial asset trades at the fair value of $\theta_2 y_2$ (Fair Pricing Equilibrium Region). For higher values of q, there is greater liquidation of the financial asset subsequent to the economic shock, and the spare liquidity of surplus-liquidity firms is stretched, resulting in an equilibrium market clearing price lower than the fair value (Fire Sale Equilibrium Region).

Proposition 2: *The Fire Sale Equilibrium Region consists of three types of equilbria, depending on the value of the bankruptcy exemption parameter (q), as discussed below.*

(i) The Real Sector Price Discrimination Equilibrium: Both the financial asset market and the real asset market are open and the real asset loans exhibit price discrimination.

$$\bar{\beta} = \mathcal{B}, \tag{16}$$

$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - \frac{4\gamma k_1}{p - \rho^*}} < \frac{R}{2}, \tag{17}$$

$$p = \rho^* + \frac{1}{q}\int_{\rho_{min}}^{\rho^*} G(\rho)d\rho + \int_{\rho^*}^{p} G(\rho)d\rho - \frac{\mathcal{B}}{q}. \tag{18}$$

(ii) The Real Sector Liquidity Crunch Equilibrium: Both the financial asset market and the real asset market are open and the real asset market experiences a fire-sale "quantity" effect. The

equilibrium price (p), the equilibrium face value (f_r) and the number of real asset loans ($\bar{\beta}$) are specified as stated below.

$$
\begin{aligned}
p &= \rho^* + \frac{4\gamma k_1}{R^2}, && (19)\\
f_r &= \frac{R}{2}, && (20)\\
\bar{\beta} &= -q(p-\rho^*) + \int_{\rho_{min}}^{\rho^*} G(\rho)d\rho + q\int_{\rho^*}^{p} G(\rho)d\rho. && (21)
\end{aligned}
$$

(iii) The Real Sector Credit Crunch Equilibrium: The real sector shuts down. Only the financial asset market is open and it is associated with a fire-sale "price" effect. The equilibrium price (p) is given as below (note, $\bar{\beta} = 0$, and although $f_r = \frac{R}{2}$):

$$
p = \rho^* + \frac{1}{q}\int_{\rho_{min}}^{\rho^*} G(\rho)d\rho + \int_{\rho^*}^{p} G(\rho)d\rho. \qquad (22)
$$

Consider what would happen as q varies over the interval $(0, 1)$ for a given level of economic shock (θ_2). For low values of q, the economy would be in the Fair Pricing Equilibrium, as discussed in Proposition (1). As q increases, there would be greater liquidation by credit-constrained firms, causing the financial asset market clearing price to drop below the fair value ($\theta_2 y_2$). There are three types of equilibria that would arise depending on the value of q. As q increases from 0 toward 1, the economy transitions from the Fair Pricing Equilibrium to the Price Discrimination Equilibrium, then to the Liquidity Crunch Equilibrium, and finally to the Credit Crunch Equilibrium. These equilibria are discussed in greater detail below.

4.7. Real Sector Price Discrimination Equilibrium

If q is higher than at the border of the Fair Pricing and Fire Sale Equilibrium Regions, there is enough liquidation of assets to cause the financial asset market clearing price to be lower than the fair value of $\theta_2 y_2$. In this region, there is a fire-sale "price" effect" in that as q increases, the price discount from fair value increases. This pricing feature is similar to the "cash-in-the-market" pricing in Gale and Allen [1994] and Allen and Gale [1998].

The fire-sale "price" effect causes the gross return from investing in the financial asset to exceed 1. Cross-market arbitraging activity would imply that the expected return from investing in the real asset must match that from investing in the financial asset. Consequently, the face value (equivalently, the effective interest rate) on the real asset loans would be increased to offer the same return as on the financial asset. We refer to this equilibrium as the Price Discrimination Equilibrium because surplus-liquidity firms will divert their resources to the real asset market only if they can earn supra-normal rents, i.e., discriminate on price to ensure that they get the same return as on the financial asset.

At a sufficiently high value of q, the economy transitions to the Real Sector Liquidity Crunch Region, as discussed below.

4.8. Real Sector Liquidity Crunch Equilibrium

There is a limit to which surplus-liquidity firms can engage in price discrimination, by increasing the face value on the real asset loan. There is an upper bound on the face value because of a moral hazard problem in the real sector. Borrowers, being residual cash flow claimants, expend less effort as the face value increases, as shown in Equation (1). Due to lower effort, the asset quality suffers as the face value increases. The expected profit from lending in the real sector is, therefore, concave in the face value of the real asset loan. The profit-maximizing face value is $\frac{R}{2}$, and surplus-liquidity traders would never find it incentive compatible to post a higher face value than $\frac{R}{2}$ because the marginal benefit from a higher face value will be lower than the marginal cost in the form of loans with lower asset quality.[15] When the upper bound in face value is hit due to an increase in q, the return from investing in the real asset hits an upper bound. The economy transitions from the Real Sector Price Discrimination Equilibrium Region to the Real Sector Liquidity Crunch Equilibrium Region.

In this Liquidity Crunch Equilibrium Region, the financial asset price remains invariant to q because the real asset return has hit an upper bound and cannot increase any further even when q increases. Cross-market arbitraging activity implies that the financial asset return is also arrested, and the price of the financial asset price stays at the same level for all value of q in this region. The financial asset price can no longer adjust to ensure market clearing. Instead, financial market clearing is now ensured by sucking out liquidity from the real sector, i.e., by a reduction in $\bar{\beta}$. This diversion of surplus-liquidity firms' resources is required to clear the financial asset market, and the real sector contracts with an increase in q in this region. This phenomenon is a fire-sale effect; however, it appears as quantity discrimination effect, and we refer to it as the fire-sale "quantity" effect.

[15] The expected profits from lending to households (ef_r) is concave in f_r and is maximized at f_r equal to $\frac{R}{2}$. It is worth highlighting that the competitive equilibrium face value (f_r) is the same as the profit-maximizing value for lenders in the real sector. Thus, the equilibrium is stable to off-equilibrium offers because surplus-liquidity firms would make lower profits at any other value of f_r.

The process of shrinking the real sector continues as q increases in this region. At a sufficiently high value of q, the real sector completely collapses. The economy now transitions to the Real Sector Credit Crunch Equilibrium Region, which is discussed below.

4.9. Real Sector Credit Crunch Equilibrium

In this region, the cross-market equilibrium return condition is irrelevant because the value of q is high enough to cause a real sector breakdown. Only the financial asset market is open and now the financial asset price can adjust freely to ensure financial asset market clearing. As in the Price Discrimination Equilibrium, there is a fire-sale "price" effect in this region. This region can arise at a high value of q. The return on the financial asset is no longer bounded by the return on the real asset; in this region, the return on the financial asset always exceeds the potential return on the real asset.

To summarize, an interaction between the moral hazard problems in the financial sector and the real sector drives the underlying economics of the model. First, risk shifting concerns constrain funding liquidity, thereby causing fire sales in the financial sector when an economic shock arises. Cross-market arbitraging activity (which ensures that the expected returns in the two markets are the same) implies that moral hazard problems in the real sector (effort aversion) is in sync with moral hazard problems in the financial sector. Depending on the severity of the moral hazard problem in the economy, the economy experiences four type of equilibria: Fair Pricing, Price Discrimination, Liquidity Crunch, and Credit Crunch

We now move to the ex-ante equilibrium, in which we will evaluate the ex-ante optimal bankruptcy parameter (q) after taking into account the ex-post fire-sale effects.

5. The Ex-Ante Model

In this section, we endogenize the debt obligations assumed by firms at Date 0. Recall that firms face varying levels of investment shortfall (s) at Date 0. For the ex-ante equilibrium analysis we assume that the investment shortfall (s) faced by financial intermediaries is uniformly distributed, i.e., U$[s_{min}, s_{max}]$. Financial intermediaries finance this investment shortfall (s) in the short-term debt market, which is subject to rollover risk at Date 1. Let the outstanding liability at Date 1 be $\rho(s)$. Lenders can refuse to roll over debt at Date 1 if they calculate that the state of the economy (realization of θ_2 at Date 1) will make it impossible for the borrowing firm to honor its outstanding liability ($\rho(s)$) at Date 1. In such an event, lenders can liquidate the financial asset, but only with a probability q; with a probability of $(1-q)$, they are forced to renegotiate a strategic write-down to ρ^*.

The key to analyzing the ex-ante model is the observation that the financial asset market clearing price at Date 1 (i.e., the liquidation price at Date 1) and the liability assumed at Date 0 are endogenously related. The initial liability structure in the economy affects the extent of liquidation at Date 1, and therefore, the liquidation price in equilibrium. Lenders anticipate the implied distribution of liquidation price levels and accordingly determine the face value of the loans to be disbursed at Date 0, i.e., the initial liability structure of financial firms in the economy.

Stated differently, while solving the ex-post model, we assumed an exogenous debt structure in the economy. Given this distribution of ρ, we derived the ex-post equilibrium outcomes, $(p, \bar{\beta}, f_r)$. In the ex-ante model, we begin with a distribution of investment shortfalls (s) at Date 0 which translates into a corresponding distribution of Date 1 liabilities (ρ). We denote the resulting distribution of liabilities as $\hat{G}(\rho(s))$. The liquidation price at Date 1 depends on the distribution of ρ across firms. In other words, $\hat{G}(\rho)$ and $p(\theta_2)$ are endogenously related in a dynamic manner.

We will eventually explore the role of the bankruptcy exemption parameter (q) in maximizing ex-ante financing while minimizing ex-post inefficient liquidation.

5.1. The Set-up

Figure (**2**) provides the basic set-up for the ex-ante model. As of Date 0, the Date 1 shock, θ_2, is unknown. For tractability, we consider a discrete two-state distribution for θ_2: with a probability, r, the state of the economy is described by θ_2^h (which we refer to as the high θ_2 state of the economy, or simply, the high state), and with a probability, $(1-r)$, the state of the economy is described by θ_2^l (which we refer to as the low θ_2 state, or simply, the low state of the economy).

We make the following assumptions regarding the high state (θ_2^h). First, we assume that the asset payoff in the high state is given by y_2^h, where $y_2^h > y_1 > y_2$. Consequently, there are no moral hazard issues in the high state. This assumption is similar to the contention in Gorton and Metrick [2010] regarding the role of adverse selection in repo markets. They rely on arguments in Gorton and Pennacchi [1990] and Dang et al. [2010] that repo securities are "information insensitive" securities druing normal times (resulting in high liquidity), but are highly "information sensitive" when the economic shock is severe (resulting in liquidity drying up).

In a similar vein, we also argue that moral hazard related financial frictions (risk-shifting in the financial asset market and effort aversion in the real asset market) are expected to kick in only in the low state of the economy. In other words, the funding liquidity of the financial asset is equal to its fair value ($p = \theta_2^h y_2^h$). Due to arbitraging activity, the real asset would also be fairly priced, i,e, $ef_r = 1$. Furthermore, since household borrowers exhibit no effort aversion ($\gamma \to 0$), the effort (e) in the high state hits the cap of 1 (it is a probability measure).[16] It follows that the face value of real asset loans (f_r) would be equal to 1.

Second, we assume that the market for real asset loans is fully satiated, i.e., the surplus-liquidity firms' supply of real asset loans in the high state meets the maximum potential aggregate loan requirements of household borrowers ($\mathcal{B}$). In other words, there is no unmet demand of household borrowers in the high state.[17]

Let us compare the high state and low state properties. In the high state, all firms will be able to roll over their debt at Date 1 because funding liquidity will be equal to the fair value of the asset. Consequently, the system is in Fair Pricing Equilibrium with the equilibrium characteristics specified as follows:[18]

$$p(\theta_2^h) = \theta_2^h y_2^h \; ; \qquad f_r(\theta_2^h) = 1 \; ; \qquad \bar{\beta}(\theta_2^h) = \mathcal{B} \tag{23}$$

However, in the low state, a subset of firms will always be credit-constrained and unable to roll over their debt without liquidating some or all of their assets. Furthermore, the real asset market is not always satiated in the low state. Consequently, any of the four equilibrium types described in Section (4.6) could exist in the low state depending on the severity of the economic shock (θ_2^l).[19] The equilibrium characteristics in the low state are as specified in Propositions (1) & (2). We focus on analyzing the dynamics of these low state equilibria and its impact on the social welfare. For simplicity in notation, we omit explicit reference of the state when referring to the equilibrium characteristics of the low state in the following sections (i.e., p refers to $p(\theta_2^l)$, f_r refers to $f_r(\theta_2^l)$, $\bar{\beta}$

[16] Lack of effort aversion for household borrowers in the high state is assumed to mirror the lack of frictions in the financial asset market. However, the results of the paper follow even in the absence of this assumption.

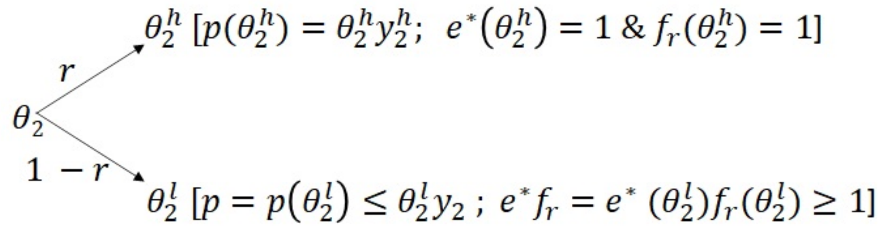
[17] In general, one can put an explicit restriction on $\mathcal{B}$ to be strictly less than an endogenously determined $\bar{\beta}$ in the high state, thereby ensuring that there will be no unmet demand. This restriction would essentially result in a constraint on θ_2^h. To avoid clutter, we express this constraint as simple assumption, which states that there is no unmet demand in the real asset market in the high state.

[18] The results for $p(\theta_2^h)$ and $\bar{\beta}(\theta_2^h)$ follow from the equilibrium characteristics of the system in the fair pricing equilibrium as obtained in Proposition (1). However, in the absence of effort aversion in households, households exert maximal effort ($e^* = 1$); implying that a fairly priced real asset loan ($e^* f_r = 1$) would have unit face value ($f_r = 1$).

[19] Note that fair pricing in the high state is not the same as fair pricing in the low state. First, there is no rollover risk in the high state whereas rollover risk always arises in the low state, even under fair pricing. This difference arises as $\rho^*(\theta_2^h) = p(\theta_2^h) = \theta_2^h y_2^h$, and all firms can roll over their debt in the high state. In the low state, $\rho^*(\theta_2^l) = \theta_2^l y_2 - k_1$ and firms having $\rho > \rho^*(\theta_2^l)$ will be unable to roll over their debt without partially (or fully) liquidating their financial asset even in the fair pricing equilibrium. Second, due to the absence of effort aversion by households in the high state, $f_r(\theta_2^h) = 1$; whereas in the low state due to non-zero effort aversion, $f_r(\theta_2^l) = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - 4\gamma}$ in the fair pricing equilibrium, as given by Proposition (1).

refers to $\bar{\beta}(\theta_2^l)$, ρ^* refers to $\rho^*(\theta_2^l)$, k_1 refers to $k_1(\theta_2^l)$ and $\bar{p}$ refers to $\bar{p}(\theta_2^l)$).[20]

Figure **2**: **Ex-ante view of the states of the economy (θ_2).** The economy is in the high state (θ_2^h) with a probability r and in the low state (θ_2^l) with a probability $1 - r$. In the high state of the economy, both the financial asset and the real asset are fairly priced. However, in the low state of the economy, both assets *could* exhibit fire-sale effects.



5.2. Borrower's Payoff Potential and Investment Shortfall Financing

As shown in Figure (**2**), the high state occurs with a probability of r and the low state with a probability of $1 - r$. Lenders take into account the borrowers payoff potential in both states of the world. In the high state (θ_2^h), borrowers enjoy a payoff potential of $p(\theta_2^h) = \theta_2^h y_2^h$. In the low state, the borrower's payoff potential is determined as follows. With a probability q, lenders take control and liquidate the assets at the market clearing price of $p(\theta_2^l)$. With a probability of $1-q$, the liability is renegotiated downward to its capacity to raise funds, or its funding liquidity, ρ^*. Thus, given an adverse shock θ_2^l at Date 1, lenders can expect a payoff, $\bar{p}$, given by

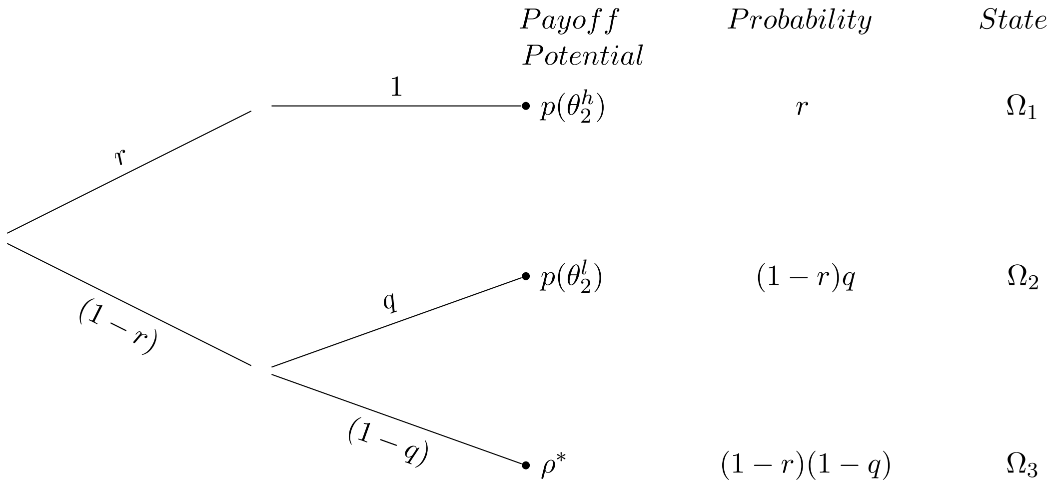
$$\bar{p}(\theta_2^l, q) = qp(\theta_2^l) + (1 - q)\rho^*. \tag{24}$$

Figure (**3**) summarizes the borrower's payoff potential, which depends on the state of the world (θ_2^h or θ_2^l) and whether the borrower faces liquidation (with a probability of q) or renegotiation (with a probability of (1-q)). This payoff potential helps determine the amount of investment shortfall that lenders would be willing to finance for a given face value (ρ).

The borrower's payoff potential depends on the state (Ω_1, Ω_2 and Ω_3) which arise with different probabilities, as shown in the last two columns of Figure (**3**). Table (1) maps the investment shortfall ($s(\rho)$) that can be financed for a given ρ. Since borrower's payoff potential depends on ρ, the investment shortfall that can be financed changes in functional form over different intervals of ρ, as can be seen in the different rows of Table (1). It can be seen that $s(\rho)$ is a piece-wise linear function of ρ.

[20] We continue to use explicit references to the high state while discussing its equilibrium characteristics, as used in Equation (23).

Figure **3**: **Ex-ante Payoff Potential.** The lender's access to borrower's cash flows (i.e., payoffs potential of borrower) for a given adverse shock (θ_2^l) in different cases is shown along with the probability of the case.



ρ	Default States	Non-default States	Investment Shortfall That is Financed by Debt ($s(\rho)$)
$\rho_{min} \leq \rho \leq \rho^*$	$\varnothing$	Ω_1, Ω_2, Ω_3	ρ
$\rho^* < \rho \leq p(\theta_2^l)$	Ω_3	Ω_1, Ω_2	$[r+(1-r)q]\rho+(1-r)(1-q)\rho^*$[21]
$p(\theta_2^l) < \rho \leq p(\theta_2^h)$	Ω_2, Ω_3	Ω_1	$r\rho+(1-r)\bar{p}(\theta_2^l)$
$p(\theta_2^h) < \rho$	Ω_1, Ω_2, Ω_3	$\varnothing$	$rp(\theta_2^h)+(1-r)\bar{p}(\theta_2^l)$

Table 1: **Mapping of the Face Value of Liability (ρ) to the Investment Shortfall ($s(\rho)$) that is financed by Debt.** This table presents mapping between the face value of debt ρ and the corresponding investment shortfall, $s(\rho)$, that can be financed at that level of ρ. $s(\rho)$ is equal to the expected ex-ante payoff (at Date 0) that the creditors would receive at that level of ρ.

5.3. Endogenous Debt Distribution at Date 0

Each financial firm creates an asset with an expected payoff of $E_{\theta_2}[\theta_2 y_2]$ by investing an amount s; thus, the surplus created by each financial firm is the NPV of the financial asset, i.e., $E_{\theta_2}[\theta_2 y_2 - s]$. Investment shortfalls of the continuum of firms are assumed to be uniformly distributed over the

[21]This implies that in the event $\rho < p(\theta_2^l)$, the lender receives ρ with a probability q in the low state. The implicit assumption is that when the price exceeds the liability, with probability q, the court enforces efficient liquidation (i.e., only liquidates the fraction of assets required to clear the liability to the lender) and the lender receives ρ. The alternative assumption could be that, with probability q, the lender is given full control of the asset. In this case, the lender enjoys the surplus of $p(\theta_2^l) - \rho$. We also evaluate the model under this alternate assumption and obtain similar results as under the current assumption.

range $[s_{min}, s_{max}]$. Note that s_{max} is the maximum investment level at which a firm still finds the investment in the financial asset to have a non-negative NPV (i.e., $s_{max} = E_{\theta_2}[\theta_2 y_2] = r\theta_2^h y_2^h + (1-r)\theta_2^l y_2)$. It follows that the NPV of acquiring the financial asset is always positive for any $s \in (s_{min}, s_{max})$.[22]

From the lender's perspective, the maximum investment shortfall (debt capacity) that can be financed based on the asset's payoff is given by $\hat{s} = rp(\theta_2^h) + (1-r)\bar{p}(\theta_2^l)$, which is always less than or equal to s_{max}. Consequently, the range of investment shortfalls that actually get financed at Date 0 is given by $[s_{min}, \hat{s}]$, i.e., debt capacity becomes the binding constraint for financing investment shortfalls. The lemma below discusses the endogenous leverage in the economy at Date 0.

