A Supply and Demand Approach to Capital Markets*

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Abstract

We model capital markets by a parsimonious system of simultaneous linear equations expressing firm-level supply and demand for financial capital. Investor preferences, biases, and risk assessments drive capital supply, while a firm's profitability and collateralizability drive demand. Firm sizes and capital costs are endogenously determined in general equilibrium. Using theoretically-motivated instruments, we estimate the supply and demand schedules of over 1,200 U.S. firms. We quantify the equilibrium sensitivity of firm size and capital cost to systematic risks, social score, profitability, and asset tangibility. The paper highlights the usefulness of empirical approaches at the intersection of corporate finance and asset pricing.

Keywords: Asset pricing, anomalies, capital allocation, general equilibrium, factor-based investing, production economy, sustainable investing.

JEL Classification: C36, G11, G12.

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Financial markets determine the cost and quantity of capital raised by individual firms from the population of investors. A firm's cost of capital drives its capital budgeting decisions and thereby its productive investment and size (Liu, Whited, and Zhang 2009; Lowry 2003; Pastor and Veronesi 2005, 2009). The portfolio allocation decisions of investors are contingent on the firms' expected returns and risk exposures and the investors' own wealth, preferences, and biases, such as an appetite for stocks with high environmental, social, and governance (ESG) scores (Merton 1987; Pastor, Stambaugh, and Taylor 2020). Firm characteristics and investor preferences and wealth interact to set the equilibrium cost and quantity of capital raised by each firm.

Despite enormous advances in the past decades, understanding the relation between the cost and quantity of capital at the firm level remains a challenge for empirical finance research. One explanation is that, partly for technical reasons, the literature tends to investigate separately capital costs and quantities. On the one hand, corporate finance studies often take the cost of capital as exogenous and focus on implications for the quantity of capital and productive investments of firms (Jorgensen and Siebert 1968, Lamont 1997). On the other hand, a major body of asset pricing research expresses a firm's cost of equity as the linear combination of factor exposures (Cochrane 2011, Harvey, Liu, and Zhu 2016, Hou, Xue, and Zhang 2015), while little attention is paid to the quantity of capital supplied to firms. To overcome these limitations, a growing family of general equilibrium models with production provide micro-foundations to specific risk factors and risk premia used to compute the cost of equity (Garleanu, Kogan, and Panageas 2012, Gomes, Kogan, and Zhang 2003, Kogan, Papanikolaou, and Stoffman 2018, Papanikolaou 2011, Parlour and Walden 2011). Until now, however, this general equilibrium approach has not been used empirically to estimate the cost of equity of individual firms, partly because of its technical sophistication.

Recently, a new strand of empirical studies focuses on modelling the interplay of prices and quantities in capital markets. In an influential study, Koijen and Yogo (2019) exploit the growing availability of investor-level holdings data and develop a new approach to estimate the price impact of different groups of investors in a setting where the number of shares outstanding and firm characteristics are exogenous (see also Gabaix and Koijen 2020a, 2020b, and Koijen, Richmond, and Yogo 2020).[1] As Koijen and Yogo write, a natural next step

[1] Other studies focus on index additions and deletions to estimate investors' price impact (Chang, Hong, and Liskovich 2015, Greenwood 2005, Hau, Massa, and Peress 2010, Pavlova and Sikorskaya 2021, Petajisto 2009, Shleifer 1986, Wurgler and Zhuravskaya 2002). Another line of empirical research focuses on the role of intermediaries who absorb shocks to the supply and demand for assets (Fontaine, Garcia, and Gungor 2021, Garleanu, Pedersen, and Poteshman 2009, Greenwood, Hanson, Stein, and Sunderam 2020).

is to "endogenize corporate policies such as investment and capital structure [to] answer a broad set of questions at the intersection of asset pricing and corporate finance."

Building on these advances, this paper develops a tractable and parsimonious empirical model of capital markets. We define general equilibrium by a system of simultaneous linear equations expressing firm-level supply and demand for financial capital. To address endogeneity issues, we develop an instrumental variable (IV) estimation methodology of the type commonly used in many fields of economics and finance (Theil 1953, Wooldridge 2002, Zellner and Theil 1962). We estimate the capital supply and demand schedules of individual firms, which allows us to measure the equilibrium relationship between the cost and quantity of capital at the firm level. A major benefit of our approach is that it solves the endogeneity problems commonly encountered with reduced-form models of capital markets. Our supply and demand system can also be used to address key policy questions, such as the impact of investor tilts or tax policies on the quantity and cost of capital at the firm level.

Our detailed contributions are the following. We begin by introducing a class of flexible supply and demand systems for capital markets. In our systems, investors drive the supply of financial capital and firms drive demand. We measure capital cost by the Sharpe ratio, i.e. the stock return compensation per unit of volatility, which we also call the price of risk. The corresponding quantity metric is *risk capital*, which we define as the dollar volatility of the firm's market capitalization. Importantly, the cross-section of firm risk capitals is isomorphic to the cross-section of stock correlations to the stock market portfolio.

We use the tight link between risk capitals and market correlations to construct supply and demand systems that have two equivalent formulations. The first formulation is most helpful for understanding the cross-section of stock returns, while the second formulation sheds light on the cross-section of firm sizes and capital costs. Specifically, in the first formulation, each supply or demand schedule expresses the stock's price of risk as a linear function of (i) its correlation to the market and (ii) variables driving either supply or demand, which we henceforth call *shifters*. This version of the system characterizes supply and demand in the Sharpe ratio-market correlation plane and is therefore helpful to relate our results to investor-based asset pricing. In the second formulation, the schedules are re-expressed in the Sharpe ratio-risk capital plane, which is helpful to understand the capital budget decisions of firms. Moreover, for strong economic reasons, a common feature of both formulations is that the firms' demand schedules only take into account their own capital budgeting decisions, while investors simultaneously consider all firms in their allocation decisions and supply schedules.

To further guide the empirical specification of the supply and demand system, we develop a tractable two-period general equilibrium model with production units, using classic building blocks from the production- and investor-based literatures. Each firm operates a decreasing-return-to-scale technology and produces a risky cash flow at the terminal date. A firm has no initial resources and raises capital by issuing stocks and bonds, which provide tax benefits. Firms are heterogeneous in profitability, cash flow volatility, and the ability to collateralize assets. Investors supply financial capital to firms and a riskless asset in zero net supply. Capital allocation decisions are driven by mean-variance portfolio optimization and by the hedging or behavioral properties of individual stocks. Relevant stock properties may include exposures to the investors' private equity risk, subjective beliefs, or preferences for socially responsible companies. These motives all cause investors to tilt away from the mean-variance efficient portfolio.

Our micro-founded model generates a linear system of simultaneous equations. Demand shifters include profitability per unit of risk, cash flow volatility, and the ratio of tangible to intangible assets, while supply shifters include firm characteristics associated with investor portfolio tilts, such as ESG scores and correlation with aggregate private equity. The demand schedule decreases with the firm's risk capital while the supply schedule increases with it. As a result, variation in demand shifters produces a *positive* relation between the quantity and price of risk, whereas variation in supply shifters produces a *negative* relation between price and quantity.

The model has rich implications for the cross-section of stock returns, as is evident when we express the system in the Sharpe ratio-market correlation plane. When investors are mean-variance optimizers, all firms have the *same* supply schedule, the cross-sectional relation between market correlation and Sharpe ratio is linear and positive, and the CAPM holds. By contrast, when investors deviate from the mean-variance efficient portfolio, firms can have heterogeneous supply schedules and the relation between market correlation and Sharpe ratio can be non-linear or take any sign. As our findings in the price-quantity plane imply, the risk-return relation is positive if firms are primarily heterogeneous in demand shifters, but is negative if heterogeneity in supply shifters dominates. Heterogeneity also drives deviations from the CAPM, as measured by alpha, in general equilibrium. Variation in supply shifters generates the betting-against-beta (BAB) and betting-against-correlation (BAC) anomalies reported in Frazzini and Pedersen (2014) and Asness et al. (2020), as well as the size, investment, and value effects. Variation in demand shifters can also generate these anomalies if supply schedules are flatter than the CAPM's security market line. The model therefore links supply and demand shifters to multiple pricing effects in the literature.

Our estimation methodology proceeds in several steps. We use time-series data to estimate the Sharpe ratio, market correlation, and risk capital of 1,243 U.S. firms over three non-overlapping five-year windows between 1999 and 2019. We then apply two- and three-stage least squares to the panel of risk prices and quantities.

Our choice of instruments is grounded in economic theory. To uncover supply schedules, we instrument demand variation by the firms' return on assets (ROA), which Hou, Xue, and Zhang (2020) use as a measure of profitability. We also use a firm's cash flow volatility and Peters and Taylor (2017)'s intangible intensity as additional demand instruments. To uncover demand schedules, we build on Hong and Kacperczyk (2009) and instrument supply variation by a firm's social score, which we obtain from MSCI. We also compute the correlation between a firm's return and aggregate private business equity, which we obtain from the Financial Accounts database (Z1). The selection of instruments is by no means exhaustive, and future research could integrate additional instruments building on the work of Ball, Sadka, and Tseng (2019), Koijen and Yogo (2019) and Gabaix and Koijen (2020a).

