THE UNIVERSITY OF CHICAGO

Subjective Learning of Trading Talent

Theory and Evidence from Individual Investors in the U.S.

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Overview

- **Research Question:** How individual investors learn about their trading talent.
- **Quantitative Result:** Learning about trading talent is about 7 times more sensitive to new signals than learning about stocks.
- **Qualitative Result:** A unifying framework for several documented puzzles
  - Timing of stock switching
  - Performance contingent attrition and trading intensity

Abstract

Recent studies show evidence that investors learn about their trading abilities. This paper focuses on understanding how investors learn about their talent and proposes a unifying framework that explains many puzzling facts about individual equity investors. In my model, the investor forms subjective beliefs both about the expected return of the current stock-in-holding and about her trading talent represented by the expected return of the next replacement stock, and updates beliefs through learning with fading memory. I calibrate the memory decay parameters to individual trading records, and show that talent learning is about 7 times more sensitive to return signals than stock-in-holding learning. Consequently, the model indicates that stock switching always happens following good performance of the current stock because switching requires a sufficiently large wedge between expected returns of the replacement stock and the current stock to cover the fixed cost, which strongly predicts disposition effect in a learning perspective. This framework also accounts for the performance-contingent trading intensity and attrition, and explains why a negative shock leads to attrition when an investor has several years of experience, which is inconsistent with the decreasing-gain updating under standard Bayesian learning.

Key Feature: Two Types of Learning

Bayesian learning about two subjects with constant-gain updating (Nagel and Xu, 2019).

- **Trading talent**
  - Learning about ex-ante distribution of a draw of the subjective mean return $\widetilde{\mu}_x$ of next replacement stock x_t
  - Normal density $\pi_t(\widetilde{\mu}_x)$: mean $\widetilde{m}_{x,t}$ and variance $\widetilde{s}^2_{x,t}$
  - Perpetual and cumulative
- **Current stock y_t held at t**
  - Learning about subjective mean return $\widetilde{\mu}_y$ of the current stock y_t
  - Normal density $\pi_t(\widetilde{\mu}_y)$: mean $\widetilde{m}_{y,t}$ and variance $\widetilde{s}^2_{y,t}$
  - Temporary and specific to stock

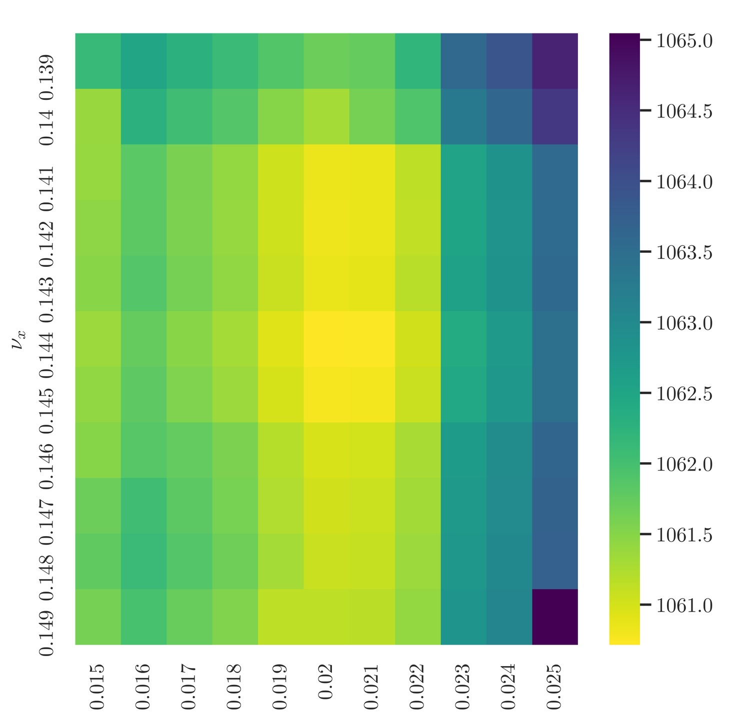

Figure 1. Estimation of Decay Parameters

- **Why two types of learning?**
  Attrition takes place learning about more than stock
- **Why only learn about mean?**
  Under-diversified portfolio + relatively long horizon (not day traders) + simplify to demonstrate the mechanism
- **Belief dynamics**
  Weighted average of prior and new signal
  $$\widetilde{m}_{x,t+1} = (1-\nu_x)\,\widetilde{m}_{x,t} + \nu_x r_{y,t+1}$$
  $$\widetilde{m}_{y,t+1} = (1-\nu)\,\widetilde{m}_{y,t} + \nu r_{y,t+1}$$
- **Memory decay parameter ν and ν_x**
  Calibrated to individual holdings data.
  $$\nu_x^\star = 0.144, \quad \nu^\star = 0.02$$