Lemma 5: *Given a uniform distribution of investment shortfalls in the economy (i.e., $H(\tilde{s})$ is $U[s_{min}, s_{max}]$), the endogenous distribution of leverage ($\rho : \rho \in [\rho_{min}, \rho_{max}]$) at Date 0 that takes into account the expected payoff to the lenders at Date 2 is specified by $\hat{G}(\rho)$, as follows:*

$$\hat{G}(\rho) = \frac{\tilde{s}(\rho) - s_{min}}{\hat{s} - s_{min}},$$

$$\text{where } \tilde{s}(\rho) = \begin{cases} \rho, & \text{if } \rho_{min} \le \rho \le \rho^* \\ \big(r + (1-r)q\big)(\rho - \rho^*) + \rho^*, & \text{if } \rho^* < \rho \le p(\theta_2^l) \\ r\rho + (1-r)\bar{p}(\theta_2^l), & \text{if } p(\theta_2^l) < \rho \le p(\theta_2^h) \end{cases} . \tag{25}$$

5.4. *Ex-ante Dynamic Equilibrium*

First, we begin with a formal definition of the ex-ante dynamic equilibrium. A dynamic equilibrium setup is (i) a pair of functions $\rho(s)$ and $p(\theta_2^l)$, which respectively give the promised face value ($\rho(s)$) for raising short-term financing of s units at Date 0 and the equilibrium price ($p(\theta_2^l)$) at Date 1 given the interim signal of asset quality of θ_2^l; and (ii) a truncation point $\hat{s}$, such that $\rho(s)$, $p(\theta_2^l)$ and $\hat{s}$ satisfy the following fixed-point recursion:

1. For a given θ_2^l, the asset's price ($p(\theta_2^l)$) is constrained by the price function described in Proposition (1) and Proposition (2).
2. The derived distribution of debt, $\hat{G}(\rho)$ depends on $s(\rho) \in [s_{min}, \hat{s}]$ where $\hat{s}$ is the maximal investment shortfall that is financed (Equation (25)). Because $s(\rho)$ depends on the asset's price ($p(\theta_2^l)$), the derived distribution, $\hat{G}(\rho)$, depends on the asset price.
3. Given the price function $p(\theta_2^l)$, for every investment shortfall $\tilde{s} \in [s_{min}, \hat{s}]$, the promised face value ρ is determined by the requirement that lenders receive in expectation the amount being lent

[22] Furthermore, $s_{min} \ge \theta_1 y_1$, otherwise risk-shifting is not a credible threat.

(individual rationality of short-term financiers).

5.5. Ex-ante Equilibrium Characteristics

The ex-ante equilibrium is defined for a given θ_2^h and θ_2^l. In the high state, the endogenous distribution of debt has no impact on the equilibrium characteristics. In the low state, the equilibrium characteristics will mirror the solution provided in Proposition (1) and Proposition (2), except that the exogenously specified distribution of leverage ($G(\rho)$) in Equations (18), (21), and (22) must now be substituted by the endogenously derived distribution ($\hat{G}(\rho)$), as described in Equation (25). From Proposition (2) it can be seen that, in the Liquidity Crunch Equilibrium, p and f_r are not functions of the distribution of debt in the economy. Consequently, they continue to be specified as stated in Proposition (2) (i.e., $p = \rho^* + 4\gamma k_1/R^2$ and $f_r = R/2$) in the ex-ante equilibrium as well. However, $\bar{\beta}$ in the Liquidity Crunch Equilibrium and p in the Price Discrimination Equilibrium and the Credit Crunch Equilibrium are functions of the distribution of debt and consequently, the specification of these terms vary from that obtained for the ex-post equilibrium (see Appendix A for closed form equilibrium solutions of $\bar{\beta}$ and p). Below, we present a proposition that evaluates the impact of the bankruptcy exemption parameter (q) on the ex-ante equilibrium characteristics.

PROPOSITION 3: *For a given combination of (θ_2^h, θ_2^l), the effect of the bankruptcy exemption parameter (q) on equilibrium variables in low state (θ_2^l), namely, the price of the financial asset (p), the Date 1 expected payoff to lenders from the financial asset ($\bar{p}$), the face value of the real asset loan (f_r) and the number of real asset loans ($\bar{\beta}$) are given as follows.*

(i) When the system is in the Price Discrimination Equilibrium Region, the ex-ante equilibrium solutions for p, $\bar{p}$, and f_r display the following properties (note, $\bar{\beta} = \mathcal{B}$):

$$\frac{dp}{dq} < 0 \qquad \frac{d\bar{p}}{dq} < 0 \qquad \frac{df_r}{dq} > 0 \qquad \frac{d\bar{\beta}}{dq} = 0. \tag{26}$$

(ii) When the system is in the Liquidity Crunch Equilibrium Region, the ex-ante equilibrium solutions for p, $\bar{p}$, f_r, and $\bar{\beta}$ display the following relationship with q (note, $f_r = \frac{R}{2}$):

$$\frac{dp}{dq} = 0 \qquad \frac{d\bar{p}}{dq} > 0 \qquad \frac{df_r}{dq} = 0 \qquad \frac{d\bar{\beta}}{dq} < 0. \tag{27}$$

(iii) When the system is in the Credit Crunch Equilibrium Region, the ex-ante equilibrium solutions for p and $\bar{p}$ display the following relationship with q (note, $\bar{\beta} = 0$ and $f_r = \frac{R}{2}$):

$$\frac{dp}{dq} < 0 \qquad \frac{d\bar{p}}{dq} < 0 \qquad \frac{df_r}{dq} = 0 \qquad \frac{d\bar{\beta}}{dq} = 0. \tag{28}$$

To understand the first and third set of results (for the Price Discrimination Equilibrium and the Credit Crunch Equilibrium), recall that $\bar{p} = qp + (1-q)\rho^*$, which is equal to $\rho^* + q(p - \rho^*)$. We differentiate $\bar{p}$ with respect to q to obtain $\frac{d\bar{p}}{dq} = (p - \rho^*) + q\frac{dp}{dq}$. The first term, which we refer to as the "pure liquidation" effect, is always positive because $p > \rho^*$. This term captures the preference of the lender for liquidation at higher price (p) rather than the renegotiated write-down to a lower value, ρ^*. It points out that an increase in q increases the likelihood of liquidations in relation to write-downs, thereby increasing the expected proceeds from lending. The second term reflects the fire-sale "price" effect associated with liquidation, i.e., how the price (p) of the financial asset, and thereby $\bar{p}$, depend on q. The combination of the pure liquidation effect and the fire-sale "price" effect determines how $\bar{p}$ changes with q. In both the Price Discrimination Equilibrium and the Credit Crunch Equilibrium, there is a fire-sale "price" effect (i.e., $\frac{dp}{dq} < 0$) and it overwhelms the pure liquidation effect. Consequently, $\bar{p}$ is decreasing in q.

We can also show that f_r in the Price Discrimination Equilibrium increases with q. This result arises due to the cross-market equilibrium return condition - an increase in q causes the fire sale "price" effect on the financial asset; the same is reflected in the real asset market in the form of a higher face value on real asset loans. In the Credit Crunch Equilibrium Region, $f_r = \frac{R}{2}$ and it is independent of q. Finally $\bar{\beta}$ is constant in both equilibrium regions and, is therefore, independent of q.

The second set of results for the Liquidity Crunch Equilibrium follow once we recognize that the financial asset market must necessarily clear. When q increases, there is a greater degree of liquidation in the financial asset market; as a consequence, the liquidity available for acquiring real assets diminishes. The reduction in $\bar{\beta}$ with q reflects the fire-sale "quantity" effect. Note that the market clearing price (p) is, however, invariant to q. Essentially, $\bar{\beta}$ absorbs the entire impact of q in the Liquidity Crunch Equilibrium. However, both price p and face value f_r on real assets ($= \frac{R}{2}$) remain unaffected by the change in q. Finally, $\frac{d\bar{p}}{dq} > 0$ because of the pure liquidation effect.

5.6. Equilibrium Regions

Keeping θ_2^h fixed, we examine the equilibrium for different realizations of θ_2^l and then analyze the relation between the equilibrium characteristics in the low state and the bankruptcy exemption parameter (q). To elaborate, we analyze the equilibrium characteristics for a given combination of the bankruptcy exemption parameter ($q \in [0, 1]$) and low-state Date 1 shock ($\theta_2^l \in [\theta_2^{min}, \theta_2^{max}]$), where the interval on θ_2^l is determined by feasibility constraints.[23]

Figure (4) shows the typical demarcation of the feasible $q - \theta_2^l$ space into the Fair Pricing (FP), as shown by the white region and the Fire Sale (FS) region, as shown by the gray shaded region. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC) and the Credit Crunch (CC) equilibria; we use increasingly darker shades of gray to represent greater fire-sale effects. For different magnitudes of the economic shock (θ_2), we see how the type of equilibrium changes with the bankruptcy parameter (q). The solid $\bar{\bar{q}}(\theta_2^l)$ curve represents the boundary between the FP and PD equilibrium regions. The long dashed $\bar{q}(\theta_2^l)$ curve represents the boundary between the PD and LC equilibrium regions. The dotted $\hat{q}(\theta_2^l)$ curve represents the boundary between the LC and CC equilibrium regions. To see how the system transitions across different types of equilibrium regions, consider the case with $\theta_2^l = 0.48$). The vertical dotted line emanating from this level of θ_2 captures how the system transitions across different types of equilibrium regions, as q increases from 0 to 1 along the dotted vertical line.

5.7. *Equilibrium Characteristics for a Given Economic Shock*

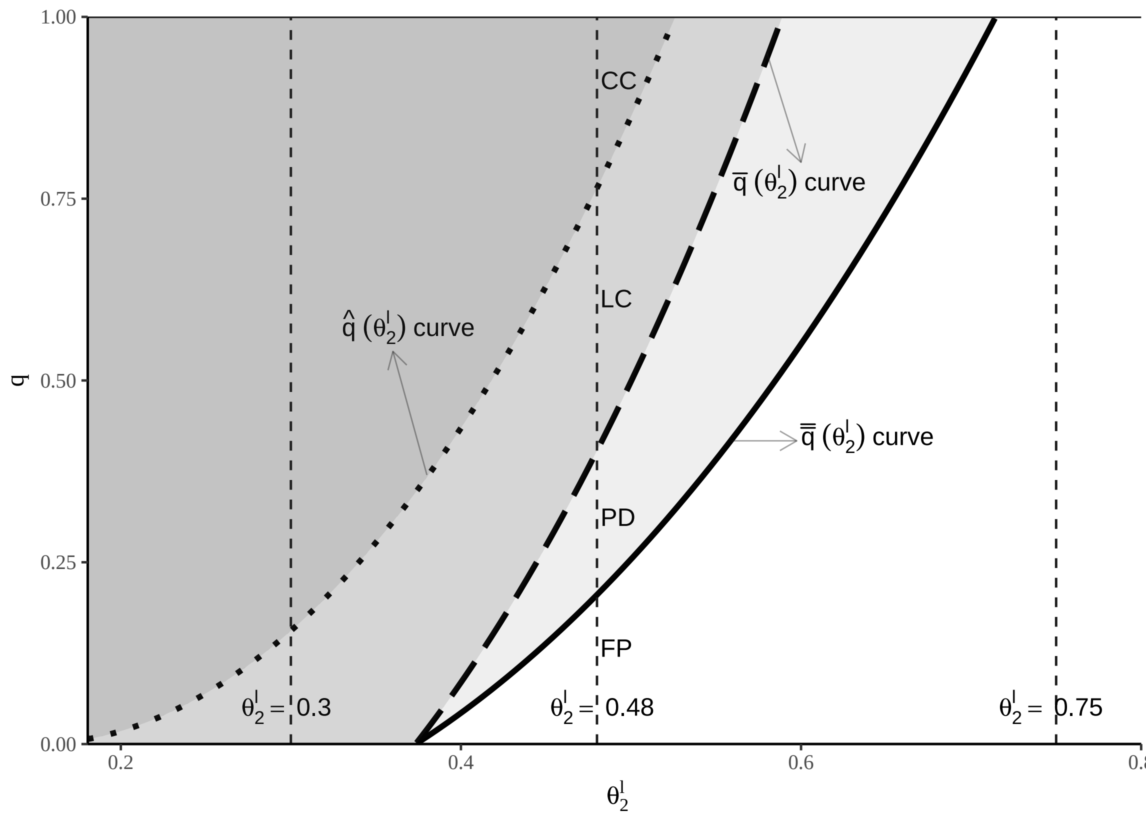
Figure (5) shows the evolution of equilibrium values of p, $\bar{p}$, $\bar{\beta}$ and f_r with q for $\theta_2^l = 0.48$. Panel A shows the evolution of p, Panel B shows the evolution of $\bar{p}$, Panel C shows the evolution of $\bar{\beta}$ and Panel D shows the evolution of f_r as q is increased from 0 to 1 at $\theta_2^l = 0.48$. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines.

The financial sector characteristics, as captured by p and $\bar{p}$, are shown in Panel A and Panel B, respectively. Although difficult to detect by observing the figures, the relation between $\bar{p}$ with q is non monotonic, as discussed in Proposition 3. Furthermore, it can be seen in Panel C that $\bar{\beta}$ is (weakly) decreasing q and in Panel D that f_r is (weakly) increasing in q. Thus, the real sector characteristics are monotonic in q.

Panel E shows the equilibrium return on the financial asset market and the real asset market. The returns in both these markets are the same in the FP, PD, and LC regions, but diverge in the CC region, where the financial asset market returns exceeds that of the real asset market, consistent with a complete shut down (credit crunch) in the real asset market. Panel F shows the decreasing relation between effort and bankruptcy exemption; it implies that the asset quality

[23] $\theta_2^l \in [\theta_2^{min}, \theta_2^{max}]$. The lower bound on θ_2^l (θ_2^{min}) ensures financial market clearing for all θ_2^l, while the upper bound on θ_2^l (θ_2^{max}) ensures that $\theta_2^l < \theta_2^h$.

Figure 4: **Equilibrium Regions.** Typical demarcation of the feasible $q - \theta_2^l$ space into the Fair Pricing (FP), as shown by the white region and the Fire Sale (FS) region, as shown by the gray shaded region. The Fire Sale region consists of the Price Discrimination (PD), the Liquidity Crunch (LC) and the Credit Crunch (CC) equilibria. The solid $\bar{\bar{q}}(\theta_2^l)$ curve is the boundary between the FP and PD equilibrium regions. The long dashed $\bar{q}(\theta_2^l)$ curve is the boundary between the PD and LC equilibrium regions. The dotted $\hat{q}(\theta_2^l)$ curve is the boundary between the LC and CC equilibrium regions. The PD, LC and CC equilibrium regions jointly constitute the Fire Sale Equilibrium Region which is indicated by the differing shades of gray (the darker shades indicate greater fire-sale effects). For a strong economic shock, indicated by $\theta_2^l = 0.48$, as q is increased from 0, the system transitions from FP equilibrium to PD equilibrium at $q = 0.21$, then from PD equilibrium to LC equilibrium at $q = 0.42$ and finally from LC equilibrium to CC equilibrium at $q = 0.79$. For a mild economic shock indicated by $\theta_2^l = 0.75$, the system remains in FP equilibrium for any q. For a severe economic shock, indicated by $\theta_2^l = 0.3$, the system starts in LC equilibrium at $q = 0$ and transitions to CC equilibrium at $q = 0.16$. $\theta_2^l = 0.30$, $\theta_2^l = 0.48$ and $\theta_2^l = 0.75$ are indicated by the three thin vertical dashed lines. Parameter Configuration used: $\theta_2^{min} = 0.18$, $\theta_2^{max} = \theta_2^h = 1$, $\theta_1 = 0.02$, $y_2 = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.



worsens as bankruptcy exemption parameter (q) increases, thereby demonstrating how the moral hazard problem in the real asset market worsens with q.

6. Welfare Analysis

In this section, we examine the welfare implications of bankruptcy exemption for a given $\theta_2^l \in (\theta_2^{min}, \theta_2^{max})$. We evaluate the economic surplus created due to lending at Date 0 and lending at Date 1 as a function of q. We show that surplus due to Date 0 lending surplus is increasing/invariant/decreasing in bankruptcy exemption, depending on the type of equilibrium; however, surplus due to Date 1 lending is always (weakly) decreasing in bankruptcy exemption. Thus, from an ex-ante overall perspective, bankruptcy exemption may create a trade-off between surplus created due to Date 0 lending and Date 1 lending, and bankruptcy exemption can be set at an optimal level to maximize social welfare. In an ex-ante framework, Date 0 lending must take into account the implications of Date 1 lending; we, therefore, begin the analysis with surplus creation due to Date 1 lending.

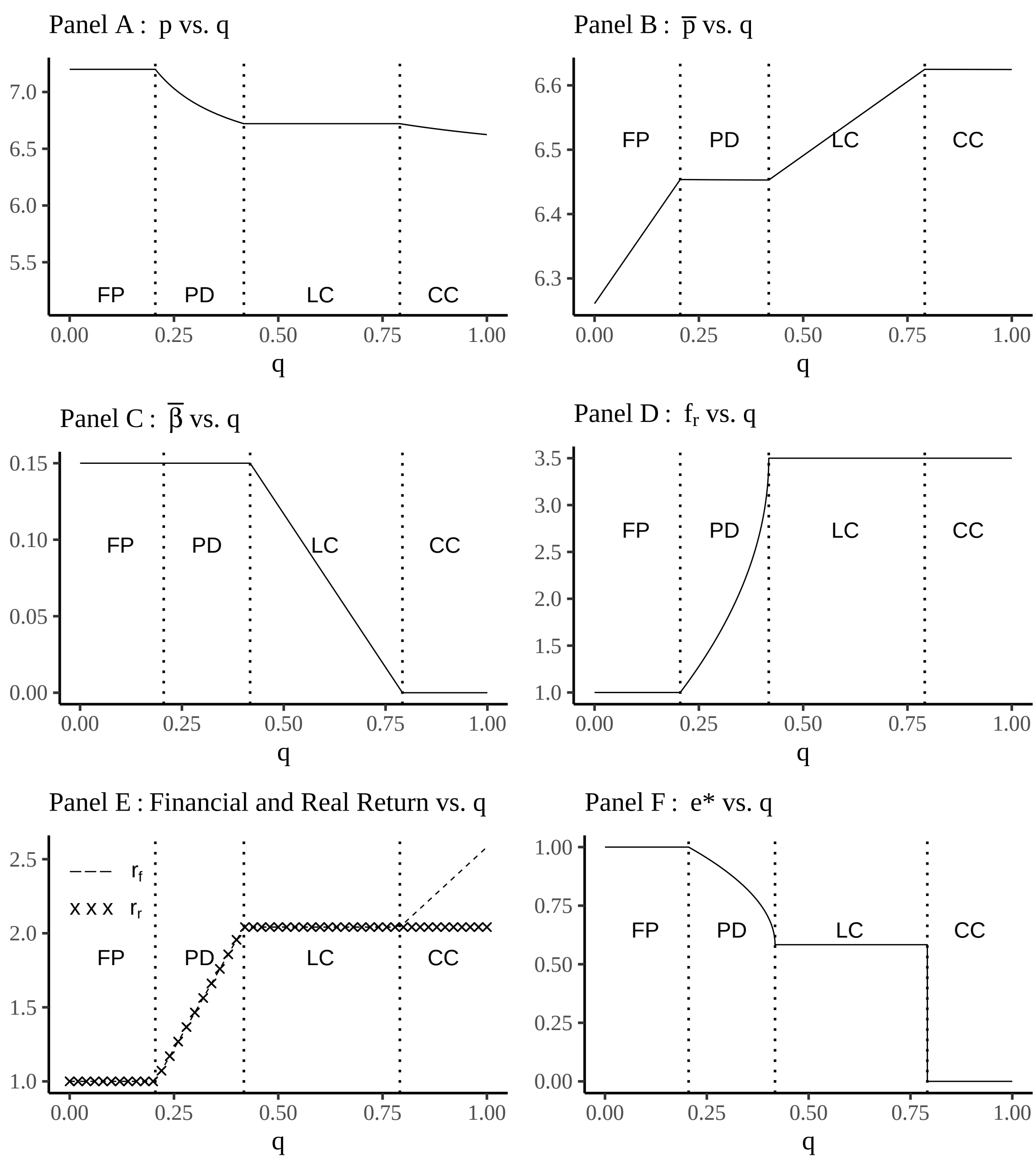
6.1. Surplus Creation Due to Date 1 Lending

The ex-ante expected surplus created at Date 1 varies for different combinations of θ_2 (which can either be θ^h or θ_2^l) and q. The Date 1 surplus, conditional on θ_2, depends on q through the number of real asset loans supplied ($\bar{\beta}(q;\theta_2)$) and the surplus created per real asset loan ($S_r(q;\theta_2)$), which is given by expected payoffs of the real asset created at Date 1, net of pecuniary equivalent of effort (e) expended by households. More specifically, in the high state, $S_r(q;\theta_2^h) = e^*(\theta_2^h)R = R$ as there is no effort aversion. In the low state $S_r(q;\theta_2^l) = e^*(\theta_2^l)R - \frac{1}{2}\gamma[e^*(\theta_2^l)]^2$, where effort, $e^*(\theta_2^l) = \frac{1}{\gamma}[R - f_r(\theta_2^l)]$, is endogenously determined because the equilibrium face vale (f_r) depends on q. Using these results for the high state (θ_2^h) and the low state (θ_2^l), we get the following expansion for the ex-ante expected Date 1 surplus, as shown below,

$$S_{D1}(q) \ = r\mathcal{B}R + (1-r)\bar{\beta}(q;\theta_2^l)S_r(q;\theta_2^l). \tag{29}$$

In the high state (θ_2^h), the face value is equal to 1. Further, there is no unmet demand in the real asset loan market, i.e., $\mathcal{B}$ loans are originated. Thus, Date 1 surplus created in the high state ($S_r(\theta_2^h)$) is equal to $\mathcal{B}R$. The first term in the above expression for S_{D1} reflects the surplus in the high state, after factoring in the probability of the high state ($=r$). Note that this term is independent of q.