The empirical estimates are strikingly in line with the predictions of the micro-founded model. The price and quantity of risk are positively related in the supply schedule and negatively related in the demand schedule of each firm, while the coefficient on each shifter has the sign predicted by the model. The coefficients are robust to alternative specifications and have reasonably high levels of significance even though we run the estimation at the stock level. The fitted model is able to produce substantial dispersion in equilibrium values of market correlation and Sharpe ratio. This dispersion is dominated by cross-sectional variation in firm demand shifters, which explains why the relation between market correlation and Sharpe ratio is generally positive. Heterogeneity in supply is nonetheless substantial and helps to explain key properties of the cross-section of stock returns.

The slopes of supply and demand schedules vary significantly depending on the plane of analysis, as theory predicts. In the Sharpe ratio-market correlation plane, which characterizes the risk-return trade-off of investors, the supply slope ranges between 0.4 and 0.6 across specifications. That is, an increase in market correlation of 0.1 produces an increase in the Sharpe ratio of 0.005. Demand slopes are strongly negative, ranging from -80 to -160 depending on the estimation. The shape of the demand schedule implies that the market correlation of a firm is nearly insensitive to its Sharpe ratio in equilibrium, so an investor can take this market correlation as exogenous. By contrast, in the Sharpe ratio-risk capital plane, the risk capital demanded by firms is reasonably sensitive to the price of risk, while investors supply capital at a price that is nearly invariant to quantity. These contrasted results help to understand why most studies in asset pricing tend to focus on a firm's price of risk and take market correlation as given, while corporate finance studies focus on quantities and take a firm's cost of capital as given.

Using our IV estimates of the supply and demand system, we next quantify the sensitivities of firm size and capital cost with respect to supply shifters in general equilibrium. A one-standard deviation improvement in a firm's social score reduces its equilibrium Sharpe ratio by 0.06 and increases its level of risk capital by $1 billion. These changes are large and the predicted increase in risk capital amounts to 26% of the average risk capital of firms in our sample. Similarly, a one-standard deviation reduction in a firm's exposure to private business risk reduces its equilibrium Sharpe ratio by 0.04 and increases risk capital by $0.7 billion. These effects highlight that investor preferences have first-order impact on the firms' capital budgeting decisions.

We also quantify how demand shifters impact a firm's cost and quantity of capital in general equilibrium. Demand shifts do not impact a firm's Sharpe ratio nearly as much as supply shifts because the firm's decisions have little impact on its level of correlation to the stock market. However, demand shifts have a substantial impact on the firm's risk capital. A one-standard deviation improvement in profitability increases a firm's risk capital by $3 billion. Similarly, a one-standard deviation decrease in intangible intensity increases a firm's risk capital by $0.6 billion, while a one-standard deviation increase in cash flow volatility increases a firm's risk capital by $1 billion.[2]

The paper highlights the usefulness of a supply and demand approach for corporate finance and asset pricing. The methodology allows researchers to investigate how a firm's capital budgeting decisions are driven by a wide range of supply and demand shifters in equilibrium. Furthermore, several patterns documented in the cross-section of stock returns receive a natural interpretation as price-quantity relations. For instance, risk and return are *negatively* related when supply shifters exhibit stronger heterogeneity than demand shifters. This result is noteworthy because the fact that portfolios of low-market beta stocks can generate higher average returns than portfolios of high-market beta stocks has long been viewed as a core violation of the fundamental trade-off between risk and return (Baker, Bradley, and Wurgler 2011).

[2]The latter result is consistent with theory because the mean profitability to cash flow volatility is taken as fixed. See Section 2.1 for further details.

The paper contributes to the growing empirical literature documenting linkages between stock prices and the supply of financial capital from institutions (Garleanu, Pedersen, and Poteshman 2009, Klinger and Sundaresan 2019, Koijen and Yogo 2019) or households (Betermier, Calvet, and Sodini 2017, Betermier et al. 2021), and to the literature documenting linkages between stock prices and firms' demand for financial capital (Liu, Whited, and Zhang 2009, Lowry 2003, Pastor and Veronesi 2005, 2009). Our paper provides an integrated framework allowing researchers to empirically investigate how supply and demand forces jointly interact to set a firm's equilibrium cost and quantity of capital.

A possible extension would be to combine the identification strategy developed in Koijen and Yogo (2019) with the empirical methodology of the present paper. In their study, Koijen and Yogo exploit exogenous sources of heterogeneity in the asset holdings of institutional investors to estimate the price impact of different investor groups. By contrast, we estimate the slope of a firm's aggregate supply of capital by exploiting variation in firm characteristics driving its demand, and we also estimate the slope of a firm's capital demand from variation in aggregate investor supply. Since our general equilibrium model includes heterogeneous investors, future research could incorporate investor-level holding data in our empirical setup to estimate how different investor groups impact a firm's price of risk and capital budgeting decisions.

The paper is organized as follows. Section 1 introduces a flexible linear supply and demand system characterizing capital markets. Section 2 micro-founds this system and studies the general equilibrium relationship between the quantity and cost of capital and the corresponding relationship between risk and return. Section 3 develops the IV estimation methodology and presents empirical results. Section 4 documents the heterogeneity of estimated supply and demand schedules, measures the equilibrium impact of each supply or demand shifter on a firm's cost and quantity of capital, and provides robustness checks. Section 5 concludes. The Internet Appendix contains proofs and additional empirical tests.

1 Capital Markets as a Supply and Demand System

1.1 Description of the Economy

We consider an economy with two periods $t \in \{0, 1\}$ and a unique good. All quantities at dates 0 and 1 are expressed in units of this good. The production sector consists of N

firms indexed by $n \in \{1, \cdots, N\}$. The investment sector is specified by H agents indexed by $h \in \{1, \cdots, H\}$. At the beginning of period 0, the investor sector holds the aggregate endowment $X_0 > 0$ and fully owns the firms. Each investor h receives an exogenous share ϕ^h of the aggregate endowment X_0 and initially owns the share ϕ^h of each firm, which determines her share of the firms' economic profit. The shares are non-negative and add up to unity: $\sum_{h=1}^{H} \phi^h = 1$.

Firms have no initial resources. During period 0, each firm n raises E_n units of the good by issuing stocks and D_n units of the good by issuing bonds. We assume for simplicity that bonds are risk-free and yield the risk-free rate r_f. Let $V_n = E_n + D_n$ denote the capital raised by the firm. The firm allocates I_n to uninstalled productive capital and pays out the economic profit, $\phi^h(V_n - I_n)$, to each agent h at the end of period 0.

Each firm operates a specific decreasing return to scale technology. An investment of I_n units of *uninstalled* capital at $t = 0$ results in $K_n(I_n)$ units of *installed* capital in the same period, where $K_n(\cdot)$ is an increasing and concave function of I_n.

At date $t = 1$, the firm generates earnings before interest and taxes (EBIT) given by:

$$EBIT_n = (a_n + z_n)\, \sigma_{\mathrm{CF},n}\, K_n. \tag{1}$$

The random variable z_n has zero mean and unit variance. The variables $\sigma_{\mathrm{CF},n}$ and a_n represent, respectively, the volatility and mean-to-volatility ratio of the firm's EBIT per unit of installed capital. We denote by $\boldsymbol{z} = (z_1, \ldots, z_N)'$ the vector of cash flows shocks and by $\boldsymbol{\rho} = (\rho_{i,j})_{1 \leq i,j \leq N}$ its correlation matrix. The technology operated by each firm, defined by $K_n(\cdot)$ and the EBIT specification (1), is exogenous to the model.

Each firm is subjected to the corporate tax rate τ. Free cash flow is the sum of after-tax earnings, $(1 - \tau) \times EBIT_n$, and a tax shield on interest payments, $\tau\, r_f\, D_n$, or equivalently:

$$FCF_n(K_n, D_n) = (1 - \tau)(a_n + z_n)\, \sigma_{\mathrm{CF},n}\, K_n + \tau\, r_f\, D_n. \tag{2}$$

Free cash flow is used to pay the principal and interest, $(1 + r_f)\, D_n$, and the dividend, $DIV_n = FCF_n - (1 + r_f)\, D_n$, to the investor sector.

By equation (1), the variable

$$Q_n = (1 - \tau)\, \sigma_{\mathrm{CF},n}\, K_n \tag{3}$$

quantifies the volatility of the free cash flow in units of the good. We call it the *risk capital* of firm n. The variable Q_n greatly simplifies notation and plays a central role throughout the analysis. This definition is familiar in the risk management literature, which increasingly measures risk in dollar amounts rather than in proportional changes (see Artzner et al. 1999).

To avoid default, the debt of the firm cannot exceed a fraction of installed capital:

$$D_n \leq b_n\,(1-\tau)\,\sigma_{\mathrm{CF},n}\,K_n = b_n\,Q_n.$$

We assume that the normalized earnings shock z_n has a lower bound $\min z_n$ strictly larger than -a_n and that b_n is at most $(a_n + \min z_n)/[1+(1-\tau)r_f]$. These conditions guarantee the absence of default on the bond and the non-negativity of the dividend.

The financial resources of agent h in period 0 consist of her initial endowment and her share of the firms' profits: $W_0^h = \phi^h\,W_0$, where aggregate financial wealth is the sum of the aggregate endowment and firm profits:

$$W_0 = X_0 + \sum_{n=1}^{N}(V_n - I_n). \tag{4}$$

The agent does not consume in period 0 and invests E_n^h units of the good in each stock n and B^h units of the good in the riskless asset, which will allow her to consume in period 1. Her budget constraint is $\sum_{n=1}^{N} E_n^h + B^h = W_0^h$.