Theory: Settings

- **Discrete time portfolio choice**
  - An investor chooses from a pool of stocks.
    - Stock i log return $r_{i,t+1} = \ln R_{i,t+1}$: normal distribution
    $$r_{i,t+1} = \mu_i + \varepsilon_{i,t+1}$$
    - $\varepsilon_{i,t+1}$ $N(0, \sigma^2)$: i.i.d. (σ^2 is a known constant)
    - μ_i: unobserved constant, not random, key parameter to learn, equivalently draw of μ_i
  - Can only hold one stock at a time.
  - Two decisions each period: (1) current vs. replacement; (2) investment consumption
- **Timing:** Within each period,
  - Realized stock return $r_{y,t}$ is observed
  - Two subjective beliefs are updated
  - Decisions on consumption, investment, and stock choice (replace or not) are made
- **Preferences:** Epstein-Zin utility function, i.e., $U_t = \left[(1-\beta)\,C_t^{1-\rho} + \beta\left(\widetilde{\mathrm{E}}_t\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\rho}{1-\gamma}}\right]^{\frac{1}{1-\rho}}$

Theory: Result

- **Methodology for analytical solution:** Techniques in Campbell and Viceira (1999)
  - Log-linearization of Euler equation, budget constraint and portfolio return
  - Guess-and-verify method (Re-derived for the unit root state variable in my settings)
- **Optimal investment and consumption:**
  $$a_t^\star = A_0 + A_1 \widetilde{m}_{y,t}$$
  $$V_t^\star = \exp\left\{\frac{B_0 - \frac{1}{\rho}\ln(1-\beta)}{1-\frac{1}{\rho}} + \frac{B_1}{1-\frac{1}{\rho}}\widetilde{m}_{y,t} + \frac{B_2}{1-\frac{1}{\rho}}\widetilde{m}_{y,t}^2\right\}$$
- **Optimal switching rule:** Compare
  $$U_t(y_t) = W_t \exp\left\{\frac{B_0 - \frac{1}{\rho}\ln(1-\beta)}{1-\frac{1}{\rho}} + \frac{B_1}{1-\frac{1}{\rho}}\widetilde{m}_{y_t,t} + \frac{B_2}{1-\frac{1}{\rho}}\widetilde{m}_{y_t,t}^2\right\}$$
  $$U_t(y_{x,t}) = (1-\theta)\,W_t \exp\left\{\frac{B_0 - \frac{1}{\rho}\ln(1-\beta)}{1-\frac{1}{\rho}} + \frac{B_1}{1-\frac{1}{\rho}}\widetilde{m}_{x,t} + \frac{B_2}{1-\frac{1}{\rho}}\widetilde{m}_{x,t}^2\right\}$$
  and switch to the new stock $y_{x,t}$ iff $U_t(y_{x,t}) > U_t(y_t)$
- **Properties:** Monotonicity ($A_1 > 0$) and Convexity ($\frac{B_2}{1-\frac{1}{\rho}} > 0$)