Figure 5: **Evolution of equilibrium p, $\bar{p}$, $\bar{\beta}$, f_r, r_f, $_r$ and e^* with q for a given θ_2^l.** Panel A depicts the price of the financial asset (p), Panel B depicts the lenders' expected payoff from the financial asset ($\bar{p}$), Panel C depicts the level of real asset loans made ($\bar{\beta}$), Panel D depicts the face value of real asset loans (f_r), Panel E depicts the returns from the financial (r_f) and real (r_r) asset and Panel F depicts the optimal effort (e^*) exerted by a borrower in the real asset market. The evolution of the equilibrium level of these variables is shown as q is increased from 0 to 1 at $\theta_2^l = 0.48$. The values of q at which the system transitions across each of the equilibrium regions are indicated by dotted vertical lines. Transition points: FP to PD at $\bar{\bar{q}} = 0.21$, PD to LC at $\bar{q} = 0.42$ and LC to CC at $\hat{q} = 0.79$. Parameter Configuration used: $\theta_2^l = 0.48$, $\theta_2^h = 1$, $\theta_1 = 0.02$, $y_2 = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.



The second term in Equation (29) reflects the Date 1 surplus, conditional on the low state (θ_2^l), after factoring in the probability of the low state ($= 1 - r$). This term depends on q through $\bar{\beta}(q; \theta_2^l)$ as well as $S_r(q; \theta_2^l)$. Furthermore, the dependence on q varies across the different types of equilibrium that may arise in the low state.

We rely on the comparative statics results discussed in Proposition 3 to show that, for a given θ_2^l and θ_2^h, $\frac{dS_{D1}}{dq}$ is invariant to q in the Fair Pricing Equilibrium and Credit Crunch Equilibrium regions but strictly decreasing in the Price Discrimination Equilibrium and the Liquidity Crunch Equilibrium regions. The relationship of S_{D1} with q can be summarized as weakly decreasing. The first set of rows in Table 2 provides specific insights for understanding this relation across all the different types of equilibrium.

In essence, fire-sale "price" effects, which affect f_r, and fire-sale "quantity" effects, which affect $\bar{\beta}$, cause $\frac{dS_{D1}}{dq}$ to be (weakly) decreasing in q. Interestingly, an important implication arising from this result is that the ex-ante expected Date 1 surplus is never increasing in q.

6.2. Surplus Creation Due to Date 0 Lending

The ex-ante expected surplus created by Date 0 lending (S_{D0}) is calculated as follows. The expression for S_{D0} is given by aggregating the expected surplus (which is given by $E_{\theta_2}(\theta_2 y_2 - s)$) across all firms that are able to secure financing at Date 0 (i.e., those firms that have investment shortfall, $\tilde{s}$, less than $\hat{s}$). Using these inputs, the expected Date 0 surplus can be stated, as follows.

$$
\begin{aligned}
S_{D0}(q) &= \int_{s_{min}}^{\hat{s}} E_{\theta_2}[\theta_2 y_2 - s] dH(s) \\
&= \frac{\hat{s} - s_{min}}{s_{max} - s_{min}} \left[E_{\theta_2}[\theta_2 y_2] - \frac{\hat{s} + s_{min}}{2} \right]. \qquad (30)
\end{aligned}
$$

$S_{D0}(q)$ simplifies to Equation (30). It can be shown that the right hand side of Equation (30) is increasing in $\hat{s}$. Furthermore, since $\hat{s}$ is increasing in $\bar{p}$, it follows that S_{D0} is increasing in $\bar{p}$. Thus, the relation between S_{D0} and q depends on the relation between $\bar{p}$ and q.

As discussed earlier, in Proposition (3), the expected financial asset price ($\bar{p}$) could be increasing or decreasing in q depending on the type of equilibrium. In the Fair Pricing Equilibrium and the Liquidity Crunch Equilibrium regions, $\bar{p}$ is increasing in q, but in the Price Discrimination Equilibrium and the Credit Crunch Equilibrium regions, $\bar{p}$ is decreasing in q because of the fire-sale "price" effect. It follows that S_{D0} is either increasing in q or decreasing depending on the type of equilibrium. The second set of rows in Table 2 provides specific insights for understanding this relation across all the different types of equilibrium.

6.3. Equilibrium Type and Welfare Trade-offs

Table (2) analyzes the welfare trade-offs under each equilibrium type in the low state, conditional on a given value of (θ_2^l). In the Fair Pricing Equilibrium region), both assets are trading at their fair value, and the real asset market is fully satiated (i.e., $\bar{\beta}(\theta_2^l) = \mathcal{B}$). Thus, the marginal effect of bankruptcy exemption (q) is that it increases the expected Date 0 lending without causing any reduction in the expected Date 1 surplus, as shown in the first column of Table 2.

Once q is sufficiently high, the system crosses over to the Fire Sale Equilibrium regions. As shown in the second and fourth columns of Table 2, the expected Date 0 surplus is decreasing in q in the Price Discrimination and the Credit Crunch regions because of fire-sale "price" effects, which adversely affect ex-ante lending at Date 0. At the same time, the expected Date 1 surplus is invariant to q in these regions because of the supply of real sector loans is constant (either $\mathcal{B}$ in the Price Discrimination region or 0 in the Credit Crunch region).

Fair Pricing Equilibrium	Price Discrimination Equilibrium	Liquidity Crunch Equilibrium	Credit Crunch Equilibrium
$\bar{p} \uparrow$ with q $\hat{s} \uparrow$ with q $\Rightarrow S_{D0} \uparrow$ with q	$\bar{p} \downarrow$ with q $\hat{s} \downarrow$ with q $\Rightarrow S_{D0} \downarrow$ with q	$\bar{p} \uparrow$ with q $\hat{s} \uparrow$ with q $\Rightarrow S_{D0} \uparrow$ with q	$\bar{p} \downarrow$ with q $\hat{s} \downarrow$ with q $\Rightarrow S_{D0} \downarrow$ with q
$\bar{\beta} \leftrightarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \leftrightarrow$ with q	$\bar{\beta} \leftrightarrow$ with q $f_r \uparrow$ with q $\Rightarrow S_{D1} \downarrow$ with q	$\bar{\beta} \downarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \downarrow$ with q	$\bar{\beta} \leftrightarrow$ with q $f_r \leftrightarrow$ with q $\Rightarrow S_{D1} \leftrightarrow$ with q
$S_{Total} \uparrow$ with q	$S_{Total} \downarrow$ with q	$S_{Total} \downarrow$ with q	$S_{Total} \downarrow$ with q

Table 2: **Equilibrium Characteristics in each Low State Equilibrium Region**. Behavior of the price of the financial asset (p), the expected Date 1 payoff to lenders from the financial asset ($\bar{p}$), the equilibrium face value of real asset loans (f_r) and the number of real asset loans ($\bar{\beta}$), as a function of the bankruptcy exemption parameter (q) in each of the low state Equilibrium Regions.

It is only in the Liquidity Crunch Equilibrium that there exists a trade-off between Date 0 surplus and Date 1 surplus. In this region, the fire-sale effect arises only as a "quantity" effect and not as a "price" effect. Date 0 lending is still increasing in q (p is invariant to q but $\bar{p}$ is strictly increasing in q). However, Date 1 surplus continues to be decreasing in q due to the fire-sale "quantity" effect. We find that the reduction in Date 1 surplus dominates the increase in Date 0 surplus.

The intuition behind this finding can be stated as follows. As q increases, the incremental lending at Date 0 is made to those borrowers who face high investment shortfalls; these borrowers are the one who were not financed at a lower q. Two implications follow: (i) the NPV of the assets originated by these borrowers is necessarily low because of the high investment requirements, and (ii) these borrowers are also the most leveraged borrowers because of the large investment requirements that they have to finance with debt. Thus, Date 0 lending results in low NPV asset origination by highly leveraged borrowers, who will face adverse fire-sale effects at Date 1 when an economic shock occurs. Since Date 0 asset origination creates low NPV assets, the loss in Date 1 surplus always dominates the low NPV gain from assets created at Date 0.

To summarize, in the Fire Sale Equilibrium region, an increase in q increases the expected Date 0 surplus, but it also inhibits the ability of surplus-liquidity firms from servicing the Date 1 real asset market, i.e., an increase in q causes financial instability in the form of an increase in interest rates, or a shrinking (and at worst, a collapsing) real asset market, resulting in a decrease in the expected Date 1 surplus. In other words, our results demonstrate that providing bankruptcy exemption in repo markets (i.e., setting $q = 1$) while creating "too much today" may also provide "too little tomorrow". There is a trade-off between these two effects that determines the socially optimal bankruptcy exemption parameter (q^{opt}), as will be discussed in the next section.

6.4. Total Surplus Creation

In this section we assess the social welfare implications of the bankruptcy exemption parameter (q) by considering the sum of the expected surplus created at Date 0 and the expected surplus created at Date 1, i.e., the expected total surplus in the economy is given by $S_{Total} = S_{D0} + S_{D1}$. Given that S_{D0} could be increasing/decreasing in q and that S_{D1} is (weakly) decreasing in q, it seems reasonable to expect that there is an optimal q that maximizes the S_{Total}.

Proposition 4: *For a given θ_2^l, the optimal q (q^{opt}) that maximizes total surplus (S_{Total}) is at the border of the Fair Pricing Equilibrium region and the Fire Sale Equilibrium region.*

As elaborated in Table 2 (bottom row), the expected total surplus (S_{Total}) is always increasing in the Fair Pricing Equilibrium region but decreasing in the Fire Sale Equilibrium regions, irrespective of the equilibrium type. Thus, the optimal q is always at the boundary of the curve demarcating the Fair Pricing Equilibrium region and the Fire Sale Equilibrium regions.

Consider a strong economic shock (as depicted by $\theta_2^l = \theta_{strong} = 0.48$) in Figure (4). As q increases, the economy transitions across different equilibrium regions. The marginal effect of bankruptcy exemption (q) is a trade-off between greater ex-ante Date 0 lending and the adverse effects of ex-post Date 1 fire-sale effects. In the Fair Pricing Equilibrium region, the latter effect is absent, and it is socially optimal to fully increase creditor rights (q) to facilitate greater ex-ante lending. In the Fire Sale Equilibrium region, fire-sale effects exist, and as demonstrated in Table 2, these effects always dominate the ex-ante lending benefits; the socially optimal q is constrained to always lie at the border of the Fair Pricing and Fire Sale Equilibrium regions.

Figure (**6**) shows the evolution of the expected Date 0 surplus (S_{D0}), the expected Date 1 surplus (S_{D1}) and the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q), conditional on a strong Date 1 shock ($\theta_2^l = 0.48$), as indicated by the marker in Figure (**4**). We see that the system transitions from the Fair Pricing to the Price Discrimination to the Liquidity Crunch and finally to the Credit Crunch equilibrium regions as q increases. In the Fair Pricing equilibrium, Date 0 surplus (S_{D0}) increases with q while Date 1 surplus (S_{D1}) is invariant in q causing the total surplus (S_{Total}) to increase in q. However, when the system transitions to the Price Discrimination equilibrium at $q = 0.21$, both S_{D0} and S_{D1} decrease with q causing S_{Total} to decrease as well. Therefore, S_{Total} has a maximum at the boundary of the Fair Pricing and Price Discrimination equilibrium regions. As q is further increased the system transitions into the Liquidity Crunch equilibrium at $q = 0.42$. While S_{D0} increases with q here, this increase is swamped by the reduction in S_{D1}, leading to an overall reduction in S_{Total} with q in the Liquidity Crunch Equilibrium. Finally, the system transitions to the Credit Crunch Equilibrium at $q = 0.79$, the real asset market shuts down, i.e., S_{D1} is again invariant in q, but S_{D0} decreases with q in this region. Consequently, S_{Total} is decreasing in the Credit Crunch Equilibrium, as well. Therefore, as can be seen Panel C, expected total surplus (S_{Total}) is maximized at the boundary of the Fair Pricing and Price Discrimination equilibrium regions ($q = 0.21$).

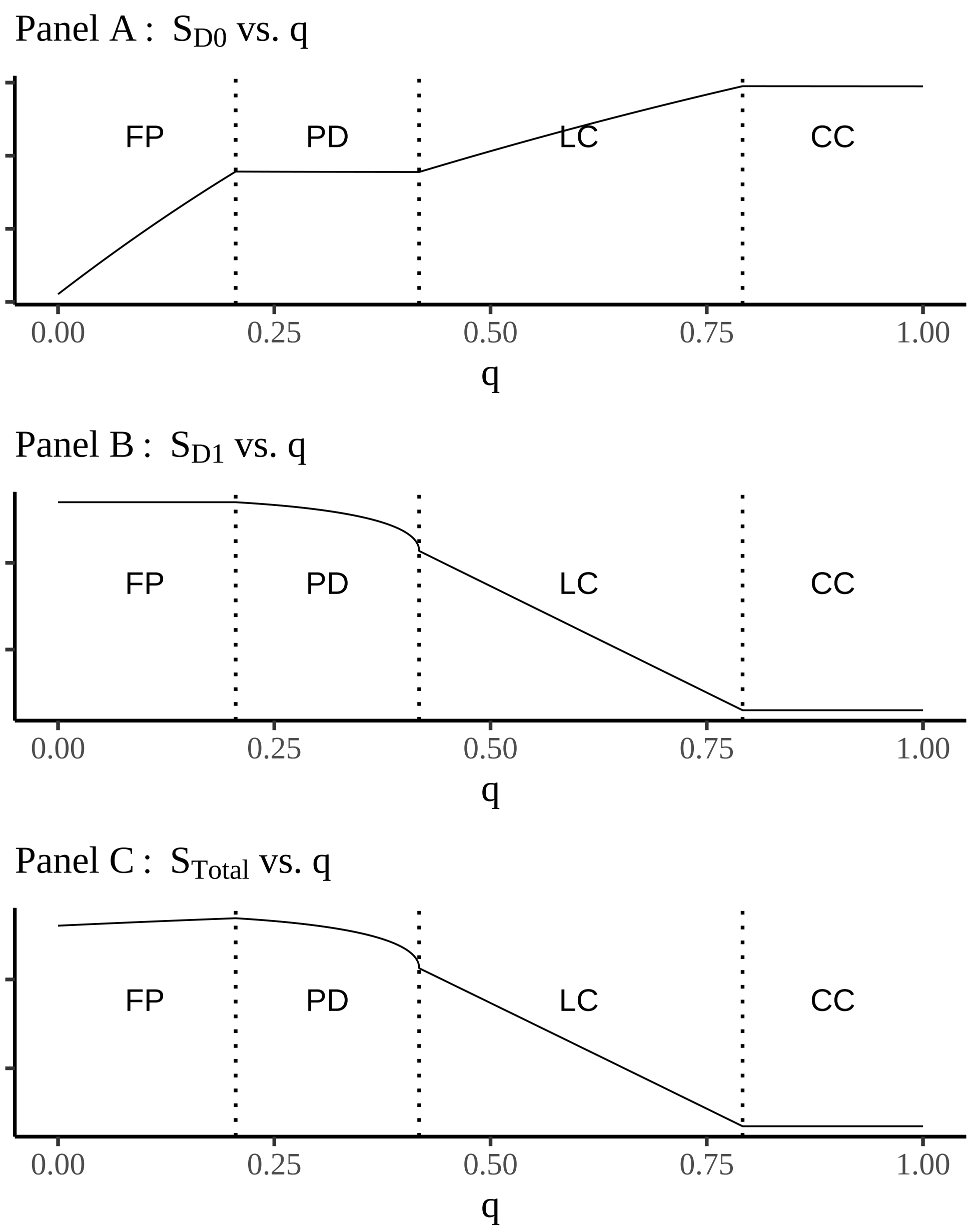
7. Bankruptcy Exemption and Systemic Risk

The views of policy makers about the impact of bankruptcy exemption on systemic risk have been well documented in the Federal Reserve Report [2011], which was crafted by the Board of Governors of the Federal Reserve System in the aftermath of the global financial crisis.[24] The report discusses polar views of researchers regarding the impact of bankruptcy exemption; some researchers argue that bankruptcy exemption increases systemic risk, while others argue that it reduces systemic risk.

The proponents of bankruptcy exemption say that it reduces systemic risk because it increases

[24] The report was prepared as a response to the requirements under Section 216(a)(2)(E) of the Dodd-Frank Wall Street Reform and Consumer Protection Act of 2010 (Dodd-Frank Act).

Figure 6: **Ex-ante Equilibrium Total Surplus Evolution.** Panel A shows the evolution of the expected Date 0 surplus (S_{D0}), Panel B shows the evolution of the expected Date 1 surplus (S_{D1}) and Panel C shows the evolution of the expected total surplus generated in the economy (S_{Total}), as a function of the bankruptcy exemption parameter (q) for a strong Date 1 shock ($\theta_2^l = 0.48$). As q increases, the system transitions from FP equilibrium to PD equilibrium at $q = 0.21$, then from PD equilibrium to LC equilibrium at $q = 0.42$ and finally from LC equilibrium to CC equilibrium at $q = 0.79$. The dotted lines represent the boundaries between the equilibrium regions. The dynamics are obtained for the same parameter configuration for which the demarcation of the feasible $q - \theta_2^l$ space is shown in Figure 4 (i.e., $\theta_2^l = 0.48$, $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2 = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.)



the liquidity of collateral (as it allows lenders to repossess collateral and liquidate it). Based on this premise, they argue that bankruptcy exemption arrests systemic risk of the form that may arise as a knock-on effect from a distressed borrower to the lending counter-party. Absent bankruptcy exemption, i.e., under an automatic stay, there would be spillover effects in the economy because lenders would face difficulty in repossessing collateral, resulting in a freeze in lending (Novikoff and Ramesh [2002], Morrison et al. [2014], and Mooney Jr [2014]). In fact, this view that bankruptcy exemption reduces systemic risk gained credence in the aftermath of the Lehman crisis. In response, Congress enacted the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 to expand the definition of repos to include mortgage loans/securities (Acharya and Öncü [2014]).[25]

We contend that the view that bankruptcy exemption mitigates systemic risk is myopic and only reflects an ex-post perspective. We show that the ex-post "benefits to lenders" in the form of assured liquidation encourages higher ex-ante leverage creation. Unlike the argument made by the proponents of bankruptcy exemption that it enhances collateral liquidity, we show that higher ex-ante leverage induced by bankruptcy exemption triggers collateral run risk due to excessive ex-post liquidation. Once the systemic risk arising from collateral runs is endogenized, lenders may find bankruptcy exemption (ex-ante) sub-optimal.However, an exception can arise under very favorable economic conditions when there is ample liquidity in the economy and fire-sale effect do not arise at any level of q. In this case, there is no conflict between financial sector growth and real sector growth, and not surprisingly, full bankruptcy exemption is optimal. In short, a myopic perspective that favors financial sector growth is not necessarily sub-optimal when economic conditions are extremely favorable (mild economic shock, good quality collateral, and small real sector).[26]

Our model is consistent with the argument that bankruptcy exemption increases systemic risk, as put forth by the opponents of bankruptcy exemption. As discussed in Section 2, several academics have pointed out that ex-post fire sales exacerbate systemic risk (Bordo and Jeanne [2002], Lorenzoni [2008], Tuckman [2010], Duffie and Skeel [2012], Stein [2012], and Acharya and Öncü [2014]). The Federal Reserve Report [2011] mentions the arguments in Skeel and Jackson [2011] and Roe [2010] to point out that bankruptcy exemption encourages collateral runs, which, (to quote the report), "*can both destabilize the debtor and have spillover effects on other creditors, other non-creditor firms*

[25] Bankruptcy exemption also applies in the case of derivative contracts but the focus of our paper is on repo markets.

[26] Some policy makers have also echoed the view that bankruptcy exemption is aimed at favoring financial sector growth (at the expense of real sector growth), i.e., for facilitating Date 0 lending. This motivation also reflects a myopic perspective. Our model shows that, even after accounting for the ex-ante Date 0 lending benefits, bankruptcy exemption is sub-optimal because of a reduction in Date 1 surplus (except when economic conditions are so favorable that there are no fire-sale effects). An ex-ante perspective, therefore, eliminates such myopic considerations.

and financial markets in general". This view is mirrored in our study.