1.2 Capital Markets

The allocation of capital to the firms takes place through competitive financial markets. We assume that there are no trading constraints and that there exists a risk-adjusted probability measure $\mathbb{Q}$, which investors use to value financial assets. The measure $\mathbb{Q}$ is endogenously determined in equilibrium.

The total market value of firm n in period 0, $V_n = D_n + E_n$, is given by

$$V_n(Q_n, D_n) = \frac{\mathbb{E}^{\mathbb{Q}}(FCF_n)}{1+r_f} = \frac{(a_n-\lambda_n)\,Q_n + \tau\,r_f\,D_n}{1+r_f}, \tag{5}$$

where $\lambda_n = -\mathbb{E}^{\mathbb{Q}}(z_n)$. The firm chooses the capital structure that maximizes the economic profit, $V_n - I_n$. For every level of risk capital Q_n, the value of the firm increases with the debt level D_n due to the presence of the tax shield. Hence the firm sets the debt level equal to its upper limit:

$$D_n = b_n \, Q_n. \tag{6}$$

The value of the firm and the value of equity at date 0 are therefore proportional to risk capital:

$$V_n = \frac{a_n - \lambda_n + \tau \, r_f \, b_n}{1 + r_f} Q_n, \tag{7}$$

$$E_n = \frac{a_n - \lambda_n - b_n \left[1 + (1 - \tau) r_f\right]}{1 + r_f} \, Q_n. \tag{8}$$

The cash flow paid to equity investors at date 1 is

$$DIV_n = \left\{a_n + z_n - b_n \left[1 + (1 - \tau) \, r_f\right]\right\} \, Q_n. \tag{9}$$

By (8) and (9), the net return on firm n's equity, $r_n = DIV_n/E_n - 1$, satisfies

$$r_n - r_f = \frac{Q_n}{E_n}(\lambda_n + z_n). \tag{10}$$

The stock's mean return, μ_n, and volatility, σ_n, follow from equations (8) and (10). The Sharpe ratio, i.e. the stock's average compensation per unit of volatility, therefore coincides with the mean of the normalized cash-flow shock under $\mathbb{Q}$: $\lambda_n = (\mu_n - r_f)/\sigma_n$.

We stack stock returns into the column vector $\boldsymbol{r} = (r_1, \cdots, r_N)'$, expected returns into $\boldsymbol{\mu} = (\mu_1, \cdots, \mu_N)'$, and Sharpe ratios into $\boldsymbol{\lambda} = (\lambda_1, \cdots, \lambda_N)'$. We also denote by $\boldsymbol{\sigma}$ the diagonal matrix with elements $\sigma_1, ..., \sigma_n$, and by $\boldsymbol{\Sigma} = \boldsymbol{\sigma} \, \boldsymbol{\rho} \, \boldsymbol{\sigma}$ the variance-covariance matrix of returns. The vectors $\boldsymbol{r}$, $\boldsymbol{\lambda}$, $\boldsymbol{\mu}$, and $\boldsymbol{\sigma}$, and the matrix $\boldsymbol{\Sigma}$ are endogenous in equilibrium.

The market portfolio is defined in the usual manner. Let $E_{\mathrm{M}} = \sum_{n=1}^{N} E_n$ denote the value of aggregate equity. The market portfolio assigns the weight E_n/E_{M} to every stock n. It earns the return $r_{\mathrm{M}} = \sum_{n=1}^{N} (E_n/E_{\mathrm{M}}) \, r_n$, which has mean μ_{M} and volatility σ_{M}.

1.3 Price and Quantity Metrics

The following price and quantity metrics conveniently characterize the capital markets equilibrium. Our price measure is the stock's Sharpe ratio λ_n, which is often called the market price of risk in the literature. The corresponding quantity metric is the risk capital:

$$Q_n = (1-\tau)\, \sigma_{\mathrm{CF},n}\, K_n = \sigma_n\, E_n = [Var(FCF_n)]^{1/2}, \tag{11}$$

where the last equalities follow from (2) and (10). The Sharpe ratio λ_n and risk capital Q_n determine the firm's value and capital structure and stock n's mean and volatility profile, as equations (6) to (10) show. The risk capitals of the firms, $Q_1, \ldots, Q_N$, also determine the risk capital of the market portfolio, $Q_{\mathrm{M}} = \sigma_{\mathrm{M}}\, E_{\mathrm{M}} = (\boldsymbol{Q}'\,\boldsymbol{\rho}\,\boldsymbol{Q})^{1/2}$.

An important feature of the quantity metric Q_n is that it is tightly linked to the stock's correlation to the market, $\rho_{\mathrm{M},n}$. In the Appendix, we derive the following identity:

$$\rho_{\mathrm{M},n} = Corr(r_{\mathrm{M}}, r_n) = \frac{\sum_{i=1}^{N} \rho_{n,i} Q_i}{Q_{\mathrm{M}}}, \tag{12}$$

where $\rho_{n,i}$ is the exogenous correlation between the cash flows of firms i and n. A firm has high market correlation $\rho_{\mathrm{M},n}$ if it has large risk capital Q_n or if it correlates strongly with firms with large levels of risk capital. To distinguish between the two channels, we will refer to Q_n/Q_{M} as the firm's *internal correlation to the market*, and we define the firm's *external market correlation* by

$$\rho^{ext}_{\mathrm{M},n} = \frac{\sum_{i \neq n} \rho_{n,i} Q_i}{Q_{\mathrm{M}}}. \tag{13}$$

The decomposition

$$\rho_{\mathrm{M},n} = \frac{Q_n}{Q_{\mathrm{M}}} + \rho^{ext}_{\mathrm{M},n} \tag{14}$$

plays an important role in setting up the supply and demand system, as we explain in the next section.

1.4 Supply and Demand System

Building on our setup, we now turn to the task of characterizing capital markets as a linear supply and demand system. Our objective is to map out the general properties one would expect from such a system and discuss implications for corporate finance and asset pricing. We provide the detailed micro-foundations of a specific system in Section 2 and develop the empirical implementation in Section 3.

The capital budgeting decisions of a firm take into account the firm's own cost of capital and its other characteristics. For this reason, we model the firm's demand schedule as

$\lambda_{\mathrm{D},n} = \boldsymbol{\beta}_{\mathrm{D}}'\boldsymbol{x}_{\mathrm{D},n} + \Delta_{\mathrm{D}}\, Q_n/Q_{\mathrm{M}}$, where Δ_{D} and $\boldsymbol{\beta}_{\mathrm{D}}$ are linear coefficients and $\boldsymbol{x}_{\mathrm{D},n}$ is a vector of firm characteristics driving demand. We will refer to the components of $\boldsymbol{x}_{\mathrm{D},n}$ as *demand shifters.* Standard corporate finance models predict that a high price of equity should lead to a low demand for capital, so that the linear coefficient Δ_{D} should be negative. In Section 2.1, we will micro-found the demand schedule and explain how to select the demand shifters.

Portfolio theory and investor-based asset pricing provide guidance on the supply of risk capital. Investor asset allocations are sensitive to the impact that a stock investment has on the risk and return of the overall portfolio. For this reason, classic asset pricing models, such as the CAPM and its extensions, predict that a firm's price of risk is a function of its exposure to the market portfolio and other forms of risk (Lintner 1965, Merton 1973, Sharpe 1964). We therefore postulate that the supply schedule is of the form $\lambda_{\mathrm{S},n} = \boldsymbol{\beta}_{\mathrm{S}}'\boldsymbol{x}_{\mathrm{S},n} + \Delta_{\mathrm{S}}\, \rho_{\mathrm{M},n}$, where Δ_{S} and $\boldsymbol{\beta}_{\mathrm{S}}$ are linear coefficients and $\boldsymbol{x}_{\mathrm{S},n}$ is a vector of *supply shifters.* In Section 2.2, we will micro-found the supply schedule and discuss the choice of shifters.

By equation (12), the system can be expressed as a set of relationships between the prices and quantities of risk:

$$\lambda_{\mathrm{S},n} = \boldsymbol{\beta}_{\mathrm{S}}'\boldsymbol{x}_{\mathrm{S},n} + \Delta_{\mathrm{S}}\, \frac{\sum_{i=1}^{N} \rho_{n,i} Q_i}{Q_{\mathrm{M}}}, \tag{15}$$

$$\lambda_{\mathrm{D},n} = \boldsymbol{\beta}_{\mathrm{D}}'\boldsymbol{x}_{\mathrm{D},n} + \Delta_{\mathrm{D}}\, \frac{Q_n}{Q_{\mathrm{M}}}. \tag{16}$$

A firm's supply schedule expresses the Sharpe ratio as a function of supply shifters, the firm's own risk capital, and the risk capital of every other firm in the economy, consistent with the view that investor decisions depend on the characteristics of all firms. By contrast, the demand schedule involves only the firm's own risk capital and shifters. The system (15)–(16) is especially convenient to analyze how supply and demand shifters impact firm size and the cost of equity. The rich general equilibrium implications of such a system are discussed in Section 2.3.2.