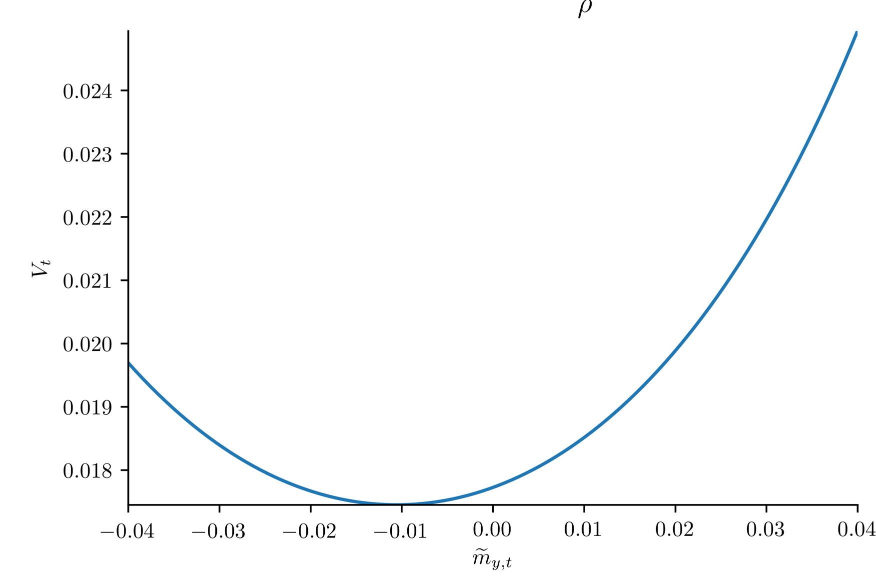

Figure 2. Value Function

Evidence: Data

- Trading records data of individual investors in the U.S. between February 1991 and November 1996
- Filter:
  - Long-only investors that start trading after February 1991
  - Quit at most once
  - Have records of at least 3 consecutive months
  - All stocks in holding matched to CRSP monthly database
- Total number of accounts: 7817

Evidence: Regressions

- **Timing of stock switching**
  - Regression: $switch_{i,t+1} = \lambda_0 + \lambda_1 \times \left(\widetilde{m}_{i,x,t} - \widetilde{m}_{i,y,t}\right) + \boldsymbol{\lambda_3 X}_{i,t} + u_i + \varepsilon_{i,t+1}$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent Variable: $switch_{i,t+1}$							
$\widetilde{m}_{i,x,t} - \widetilde{m}_{i,y,t}$	0.088***	0.107***	0.051***	0.110***	0.101***	0.080***	0.086***	0.084***
	(0.014)	(0.026)	(0.013)	(0.032)	(0.015)	(0.025)	(0.016)	(0.028)
$\widetilde{m}_{i,y,t}$			0.318***	−0.026			0.381***	−0.090
			(0.052)	(0.099)			(0.058)	(0.111)
$cmlt\ return_{i,t}$					−0.001*	0.003***	−0.003***	0.004***
					(0.001)	(0.001)	(0.001)	(0.001)
Jan_{t+1}							0.004*	0.006**
							(0.003)	(0.002)
Dec_{t+1}							−0.003	−0.003
							(0.003)	(0.002)
Account FE		Y		Y		Y		Y
Obs.	184,004	184,004	184,004	184,004	183,994	183,994	183,994	183,994

  - Takeaway: As implied by the model, stock switching is more likely to happen when the wedge between the subjective beliefs about trading talent and current stock is large enough to cover the transaction cost.
- **Attrition and performance**
  - Regression: $Y_i = \phi_0 + \phi_1 \bar{\widetilde{m}}_{i,x} + \phi_2 \bar{\widetilde{m}}_{i,y} + \phi_3 num\ switch_i + \epsilon_i$

	(1)	(2)	(3)	(4)	(5)	(6)
	Dependent Variable: $not\ quit_i$			Dependent Variable: $\overline{return_i}$		
$\bar{\widetilde{m}}_{i,x}$	2.934***	2.477***	2.717***			1.5047***
	(0.299)	(0.428)	(0.299)			(0.0340)
$\bar{\widetilde{m}}_{i,y}$		1.356			2.0317***	
		(0.955)			(0.0752)	
$num\ switch_i$			0.048***	0.0020***	−0.0016***	−0.0009***
			(0.007)	(0.0003)	(0.004)	(0.002)
Obs.	7,817	7,817	7,817	7,817	7,817	7,817

  - Takeaway:
    - Higher subjective mean of talent distribution pushes the investor further away from dropping out of the market as implied by the model.
    - More frequent stock switching hurts investment performance (Barber and Odean, 2000).

References

[1] Brad M Barber and Terrance Odean. Trading is hazardous to your wealth: The common stock investment performance of individual investors. *The Journal of Finance*, 55(2):773–806, 2000.

[2] John Y Campbell and Luis M Viceira. Consumption and portfolio decisions when expected returns are time varying. *Quarterly Journal of Economics*, 114(2):433–495, 1999.

[3] Stefan Nagel and Xu Zhengyang. Asset pricing with fading memory. *Review of Financial Studies*, 2021.

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