Bankruptcy exemption facilitates excess leverage creation because lenders are more assured of liquidation proceeds.[27] On the other hand, under an automatic stay, lenders would employ a more conservative policy and would consequently lend less and create fewer assets at Date 0. In an ex-ante optimization exercise, we not only consider the benefits of asset creation at Date 0 but also account for the fire-sale effects. We show that bankruptcy exemption is sub-optimal when the economic shock is sufficiently adverse (and/or other factors, as will be discussed in the following section, Section 8). However, under favorable parameter values such as a mild economic shock (and/or other factors, as will be discussed in the following section, Section 8), the adverse ex-post fire-sale effects are overwhelmed by the benefits of ex-ante asset creation, and full bankruptcy exemption may still be optimal. It is important to note that full bankruptcy exemption is optimal in our model only when favorable conditions arise. It does not arise because of reduction in systemic risk; it arises only because the ex-ante lending benefits overwhelm the increase in systemic risk under these favorable conditions. To conclude, our model establishes that bankruptcy increases systemic risk and thereby sheds light on the debate about the impact of bankruptcy exemption on systemic risk.

8. Determinants of the Optimal Bankruptcy Exemption parameter (q)

Proposition 4 establishes that the optimal level of bankruptcy exemption lies at the boundary of the Fair Pricing and Fire Sale regions. The optimal q reflects surplus creation due to asset origination at both Date 0 and Date 1. In this section, we relate the level of the optimal bankruptcy exemption parameter (q) to three key parameters of the model: (i) magnitude of the economic shock, (ii) collateral quality, and (iii) the size of the real economy.

8.1. The Impact of Economic Shocks

PROPOSITION 5: *The optimal bankruptcy exemption (q^{opt}) is decreasing in the severity of the economic shock (θ_2^l).*

As the severity of the economic shock increases, there is less spare liquidity in the economy and

[27]Our model relies on a direct cash flow effect in that the lender's expected proceeds increase with bankruptcy exemption. This direct cash effect facilitates a more liberal lending policy resulting in excess leverage in the economy. An indirect effect can also arise due to a moral hazard problem when lenders underinvest in due diligence of borrowers because their collateral holdings are immune to borrower default (Skeel and Jackson [2011]) and Roe [2010]). The associated incentives would also help create excess leverage.

the fire-sale effects are triggered at lower levels of q, than otherwise. In essence, the border of the Fair Pricing and the Fire Sale Equilibrium regions shifts downward, if the economic shock is severe (i.e., θ_2 is low). In such a situation, the optimal q decreases. This effect can even result in the optimal q taking a value of 0. While it is quite reasonable to expect that the *ex-post* the preferred optimal value of q could be 0, it is not at all obvious that the *ex-ante* optimal bankruptcy exemption parameter could also be equal to zero.

Figure (4) illustrates this situation. Consider the case of a severe economic shock ($\theta_2^l = \theta_{severe} = 0.30$). In this case, there is an acute shortage of funding liquidity due to the severity of the economic shock. The economy will be in a Liquidity Crunch Equilibrium even at the lowest feasible value of $q = 0$ (which induces the least amount of ex-post liquidation). The solid curve representing the boundary of the Fair Pricing region and the Fire Sale region (depicted by the $\bar{\bar{q}}(\theta_2^l)$ *curve*) does not arise in the vertical line drawn at $\theta_2 = 0.30$, i.e., both the Fair Pricing Equilibrium region and the Price Discrimination Equilibrium region vanish for the given level of economic shock. For such a severe economic shock, the economy is always in the Fire Sale Equilibrium region for the entire range of feasible $q \in (0, 1)$. This situation arises because the financial market cannot clear without reducing the supply of loans to the real sector, i.e., the system will in the Liquidity Crunch Equilibrium region, and there will some unmet demand in the real sector ($\bar{\beta} < \mathcal{B}$). The system transitions to a Credit Crunch Equilibrium at higher values of q.

Interestingly, the ex-ante optimal q^{opt} is equal to 0. This result leads to an interesting conclusion that is at variance with observed q levels of 1 that provide safe harbor to certain asset classes.

Figure (4) also depicts the situation in which the optimal bankruptcy exemption can be equal to 1. Consider the case of a mild economic shock ($\theta_2^l = \theta_{mild} = 0.75$). In this case, there is sufficient liquidity in the economy that there are no ex-post fire-sale effects. Both the financial asset and the real asset trade at fair value for any level of q. Since there is no negative externality of ex-post liquidation, it is optimal to employ full bankruptcy exemption, which facilitates ex-ante lending that maximizes total surplus in the economy.

8.2. The Role of Collateral Quality

PROPOSITION 6: *The optimal bankruptcy exemption (q^{opt}) is decreasing in collateral quality.*

The level of illiquidity discount faced by an asset (k_1) can be seen as a measure of the quality of underlying collateral. As seen in Lemma (1), assets with higher low-state payoff (y_2) have higher collateral quality (i.e., lower k_1). In our model, collateral quality (k_1) and bankruptcy exemption parameter (q) jointly determine the type of equilibrium. We map the equilibria in the system in the (k_1, q) space, which is defined over $k_1 \in [k_{min}, k_{max}]$ and $q \in [0, 1]$. Similar to the analysis behind Figure **4**, the $\bar{\bar{q}}(k_1)$ curve in Figure **7** divides the feasible (k_1, q) space into two regions (the Fair Pricing and the Fire Sale Equilibrium region) for any given (k_1, q) combination. Based on Proposition (4), the $\bar{\bar{q}}(k_1)$ curve represents the q^{opt} for a given k_1.[28]

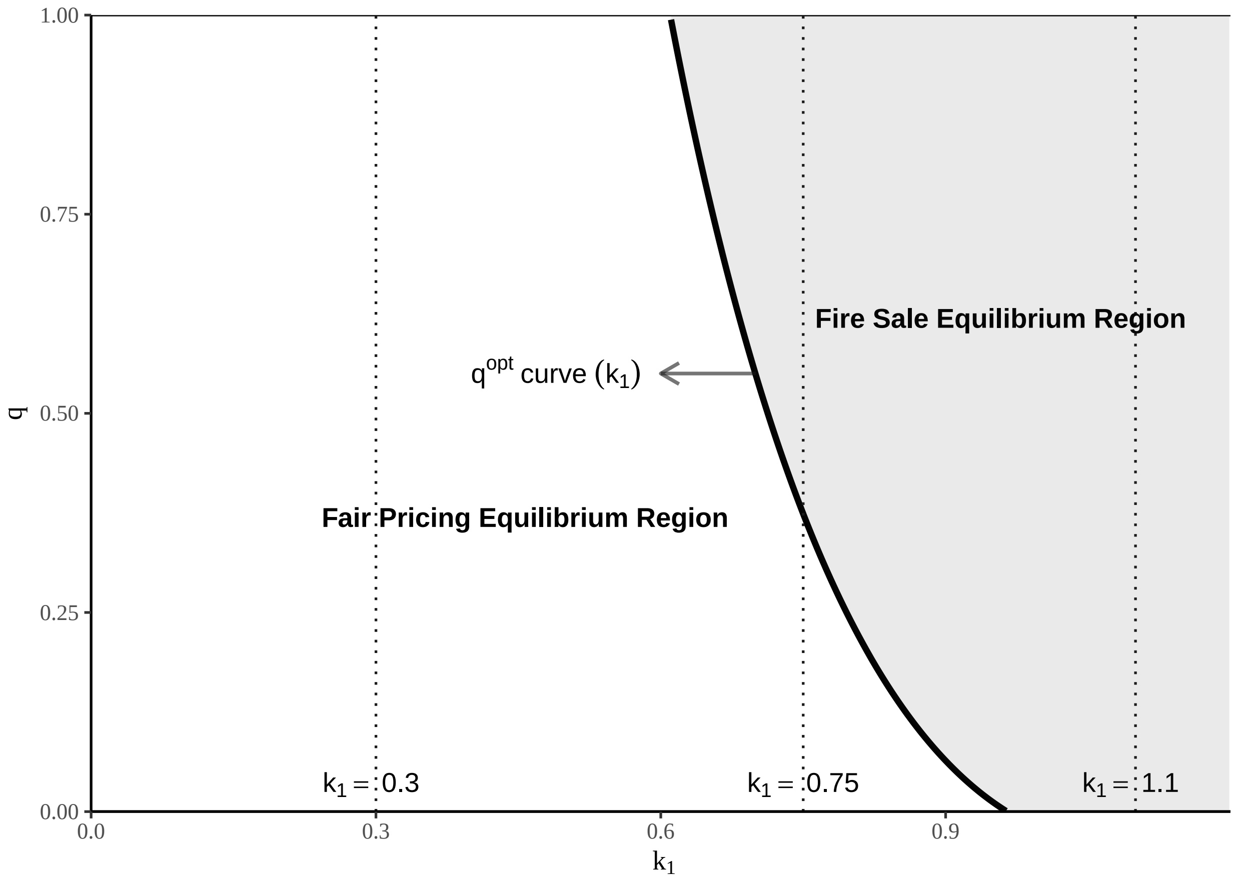
Funding liquidity is high when collateral quality is high (k_1 is low); the economy is endowed with higher spare liquidity. As a consequence, the curve representing the border of the Fair Pricing and Fire Sale Equilibrium regions is downward sloping in the feasible (k_1, q) space.[29] If collateral quality is sufficiently high, the optimal q can be as high as 1 (see $k_1 = 0.3$ in Figure **7**). On the other hand, for low quality collateral, the optimal q is 0 (see $k_1 = 1.1$ in Figure **7**).

Our model points out that full bankruptcy exemption ($q = 1$) is sub-optimal when collateral quality is low. However, full bankruptcy exemption ($q = 1$) can be optimal when the quality of collateral is good. These results suggest that there are nuanced implications of bankruptcy exemption, and extreme policies, either making bankruptcy exemption mandatory or invoking full repeal of bankruptcy exemption, are not always desirable. Consistent with the arguments in this model, the Federal Reserve Report [2011] presented in the aftermath of the global financial crisis of 2008, cites arguments made in Edwards and Morrison [2005], Jackson [2009], and Skeel and Jackson [2011], to point out that full repeal of the safe harbor provisions is not desirable. These authors argue that bankruptcy exemption should be continued for Qualified Financial Contracts (QFCs) in which collateral is in the form of cash or cash-equivalent assets but should be removed for QFCs with less liquid assets. Separately, Duffie and Skeel [2012] also make the same argument. Our findings in this section echo the views expressed in Federal Reserve report arguing for a nuanced repeal of safe harbor provisions. We show that the optimal q is equal to 1 when the collateral is of very high quality but is strictly less than 1 when collateral quality is low, i.e., bankruptcy exemption from the perspective of promoting financial sector growth is conditional on collateral quality - more specifically, on the illiquidity of collateral.

[28]The difference from Figure (**4**) is that in Figure (**7**) θ_2^l is fixed, whereas in the former k_1 is fixed; thus the limits of k_1 in the latter figure are determined in the same manner as the limits on θ_2^l in the former figure.

[29]In the (q, θ_2^l) space shown in Figure (**4**) the q^{opt} curve given by $\bar{\bar{q}}(k_1)$ shifts toward the northwest of the figure, and the optimal q increases as collateral quality is improved (i.e., k_1 is reduced).

Figure **7**: q^{opt} **variation with** k_1**.** Typical demarcation of the feasible $k_1 - q$ space into the Fair Pricing (FP) and Fire Sale (FS) equilibria. The plot is obtained by evaluating the model for assets varying in their payoffs (y_2) leading to variation in their collateral quality (k_1). The solid $q^{opt}(k_1)$ curve represents the boundary between the two equilibrium regions. For a moderate quality asset, indicated by $k_1 = 0.75$, as q is increased from 0, the system transitions from FP equilibrium to FS equilibrium at $q = 0.37$. For a high quality asset indicated by $k_1 = 0.3$, the system remains in FP equilibrium for any q. For a low quality asset, indicated by $k_1 = 1.1$, the system remains in FS equilibrium for any q. $k_1 = 0.30$, $k_1 = 0.75$ and $k_1 = 1.1$ are indicated by the three thin vertical dashed lines. Parameter Configuration used: $\theta_2^l = 0.48$, $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$, $r = 0.6$ and $\mathcal{B} = 0.15$.



8.3. The Size of the Real Economy ($\mathcal{B}$)

Proposition 7: *The optimal bankruptcy exemption (q^{opt}) is decreasing in the size of the real sector ($\mathcal{B}$).*

The above proposition points out an interesting aspect of the model. In Figure **8**, we map the Fair Pricing and the Fire Sale boundary (shown by the $q^{opt}(\mathcal{B} = 0)$ *curve*) in the (θ_2, q) space for different values of the size of the real sector ($\mathcal{B}$). As the size of the real sector ($\mathcal{B}$) increases, the border of the Fair Pricing Equilibrium and the Fire Sale Equilibrium regions shifts downward (and to the right), as can be seen in Figure (**8**). This shift causes the optimal q to decrease with $\mathcal{B}$.

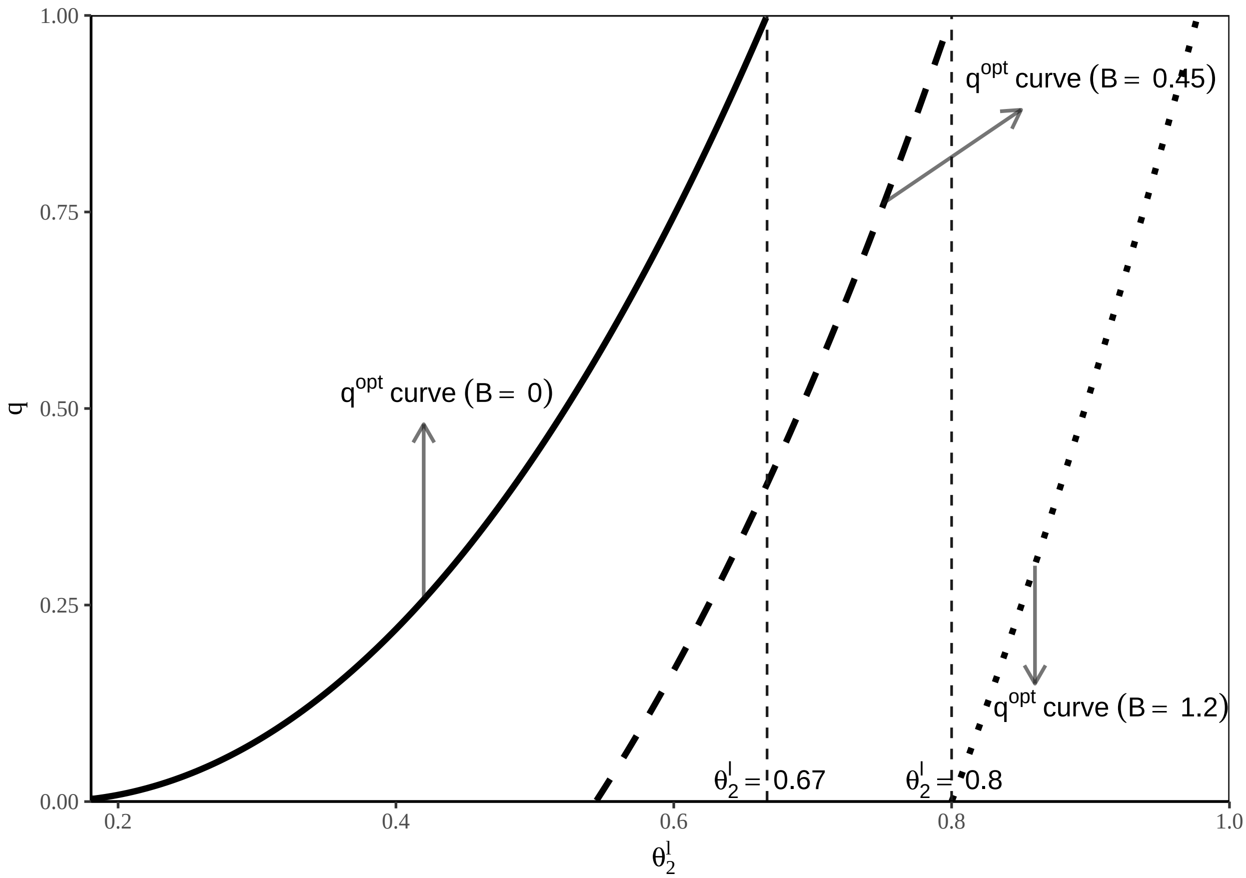
In general, as $\mathcal{B}$ increases, it is less likely that the real asset market will be fully satiated. Note that, in equilibrium, the extent to which the real sector loans are offered depends on the liquidity in the economy. At the extreme, when $\mathcal{B}$ is sufficiently high, even at $q = 0$ when there is no ex-post liquidation, the spare liquidity is insufficient to satisfy the real asset demand. Consequently, the system lies in a Liquidity Crunch Equilibrium region even at the lowest feasible value $q = 0$, and both the Fair Pricing Equilibrium region and the Price Discrimination Equilibrium region vanish for the given level of economic shock. This can be seen in Figure (**8**), where for $\theta_2^l = 0.8$ and for $\mathcal{B} = 1.2$, $q^{opt} = 0$. For any higher $\mathcal{B}$, the optimal q for the given economic shock ($\theta_2^l = 0.8$) will continue to be 0.

Conversely, as $\mathcal{B}$ decreases, the curve moves toward the northwest of (q, θ_2^l) space. However, this leftward movement is bounded when $\mathcal{B}$ hits 0, i.e., when the real sector is absent. This situation corresponds to the special case examined in Acharya and Viswanathan (2011). In their model, q is assumed to be rigid at 1. However, using the setup in this model, we can show that the optimal q could either be an interior value or equal to 1 (i.e. $0 < q^{opt}(\mathcal{B} = 0) \leq 1$ for any θ_2^l), as can be seen from the $q^{opt}(\mathcal{B} = 0)$ *curve* in Figure (**8**).

Table (3), shows how the optimal q varies with $\mathcal{B}$ for a given economic shock.[30] An important implication of the model is that bankruptcy exemption is costlier for larger economies with economies (with a larger real sector) because the socially optimal choice could be to provide an automatic stay. Bigger economies should therefore be more cautious about extending bankruptcy exemption.

[30] As shown in Appendix A16, $\theta_2^{l,B0} = \frac{s_{min}+(1-r)k_1+\sqrt{(1-r)^2k_1^2+2rk_1(\theta_2^h y_2^h - s_{min})}}{y_2}$, $\mathcal{B}_1 = \frac{(\theta_2^l y_2 - s_{min})^2}{2[r\theta_2^h y_2^h + (1-r)\theta_2^l y_2 - s_{min}]}$ and $\mathcal{B}_2 = \frac{(\theta_2^l y_2 - k_1 - s_{min})^2}{2[r\theta_2^h y_2^h + (1-r)(\theta_2^l y_2 - k_1) - s_{min}]}$.

Figure **8**: q^{opt} **variation with** $\mathcal{B}$ Optimal bankruptcy exemption parameter (q^{opt}) curve displayed for three different levels of $\mathcal{B}$. The solid curve shows q^{opt} for $\mathcal{B} = 0$, the dashed curve shows q^{opt} for $\mathcal{B} = 0.45$ and the dotted curve shows q^{opt} for $\mathcal{B} = 1.2$. The vertical dashed line at $\theta_2^l = 0.67$ indicates the value of θ_2^l at which $q^{opt}(\mathcal{B} = 0) = 1$. The values of $\mathcal{B}$ used to obtain the dashed and dotted q^{opt} curves are chosen such that for $\theta_2^l = 0.8$ (indicated by the second vertical dashed line), we have $q^{opt}(\mathcal{B} = 0.45) = 1$ and $q^{opt}(\mathcal{B} = 1.2) = 0$. Parameter configuration is the same as that used in Figure 4 (i.e. $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2 = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.



θ_2^l Range	$\mathcal{B}$ Range	Implication for q^{opt}
$\theta_2^{min} \leq \theta_2^l < \theta_2^{l,B0}$	$\mathcal{B} = 0$	$0 < q^{opt} < 1$
	$\mathcal{B} > 0$	$0 \leq q^{opt} < 1$
$\theta_2^{l,B0} \leq \theta_2^l \leq \theta_2^{max}$	$0 \leq \mathcal{B} \leq \mathcal{B}_1$	$q^{opt} = 1$
	$\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$	$0 < q^{opt} < 1$
	$\mathcal{B}_2 \leq \mathcal{B}$	$q^{opt} = 0$

Table 3: **Impact of $\mathcal{B}$ on** q^{opt}. Implication of the size of the real asset market ($\mathcal{B}$) on the optimal bankruptcy exemption parameter (q^{opt}) for a given level of the economic shock (θ_2^l) is presented.