We can alternatively express the supply and demand system in terms of market correlations instead of quantities. This formulation is especially useful to understand the cross-section of stocks returns and its links to supply and demand shifters. By equation (14), we obtain:

$$\lambda_{\mathrm{S},n} = \boldsymbol{\beta}_{\mathrm{S}}'\boldsymbol{x}_{\mathrm{S},n,t} + \Delta_{\mathrm{S}}\, \rho_{\mathrm{M},n}, \tag{17}$$

$$\lambda_{\mathrm{D},n} = \boldsymbol{\beta}_{\mathrm{D}}'\boldsymbol{x}_{\mathrm{D},n} + \Delta_{\mathrm{D}}\, (\rho_{\mathrm{M},n} - \rho_{\mathrm{M},n}^{ext}). \tag{18}$$

The key implications of such a system for the security market line and equilibrium pricing anomalies are discussed in Section 2.3.3. The system (17)– (18) is also the basis of the empirical estimation in Section 3.

The equivalence between the two systems helps to connect corporate finance and asset pricing. For the typical firm that represents a small fraction of the market portfolio, the slopes of supply and demand schedules for risk capital are scaled by Q_{M}, which is large in the data. Consider a change in the firm's cost of capital λ_n, triggered for instance by a supply shift specific to firm n. The change in the capital cost can have a small impact on the market correlation, $d\rho_{\mathrm{M},n} = d(\lambda_n)/\Delta_{\mathrm{D}}$, but a large proportional impact on the firm's risk capital: $d(Q_n)/Q_n = (Q_{\mathrm{M}}/Q_n)d\rho_{\mathrm{M},n}$. A firm's demand schedule will therefore appear elastic in the $\lambda_n - Q_n$ plane that matters most for the capital budgeting decisions of firms, but will appear inelastic in the $\lambda_n - \rho_{\mathrm{M},n}$ plane that matter most for asset pricing. We will investigate this point more closely in Section 3.

The supply and demand systems defined above are parsimonious and empirically tractable. These properties originate from the equivalence between $\rho_{\mathrm{M},n}$ and Q_n and the reduction in dimensionality associated with the choice of λ_n, $\rho_{\mathrm{M},n}$, and Q_n metrics. By definition, a stock's Sharpe ratio and market correlation are both normalized by the stock's volatility. In general equilibrium, volatility is generally endogenous and creates non-linearities in the stock's expected return and market beta, as we show in the Appendix. It is therefore simpler to specify a system based on the Sharpe ratio and correlation. In the next section, we explain how to derive such a system from classic assumptions on firm and investor behavior.

2 Micro-Foundations of Capital Supply and Demand

This section develops a micro-founded supply and demand system and investigates its implications for the cross-section of capital costs, risk capitals, and stock returns.

2.1 Demand

We endogenize a firm's demand schedule and capital structure by considering a production process with both tangible and intangible capital. In period 0, the firm allocates the uninstalled capital, I_n, defined in Section 1.1 to (i) uninstalled tangible capital, I_n^{tan}, and (ii) uninstalled intangible capital, I_n^{int}. By the end of period 0, these investments produce

$K_n(I_n^{tan})$ units of installed tangible capital and $K_n(I_n^{int})$ units of installed intangible capital, where $K_n(\cdot)$ is the decreasing return to scale technology:

$$K_n(I) = \left(\frac{\eta+1}{\eta} I\right)^{\frac{\eta}{\eta+1}}. \tag{19}$$

A high value of the parameter $\eta \in (0, \infty)$ corresponds to near constant returns to scale, whereas a low η implies fast decreasing returns to scale. We assume that η is homogeneous across firms.[3]

Installed capital, K_n, is a composite of tangible and intangible capital, given by the Leontief technology:

$$K_n = \min\left(\frac{K_n^{tan}}{d_{1,n}}, \frac{K_n^{int}}{d_{2,n}}\right).$$

The technical coefficients satisfy $d_{1,n}^{1+\frac{1}{\eta}} + d_{2,n}^{1+\frac{1}{\eta}} = 1$, which guarantees that total installed capital satisfies $K_n = K_n(I_n)$, consistent with the setup outlined in Section 1.1.

This production process has several important implications for the capital structure. The installed capital stock K_n represents the book value of the firm. If tangible installed capital is the only form of capital that can be collateralized, $D_n \leq K_n^{tan} = d_{1,n}\, K_n$, the level of debt can be rewritten as $D_n = b_n\, Q_n$, where

$$b_n = \frac{d_{1,n}}{\sigma_{\mathrm{CF},n}(1-\tau)}. \tag{20}$$

The debt limit is determined by exogenous quantities and increases with the ratio of tangible to intangible assets.

The firm chooses the amount of investment that maximizes the economic profit $V_n - I_n$. By (5) and (19), the optimal level of risk capital is

$$Q_n = (1-\tau)\,\sigma_{\mathrm{CF},n}\, \underbrace{\left[\frac{a_n - \lambda_n + \tau\, r_f\, b_n}{1+r_f}(1-\tau)\,\sigma_{\mathrm{CF},n}\right]^{\eta}}_{K_n} \tag{21}$$

if $\lambda_n \leq a_n + \tau\, r_f\, b_n$ and zero otherwise. The firm invests more if it is highly profitable and has a low price of risk. Furthermore, the demand for risk capital increases with the debt limit b_n and therefore the ratio of tangible to intangible assets.

[3] In the Appendix we extend the model to the case of firms with heterogeneous η.

By equations (14) and (21), the demand schedule for risk capital of firm n is:

$$\lambda_{\mathrm{D},n} = a_n + \tau\, r_f\, b_n - \frac{1+r_f}{[(1-\tau)\,\sigma_{\mathrm{CF},n}]^{1+1/\eta}}\, Q_{\mathrm{M}}^{1/\eta}\, (\rho_{\mathrm{M},n} - \rho_{\mathrm{M},n}^{ext})^{1/\eta}. \tag{22}$$

The firm is willing to pay a high Sharpe ratio if it is profitable or raises a small amount of risk capital. We linearize $\lambda_{\mathrm{D},n}$ around the cross-sectional mean of $(a_n, b_n, \sigma_{\mathrm{CF},n}, \rho_{\mathrm{M},n} - \rho_{\mathrm{M},n}^{ext})$ and obtain

$$\lambda_{\mathrm{D},n} = \beta_{\mathrm{D},0} + \beta_{\mathrm{D},1}\, a_n + \beta_{\mathrm{D},2}\, \sigma_{\mathrm{CF},n} + \beta_{\mathrm{D},3}\, b_n + \Delta_{\mathrm{D}}(\rho_{\mathrm{M},n} - \rho_{\mathrm{M},n}^{ext}), \tag{23}$$

where $\beta_{\mathrm{D},1} = 1$, $\beta_{\mathrm{D},2} > 0$, $\beta_{\mathrm{D},3} = \tau\, r_f > 0$, and $\Delta_{\mathrm{D}} < 0$. This equation micro-founds the demand schedule (18). Technological parameters, such as profitability per unit of risk, cash flow volatility, and the ratio of tangible to intangible assets drive the demand for capital and will henceforth be called demand shifters.

2.2 Supply

As in Fama and French (2007), we assume that the portfolio chosen by investors is the linear combination of a mean-variance efficient portfolio and a deviation portfolio driven by hedging needs or behavioral traits. Let $\boldsymbol{\omega^h} = (E_1^h/W_0^h, \ldots, E_N^h/W_0^h)'$ denote the $N \times 1$ the vector of shares of wealth invested in the stocks by investor h. Her financial wealth at $t = 1$ is equal to

$$W_1^h = W_0^h\, [1 + r_f + \boldsymbol{\omega^{h}}'(\boldsymbol{r} - r_f\, \mathbf{1})], \tag{24}$$

where $\mathbf{1} = (1, \ldots, 1)'$.

Assumption 1 (Portfolio of Supplied Capital) *The portfolio of capital supplied to firms by investor h is given by:*

$$\boldsymbol{\omega^h} = \frac{1}{\gamma^h}\, \boldsymbol{\omega_{max}} - \boldsymbol{\omega_{\delta}^h}, \tag{25}$$

where γ^h quantifies the investor's risk aversion and the vector $\boldsymbol{\omega_{max}} = \mathbf{\Sigma^{-1}}\,(\boldsymbol{\mu} - r_f\, \mathbf{1})$ is a portfolio with maximum Sharpe ratio.

The portfolio $\boldsymbol{\omega_{\delta}^h}$, which we call the *deviation portfolio*, contains stocks that are least desirable to the investor for risk-based or behavioral reasons that will be examined below.

The aggregate portfolio, $\boldsymbol{\omega} = \sum_{h=1}^{H} \phi^h \boldsymbol{\omega^h}$, satisfies

$$\boldsymbol{\omega} = \frac{1}{\gamma}\boldsymbol{\omega_{max}} - \boldsymbol{\omega_\delta}$$

where $\gamma = (\sum_{h=1}^{H} \phi^h/\gamma^h)^{-1}$ is the aggregate risk aversion coefficient and $\boldsymbol{\omega_\delta} = \sum_{h=1}^{H} \phi^h \boldsymbol{\omega_\delta^h}$ is the aggregate deviation portfolio.

The return on the aggregate deviation portfolio, $r_\delta = \boldsymbol{\omega_\delta'}\boldsymbol{r}$, plays a central role in the analysis. For every firm n, we consider the covariance between the stock's normalized return and the return of the deviation portfolio:

$$\delta_n = Cov(r_\delta, r_n)/\sigma_n. \tag{26}$$

The stock has a high δ_n if it is unappealing to investors or if it strongly correlates with unappealing stocks. We stack the covariances δ_n into the column vector $\boldsymbol{\delta} = (\delta_1, ..., \delta_N)'$. By (26), this vector satisfies $\boldsymbol{\delta} = \boldsymbol{\sigma^{-1}}\boldsymbol{\Sigma}\boldsymbol{\omega_\delta} = \boldsymbol{\rho}\boldsymbol{\sigma}\boldsymbol{\omega_\delta}$. The investor-level analog is $\boldsymbol{\delta^h} = \boldsymbol{\rho}\boldsymbol{\sigma}\boldsymbol{\omega_\delta^h}$, $h = 1, \ldots, H$, which satisfies $\boldsymbol{\delta} = \sum_{h=1}^{H} \phi^h \boldsymbol{\delta^h}$.

The aggregate supply of risk capital follows directly from the portfolio rule:

$$\lambda_{\text{S},n} = \beta_{\text{S},1}\,\delta_n + \Delta_\text{S}\,\rho_{\text{M},n}. \tag{27}$$

where $\beta_{\text{S},1} = \gamma$ and $\Delta_\text{S} = \gamma Q_\text{M}/W_0$ are strictly positive constants. The Sharpe ratio of stock n is high if it strongly correlates with either the market or the deviation portfolio. Equation (27) micro-founds the supply schedule (17).

In particular specifications, a firm's exposure to the deviation portfolio is an exogenous supply shifter.

Assumption 2 (Exogeneity of Supply Shifters) *The vectors* $\boldsymbol{\delta^1}, \ldots, \boldsymbol{\delta^H}$ *and therefore the aggregate exposure* $\boldsymbol{\delta} = \sum_{h=1}^{H} \phi^h \boldsymbol{\delta^h}$ *are exogenous.*

The use of the deviation portfolio and Assumption 2 are motivated by the following examples.

Exposure to Non-Financial Risk. Consider an investor exposed to background risk in the form of an exogenous shock $R_\theta^h W_0^h$ to non-financial wealth at $t = 1$, where R_θ^h is an exogenous random variable. Background risk may stem from human capital, private business holdings, housing, inflation, or a combination thereof. The investor's total wealth at $t = 1$ is the sum of financial and nonfinancial wealth: $W_{T,1}^h = W_1^h + R_\theta^h W_0^h$. We verify in the Appendix that (25) holds exactly if the investor has rational expectations and quadratic utility:

$$U^h = \mathbb{E}(W_{T,1}^h) - \gamma^h Var(W_{T,1}^h)/(2 W_0^h),$$

as in Merton (1987). The deviation portfolio $\boldsymbol{\omega_\delta^h} = \boldsymbol{\Sigma^{-1}} Cov(R_\theta^h, \boldsymbol{r})$ is the portfolio with the highest correlation with R_θ^h. In this example, investors use the stock market to hedge a fixed quantity of nonfinancial risk, R_θ^h, per unit of initial wealth. As a result, the vector $\boldsymbol{\delta} = \boldsymbol{\rho}\,\boldsymbol{\sigma}\,\boldsymbol{\omega_\delta}$ reduces to $\boldsymbol{\delta} = \sum_{h=1}^H \phi^h Cov(R_\theta^h, \boldsymbol{z})$ and is therefore exogenous to the model. A stock has a high exposure to the deviation portfolio, δ_n, if it has a strong positive correlation with non-financial wealth.

Sentiment. Portfolio tilts may also arise from subjective beliefs about expected stock returns. Consider that the subjective assessment of the price of risk is $\boldsymbol{\lambda} + \boldsymbol{\pi^h}$, where $\boldsymbol{\pi^h}$ is the fixed subjective bias of investor h. In the Appendix, we show that the investor with quadratic utility selects $\boldsymbol{\omega_\delta^h} = (\gamma^h)^{-1} \boldsymbol{\sigma^{-1}}\,\boldsymbol{\rho^{-1}}\,\boldsymbol{\pi^h}$ as in the Black-Litterman (1992) model. The vector $\boldsymbol{\delta} = \gamma^{-1} \sum_{h=1}^H \phi^h \boldsymbol{\pi^h}$ is once again exogenous to the model. A stock has a high exposure to the deviation portfolio, δ_n, if it correlates strongly with stocks about which investors are irrationally exuberant.

Preferences for ESG Stocks or Other Types of Stocks. Behavioral motives can also drive the deviation portfolio $\boldsymbol{\omega_\delta}$. These motives may arise from preferences for certain types of firms, such as environmentally or socially responsible companies, as in Hong and Kacperczyk (2009) and Pastor, Stambaugh, and Taylor (2020). We consider that Assumption 2 also holds in this case. A sin stock, or a stock that co-moves positively with sin stocks, has a high exposure to the deviation portfolio, δ_n.

ESG tilts, a stock's covariance with non-financial wealth, or measures of sentiment are therefore plausible candidate supply shifters. We remain agnostic on the nature of the shifters, so that the model can be used to study the impact of any type of aggregate portfolio tilt. We also emphasize that Assumption 2 is not an intrinsic limitation of our approach. If the deviation in risk capital is price-sensitive, a first-order Taylor expansion with respect to $\boldsymbol{\lambda}$ allows us to obtain closed-form equilibrium conditions, as we show in the Appendix.

2.3 General Equilibrium

Sections 2.1 and 2.2 define a supply and demand system that can be applied to the data. In this section, we now investigate general equilibrium and its implications for the cross-section of firm sizes, capital costs, and market risk exposures.

2.3.1 Definition, Existence, and Uniqueness

To provide a concise definition of equilibrium, it is useful to state a few results on the firm's decision problem. The risk capital Q_n and price of risk λ_n fully pin down the firm's installed capital $K_n = Q_n/(1-\tau)\sigma_{\text{CF},n}$, and uninstalled capital

$$I_n = \frac{\eta}{\eta+1} K_n^{1+\frac{1}{\eta}}. \tag{28}$$

Furthermore, the firm's economic profit satisfies:

$$V_n - I_n = \eta^{-1} I_n. \tag{29}$$

These results facilitate the analysis of general equilibrium in this economy.

A general equilibrium consists of an initial aggregate financial wealth W_0, firms' capital structure $\{(Q_n, D_n)\}_{n=1}^N$, investor holdings $\{(E_1^h, \ldots, E_N^h, B^h)\}_{h=1}^H$, risk prices $\lambda_1, \ldots, \lambda_N$, and interest rate r_f such that:

(i) the aggregate initial wealth satisfies $W_0 = X_0 + \sum_{n=1}^N (V_n - I_n)$, where V_n and I_n are specified by (7) and (28);

(ii) the market for each stock n clears: $\sum_{h=1}^H E_n^h = E_n$, where the holding of each investor h is $E_n^h = W_0\,\omega_n^h$ and the firm's equity E_n satisfies (8);

(iii) the market for the risk-free asset clears: $\sum_{h=1}^H B^h = \sum_{n=1}^N D_n$;

(iv) each investor h satisfies the budget constraint: $\sum_{n=1}^N E_n^h + B^h = \phi^h W_0$.

In general equilibrium, the aggregation of individual budget constraints implies that aggregate financial wealth is equal to the aggregate value of firms: $W_0 = \sum_{n=1}^N V_n$. Correspondingly, the exogenous endowment is the aggregate uninstalled investment of the firms:

$X_0 = \sum_{n=1}^{N} I_n$, consistent with the absence of consumption in period 0 and the zero net supply of the risk-free asset. By (29), the aggregate profit is therefore $\sum_{n=1}^{N}(V_n - I_n) = \eta^{-1} X_0$, so that aggregate financial wealth is

$$W_0 = (1+\eta^{-1})X_0. \tag{30}$$

One can compute the general equilibrium by equating the supply and demand schedule of each firm: $\lambda_{\mathrm{S},n} = \lambda_{\mathrm{D},n}$ conditional on r_f, and then obtaining r_f from the market clearing of the bond market.

The economy with frictionless financial markets has a unique general equilibrium under the sufficient conditions stated in the Appendix. In the rest of the analysis, we focus on a given equilibrium, so that r_f is known. Furthermore, the stochastic discount factor SDF and the risk-adjusted probability measure $\mathbb{Q}$ defined by

$$SDF = \frac{1}{1+r_f}\frac{d\,\mathbb{Q}}{d\,\mathbb{P}} = \frac{1-\boldsymbol{\lambda}'\boldsymbol{\rho}^{-1}\boldsymbol{z}}{1+r_f}$$

price all financial assets, where $d\,\mathbb{Q}/d\,\mathbb{P}$ denotes the Radon-Nikodym derivative of $\mathbb{Q}$ with respect to $\mathbb{P}$.

2.3.2 Cross-Section of Prices and Quantities

We next investigate the cross-section of the equilibrium price and quantity of risk capital.

Proposition 1 (Equilibrium Impact of Supply and Demand Shifters) *Consider two firms ℓ and n with equal cash flow volatilities: $\sigma_{\mathrm{CF},\ell} = \sigma_{\mathrm{CF},n}$ and equal correlations to other firms: $\rho_{\ell,i} = \rho_{n,i}$ for all $i \notin \{\ell, n\}$. If firm ℓ is more profitable than firm n ($a_\ell > a_n$) and both firms have identical exposures to the deviation portfolio ($\delta_\ell = \delta_n$), then firm ℓ has higher risk capital, a higher Sharpe ratio, and a higher market correlation than firm n :*

$$Q_\ell > Q_n, \quad \lambda_\ell > \lambda_n, \quad \textit{and} \quad \rho_{\mathrm{M},\ell} > \rho_{\mathrm{M},n}.$$

Similarly, if firm ℓ is more exposed to the deviation portfolio than firm n ($\delta_\ell > \delta_n$) and both firms have identical profitabilities ($a_\ell = a_n$), then firm ℓ has lower risk capital, a higher Sharpe ratio, and a lower market correlation than firm n :

$$Q_\ell < Q_n, \quad \lambda_\ell > \lambda_n, \quad \textit{and} \quad \rho_{\mathrm{M},\ell} < \rho_{\mathrm{M},n}.$$

All else equal, firms with a strong supply of capital are large and deliver low Sharpe ratios, while firms with a strong demand for capital are large but deliver high Sharpe ratios. Analogous results apply to the other demand shifters.