8.4. When is Bankruptcy Exemption/Automatic Stay Likely to be Optimal?

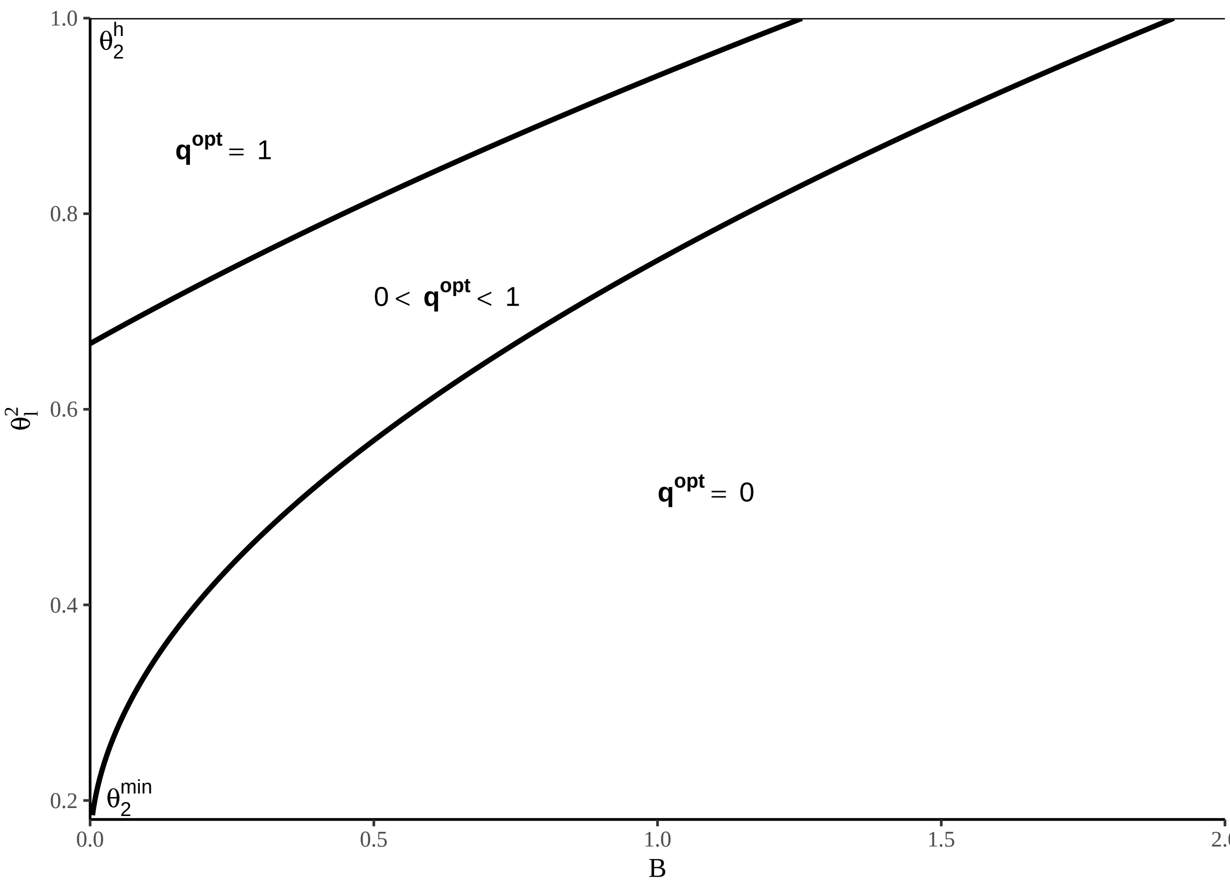
Figure (**9**) displays the results of Table 3 in graphical form by presenting the joint impact of the level of the magnitude of the economic shock (θ_2^l) and the size of the real asset market ($\mathcal{B}$) on q^{opt}. We consider the $(\mathcal{B}, \theta_2)$ space and map the three regions of optimal q ($q^{opt} = 0$, an interior q^{opt}, and $q^{opt} = 1$). We see that when the magnitude of the economic shock is mild and the size of the real asset market is small (i.e., top left corner of Fig(**9**)), $q^{opt} = 1$. As the size of real asset market increases or the severity of the economic shock increases, q^{opt} falls below 1 and moves towards 0 (i.e., bottom right corner of Fig(**9**)).

9. Conclusion

We examine the role of bankruptcy exemption in determining the extent of leverage in the economy, and thereby its consequent impact on the financial stability of the economy. While bankruptcy exemption is usually seen as facilitating financial sector growth in the hope of priming real sector growth, our model highlights that such a prescription must be viewed with caution. We show that bankruptcy exemption creates leverage-inducing growth, which can cause financial instability in the form of fire-sale effects in the real sector. In other words, bankruptcy exemption can sow the seeds of financial instability in the future and should, therefore, be carefully applied for repo collateral whose quality is highly sensitive to economic shocks.

The recent Treasury fire sales and repo rate spikes observed in the Treasury repo market during September 2019 and March 2020 suggest that our conclusions, while derived in the context of risky underlying collateral, may carry over to relatively safe collateral such as Treasuries too. As Barth et al. [2021] note, some of this stress can be attributed to a liquidation of speculative positions in the cash-futures basis trades held by hedge funds and the growing build-up of such positions

Figure **9**: q^{opt} **in** $\mathcal{B} - \theta_2^l$ **space.** Demarcation of the $\mathcal{B} - \theta_2^l$ space into regions where $q^{opt} = 0$, $0 < q^{opt} < 1$ and $q^{opt} = 1$. Parameter Configuration used: $\theta_1 = 0.02$, $\theta_2^h = 1$, $y_2 = 15$, $y_1 = 60$, $y_2^h = 65$, $R = 7$, $\gamma = 6$, $s_{min} = 1.2$ and $r = 0.6$.



in the first place. To the extent that bankruptcy exemption in repo markets encourages purely speculative positions, without attendant real intermediation benefits, there might be a possible case for rethinking safe harbor provisions in Treasury (and Agency) repo markets as well.

Finally, while our work endogenizes the impact of bankruptcy exemption on leverage in the economy, an interesting area of future research would be to consider the role of central bank as a lender of last resort in averting a financial crisis. Such expectations about central bank interventions may influence economic agents' behavior, and regulators must take into account such actions. In particular, while the lender of last resort might be able to diminish the ex-post fire-sale induced spillovers to the real economy, its expectation might raise even greater ex-ante leverage in intermediaries. How such moral hazard would interact with safe harbor provisions in repo financing seems a fruitful area for future inquiry.

References

Viral V Acharya and Sabri T Öncü. The repurchase agreement (repo) market. In Viral V Acharya, Thomas F Cooley, Matthew Richardson, and Ingo Walter, editors, *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, chapter 11, pages 319–350. New York University Stern School of Business and Wiley Online Library, 2011.

Viral V Acharya and Sabri T Öncü. A proposal for the resolution of systemically important assets and liabilities: The case of the repo market. In *The Social Value of the Financial Sector: Too Big to Fail or Just Too Big?*, pages 159–214. World Scientific, 2014.

Viral V Acharya and S Viswanathan. Leverage, moral hazard, and liquidity. *The Journal of Finance*, 66(1):99–138, 2011.

Viral V Acharya, Barry Adler, Matthew Richardson, and Nouriel Roubini. Resolution authority. In *Regulating Wall Street: The Dodd-Frank Act and the New Architecture of Global Finance*, chapter 8, pages 213–240. New York University Stern School of Business and Wiley Online Library, 2011.

Tobias Adrian and Hyun Song Shin. Liquidity and leverage. *Journal of financial intermediation*, 19(3):418–437, 2010.

Philippe Aghion and Patrick Bolton. An incomplete contracts approach to financial contracting. *The review of economic Studies*, 59(3):473–494, 1992.

Franklin Allen and Douglas Gale. Optimal financial crises. *The Journal of Finance*, 53(4):1245–1284, 1998.

Ronald W Anderson and Suresh Sundaresan. Design and valuation of debt contracts. *The Review of Financial Studies*, 9(1):37–68, 1996.

Gaetano Antinolfi, Francesca Carapella, Charles Kahn, Antoine Martin, David C Mills, and Ed Nosal. Repos, fire sales, and bankruptcy policy. *Review of Economic Dynamics*, 18(1):21–31, 2015.

Daniel Barth, R Jay Kahn, et al. Hedge funds and the treasury cash-futures disconnect. 2021.

Michael D Bordo and Olivier Jeanne. Monetary policy and asset prices: Does 'benign neglect'make sense? *International Finance*, 5(2):139–164, 2002.

Adam Copeland, Antoine Martin, and Michael Walker. Repo runs: Evidence from the tri-party repo market. *The Journal of Finance*, 69(6):2343–2380, 2014.

Adam Copeland, Darrell Duffie, and Yilin Yang. Reserves were not so ample after all. 2021.

Adam M Copeland, Antoine Martin, and Michael Walker. The tri-party repo market before the 2010 reforms. *FRB of New York Staff Report*, (477), 2010.

Tri Vi Dang, Gary Gorton, and Bengt Holmström. Financial crises and the optimality of debt for liquidity provision. *Working paper, University of Chicago*, 2010. URL http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.183.7960&rep=rep1&type=pdf.

Adrien d'Avernas and Quentin Vandeweyer. Intraday liquidity and money market dislocations. *Available at SSRN*, 2020.

Douglas W Diamond. Reputation acquisition in debt markets. *Journal of political Economy*, 97(4): 828–862, 1989.

Douglas W Diamond. Debt maturity structure and liquidity risk. *The Quarterly Journal of Economics*, 106(3):709–737, 1991.

Douglas W Diamond. Presidential Address, Committing to commit: Short-term debt when enforcement is costly. *The Journal of Finance*, 59(4):1447–1479, 2004.

Douglas W Diamond and Raghuram G Rajan. Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy*, 109(2):287–327, 2001.

Douglas W Diamond and Raghuram G Rajan. Fear of fire sales, illiquidity seeking, and credit freezes. *The Quarterly Journal of Economics*, pages 557–591, 2011.

Darrell Duffie and David A Skeel. A dialogue on the costs and benefits of automatic stays for derivatives and repurchase agreements. *University of Pennsylvania, Institute for Law & Economics Research Paper*, (12-02), 2012.

Franklin R Edwards and Edward R Morrison. Derivatives and the bankruptcy code: Why the special treatment. *Yale Journal on Regulation*, 22:91, 2005. URL https://digitalcommons.law.yale.edu/yjreg/vol22/iss1/4.

Bryan G Faubus. Narrowing the bankruptcy safe harbor for derivatives to combat systemic risk. *Duke Law Journal*, pages 801–842, 2010.

Federal Reserve Report. Study on the resolution of financial companies under the bankruptcy code, 2011.

Michael J Fleming and Kenneth Garbade. The repurchase agreement refined: Gcf repo. *Current Issues in Economics and Finance*, 9(6), 2003.

Douglas Gale and Franklin Allen. Liquidity preference, market participation and asset price volatility. *American Economic Review*, 84:933–955, 1994.

Gary Gorton and George Pennacchi. Financial intermediaries and liquidity creation. *The Journal of Finance*, 45(1):49–71, 1990.

Gary Gorton, Andrew Metrick, Andrei Shleifer, and Daniel K Tarullo. Regulating the shadow banking system. *Brookings Papers on Economic Activity*, pages 261–312, 2010.

Gary Gorton, Toomas Laarits, and Andrew Metrick. The run on repo and the fed's response. *Journal of Financial Stability*, page 100744, 2020a.

Gary B Gorton. *Slapped By The Invisible Hand: The Panic Of 2007*. Oxford University Press, 2010.

Gary B Gorton and Andrew Metrick. Haircuts. Technical Report 6, Federal Reserve Bank of St. Louis Review, 2010.

Gary B Gorton and Andrew Metrick. Securitized banking and the run on repo. *Journal of Financial Economics*, 104(3):425–451, 2012.

Gary B Gorton, Andrew Metrick, and Chase P Ross. Who ran on repo? In *AEA Papers and Proceedings*, volume 110, pages 487–92, 2020b.

Samuel G Hanson, Anil K Kashyap, and Jeremy C Stein. A macroprudential approach to Financial Regulation. *Journal of Economic Perspectives*, 25(1):3–28, 2011.

Oliver Hart and John Moore. A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics*, 109(4):841–879, 1994.

Hugo A Hopenhayn. Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150, 1992.

Thomas H Jackson. Chapter 11F: A proposal for the use of bankruptcy to resolve financial institutions. In Kenneth E Scott, George P Shultz, and John B Taylor, editors, *Ending Government Bailouts As We Know Them*, chapter 11, pages 217–252. Hoover Press, 2009.

Michael C Jensen and William H Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3(4):305–360, 1976.

Guido Lorenzoni. Inefficient credit booms. *The Review of Economic Studies*, 75(3):809–833, 2008.

Xuyang Ma. Safe harbor or automatic stay? A structural model of repo runs. *Working Paper, NYU*, 2017.

Harvey Miller. Too Big to Fail: The role of bankruptcy and antitrust law in financial regulation reform. testimony before the subcommittee on commercial and administrative law of the house of representatives committee on the judiciary, 2009.

Charles W Mooney Jr. The bankruptcy code's safe harbors for settlement payments and securities contracts: When is safe too safe. *Tex. Int'l LJ*, 49:245, 2014.

Edward Morrison, Mark J. Roe, and Christopher S. Sontchi. Rolling back the repo safe harbors. *Business Lawyer*, 69:1015, 2014.

Harold S Novikoff and Sandeep C Ramesh. *Special Bankruptcy Code Protections for Derivative and Other Financial Market Transactions.* 2002.

The Federal Reserve Bank of New York. Tri-party repo infrastructure reform. *FRBNY White Paper*, 2010.

Enrico Perotti. Systemic liquidity risk and bankruptcy exceptions. *VoxEU. org*, 13, 2010.

Mark Roe. End bankruptcy priority for derivatives, repos and swaps. *Financial Times*, 16, 2009.

Mark J Roe. Bankruptcy's Financial Crisis Accelerator: The Derivatives Players' Priorities in Chapter 11. Technical report, Harvard Public Law Working Paper No. 10, 2010.

Andrei Shleifer and Robert W Vishny. Liquidation values and debt capacity: A market equilibrium approach. *The Journal of Finance*, 47(4):1343–1366, 1992.

David Skeel and Thomas Jackson. Transaction consistency and the new finance in bankruptcy. *U. Penn. Inst. for Law and Econ.*, 2011.

Jeremey C Stein. Monetary policy as financial stability regulation. *The Quarterly Journal of Economics*, pages 57–95, 2012.

Joseph E Stiglitz and Andrew Weiss. Credit rationing in markets with imperfect information. *The American economic review*, 71(3):393–410, 1981.

Bruce Tuckman. Amending safe harbors to reduce systemic risk in otc derivatives markets. *CFS Policy paper*, 2010.

Bruce Tuckman. Flash: Tri-party repo infrastructure reform. *CFS Policy paper*, 2012.

Appendix A: Key Results

A1. Proof of Lemma (1)

Differentiating k_1 with respect to θ_2 and with respect to y_2, we get:

$$\frac{dk_1}{d\theta_2} = -\frac{\theta_1^2(y_1 - y_2)}{\theta_2 - \theta_1} < 0 \tag{A1}$$

$$\frac{dk_1}{dy_2} = -\frac{\theta_2\theta_1}{\theta_2 - \theta_1} < 0 \tag{A2}$$

Denoting $\kappa = \frac{\theta_1^2(y_1-y_2)}{\theta_2-\theta_1} > 0$ and $\iota = \frac{\theta_2}{\theta_2-\theta_1} > 0$, we obtain $\frac{d\rho^*}{d\theta_2} = y_2 + \kappa > 0$ and $\frac{d\rho^*}{dy_2} = \theta_2\iota > 0$.

A2. Proof of Lemma (2)

Given the result in (1), it follows that optimal effort is decreasing in f_r. The expected profits of the lender ef_r is equal to $\frac{1}{\gamma}(R - f_r)f_r$ is quadratic in f_r with a negative coefficient on $(f_r)^2$, implying a concave relationship. The first order condition yields $\frac{1}{\gamma}(R - f_r - f_r) = 0$, i.e., $f_r = \frac{R}{2}$, i.e, the expected profit function is maximized at $f_r = \frac{R}{2}$.

A3. Proof of Lemma (3)

Step 1. Restate the optimization problem of surplus-liquidity firms.

Using the results in Lemma (1), namely, $\theta_2 y_2 - \rho^* = k_1$, and Lemma (2), namely, $e = \frac{1}{\gamma}(R - f_r)$, we can re-formulate the optimization problem in (4) - (5) as a Lagrangian optimization problem with μ, η, and ν as Lagrangian parameters. μ is the Lagrangian parameter for the budget constraint, whereas η and ν are the the Lagrangian parameters employed for the non-negativity constraints, $\alpha \geq 0$ and $\beta \geq 0$, respectively.

$$\max_{\alpha>0,\beta\geq 0}(1+\alpha)k_1 + \beta e f_r - \mu\left[\alpha(p-\rho^*) + \beta - (\rho^* - \rho)\right] - \eta\alpha - \nu\beta \tag{A3}$$

The solution depends on the following first order condition for α, β, μ, η, and ν, respectively.

$$k_1 - \mu(p - \rho^*) - \eta = 0 \tag{A4}$$

$$ef_r - \mu - \nu = 0 \tag{A5}$$

$$\alpha(p - \rho^*) + \beta = (\rho^* - \rho) \tag{A6}$$

$$\alpha = 0 \tag{A7}$$

$$\beta = 0 \tag{A8}$$

Since the secondary market for legacy financial assets must necessarily clear, we impose the condition that $\alpha > 0$, which implies that the Lagrangian parameter $\eta = 0$. It follows from Equation (A4) that

$$\mu = \frac{k_1}{p - \rho^*}. \tag{A9}$$

The real asset market is a primary market and we must account for the possibility of the market being closed ($\beta = 0$) and the market being open ($\beta > 0$); these cases correspond to the Lagrangian parameter, ν, being strictly greater than or equal to 0, respectively. From Equation (A5), we get

$\nu = ef_r + \mu$. Thus, after incorporating the result in Equation (A9), we can conclude that when $\nu = 0$,

$$\frac{k_1}{p - \rho^*} = ef_r, \tag{A10}$$

and when $\nu > 0$, we get

$$\frac{k_1}{p - \rho^*} > efr. \tag{A11}$$

Note that $\mu > 0$ holds because the budget constraint in (5) is always binding due to non-satiation, i.e.,surplus-liquidity firms will always have incentive to deploy their spare liquidity fully in either of the two markets).

A4. Proof of Lemma (4):

We start with the aggregate budget constraint, which equates aggregate supply and demand as shown in Equation (11), restated below:

$$\int_{\rho^*}^{\rho_{max}} q \min\left[\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right] g(\rho) d\rho + \frac{\bar{\beta}}{p - \rho^*} = \int_{\rho_{min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*} g(\rho) d\rho \tag{A12}$$

Since $\frac{(\rho^* - \rho)}{(p - \rho^*)} > 0$ for $\rho < \rho^*$, it follows trivially that the RHS of (A12), $\int_{\rho_{min}}^{\rho^*} \frac{\rho^* - \rho}{p - \rho^*} g(\rho) d\rho$, can be written as $\int_{\rho_{min}}^{\rho^*} \max\left[\frac{(\rho^* - \rho)}{(p - \rho^*)}, -1\right] g(\rho) d\rho$. Thus, we obtain:

$$\int_{\rho^*}^{\rho_{max}} q \min\left[\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right] g(\rho) d\rho + \frac{\bar{\beta}}{p - \rho^*} = \int_{\rho_{min}}^{\rho^*} \max\left[\frac{\rho^* - \rho}{p - \rho^*}, -1\right] g(\rho) d\rho \tag{A13}$$

Adding $-\int_{\rho^*}^{\rho_{max}} \min\left[\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right] g(\rho) d\rho$ to the LHS of the above equation, and adding an equivalent term, $\int_{\rho^*}^{\rho_{max}} \max\left[\frac{(\rho^* - \rho)}{(p - \rho^*)}, -1\right] g(\rho) d\rho$ to the RHS of the above equation, we obtain:[31]

$$-(1 - q) \int_{\rho^*}^{\rho_{max}} \min\left[\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right] \; g(\rho) d\rho + \frac{\bar{\beta}}{p - \rho^*} = \int_{\rho_{min}}^{\rho_{max}} \max\left[\frac{\rho^* - \rho}{p - \rho^*}, -1\right] \; g(\rho) d\rho$$

Integration by parts and rearranging the terms yields:

$$\begin{aligned}
\frac{\bar{\beta}}{p - \rho^*} =& -1 + \frac{1}{(p - \rho^*)} \int_{\rho_{\min}}^{p} G(\rho) d\rho + (1 - q) \left[1 - \frac{1}{(p - \rho^*)} \int_{\rho^*}^{p} G(\rho) d\rho\right] \\
\bar{\beta} =& -q(p - \rho^*) + \int_{\rho_{min}}^{p} G(\rho) d\rho - (1 - q) \int_{\rho^*}^{p} G(\rho) d\rho \\
\bar{\beta} =& -q(p - \rho^*) + \int_{\rho_{min}}^{\rho^*} G(\rho) d\rho + q \int_{\rho^*}^{p} G(\rho) d\rho
\end{aligned} \tag{A14}$$

[31]Note that, $-\int_{\rho^*}^{\rho_{max}} \min\left[\frac{(\rho - \rho^*)}{(p - \rho^*)}, 1\right] g(\rho) d\rho = \int_{\rho^*}^{\rho_{max}} \max\left[\frac{-(\rho - \rho^*)}{(p - \rho^*)}, -1\right] g(\rho) d\rho = \int_{\rho^*}^{\rho_{max}} \max\left[\frac{(\rho^* - \rho)}{(p - \rho^*)}, -1\right] g(\rho) d\rho$.

A5. *Proof of Proposition (1)*

For parsimony, we characterize the equilibrium in terms of the triplet $(p, \bar{\beta}, f_r)$. In the Fair Pricing (FP) Equilibrium both financial and real assets are fairly priced, i.e., price of an asset is equal to the expected payoff from the asset ($p = E(y_2)$ and $ef_r = 1$) and expected return on investment for liquidity surplus firms is 0. This outcome results when the supply of liquidity exceeds the demand for liquidity leading to the satiation of the real asset market even when the price of the financial asset (p) is at its highest possible value of $\theta_2^l y_2$. Consequently, in the FP equilibrium, we have:

$$p = E(y_2) = \theta_2 y_2 \tag{A15}$$

$$ef_r = 1 \Rightarrow \frac{1}{\gamma}(R - f_r)f_r = 1 \Rightarrow f_r = \frac{R}{2} - \frac{\sqrt{R^2 - 4\gamma}}{2} \tag{A16}$$

$$\bar{\beta} = \mathcal{B} \tag{A17}$$

A6. *Proof of Proposition (2):*

A6.1. *Real Asset Price Discrimination Equilibrium (PD)*

Conditional on a given θ_2, the system transitions from the Fair Pricing Equilibrium Region to the Fire Sale Equilibrium Region as q increases and there is too much liquidation of assets at Date 1. In this situation, the market clearing price (p) falls below the fair value ($\theta_2 y_2$). The real asset market continues to remain fully satiated ($\bar{\beta} = \mathcal{B}$), as in the Fair Pricing region. The price of the financial asset is obtained by substituting for $\bar{\beta} = \mathcal{B}$ in Equation (A14).

The cross-market equilibrium return condition implies that the face value of the real asset loan (f_r) increases to ensure that the returns on both assets are equal. Equation (8) reflects the cross-market equilibrium return condition, yielding:

$$\bar{\beta} > 0 \implies \frac{k_1}{p - \rho^*} = ef_r > \mu > 0. \tag{A18}$$

(A18) can be simplified into a quadratic equation in f_r, after recognizing that $e^* = \frac{1}{\gamma}(R - f_r)$ and $\rho^* = \theta_2 y_2 - k_1$. Note that, in equilibrium, the larger root greater than $\frac{R}{2}$ can be ignored due to constraints expressed in Section (4.4.1), yielding:[32]

$$f_r = \frac{R}{2} - \frac{1}{2}\sqrt{R^2 - \frac{4\gamma k_1}{p - \rho^*}} \tag{A19}$$

[32] $f_r^m = R/2$ is the face value of the loan at which the lender's profit is maximized when effort level of the households is endogenously determined. Consequently, it is never in the interest of lenders to charge a face value higher than f_r^m, implying $f_r < f_r^m = R/2$.

A6.2. Real Asset Liquidity Crunch Equilibrium (LC)

As q increases in the Price Discrimination region, the face value (f_r) increases in equilibrium (a result that will be shown further down). The maximum value of f_r is equal to $\frac{R}{2}$, as discussed in Section (4.4.1). If the demand for liquidity exceeds supply when f_r is at its highest possible value of $R/2$, supply-demand equilibrium is achieved through the rationing of the real asset market with the aggregate number of real asset loans extended ($\bar{\beta}$) falling below $\mathcal{B}$. In the LC equilibrium, $f_r = R/2$, $\bar{\beta}$ is given by Equation (A14). p can be obtained as follows from the cross-market equilibrium return condition in Equation (A18) while noting that when $f_r = R/2$, $ef_r = \frac{R^2}{4\gamma}$:

$$\begin{aligned} p - \rho^* =& \frac{k_1}{ef_r} = \frac{4\gamma k_1}{R^2} \\ \Rightarrow p =& \rho^* + \lambda \quad where \;\; \lambda = \frac{4\gamma k_1}{R^2} \end{aligned} \tag{A20}$$

A6.3. Real Asset Credit Crunch Equilibrium (CC)

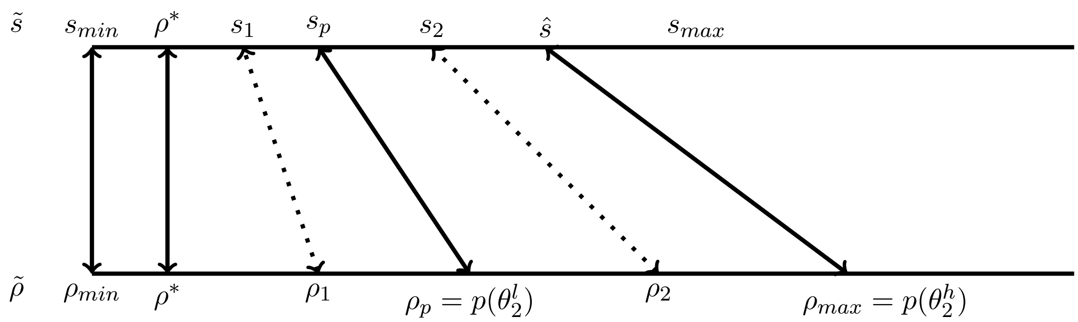
Note that $\bar{\beta}$ is decreasing in q in the Liquidity Crunch region (a result that will be established further down). Thus, as q increases, $\bar{\beta}$ will decrease and at a sufficiently high value of q, $\bar{\beta}$ will be equal to 0, and the system will transition to the Credit Crunch region. In this case, the equilibrium price, p, is given by the solution of Equation (12), in which $\bar{\beta}$ is set equal to 0. Furthermore, the cross-market equilibrium return condition is irrelevant. The equilibrium should satisfy (9) and (12) evaluated at $\bar{\beta} = 0$. The equilibrium triplet (p, $\bar{\beta}$, f_r) will now be reduced to singleton, $p(0)$, because $\bar{\beta}$ and f_r are irrelevant when the real asset market is closed.

A7. Proof of Lemma (5): Derived Distribution of Debt

Figure (**10**) presents a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ. To obtain the derived distribution of $\hat{G}(.) = G(\rho|s \leq \hat{s})$, we first note that for $\rho_{min} \leq \rho \leq \rho^*$, $\tilde{\rho} = \tilde{s}$ is uniform over $[\rho_{min}, \rho^*]$ because $\tilde{s}$ is uniformly distributed over $[s_{min}, \rho^*]$. Then, as shown in the adjoining figure, consider $\rho_1 \in (\rho^*, \rho_p]$, where ρ_1 is the face value that finances an investment shortfall of s_1, and $\rho_p = p(\theta_2)$. We obtain:

$$\begin{aligned} \hat{G}(\rho_1) &= G(\tilde{\rho} \leq \rho_1 | \tilde{s}(\rho_1) \leq \hat{s}) = Prob(\tilde{s}(\rho_1) \leq s_1 | \tilde{s}(\rho_1) \leq \hat{s}) \\ &= Prob(\tilde{s}(\rho_1) \leq \rho^* | \tilde{s}(\rho_1) \leq \hat{s}) + Prob(\rho^* < \tilde{s}(\rho_1) \leq s_1 | \tilde{s}(\rho_1) \leq \hat{s}) \\ &= \frac{\rho^* - s_{min}}{\hat{s} - s_{min}} + \frac{s_1 - \rho^*}{\hat{s} - s_{min}} = \frac{s_1 - s_{min}}{\hat{s} - s_{min}} \end{aligned}$$

Figure **10**: **Derived Distribution of Debt.** The figure below shows a pictorial representation of the mapping between support for $s(\rho)$ and the support for ρ. The full double arrow lines indicate borders around which the ρ function changes and the dotted double arrow lines are specific values of ρ and s used to derive the distribution of ρ given $s \leq \hat{s}$.



Now, consider $\rho_2 \in (\rho_p, \rho_{max}]$. In this case, we obtain:

$$\begin{aligned}
\hat{G}(\rho_2) &= G(\tilde{\rho} \leq \rho_2 | \tilde{s}(\rho_2) \leq \hat{s}) = Prob(\tilde{s}(\rho_2) \leq s_2 | \tilde{s}(\rho_2) \leq \hat{s}) \\
&= Prob(\tilde{s}(\rho_2) \leq \rho^* | \tilde{s}(\rho_2) \leq \hat{s}) + Prob(\rho^* < \tilde{s}(\rho_2) \leq s_p | \tilde{s}(\rho_2) \leq \hat{s}) + Prob(s_p < \tilde{s}(\rho_2) \leq s_2 | \tilde{s}(\rho_2) \leq \hat{s}) \\
&= \frac{\rho^* - s_{min}}{\hat{s} - s_{min}} + \frac{s_p - \rho^*}{\hat{s} - s_{min}} + \frac{s_2 - s_p}{\hat{s} - s_{min}} = \frac{s_2 - s_{min}}{\hat{s} - s_{min}}
\end{aligned}$$

In general, we have $\hat{G}(\rho)$ specified as follows for $\rho_{min} \leq \rho \leq p$ (where $m = r + (1-r)q$):

$$\hat{G}(\rho) = \frac{s(\rho) - s_{min}}{\hat{s} - s_{min}} \text{ where } s(\rho) = \begin{cases} \rho, & \text{if } \rho_{min} \leq \rho \leq \rho^* \\ m(\rho - \rho^*) + \rho^*, & \text{if } \rho^* < \rho \leq p(\theta_2^l) \end{cases} \tag{A21}$$

A8. Model Parameter Space Restrictions

A well defined model parameter space should jointly satisfy the following constraints where $\omega = \frac{4\gamma}{R^2}$ and consequently $\lambda = \omega k_1$.[33]

$$\theta_2^{min} = (s_{min} + k_1 + \lambda)/y_2 \tag{A22}$$

$$\Rightarrow \theta_2^{min} = \frac{\theta_1 y_1 + \omega\theta_1(y_1 - y_2) + s_{min} + \sqrt{[\theta_1 y_1 + \omega\theta_1(y_1 - y_2) + s_{min}]^2 - 4\theta_1 y_2 s_{min}}}{2y_2} \tag{A23}$$

$$\theta_2^{max} < \theta_2^h \tag{A24}$$

Equation (A22) ensures financial market clearing for any $\theta_2^l \in [\theta_2^{min}, \theta_2^{max}]$ and any $q \in [0, 1]$.

[33] The result in Equation (A23) follows from solving the quadratic equation obtained by substituting $k_1(\theta_2^{min}) = \frac{\theta_2^{min}\theta_1(y_1 - y_2)}{\theta_2^{min} - \theta_1}$ and $\lambda(\theta_2^{min}) = \omega k_1(\theta_2^{min})$ in Equation (A22). The smaller root of the quadratic can be ignored as it does not satisfy the constraint $\theta_2^{min} y_2 > \theta_1 y_1$.

A9. Expression for $\bar{\beta}$ in the Ex Ante Equilibrium

In the ex-ante equilibrium, we use the endogenous distribution of debt obtained in Lemma (5) along with Equation (A14) to solve for $\bar{\beta}$ in the LC equilibrium. We obtain:

$$\begin{aligned}\bar{\beta} =& -q(p-\rho^*) + \int_{\rho_{min}}^{\rho^*} \hat{G}(\rho)d\rho + q\int_{\rho^*}^{p} \hat{G}(\rho)d\rho \\ =& -q(p-\rho^*) + \int_{\rho_{min}}^{\rho^*} \frac{\rho - s_{min}}{\hat{s} - s_{min}}d\rho + q\int_{\rho^*}^{p} \frac{m(\rho-\rho^*) + \rho^* - s_{min}}{\hat{s} - s_{min}}d\rho\end{aligned}$$

Noting that, $\rho_{min} = s_{min}$ and $p - \rho^* = \lambda$ and notating $\rho^* - s_{min} = \phi$, $r(\theta_2^h y_2^h - p) = \pi$ and $\hat{s} - s_{min} = \Delta\hat{s}$, we get:[34]

$$\bar{\beta} = -q\lambda + \frac{\phi^2}{2\Delta\hat{s}} + \frac{q\lambda}{2\Delta\hat{s}}\left[m\lambda + 2\phi\right] \tag{A25}$$

A10. Proof of Proposition (3):

A10.1. Real Asset Price Discrimination Equilibrium (PD)

Price in the PD region is obtained by using the endogenous distribution of debt from Lemma (5) in Equation (18) and solving for p. Using earlier notations of $\Delta\hat{s} = \hat{s} - s_{min}$, $\phi = \rho^* - s_{min}$ and $m = r + (1-r)q$ and denoting $p - \rho^* = \lambda_{PD}$, we obtain:

$$\begin{aligned}\mathcal{B} =& \; -q(p-\rho^*) + \int_{\rho_{min}}^{\rho^*} \hat{G}(\rho)d\rho + q\int_{\rho^*}^{p} \hat{G}(\rho)d\rho \\ (\mathcal{B} + q\lambda_{PD})\Delta\hat{s} =& \int_{\rho_{min}}^{\rho^*} (\rho - s_{min})d\rho + q\int_{\rho^*}^{p} [m(\rho-\rho^*) + (\rho^* - s_{min})]d\rho \\ 2(\mathcal{B} + q\lambda_{PD})\Delta\hat{s} =& \; \phi^2 + qm\lambda_{PD}^2 + 2q\phi\lambda_{PD}\end{aligned} \tag{A26}$$

The above quadratic in λ_{PD} can be solved to obtain the $p = \rho^* + \lambda_{PD}$. To evaluate the impact of q, we note that $\frac{d\bar{p}}{dq} = \frac{d(q\lambda_{PD})}{dq} = \lambda_{PD} + q\frac{d\lambda_{PD}}{dq}$, therefore, $\frac{d\hat{s}}{dq} = (1-r)\frac{d\bar{p}}{dq} = (1-r)\frac{d(q\lambda_{PD})}{dq}$. Now, using $m = r + (1-r)q = q + r - qr$, Equation (A26) can be rearranged as follows:

$$2(\mathcal{B} + q\lambda_{PD})\Delta\hat{s} = (\phi + q\lambda_{PD})^2 - r(q\lambda_{PD})^2 + rq\lambda_{PD}^2$$

Differentiating the above expression with respect to q and noting that $\Delta\hat{s} = \pi + \phi + m\lambda_{PD}$, we obtain:

$$2(\mathcal{B} + q\lambda_{PD})(1-r)\frac{d\bar{p}}{dq} + 2\Delta\hat{s}\frac{d\bar{p}}{dq} = 2(\phi + q\lambda_{PD})\frac{d\bar{p}}{dq} - 2rq\lambda_{PD}\frac{d\bar{p}}{dq} + r(q\lambda_{PD}\frac{d\lambda_{PD}}{dq} + \lambda_{PD}\frac{d\bar{p}}{dq})$$

[34] $\Delta\hat{s} = r\theta_2^h y_2^h + (1-r)\rho^* + (1-r)q(p-\rho^*) - s_{min} = r(\theta_2^h y_2^h - \rho^* - (p-\rho^*)) + \big(r + (1-r)q\big)(p-\rho^*) + \rho^* - s_{min} = \pi + m(p-\rho^*) + \phi$ where $\pi = r(r\theta_2^h y_2^h - p)$ and $\phi = \rho^* - s_{min}$. This general result is valid across equilibrium regions. In the PD, LC and CC regions, $p - \rho^*$ is replaced by λ_{PD}, λ and λ_{CC}, respectively.

$$\Rightarrow 2\left[(1-r)\mathcal{B}+(1-r)q\lambda_{PD}+\Delta\hat{s}\right]\frac{d\bar{p}}{dq}=2\left[\phi+(1-r)q\lambda_{PD}+\frac{r\lambda_{PD}}{2}\right]\frac{d\bar{p}}{dq}+r\lambda_{PD}\left[\frac{d\bar{p}}{dq}-\lambda_{PD}\right]$$

$$\Rightarrow \left[(1-r)\mathcal{B}+\pi+(1-r)q\lambda_{PD}\right]\frac{d\bar{p}}{dq}=\frac{-r\lambda_{PD}^2}{2}$$

$$\Rightarrow \left.\frac{d\bar{p}}{dq}\right|_{PD}=\frac{-r\lambda_{PD}^2}{2[(1-r)\mathcal{B}+\pi+(1-r)q\lambda_{PD}]}<0 \tag{A27}$$

$$\Rightarrow \left.\frac{dp}{dq}\right|_{PD}=\frac{d\lambda_{PD}}{dq}=\frac{1}{q}\left[\left.\frac{d\bar{p}}{dq}\right|_{PD}-\lambda_{PD}\right]<0 \tag{A28}$$

f_r in the PD region is a function of p and therefore varies with q. Differentiating Equation (17) with respect to q, we obtain:

$$\begin{aligned}\left.\frac{df_r}{dq}\right|_{PD} &= -\frac{1}{4}\left[R^2-\frac{4\gamma k_1}{p-\rho^*}\right]^{-\frac{1}{2}}(-4\gamma k_1)\left[\frac{-1}{(p-\rho^*)^2}\right]\left[\left.\frac{dp}{dq}\right|_{PD}\right]\\ &= -\left[R^2-\frac{4\gamma k_1}{p-\rho^*}\right]^{-\frac{1}{2}}\left[\frac{\gamma k_1}{(p-\rho^*)^2}\right]\left[\left.\frac{dp}{dq}\right|_{PD}\right]>0\end{aligned} \tag{A29}$$

Finally, as $\bar{\beta}=\mathcal{B}$ in the PD region, $\left.\frac{d\bar{\beta}}{dq}\right|_{PD}=0$.

A10.2. Real Asset Liquidity Crunch Equilibrium (LC)

In the LC Equilibrium, $p=\rho^*+\lambda$, $\bar{p}=\rho^*+q\lambda$, $f_r=R/2$ and $\bar{\beta}$ is given by Equation (A14). Therefore, $\left.\frac{dp}{dq}\right|_{LC}=0$, $\left.\frac{d\bar{p}}{dq}\right|_{LC}=\lambda>0$ and $\left.\frac{df_r}{dq}\right|_{LC}=0$. We restate the expression for $\bar{\beta}$ in Equation (A14) as follows:

$$(\bar{\beta}+q\lambda)(\hat{s}-s_{min})=\int_{\rho_{min}}^{\rho^*}(s(\rho)-s_{min})d\rho+q\int_{\rho^*}^{p}(s(\rho)-s_{min})d\rho$$

We differentiate the above equation with respect to q (noting that $\frac{d\hat{s}}{dq}=(1-r)\lambda$ in the LC equilibrium and using the result from Equation (A32) for the RHS) to obtain (using the notational simplifications of m, π, ϕ and $\Delta\hat{s}$ developed earlier):

$$LHS=(\bar{\beta}+q\lambda)\frac{d\hat{s}}{dq}+(\frac{d\bar{\beta}}{dq}+\lambda)(\hat{s}-s_{min})=\Delta\hat{s}\frac{d\bar{\beta}}{dq}+\Delta\hat{s}\lambda+(\bar{\beta}+q\lambda)(1-r)\lambda$$

$$RHS=\Big(s(p)-s_{min}\Big)\frac{d\bar{p}}{dq}-\frac{r(p-\rho^*)^2}{2}=(\phi+m\lambda)\lambda-\frac{r\lambda^2}{2}$$

Combining the two and noting that $\Delta\hat{s}=\pi+\phi+m\lambda$, we obtain:

$$\Delta\hat{s}\frac{d\bar{\beta}}{dq}=(\phi+m\lambda-\Delta\hat{s})\lambda-(1-r)(\bar{\beta}+q\lambda)\lambda-\frac{r\lambda^2}{2}$$

$$\Rightarrow \frac{d\bar{\beta}}{dq}=-\frac{\Big(\pi+(1-r)\bar{\beta}+(m-r/2)\lambda\Big)\lambda}{\Delta\hat{s}}<0 \tag{A30}$$

As π, ϕ, $\Delta\hat{s}$, $\bar{\beta}$, $(m-r/2)$ and λ are all positive, $\frac{d\bar{\beta}}{dq}<0$.

A10.3. Real Asset Credit Crunch Equilibrium (CC)

In the CC Equilibrium, we use $\bar{p} = \rho^* + q(p - \rho^*)$ to rewrite Equation (B7) as follows:

$$\bar{p} - \rho^* = \int_{\rho_{min}}^{\rho^*} \hat{G}(\rho) d\rho + q \int_{\rho^*}^{p} \hat{G}(\rho) d\rho \tag{A31}$$

$$(\hat{s} - s_{min})(\bar{p} - \rho^*) = \int_{\rho_{min}}^{\rho^*} (s(\rho) - s_{min}) d\rho + q \int_{\rho^*}^{p} (s(\rho) - s_{min}) d\rho$$

Noting that $\frac{d\hat{s}}{dq} = (1-r)\frac{d\bar{p}}{dq}$, $\frac{ds(\rho)}{dq} = 0$ for $\rho_{min} \le \rho \le \rho^*$, and $\frac{ds(\rho)}{dq} = (1-r)(\rho - \rho^*)$ for $\rho^* \le \rho \le p$, we separately differentiate the LHS and the RHS of the above equation with respect to q, while notating $p - \rho^* = \lambda_{CC}$ in the CC equilibrium, to obtain:

$$LHS = (\hat{s} - s_{min})\frac{d\bar{p}}{dq} + (\bar{p} - \rho^*)(1-r)\frac{d\bar{p}}{dq} = \frac{d\bar{p}}{dq}\left[\hat{s} - s_{min} + q(1-r)\lambda_{CC}\right]$$

$$\begin{aligned}
RHS &= \int_{\rho^*}^{p} (s(\rho) - s_{min}) d\rho + q\left[\Big(s(p) - s_{min}\Big)\frac{dp}{dq} + \int_{\rho^*}^{p} \frac{ds(\rho)}{dq} d\rho\right] \\
&= \Big[s(p) - s_{min}\Big]\left[\frac{d\bar{p}}{dq} - (p - \rho^*)\right] + \int_{\rho^*}^{p} \Big[\big(m + q(1-r)\big)(\rho - \rho^*) + (\rho^* - \rho_{min})\Big] d\rho \\
&= \Big[s(p) - s_{min}\Big]\frac{d\bar{p}}{dq} - \frac{r(p - \rho^*)^2}{2}
\end{aligned} \tag{A32}$$

Rearranging and noting that $\hat{s} - s(p) = \pi > 0$, yields:

$$\left.\frac{d\bar{p}}{dq}\right|_{CC} = -\frac{r\lambda_{CC}^2}{2\Big(\pi + q(1-r)\lambda_{CC}\Big)} < 0 \tag{A33}$$

By extension $\left.\frac{dp}{dq}\right|_{CC} = (\frac{1}{q})(\frac{d\bar{p}}{dq} - (p - \rho^*)) < 0$ in the CC Equilibrium. Further, as $\bar{\beta} = 0$ and $f_r = R/2$ in the CC equilibrium, it follows that $\left.\frac{d\bar{\beta}}{dq}\right|_{CC} = 0$ and $\left.\frac{df_r}{dq}\right|_{CC} = 0$

A11. Variation of expected Surplus at Date 1 (S_{D1}) with q

In Equation (29), the first term is a constant while both $\bar{\beta}$ and $S_r(\theta_2^l)$ could potentially vary with q. By noting that $e^* = \frac{1}{\gamma}(R - f_r)$, we obtain $S_r(\theta_2^l) = \frac{1}{2\gamma}(R^2 - f_r^2)$. Therefore, we have:

$$\frac{dS_{D1}}{dq} = S_r(\theta_2^l)\frac{d\bar{\beta}}{dq} + \bar{\beta}\frac{dS_r(\theta_2^l)}{dq} = S_r(\theta_2^l)\frac{d\bar{\beta}}{dq} - \bar{\beta} f_r \frac{df_r}{dq} \tag{A34}$$

As both $\bar{\beta}$ and $S_r(\theta_2^l)$ are always positive, using results from Propositions (1) & (3), we obtain:

(i) FP equilibrium: $\left.\frac{dS_{D1}}{dq}\right|_{FP} = 0$; as $\left.\frac{d\bar{\beta}}{dq}\right|_{FP} = 0$ and $\left.\frac{df_r}{dq}\right|_{FP} = 0$.