Example. To develop intuition, we construct an example in which general equilibrium has a convenient graphical representation. Consider a symmetric economy in which firms have identical cash-flow correlations: $\rho_{i,j} = \bar{\rho}$ for every $i \neq j$. [4]

Let $\bar{Q} = N^{-1} \sum_{n=1}^{N} Q_n$ denote the average risk capital across firms. By (12), a firm's correlation to the market reduces to $\rho_{\mathrm{M},n} = [(1-\bar{\rho})\, Q_n + \bar{\rho} N \bar{Q}]/Q_{\mathrm{M}}$. The supply schedule (27) is then an affine function of the firm's risk capital and exogenous supply shifter:

$$\lambda_{\mathrm{S},n} = \beta_{\mathrm{S},0} + \beta_{\mathrm{S},1}\, \delta_n + (1-\bar{\rho})\, \Delta_{\mathrm{S}}\, \frac{Q_n}{Q_{\mathrm{M}}}, \tag{31}$$

where $\beta_{\mathrm{S},0} = \Delta_{\mathrm{S}}\, \bar{\rho}\, N\, \bar{Q}/Q_{\mathrm{M}}$. The cross-sectional constant $\beta_{\mathrm{S},0}$ quantifies crowding out due to investment in other firms. It is substantial if investor wealth is small compared to aggregate risk capital or if firm cash flows are highly correlated to other firms. The demand schedule is given by (23). The intersection of the schedules (23) and (31) determines the firm's outcome. In the rest of the example, we assume for simplicity that firms only differ in their ESG scores (supply shifter) and profitability (demand shifter).

Panel A of Figure 1 illustrates the demand channel by comparing two firms that differ only in profitability. The more profitable firm demands more risk capital. Because investors require compensation to supply the additional risk capital, the more profitable firm has a higher Sharpe ratio than the less profitable firm in general equilibrium.

Panel B of Figure 1 illustrates the supply channel by comparing two firms that differ only in the ESG score. The firm with a high ESG score can raise capital more cheaply than the other firm, which leads it to choose higher levels of investment and production. Consequently, the firm with a high ESG score has a higher quantity of risk but a lower Sharpe ratio than the firm with a low ESG score.

The equilibrium cross-sectional relation between firm quantities and risk prices depends on the heterogeneity of supply and demand shifters. Among firms with identical supply curves, cross-sectional variation in demand shifters induces a *positive* cross-sectional rela-

[4] In the Appendix, we show that general equilibrium is available in closed form whether cash-flow correlations are homogeneous or heterogeneous.

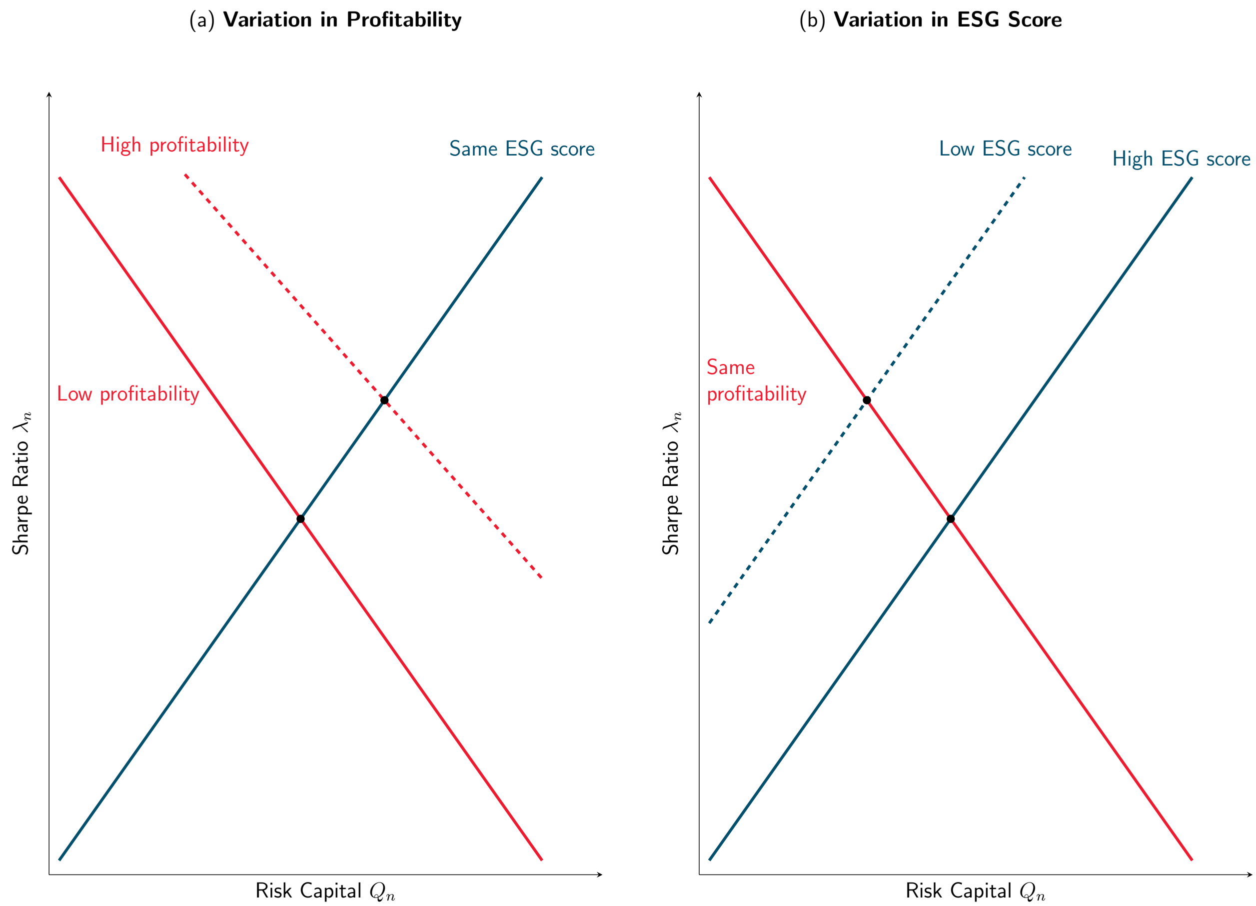

Figure 1: **Cross-Sectional Equilibrium Relation of Sharpe Ratio and Risk Capital.** This figure illustrates the supply and demand schedules of two firms that differ only in profitability, a_n (Panel A) or only in the ESG score (Panel B). The figure is based on the symmetric version of the model in which firms have equal cash-flow correlation.

tionship between quantities and prices because firm outcomes are all located on the common supply curve. Similarly, among firms with similar profitability and therefore identical demand schedules, cross-sectional variation in supply shifters induces a *negative* relationship between quantities and prices. Indeed, among similarly profitable firms, large firms are the firms with a low price of risk. We measure the heterogeneity of shifters and the resulting price-quantity relationship in Section 4.

Price-quantity relationships map directly into relationships between risk and return. Since a stock's correlation $\rho_{\mathrm{M},n}$ is an affine function of the firm's risk capital Q_n, the cross-sectional relation between market correlation and the Sharpe ratio is positive among stocks that have different profitability but similar ESG scores. Conversely, the cross-sectional relation between market correlation and the Sharpe ratio is negative among stocks that have similar profitability but heterogeneous scores, because firm outcomes are now located along the common demand curve. In the Appendix, we show that similar relationships between the risk premia and market betas of stocks arise in general equilibrium.[5] In the next section, we return to the general version of the economy (with unconstrained parameters) and we explore the asset pricing implications of our supply and demand system.

2.3.3 Cross-Section of Stock Returns

In the absence of background risk and behavioral tilts ($\delta_n = 0$ for all n), the supply schedule of each firm, given by equation (27), reduces to

$$\lambda_n = \lambda_{\mathrm{M}}\, \rho_{\mathrm{M},n}. \tag{32}$$

All firms have the same supply schedule. The CAPM holds in this special setting, which can be immediately seen by multiplying (32) by the stock's volatility. Figure 2, Panel A, illustrates the CAPM equilibrium in the $\lambda_n - \rho_{\mathrm{M},n}$ plane. The common supply schedule intersects all demand schedules and therefore produces a cross-sectional relation between market correlation and Sharpe ratio given by (32).

In the presence of investor portfolio tilts, a firm's supply schedule can be rewritten as:

$$\lambda_{\mathrm{S},n} = \rho_{\mathrm{M},n}\, \lambda_{\mathrm{M}} + \gamma(\delta_n - \rho_{\mathrm{M},n}\, \delta_M), \tag{33}$$

where $\delta_{\mathrm{M}} = Cov(r_\delta, r_{\mathrm{M}})/\sigma_{\mathrm{M}} = (\sum_{n=1}^{N} \delta_n\, \sigma_n\, E_n)/(\sigma_{\mathrm{M}}\, E_{\mathrm{M}})$ quantifies the exposure of the market portfolio to the deviation portfolio.[6] Investors require compensation for exposure to market risk and for exposure to the deviation portfolio, suggesting a two-factor pricing model. The compensation for deviation risk, $\gamma(\delta_n - \rho_{\mathrm{M},n}\, \delta_M)$, accounts for the fact that the market risk premium partly compensates for deviation risk in the market portfolio. In the Appendix, we verify that the cross-section of equity returns is represented by a two-factor model based on the market return, r_m, and the deviation portfolio return, r_δ. These properties are purely driven by the supply side of capital markets.