(ii) PD equilibrium: $\frac{dS_{D1}}{dq}\Big|_{PD} = -\bar{\beta} f_r \frac{df_r}{dq}\Big|_{PD} < 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{PD} = 0$ and $\frac{df_r}{dq}\Big|_{PD} > 0$.

(iii) LC equilibrium: $\frac{dS_{D1}}{dq}\Big|_{LC} = S_r(\theta_2^l) \frac{d\bar{\beta}}{dq}\Big|_{LC} < 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{LC} < 0$ and $\frac{df_r}{dq}\Big|_{LC} = 0$.

(iv) CC equilibrium: $\frac{dS_{D1}}{dq}\Big|_{CC} = 0$; as $\frac{d\bar{\beta}}{dq}\Big|_{CC} = 0$ and $\frac{df_r}{dq}\Big|_{CC} = 0$.

A12. Variation of expected Surplus at Date 0 (S_{D0}) with q

We differentiate Equation (30) with respect to q, to obtain:

$$\frac{dS_{D0}}{dq} = \frac{[(1-r)\theta_2^l y_2 + r\theta_2^h y_2^h] - \hat{s}}{s_{max} - s_{min}} \frac{d\hat{s}}{dq} = \frac{(1-r)[k_1 - q(p-\rho^*)]}{s_{max} - s_{min}} \frac{d\hat{s}}{dq} \tag{A35}$$

As the first term on the RHS in above expression is positive, the sign of $\frac{dS_{D0}}{dq}$ depends only on the sign of $\frac{d\hat{s}}{dq}$. Therefore, using results from Propositions (1) & (3), we obtain

(i) FP equilibrium: $\frac{dS_{D0}}{dq}\Big|_{FP} > 0$; as $\frac{d\bar{p}}{dq}\Big|_{FP} > 0$.

(ii) PD equilibrium: $\frac{dS_{D0}}{dq}\Big|_{PD} < 0$; as $\frac{d\bar{p}}{dq}\Big|_{PD} < 0$.

(iii) LC equilibrium: $\frac{dS_{D0}}{dq}\Big|_{LC} > 0$; as $\frac{d\bar{p}}{dq}\Big|_{LC} > 0$.

(iv) CC equilibrium: $\frac{dS_{D0}}{dq}\Big|_{CC} < 0$; as $\frac{d\bar{p}}{dq}\Big|_{CC} < 0$.

A13. Proof of Proposition (4): q^{opt} is on the boundary of FP and PD equilibrium

Noting that $S_{Total} = S_{D0} + S_{D1}$, using results from Sub Sections (A11) and (A12) we easily obtain that $\frac{dS_{Total}}{dq}\Big|_{FP} > 0$, $\frac{dS_{Total}}{dq}\Big|_{PD} < 0$ and $\frac{dS_{Total}}{dq}\Big|_{CC} < 0$. In the LC equilibrium, we use $p - \rho^* = \lambda$, $\frac{d\hat{s}}{dq} = (1-r)\lambda$, $\frac{d\bar{\beta}}{dq}$ given by Equation (A30), $f_r = R/2$ and $S_r(\theta_2^l) = \frac{3R^2}{8\gamma} = \frac{3k_1}{2\lambda}$, to obtain:

$$\begin{aligned}\frac{dS_{Total}}{dq} &= \frac{(1-r)[k_1 - q(p-\rho^*)]}{s_{max} - s_{min}} \frac{d\hat{s}}{dq} + \frac{3(1-r)k_1}{2\lambda} \frac{d\bar{\beta}}{dq} \\ &= \frac{(1-r)(k_1 - q\lambda)}{s_{max} - s_{min}}(1-r)\lambda - \frac{3(1-r)k_1}{2\lambda} \frac{\Big(\pi + (1-r)\bar{\beta} + (m - r/2)\lambda\Big)\lambda}{\Delta\hat{s}}\end{aligned}$$

In the above expression, both terms are positive, and therefore, the sign of $\frac{dS_{Total}}{dq}$ depends on whether the first term exceeds the second term. The first term (i.e. $\frac{dS_{D0}}{dq}$) is maximized at the lowest possible value of $s_{max} - s_{min}$, i.e. 2λ.[35] Therefore, by substituting 2λ for $s_{max} - s_{min}$, the

[35] The lowest value of $s_{max} - s_{min} = r\theta_2^h y_2^h + (1-r)\theta_2^l y_2 - s_{min}$ is obtained at $r = 0$ and $\theta_2^l = \theta_2^{min}$ as $k_1 + \lambda$. Further, as $k_1 \geq \lambda$, we obtain, $s_{max} - s_{min} \geq 2\lambda$.

expression can be written as an inequality as follows:

$$
\begin{aligned}
\frac{dS_{Total}}{dq} &\leq \frac{(1-r)^2(k_1-q\lambda)\lambda}{2\lambda} - \frac{3(1-r)k_1\Big(\pi+(1-r)\bar{\beta}+(m-r/2)\lambda\Big)}{2\Delta\hat{s}} \\
\Rightarrow \frac{2\Delta\hat{s}}{(1-r)}\frac{dS_{Total}}{dq} &\leq (1-r)(k_1-q\lambda)\Delta\hat{s} - 3k_1\Big(\pi+(1-r)\bar{\beta}+(m-r/2)\lambda\Big) \\
\Rightarrow \frac{2\Delta\hat{s}}{(1-r)k_1}\frac{dS_{Total}}{dq} + \frac{(1-r)q\lambda\Delta\hat{s}}{k_1} &\leq (1-r)\Delta\hat{s} - 3\pi - 3(1-r)\bar{\beta} - 3(m-r/2)\lambda
\end{aligned} \tag{A36}
$$

As the second term in the LHS of Equation (A36) is positive, to show $\frac{dS_{Total}}{dq} < 0$, it suffices to show that the RHS is negative. Noting that $\Delta\hat{s} = \pi + \phi + m\lambda$, we obtain the RHS of Equation (A36) as:

$$
\begin{aligned}
RHS &= (1-r)(\pi+\phi+m\lambda) - 3\pi - 3(1-r)\bar{\beta} - 3[m/2+(1-r)q/2]\lambda \\
&= -(2+r)\pi - (1-r)\left(\frac{3[\phi^2-q\lambda(2\pi+m\lambda)]}{2(\pi+\phi+m\lambda)} - \phi\right) - \left(\frac{(1+2r)m}{2} + \frac{3(1-r)q}{2}\right)\lambda \\
\Rightarrow 2\Delta\hat{s}(RHS) &= -(4+2r)\pi(\pi+\phi+m\lambda) - (1-r)\left(3\phi^2 - 6q\lambda\pi - 3qm\lambda^2 - 2\phi\pi - 2\phi^2 - 2m\phi\lambda\right) \\
&\quad - [(1+2r)m + 3(1-r)q]\,\lambda(\pi+\phi+m\lambda) \\
&= -(4+2r)\pi^2 - [4+2r-2(1-r)]\,\pi\phi - [4m+2rm-6(1-r)q]\,\pi\lambda - (1-r)(\phi^2-2m\phi\lambda) \\
&\quad - [(1+2r)m+3(1-r)q]\,\pi\lambda - [(1+2r)m+3(1-r)q]\,\phi\lambda - [(1+2r)m]\,m\lambda^2 \\
&= -\Bigg[(4+2r)\pi^2 + [2+4r]\,\pi\phi + [2m+4rm+3r]\,\pi\lambda + (1-r)\phi(\phi-m\lambda) \\
&\quad + 3\,[rm+(1-r)q]\,\phi\lambda + [(1+2r)m]\,m\lambda^2\Bigg] \\
&< 0
\end{aligned}
$$

The final result follows as all the terms in the large square brackets are positive.[36] Consequently, we have $\frac{dS_{Total}}{dq} < 0$ in the LC equilibrium. As $\frac{dS_{Total}}{dq}$ is strictly increasing in the FP equilibrium and strictly decreasing in the PD, LC and CC equilibria, it follows that S_{Total} is maximized at the boundary between FP and PD equilibrium. For a given set of parameters, we denote the value of q at which the system transitions from FP to PD equilibrium as $\bar{\bar{q}}$. $\bar{\bar{q}}$ can be obtained by solving for q in Equation (18) after setting $\lambda_{PD} = p - \rho^* = k_1$ on the FP-PD boundary. Therefore, we obtain:

$$
\begin{aligned}
&2(\mathcal{B}+qk_1)\Big[r(\theta_2^h y_2^h - \rho^*) + \phi + (1-r)qk_1\Big] - \phi^2 - qmk_1^2 - 2q\phi k_1 = 0 \\
\Rightarrow &(1-r)k_1^2q^2 - \Big[rk_1 - 2r(\theta_2^h y_2^h - \rho^*) - 2(1-r)\mathcal{B}\Big]\,k_1 q - \Big[\phi^2 - 2\mathcal{B}\phi - 2r\mathcal{B}(\theta_2^h y_2^h - \rho^*)\Big] = 0
\end{aligned} \tag{A37}
$$

[36] Note that $\phi \geq \lambda$ even at the lowest value of ϕ evaluated at θ_2^{min}. Therefore, $\phi \geq m\lambda$ as $0 \leq m \leq 1$.

Solving the above quadratic for q^{opt}, we obtain:[37]

$$q^{opt} = \frac{\left[rk_1 - 2r(\theta_2^h y_2^h - \rho^*) - 2(1-r)\mathcal{B}\right]}{2(1-r)k_1} + \frac{\sqrt{\left[rk_1 - 2r(\theta_2^h y_2^h - \rho^*) - 2(1-r)\mathcal{B}\right]^2 + 4(1-r)\left[\phi^2 - 2\mathcal{B}\phi - 2r\mathcal{B}(\theta_2^h y_2^h - \rho^*)\right]}}{2(1-r)k_1} \tag{A38}$$

A14. Proof of Proposition (5)

We first evaluate the impact of θ_2^l on λ_{PD}. Noting that $\frac{d\hat{s}}{d\theta_2^l} = (1-r)\frac{d\bar{p}}{d\theta_2^l} = (1-r)(y_2 + \kappa + q\frac{d\lambda_{PD}}{d\theta_2^l})$ while differentiating Equation (A26) with respect to θ_2^l to obtain:[38]

$$2(\mathcal{B} + q\lambda_{PD})(1-r)(y_2 + \kappa + q\frac{d\lambda_{PD}}{d\theta_2^l}) + 2q\Delta\hat{s}\frac{d\lambda_{PD}}{d\theta_2^l} = 2(y_2+\kappa)\phi + 2qm\lambda_{PD}\frac{d\lambda_{PD}}{d\theta_2^l} + 2q\phi\frac{d\lambda_{PD}}{d\theta_2^l} + 2q(y_2+\kappa)\lambda_{PD}$$

$$\Rightarrow q\left[(\mathcal{B} + q\lambda_{PD})(1-r) + \Delta\hat{s} - m\lambda_{PD} - \phi\right]\frac{d\lambda_{PD}}{d\theta_2^l} = \left[\phi + q\lambda_{PD} - (\mathcal{B} + q\lambda_{PD})(1-r)\right](y_2+\kappa)$$

$$\Rightarrow \frac{d\lambda_{PD}}{d\theta_2^l} = \frac{\left[\phi + qr\lambda_{PD} - (1-r)B\right](y_2+\kappa)}{q\left[(1-r)(\mathcal{B} + q\lambda_{PD}) + \pi\right]} > 0 \tag{A39}$$

$$\Rightarrow \left.\frac{dp}{d\theta_2^l}\right|_{PD} = \frac{d\lambda_{PD}}{d\theta_2^l} + (y_2+\kappa) > 0 \tag{A40}$$

Denoting the FP-PD boundary in the $\theta_2^l - q$ space as $\bar{\bar{q}}(\theta_2^l)$, we write the boundary as $\lambda_{PD}(\theta_2^l, \bar{\bar{q}}(\theta_2^l)) = k_1$ and differentiate this expression with respect to θ_2^l to obtain:

$$\frac{\partial\lambda_{PD}}{\partial\theta_2^l} + \frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}(\theta_2^l)}\frac{d\bar{\bar{q}}(\theta_2^l)}{d\theta_2^l} = 0$$

$$\Rightarrow \frac{d\bar{\bar{q}}(\theta_2^l)}{d\theta_2^l} = -\frac{\frac{\partial\lambda_{PD}}{\partial\theta_2^l}}{\frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}(\theta_2^l)}} > 0 \tag{A41}$$

The result follows from noting that in the fraction in the RHS of Equation (A41), the numerator is positive while the denominator is negative as $\frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}} = \left.\frac{\partial p}{\partial q}\right|_{PD} < 0$ (see Proposition 3). Thus, the FP-PD boundary is positively sloped in the $\theta_2^l - q$ space implying that q^{opt} decreases with the severity of the economic shock.

A15. Proof of Proposition (6)

Collateral quality improves with asset payoff (y_2) as k_1 is decreasing in y_2. We first evaluate the impact of y_2 on λ_{PD}. Noting that $\frac{d\hat{s}}{dy_2} = (1-r)\frac{d\bar{p}}{dy_2} = (1-r)(\theta_2^l\iota + q\frac{d\lambda_{PD}}{dy_2})$ while differentiating

[37] The smaller root can be ignored as it is less than 0.

[38] The result in Equation (A39) obtains as the numerator of the fraction in Equation (A39) is positive in the PD region. PD region exists at a given θ_2^l for some q only if $\mathcal{B} < \bar{\beta}(q=0)$ which implies $\mathcal{B} < \phi$.

Equation (A26) with respect to y_2 to obtain:

$$2(\mathcal{B}+q\lambda_{PD})(1-r)(\theta_2^l\iota+q\frac{d\lambda_{PD}}{dy_2})+2q\Delta\hat{s}\frac{d\lambda_{PD}}{dy_2}=2\phi\theta_2^l\iota+2qm\lambda_{PD}\frac{d\lambda_{PD}}{dy_2}+2q\phi\frac{d\lambda_{PD}}{dy_2}+2q\lambda_{PD}\theta_2^l\iota$$

$$\Rightarrow q\left[(1-r)(\mathcal{B}+q\lambda_{PD})+\Delta\hat{s}-m\lambda_{PD}-\phi\right]\frac{d\lambda_{PD}}{dy_2}=[\phi+q\lambda_{PD}-(1-r)(\mathcal{B}+q\lambda_{PD})]\theta_2^l\iota$$

$$\Rightarrow \frac{d\lambda_{PD}}{dy_2}=\frac{[\phi+rq\lambda_{PD}-(1-r)\mathcal{B}]\theta_2^l\iota}{q\left[(1-r)(\mathcal{B}+q\lambda_{PD})+\pi\right]}>0 \tag{A42}$$

$$\Rightarrow \frac{\partial\lambda_{PD}}{\partial k_1}=\frac{\frac{d\lambda_{PD}}{dy_2}}{\frac{dk_1}{dy_2}}=-\frac{\theta_2^l[\phi+rq\lambda_{PD}-(1-r)\mathcal{B}]}{q\theta_1\left[(1-r)(\mathcal{B}+q\lambda_{PD})+\pi\right]}<0 \tag{A43}$$

$$\Rightarrow \left.\frac{\partial p}{\partial k_1}\right|_{PD}=\frac{d\lambda_{PD}}{dk_1}-\frac{\theta_2^l}{\theta_1\iota}<0 \tag{A44}$$

Denoting the PD-FP boundary in the k_1-q space as $\bar{\bar{q}}(k_1)$, we write the boundary as $\lambda_{PD}(k_1,\bar{\bar{q}}(k_1))=k_1$ and differentiate this expression with respect to k_1 to obtain:

$$\frac{\partial\lambda_{PD}}{\partial k_1}+\frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}(k_1)}\frac{d\bar{\bar{q}}(k_1)}{dk_1}=1$$

$$\Rightarrow \frac{d\bar{\bar{q}}(k_1)}{dk_1}=\frac{1-\frac{\partial\lambda_{PD}}{\partial k_1}}{\frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}(k_1)}}<0 \tag{A45}$$

The result follows from noting that in the fraction in the RHS of Equation (A45), the numerator is positive (see Equation (A43) while the denominator is negative (see Proposition (3)). Thus, the PD-FP boundary is negatively sloped in the k_1-q space implying that q^{opt} is decreasing in collateral quality.

A16. Proof of Proposition (7)

We first evaluate the impact of $\mathcal{B}$ on λ_{PD}. Noting that $\frac{d\hat{s}}{d\mathcal{B}}=(1-r)\frac{d\bar{p}}{d\mathcal{B}}=(1-r)q\frac{d\lambda_{PD}}{d\mathcal{B}}$ while differentiating Equation (A26) with respect to $\mathcal{B}$, we obtain:

$$2(\mathcal{B}+q\lambda_{PD})(1-r)q\frac{d\lambda_{PD}}{d\mathcal{B}}+2\Delta\hat{s}(1+q\frac{d\lambda_{PD}}{d\mathcal{B}})=2qm\lambda_{PD}\frac{d\lambda_{PD}}{d\mathcal{B}}+2q\phi\frac{d\lambda_{PD}}{d\mathcal{B}}$$

$$\Rightarrow q\left[(\mathcal{B}+q\lambda_{PD})(1-r)+\Delta\hat{s}-\phi-m\lambda_{PD}\right]\frac{d\lambda_{PD}}{d\mathcal{B}}=-\Delta\hat{s}$$

$$\Rightarrow \frac{d\lambda_{PD}}{d\mathcal{B}}=-\frac{\Delta\hat{s}}{q\left[(\mathcal{B}+q\lambda_{PD})(1-r)+\Delta\pi\right]}<0 \tag{A46}$$

$$\Rightarrow \left.\frac{dp}{d\mathcal{B}}\right|_{PD}=\frac{d\lambda_{PD}}{d\mathcal{B}}<0 \tag{A47}$$

Denoting the FP-PD boundary in the $\mathcal{B}-q$ space as $\bar{\bar{q}}(\mathcal{B})$, we write the boundary as $\lambda_{PD}(\mathcal{B},\bar{\bar{q}}(\mathcal{B}))=k_1$ and differentiate this expression with respect to $\mathcal{B}$ to obtain:

$$\frac{\partial\lambda_{PD}}{\partial\mathcal{B}}+\frac{\partial\lambda_{PD}}{\partial\bar{\bar{q}}(\mathcal{B})}\frac{d\bar{\bar{q}}(\mathcal{B})}{d\mathcal{B}}=0$$

$$\Rightarrow \frac{d\bar{\bar{q}}(\mathcal{B})}{d\mathcal{B}} = -\frac{\frac{\partial \lambda_{PD}}{\partial \mathcal{B}}}{\frac{\partial \lambda_{PD}}{\partial \bar{\bar{q}}(\mathcal{B})}} < 0 \tag{A48}$$

The result follows from noting that in the fraction in the RHS of Equation (A48), both the numerator and the denominator are negative (see Equation A46 and Proposition 3)). Thus, the FP-PD boundary is negatively sloped in the $\mathcal{B} - q$ space implying that the optimal q^{opt} is decreasing in the size of the real sector.