When the aggregate deviation portfolio is not zero, the sign of the cross-sectional relation

[5] The results are less clean due to a mitigating volatility channel, which is weak under empirically relevant values of η.

[6] The aggregation of equation (27) implies that $\sum_{n=1}^{N}(E_n/E_{\mathrm{M}})\, \sigma_n\, \lambda_{\mathrm{S},n} = \gamma\, \delta_{\mathrm{M}}\, \sigma_{\mathrm{M}} + \Delta_{\mathrm{S}}\, \sigma_{\mathrm{M}}$ and therefore equation (33) holds.

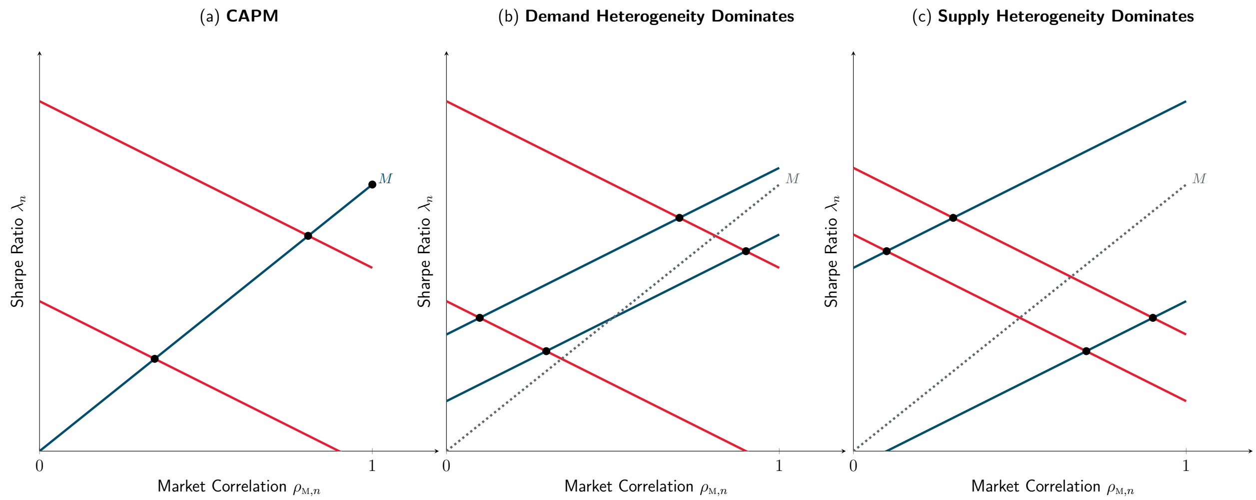

Figure 2: **Cross-Section of Stock Returns** This figure illustrates the cross-sectional equilibrium relation between firms' Sharpe ratio and correlation to the market. Panel A considers the CAPM special case where investors do not deviate from the tangency portfolio ($\delta_n = 0$). Panels C and D each consider two supply curves and two demand curves, albeit with different levels of heterogeneity. The grey dotted line corresponds to CAPM's security market line.

between risk and return depends on the dominant source of heterogeneity. Panels B and C of Figure 2 compare firms that differ primarily in profitability (Panel B) or in exposure to the deviation portfolio (Panel C). The cross-sectional relation between market correlation and Sharpe ratio is positive if firms are primarily heterogeneous in profitability (Panel B), and negative if heterogeneity in exposure to deviation risk dominates (Panel C). The exact slope of this relation also depends on the correlation between supply and demand shifters. For instance in Panel B, the cross-sectional relation between risk and return is relatively steep if firms with a high demand for capital face a low supply of capital from investors, while firms with a low demand for capital face a high supply.

An empirical implication of these results is that a univariate regression of average performance on market sensitivity can be downward sloping or reveal nonlinearities, consistent with the empirical findings of Fama and French (1992), De Giorgi, Post, and Yalçın (2019), and Hong and Sraer (2016). The negative relation between market correlation and Sharpe ratio is noteworthy because, historically, the fact that some stocks with low market risk generate higher returns than stocks with high market risk has been viewed as a core violation of the trade-off between risk and return.[7]

[7] In the Appendix, we show that, if production technologies are linear ($\eta = +\infty$) or fixed as in an endowment economy ($\eta = 0$), a downward-sloping relation is possible only if it is hard-wired into the model. By contrast, a downward-sloping relation arises naturally when firms exhibit decreasing returns to scale.

We define a stock's *normalized alpha* α_n as the Sharpe ratio earned on the stock in excess of the compensation for market risk exposure:

$$\alpha_n = \lambda_n - \rho_{\mathrm{M},n}\, \lambda_{\mathrm{M}}. \tag{34}$$

The following proposition explains how supply and demand shifters impact a stock's normalized alpha.

Proposition 2 (Supply and Demand Drivers of Alpha) *Consider two firms ℓ and n with equal cash flow volatilities: $\sigma_{\mathrm{CF},\ell} = \sigma_{\mathrm{CF},n}$ and equal correlations to other firms: $\rho_{\ell,i} = \rho_{n,i}$ for all $i \notin \{\ell, n\}$.*

If firm ℓ is less exposed to the deviation portfolio than firm n ($\delta_\ell > \delta_n$) and both firms have identical profitabilities ($a_\ell = a_n$), then firm ℓ has lower normalized alpha than firm $n : \alpha_\ell < \alpha_n$.

Moreover, if firm ℓ is more profitable than firm n ($a_\ell > a_n$) and both firms have identical exposures to the deviation portfolio ($\delta_\ell = \delta_n$), then firm ℓ has lower alpha if and only if the market has positive exposure to the deviation portfolio ($\delta_M > 0$). Otherwise, if the market has negative exposure to the deviation portfolio, firm ℓ has greater alpha.

Analogous results apply to other demand shifters.

3 illustrates the proposition. Panel A compares the normalized alpha for two firms with heterogeneous supply schedules. For each firm, the normalized alpha corresponds to the vertical difference between the firm's equilibrium Sharpe ratio and that predicted by the CAPM. Because the firm with high supply has a higher market correlation but a *lower* Sharpe ratio than the firm with low supply, it must have a lower alpha. We therefore obtain the betting against beta (BAB) and betting-against-correlation (BAC) strategies, which have been the subject of an extensive empirical literature.[8]

The impact of demand shifters on alpha depends on whether the market has positive or negative exposure to the deviation portfolio. In Panel B of Figure 3, the market has positive

[8]Portfolios holding low-market sensitivity stocks and shorting high-market sensitivity stocks tend to generate positive alphas, whether market sensitivity is measured by beta or correlation (Asness et al. 2020, Black 1993, Black, Jensen, and Scholes 1972, Hong and Sraer 2016). These observations prompted the development of betting-against-beta (BAB) and betting-against-correlation (BAC) strategies (Asness, Moskowitz, and Pedersen 2013, Frazzini and Pedersen 2014).

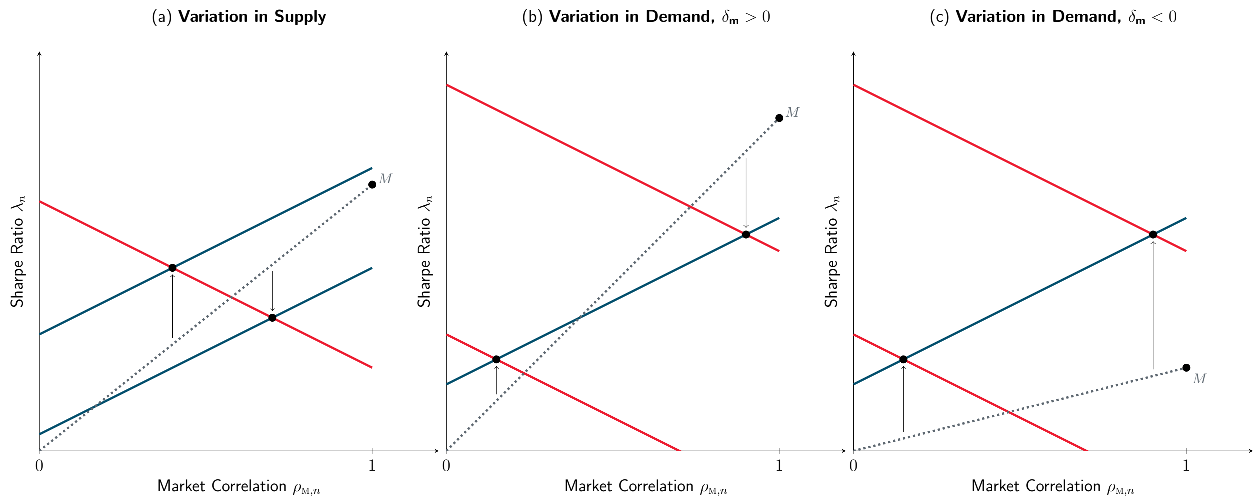

Figure 3: **Supply and Demand Drivers of CAPM-Alpha.** This figure illustrates how variation in supply and demand shifters drives CAPM alpha. Panel A considers firms that only differ in deviation portfolio exposure. Panels B and C each consider firms that only differ in profitability. In Panel B, the market has positive exposure to the deviation portfolio, while in Panel C the market has negative exposure. The grey dotted line corresponds to CAPM's security market line.

exposure to the deviation portfolio ($\delta_M > 0$). Because of this positive exposure, the market has a price of risk that exceeds the slope of the firms' supply schedule. Consequently, because supply is "too" flat relative to the CAPM's security market line, a firm with high demand has low alpha. By contrast, in Panel C the market has negative exposure to the deviation portfolio and therefore a low price of risk. A supply schedule that is relatively steep in turn implies that firms with high demand have high alpha.