In the special case when $\mathcal{B} = 0$, it can be seen from Equation (A38) that $q^{opt} > 0$ when $\phi > 0$. Therefore, as $\phi \geq \lambda$ in the model parameter space given by $(\theta_2^{min}, \theta_2^{max})$, we have q^{opt} to be strictly greater than 0 when $\mathcal{B} = 0$. Further, denoting the value of θ_2^l for which $q^{opt} = 1$ when $\mathcal{B} = 0$ as $\theta_2^{l,B0}$, we obtain $\theta_2^{l,B0} = \frac{s_{min}+(1-r)k_1+\sqrt{(1-r)^2k_1^2+2rk_1(\theta_2^h y_2^h - s_{min})}}{y_2}$.[39] Therefore, as q^{opt} is increasing in θ_2^l, for any $\theta_2^l < \theta_2^{l,B0}$, $q^{opt} < 1$. In addition, as q^{opt} is decreasing in $\mathcal{B}$, $q^{opt} < 1$ for any $\mathcal{B} \geq 0$ for any $\theta_2^l < \theta_2^{l,B0}$. For $\theta_2^l \geq \theta_2^{l,B0}$, we denote the value of $\mathcal{B}$ at which $q^{opt} = 1$ as $\mathcal{B}_1$ and the value of $\mathcal{B}$ at which $q^{opt} = 0$ as $\mathcal{B}_2$. Again as q^{opt} is decreasing in $\mathcal{B}$, for a given $\theta_2^l \geq \theta_2^{l,B0}$, we conclude that:

(i) $q^{opt} = 1$ for $\mathcal{B} \leq \mathcal{B}_1$

(ii) $0 < q^{opt} < 1$ for $\mathcal{B}_1 < \mathcal{B} < \mathcal{B}_2$

(iii) $q^{opt} = 0$ for $\mathcal{B} \geq \mathcal{B}_1$

We obtain $\mathcal{B}_1 = \frac{(\theta_2^l y_2 - s_{min})^2}{2[r\theta_2^h y_2^h + (1-r)\theta_2^l y_2 - s_{min}]}$ and $\mathcal{B}_2 = \frac{(\theta_2^l y_2 - k_1 - s_{min})^2}{2[r\theta_2^h y_2^h + (1-r)(\theta_2^l y_2 - k_1) - s_{min}]}$.[40]

[39]To obtain $\theta_2^{l,B0}$, we solve Equation (A37) for θ_2^l after setting $q = 1$ and $\mathcal{B} = 0$.

[40]To obtain $\mathcal{B}_1$ and $\mathcal{B}_2$, we solve Equation (A37) for $\mathcal{B}$ for which $q^{opt} = 0$ and $q^{opt} = 1$, respectively, at a given θ_2^l.

Appendix B: Other Results

B1. Proof of Existence and Uniqueness of Equilibrium Solution

We can rewrite Equation (A14) that describes the dynamics of the supply and demand for liquidity in the financial asset and real asset market as follows:

$$\bar{\beta} + q(p - \rho^*) - q \int_{\rho^*}^{p} G(\rho) d\rho = \int_{\rho_{min}}^{\rho^*} G(\rho) d\rho \tag{B1}$$

The left hand side of Equation (B1) reflects the supply ($S(p)$) of assets liquidated by credit-constrained firms as well as the real asset loans absorbed in the primary market for mortgage loans. We denote this as the supply side requirement ($S(p)$). On the other side, the demand side ($D(p)$) is given by the right hand side of Equation (B1). It reflects the demand of surplus-liquidity traders to acquire financial assets and real asset loans. The excess demand, $ED(p) = D(p) - S(p)$, when set equal to 0, yields the financial asset market price and the quantity of real asset loans, as stated in Equation (12). Below, we present a precise expression for excess demand, which can also be inferred from Equation (12):

$$ED(p) = \frac{-q(p - \rho^*) + \int_{\rho_{min}}^{\rho^*} G(\rho) d\rho + q \int_{\rho^*}^{p} G(\rho) d\rho - \bar{\beta}}{p - \rho^*}. \tag{B2}$$

For $p = \rho^*$, $D(p)$ is infinite while $S(p)$ is finite and therefore $ED(p)$ is positive. At the other end, for $p > \theta_2 y_2$, $D(p)$ is 0 while $S(p)$ is finite and therefore $ED(p)$ is negative. Consequently, there always exists at least one solution to Equation (B2) that corresponds to a price in the range ρ^* to $\theta_2 y_2$.

Note that although there are multiple combinations of $(p, \bar{\beta})$ that satisfy the market clearing condition, once we factor in the equilibrium requirements of cross-market arbitraging activity, we can see that it is only at the specific combination of $p = \rho^* + \lambda$ and $\bar{\beta}$ evaluated at $p = \rho^* + \lambda$ that the equilibrium return to surplus-liquidity traders is at its maximum. At any other feasible $(p, \bar{\beta})$ combination, the equilibrium return would be lower; thus, the equilibrium is robust to off-equilibrium conjectures.[41]

Now, we show that $\frac{dED(p)}{dp} < 0$. First, given that the denominator of the expression for excess demand in Equation (B2) is increasing in p, it suffices to show that the numerator of the expression

[41] A lower price than $\rho^* + \lambda$ would cause the return from investing in the financial asset market to exceed that from investing in the real asset market, resulting in a market shut down in the real asset market ($\bar{\beta} = 0$). On the other hand if the price is greater than $\rho^* + \lambda$, surplus-liquidity firms would be operating in region where the marginal return from investing in the real asset market is greater than the marginal cost, thereby making this conjecture infeasible.

is decreasing in p in order to make the inference that $\frac{dED(p)}{dp} < 0$ i.e., the excess demand curve intersects the x-axis only once over the interval $(\rho^*, \theta_2 y_2)$.

After recognizing that $G(\rho) = \frac{H(s(\rho))-s_{min}}{H(\hat{s})-s_{min}}$, and that $G(\rho)$ depends on p only through $\hat{s}$, it follows that $\frac{dG(\rho)}{dp} = -\frac{h(s(\rho))-s_{min}}{(H(\hat{s})-s_{min})^2}\frac{d\hat{s}}{dp} < 0$, since $\hat{s}$ is increasing in p. Differentiating the numerator in Equation (B2), we get

$$\frac{dNUM}{dp} = -q(1-G(p)) - \frac{h(s(\rho))-s_{min}}{(H(\hat{s})-s_{min})^2}\frac{d\hat{s}}{dp}\left[\int_{\rho_{min}}^{\rho^*} G(\rho)d\rho + q\int_{\rho^*}^{p} G(\rho)d\rho\right] - \frac{d\bar{\beta}}{dp} \tag{B3}$$

Note that the first two terms in the above equation are negative, but the sign of the third term depends on the sign of $\frac{d\bar{\beta}}{dp}$. It can be shown that $\frac{d\bar{\beta}}{dp} \geq 0$. To see this, recall that, in equilibrium, $\bar{\beta}$ is given by the following relation with price, p:

- CC region: $\forall p < \rho^* + \lambda$, $\bar{\beta} = 0$,
- FP and PD region: $\forall p > \rho^* + \lambda$, $\bar{\beta} = \mathcal{B}$,
- LC region: $\forall p = \rho^* + \lambda$, $\bar{\beta} \in (0, \mathcal{B})$.

It can be seen $\bar{\beta}$ is either 0 or a specific value $\in (0, \mathcal{B})$, i.e., $\bar{\beta}$ is a step function of p. The derivative is 0 for all p not equal to $\rho^* + \lambda$ and is equal to the Dirac Delta function (which is positive) at $p = \rho^* + \lambda$. In short, $\frac{d\bar{\beta}}{dp} \geq 0$.

It follows that $\frac{dED(p)}{dp} < 0$ $\forall p \in (\rho^*, \theta_2 y_2)$. Hence the excess demand curve intersects the x-axis only once. This result establishes the existence and uniqueness proof.

B2. Rationale for Model Parameter Space Constraints

The first constraint which establishes θ_2^{min} (Equation (A22)) ensures that the financial market clears for even the lowest possible value of the Date 1 shock. As the financial market needs to clear for all values of q, we evaluate financial market clearing when the demand for liquidity is highest i.e. $q = 1$. At $q = 1$, the condition for financial market clearing can be expressed by setting $\bar{\beta} = 0$ and $RHS \geq 0$ in Equation (A14), to obtain:

$$\begin{aligned} 0 \leq & \; -(p-\rho^*) + \int_{\rho_{min}}^{\rho^*} \hat{G}(\rho)d\rho + \int_{\rho^*}^{p} \hat{G}(\rho)d\rho \\ \Rightarrow (p-\rho^*) \leq & \int_{\rho_{min}}^{p} \hat{G}(\rho)d\rho \end{aligned}$$

Noting that at $q = 1$, $\hat{G}(\rho) = (\rho - \rho_{min})/(\hat{s} - s_{min})$ for the entire range of $\rho_{min} < \rho < p$, we can restate above equation after integration as:

$$\begin{aligned}(p - \rho^*) &\leq \frac{(p - \rho_{min})^2}{2(\hat{s} - s_{min})} \\ \Rightarrow 2(p - \rho^*) &\leq (p - \rho_{min})\hat{G}(p)\end{aligned}$$

As $\hat{G}(p) \leq 1$, for the above constraint to hold, we need:

$$\begin{aligned}2(p - \rho^*) &\leq (p - \rho_{min}) \\ \Rightarrow (p - \rho^*) &\leq (\rho^* - \rho_{min})\end{aligned}$$

For this constraint to always be satisfied, we need it to hold for the least possible value of the RHS and the maximum possible value for the LHS, λ (thus ensuring that the Liquidity Crunch Equilibrium is feasible at θ_2^{min} for $q = 1$).[42] Therefore, the model parameter space constraint is obtained follows (while noting that $\rho_{min} = s_{min}$):

$$\begin{aligned}\lambda &\leq \theta_2^l y_2 - k_1 - s_{min} \\ \Rightarrow \theta_2^l &\geq \frac{k_1 + s_{min} + \lambda}{y_2} \\ \Rightarrow \theta_2^{min} &= \frac{k_1 + s_{min} + \lambda}{y_2} \qquad \text{(B4)}\end{aligned}$$

B3. Price in the Credit Crunch Equilibrium

We evaluate Equation (22) using the ex-ante definition of $\hat{G}(\rho)$ from Equation (A21) to obtain p in the CC equilibrium. Noting that $s_{min} = \rho_{min}$ and using the notations $\phi = \rho^* - s_{min}$, $\Delta\hat{s} = \hat{s} - s_{min}$ and $m = r + (1 - r)q$, we obtain:

$$\begin{aligned}p &= \rho^* + \frac{1}{q}\int_{\rho_{min}}^{\rho^*} \frac{\rho - s_{min}}{\Delta\hat{s}} d\rho + \int_{\rho^*}^{p} \frac{m(\rho - \rho^*) + \rho^* - s_{min}}{\Delta\hat{s}} d\rho \\ \Rightarrow 2\Delta\hat{s}(p - \rho^*) &= \phi^2/q + m(p - \rho^*)^2 + 2\phi(p - \rho^*) \\ \Rightarrow 0 &= m(p - \rho^*)^2 - 2(\hat{s} - \rho^*)(p - \rho^*) + \phi^2/q \qquad \text{(B5)}\end{aligned}$$

Substituting for $\hat{s} - \rho^*$ in Equation (B5) and rearranging, we obtain:

$$[r - (1 - r)q](p - \rho^*)^2 - 2r(\theta_2^h y_2^h - \rho^*)(p - \rho^*) + \phi^2/q = 0$$

[42] Note that the financial market always clears in the Credit Crunch Equilibrium for any q so long as the surplus liquidity in the economy (ϕ) is positive, which implies $\theta_2^{min} > \frac{k_1 + s_{min}}{y_2}$. However, we use the more restrictive condition on θ_2^{min} presented by Equation (B4) which ensures that the surplus liquidity in the economy (ϕ) exceeds the a minimal amount (λ). We rely on this assumption as it affords greater tractability; our numerical simulations indicate that the results of our model carryover even if we relax this assumption.

The above quadratic equation can be solved to obtain the value of p as follows:[43]

$$.p(\theta_2^l) = \rho^* + \frac{r(\theta_2^h y_2^h - \rho^*) \pm \sqrt{r^2(\theta_2^h y_2^h - \rho^*)^2 - [r-(1-r)q]\,\phi^2/q}}{r-(1-r)q} \tag{B6}$$

B4. $\left.\frac{dp}{d\theta_2^l}\right|_{LC} > 0$ *and* $\left.\frac{d\bar{p}}{d\theta_2^l}\right|_{LC} > 0.$

In the LC Equilibrium, p is given by $p = \rho^* + \lambda$, and therefore, $\left.\frac{dp}{d\theta_2^l}\right|_{LC} = y_2 + (1-\omega)\kappa > 0$. Further, as $\bar{p} = \rho^* + q\lambda$, we have $\left.\frac{d\bar{p}}{d\theta_2^l}\right|_{LC} = y_2 + (1-q\omega)\kappa > 0$.

B5. $\left.\frac{dp}{d\theta_2^l}\right|_{CC} > 0$ *and* $\left.\frac{d\bar{p}}{d\theta_2^l}\right|_{CC} > 0.$

Rearranging Equation (B5) and denoting $\lambda_{CC} = p - \rho^*$ we obtain for CC Equilibrium:

$$2q(\hat{s} - \rho^*)\lambda_{CC} = \phi^2 + mq\lambda_{CC}^2 \tag{B7}$$

As $\left.\frac{d\hat{s}}{d\theta_2^l}\right|_{CC} = (1-r)\left[y_2 + \kappa + q\frac{d\lambda_{CC}}{d\theta_2^l}\right]$, we differentiate Equation (B7) with respect to θ_2^l to obtain:

$$2q\left[(1-r)\left(y_2 + \kappa + q\frac{d\lambda_{CC}}{d\theta_2^l}\right) - (y_2+\kappa)\right]\lambda_{CC} + 2q(\hat{s}-\rho^*)\frac{d\lambda_{CC}}{d\theta_2^l} = 2\phi(y_2+\kappa) + 2mq\lambda_{CC}\frac{d\lambda_{CC}}{d\theta_2^l}$$

Rearranging, while noting that $\hat{s} - \rho^* = \pi + m\lambda_{CC}$, we obtain:

$$q\left[\pi + (1-r)q\lambda_{CC}\right]\frac{d\lambda_{CC}}{d\theta_2^l} = (\phi + rq)(y_2+\kappa)$$

$$\Rightarrow \frac{d\lambda_{CC}}{d\theta_2^l} = \frac{(\phi+rq)(y_2+\kappa)}{q[\pi + (1-r)q\lambda_{CC}]} > 0 \tag{B8}$$

We also have:

$$\left.\frac{dp}{d\theta_2^l}\right|_{CC} = \frac{d\rho^*}{d\theta_2^l} + \frac{d\lambda_{CC}}{d\theta_2^l} = y_2 + \kappa + \frac{d\lambda_{CC}}{d\theta_2^l} > \left.\frac{dp}{d\theta_2^l}\right|_{LC} > 0 \tag{B9}$$

$$\left.\frac{d\bar{p}}{d\theta_2^l}\right|_{CC} = \frac{d\rho^*}{d\theta_2^l} + q\frac{d\lambda_{CC}}{d\theta_2^l} = y_2 + \kappa + q\frac{d\lambda_{CC}}{d\theta_2^l} > \left.\frac{d\bar{p}}{d\theta_2^l}\right|_{LC} > 0 \tag{B10}$$

B6. $\frac{d\bar{\beta}}{d\theta_2^l} > 0$

Restating Equation (A25) we obtain:

$$2(\bar{\beta} + q\lambda)\Delta\hat{s} = \phi^2 + 2q\phi\lambda + mq\lambda^2 \tag{B11}$$

[43]When $r - (1-r)q > 0$, both roots of the quadratic are positive. However, it can be the seen that for the larger root, $p > \theta_2^h y_2^h$ which violates economic feasibility constraints, and therefore, this larger root can be ignored. On the other hand, when $r - (1-r)q < 0$, for the smaller root, it can be seen that $p - \rho^* < 0$ which is also in violation of the economic feasibility constraints, and therefore, this smaller root can be ignored.

As $\frac{d\hat{s}}{d\theta_2^l}\Big|_{LC} = (1-r)[y_2 + (1-q\omega)\kappa]$, differentiating Equation (B11) with respect to θ_2^l, yields:

$$2(\bar{\beta} + q\lambda)(1-r)[y_2 + (1-q\omega)\kappa] + 2\Delta\hat{s}\left[\frac{d\bar{\beta}}{d\theta_2^l} - q\omega\kappa\right] = 2\phi(y_2+\kappa) + 2q\lambda(y_2+\kappa) - 2q\phi\omega\kappa - 2mq\lambda\omega\kappa$$

Rearranging, we obtain:

$$\frac{d\bar{\beta}}{d\theta_2^l} = \frac{[\phi - (1-r)\bar{\beta} + qr\lambda](y_2+\kappa) + [\pi + (1-r)(\bar{\beta} + q\lambda)]q\omega\kappa}{\Delta\hat{s}} > 0 \tag{B12}$$

B7. Proof: $\frac{d\hat{\theta}_2^l(q)}{dq} > 0$

For a given $q \in [0,1]$, at $\theta_2^l = \theta_2^{min}$, the system could be in either CC or LC equilibria. When the system is in CC Equilibrium, price is given by Equation (B6). Due to the fire sale effect in the CC Equilibrium, this equilibrium price ($p(q, \theta_2^{min})\big|_{CC}$) is lower than the price that would have prevailed at θ_2^{min} in the absence of a fire sale effect (which is given by $p(q, \theta_2^{min})\big|_{LC} = \rho^*(\theta_2^{min}) + \lambda$). This is alternatively expressed as $p(\theta_2^{min})\big|_{CC} - \rho^*(\theta_2^{min}) \leq \lambda$. Now as θ_2^l increases, liquidity in the system increases, the fire sale effect reduces, resulting in a lower discount on price in the CC Equilibrium (mathematically seen by the result $\frac{d(p-\rho^*)}{d\theta_2^l}\Big|_{CC} > 0$). At $\theta_2^l = \hat{\theta}_2^l(q)$, $p(\hat{\theta}_2^l(q))\Big|_{CC} - \rho^*(\hat{\theta}_2^l(q)) = \lambda$, the system crosses from CC equilibrium to LC equilibrium and there is no fire sale discount. Defining $\lambda_{CC} = p|_{CC} - \rho^*$ and incorporating in the above result, we obtain:

$$\lambda_{CC}\big(q, \hat{\theta}_2^l(q)\big) = \lambda \tag{B13}$$

Based on earlier results in Proposition (3) and Section (B5), we know that $\frac{\partial\lambda_{CC}}{\partial\theta_2^l} > 0$ and $\frac{\partial\lambda_{CC}}{\partial q} \leq 0$. Differentiating Equation (B13) with respect to q yields:

$$\frac{\partial\lambda_{CC}}{\partial q} + \frac{\partial\lambda_{CC}}{\partial\hat{\theta}_2^l(q)}\frac{d\hat{\theta}_2^l(q)}{dq} = 0$$

Rearranging, one obtains

$$\frac{d\hat{\theta}_2^l(q)}{dq} = -\frac{\partial\lambda_{CC}/\partial q}{\partial\lambda_{CC}/\partial\theta_2^l} > 0 \tag{B14}$$

B8. List of Proofs

No.	Description of Proof	Appendix Reference	Main Text Reference
1	Funding Liquidity Lemma	Appendix A, SS A1	Lemma (1)
2	Effort Lemma	Appendix A, SS A2	Lemma (2)
3	Optimization Lemma	Appendix A, SS A3	Lemma (3)
4	Market Clearing Lemma	Appendix A, SS A4	Lemma (4)
5	Ex Post: Equilibrium p, f_r and $\bar{\beta}$ in FP Region	Appendix A, SS A5	Proposition (1)
6	Ex Post: Equilibrium p, f_r and $\bar{\beta}$ in FS Region	Appendix A, SS A6	Proposition (2)
7	Ex Ante: Derived Distribution of Debt	Appendix A, SS A7	Section 5, Lemma (5)
8	Ex Ante: Model Restrictions	Appendix A, SS A8	Section 5, SS 5.5
9	Ex Ante: Equilibrium $\bar{\beta}$	Appendix A, SS A9	Proposition (3)
10	Ex Ante: PD Region Dynamics	Appendix A, SS A10.1	Proposition (3)
11	Ex Ante: LC Region Dynamics	Appendix A, SS A10.2	Proposition (3)
12	Ex Ante: CC Region Dynamics	Appendix A, SS A10.3	Proposition (3)
13	Ex Ante: S_{D1} Dynamics	Appendix A, SS A11	Proposition (4)
14	Ex Ante: S_{D0} Dynamics	Appendix A, SS A12	Proposition (4)
15	Ex Ante: q^{opt}	Appendix A, SS A13	Proposition (4)
16	Ex Ante: q^{opt} is increasing in θ_2^l	Appendix A, SS A14	Proposition (5)
17	Ex Ante: q^{opt} is decreasing in k_1	Appendix A, SS A15	Proposition (6)
18	Ex Ante: q^{opt} is decreasing in $\mathcal{B}$	Appendix A, SS A16	Proposition (7)
19	Existence and Uniqueness of Solution	Appendix B, SS B1	Section 5, SS 5.5
20	Rationale for Model Constraints	Appendix B, SS B2	Section 5, SS 5.5
21	Expression for p_{CC}	Appendix B, SS B3	Proposition (3)
22	Ex Ante: $\frac{dp}{d\theta_2^l}\Big\vert_{LC} > 0$ and $\frac{d\bar{p}}{d\theta_2^l}\Big\vert_{LC} > 0$	Appendix A, SS B4	Proposition (3)
23	Ex Ante: $\frac{dp}{d\theta_2^l}\Big\vert_{CC} > 0$ and $\frac{d\bar{p}}{d\theta_2^l}\Big\vert_{CC} > 0$	Appendix A, SS B5	Proposition (3)
24	Ex Ante: $\frac{d\bar{\beta}}{d\theta_2^l} > 0$	Appendix A, SS B6	Proposition (3)
25	Ex Ante: $\frac{d\bar{\theta}_2^l(q)}{dq} > 0$	Appendix B, SS B7	Section 5, SS 5.5

Table 4: **List of Proofs in Appendices**