Firm Characteristics and Anomalies Since supply and demand shifters jointly impact firm valuation and alpha, we can relate them to deviations from the CAPM such as value, size, and investment. By equations (5) and (21), the market-to-book ratio of a stock is:

$$\frac{V_n}{K_n} = \frac{(a_n - \lambda_n)\,\sigma_{\mathrm{CF},n}}{1 + r_f} = K_n^{\frac{1}{\eta}}. \tag{35}$$

A growth stock, that is a stock with a high market-to-book ratio, is highly profitable, has a low price of risk, has high investment K_n, and has high market value $V_n = (V_n/K_n)^{\eta+1}$. These results are consistent with Fama and French (1992) and Loughran (1997), who document that stocks with high market capitalizations have on average high market-to-book ratios.[9]

Stocks with high capitalization, high market-to-book ratio, and high investment persistently generate negative alpha (see Fama and French 1993, 2015, and Cooper, Gulen, and Schill 2008, among many others). These patterns arise in our model from both supply and demand channels. Firms with high demand or high supply tend to be large growth firms with high investment. We know from the previous section that stocks with high supply generate low alpha. Similarly, stocks with high demand generate low alpha when the supply schedules are relatively flat (Panel B of Figure 3).

These supply and demand channels also account for the linkages between anomalies. For instance, large firms with high investment and high market-to-book ratios tend to have high market correlations in general equilibrium. The model therefore predicts that the BAC and BAB strategies should be negatively related to size, value, and investment effects, consistent with the empirical results of De Giorgi, Post, and Yalçın (2019), Frazzini and Pedersen (2014), and Liu (2018).

3 Empirical Estimation

This section estimates our supply and demand system on a large panel of U.S. stocks. Section 3.1 develops the econometric specification. Section 3.2 presents the data and construction of variables. Section 3.3 provides baseline estimates of the supply and demand schedules. Section (3.4) reports additional estimates and robustness checks.

3.1 Econometric Specification of Supply and Demand Schedules

Building on equations (17) and (18), we consider the following econometric specification of supply and demand:

$$\lambda_{\mathrm{S},n,t} = \boldsymbol{\beta}'_{\mathrm{S}}\boldsymbol{x}_{\mathrm{S},n,t} + \Delta_{\mathrm{S}}\,\rho_{\mathrm{M},n,t} + u_{\mathrm{S},n,t}, \tag{36}$$

$$\lambda_{\mathrm{D},n,t} = \boldsymbol{\beta}'_{\mathrm{D}}\boldsymbol{x}_{\mathrm{D},n,t} + \Delta_{\mathrm{D}}\,(\rho_{\mathrm{M},n,t} - \rho^{ext}_{\mathrm{M},n,t}) + u_{\mathrm{D},n,t}, \tag{37}$$

where $u_{\mathrm{S},n,t}$ and $u_{\mathrm{D},n,t}$ are stochastic residuals and other variables are defined as in Section 1.

[9] Jurek and Viceira (2011) also find that growth stocks have accounted for 70% to 85% of the U.S. aggregate stock market since 1927.

We make several observations about the system of simultaneous equations (36)–(37). First, despite the static nature of the general equilibrium model developed in Section 2, the supply and demand schedules (36)–(37) can be applied to firms and investors solving multi-period decision problems, as we explain in the Appendix.

Second, the supply and demand systems commonly used in other fields of economics typically contain a number of parameters that grows with the number of markets, and estimation proceeds from repeated observations over time.[10] By contrast, the system (36)–(37) parsimoniously specifies N distinct markets with a number of parameters that does depend on N. It can therefore be estimated on a single cross-section of firms. Identification stems from imposing homogeneous linear coefficients Δ_{S}, Δ_{D}, $\boldsymbol{\beta}_{\mathbf{D}}$, and $\boldsymbol{\beta}_{\mathbf{D}}$ across firms, as well as a tight correlation-based structure.

Third, the price and market correlation metrics $\lambda_{n,t}$, $\rho_{\mathrm{M},n,t}$, and $\rho^{ext}_{\mathrm{M},n,t}$ are not directly observable and must be estimated. We do so by relying on a system of rolling windows similar to Fama and MacBeth (1973). For each cross-section at date t, we consider two estimation periods: (i) the period prior to date t, which we use to estimate market correlation $\rho_{\mathrm{M},n,t}$, internal market correlation $\rho_{\mathrm{M},n,t} - \rho^{ext}_{\mathrm{M},n,t}$, and the supply and demand shifters defined below, and (ii) the period after date t, which we use to obtain the forward-looking Sharpe ratio $\lambda_{n,t}$. Using separate estimation periods addresses the issue of possible correlation in the measurement errors of a firm's price and quantity of risk.

To address endogeneity issues, we choose characteristics that satisfy the following orthogonality conditions.

Identifying Restriction 1. The vector of supply shifters $\boldsymbol{x}_{\mathrm{S},n,t}$ is uncorrelated to the demand residual $u_{\mathrm{D},n,t}$.

Identifying Restriction 2. The vector of demand shifters $\boldsymbol{x}_{\mathrm{D},n,t}$ is uncorrelated to the supply residual $u_{\mathrm{S},n,t}$.

To satisfy Identifying Restriction 1, we must select a vector of shifters $\boldsymbol{x}_{\mathrm{S},n,t}$ that captures firm-level variation in capital supply and can therefore act as a valid instrument for the internal market correlation, $\rho_{\mathrm{M},n,t} - \rho^{ext}_{\mathrm{M},n,t}$, in the demand schedule (37). As the discussion of the general equilibrium model in Section 2.2 suggests, we can use a firm's ESG score, or alternatively its stock's correlation with private business equity, as supply shifters.

[10] See Hamilton (1994) for a textbook reference.

To satisfy Identifying Restriction 2, the vector of shifters $\boldsymbol{x}_{\mathrm{D},n,t}$ should capture firm-level variation in demand and therefore allow us to instrument the endogenous market correlation $\rho_{\mathrm{M},n,t}$ in the supply equation (36). Equation (23) from the general equilibrium model suggests that we use measures of profitability, cash flow volatility, and intangible intensity as demand shifters.

The system (36)–(37) permits the flexible choice of supply and demand instruments. The granular instrumental variables approach by Koijen and Yogo (2019) and Gabaix and Koijen (2020a) could be applied to our setting. In the present paper, however, we restrict attention to a simple set of instruments grounded in the equilibrium model of Section 2.

3.2 Data and Construction of Variables

The empirical analysis is based on U.S. stocks from the merged CRSP and Compustat database over the 1995 to 2019 period. We apply to firms a number of filters. We eliminate financial and utility stocks (SIC codes 4900-4949 and 6000-6999), as in Fama and French (1992, 1993). We calculate book equity as the sum of shareholders' equity, deferred taxes, and investment tax credit, minus preferred stock, as in Hou, Xue, and Zhang (2015). We exclude stocks with either negative book equity or less than a year of daily returns. Data on daily market factors and risk-free rates are retrieved from Ken French's data library.

A stock's Sharpe ratio, market correlation, and internal market correlation are calculated in annual units from daily excess returns over five years. The forward-looking Sharpe ratio is estimated from years t to $t+5$, whereas the market correlation metrics are estimated from years $t-5$ to t. A stock's internal market correlation is calculated as the ratio of its risk capital, Q_n, to the risk capital of the market portfolio, Q_{M}. Risk capital itself is measured as the annualized volatility of an asset's dollar daily excess return from $t-5$ to t.[11] Details of variable construction are provided in the Appendix.

Measuring Demand Variation. Profitability drives variation in the demand for capital under a large set of financial models, including the specification developed in Section 2.1. To uncover the supply schedules (36), we therefore instrument a firm's market correlation by a measure of profitability per unit of risk, $a_{n,t}$. Our measure of $a_{n,t}$ is based on the

[11] We measure the dollar excess return as the product of the stock's market capitalization (lagged by one day) and daily excess returns. Similarly, the risk capital of the market portfolio, Q_{M}, is calculated from the daily dollar excess returns of the value weighted market portfolio.

return on asset (ROA), which Hou, Xue, and Zhang (2020) use as a measure of profitability. Our measure of ROA is the ratio of the firm's EBIT to lagged book asset value.[12] We then calculate the firm's average profitability per unit of risk, $a_{n,t}$, as the quarterly ROA's historical mean-to-volatility ratio over ten years from $t-10$ to t. We require a minimum of 32 quarterly observations of ROA and winsorize the ROA mean-to-volatility ratio to minimize the impact of outliers.[13]

We consider two additional demand instruments motivated by the micro-founded model in Section 2. First, the volatility of the ROA is a natural candidate, which we henceforth call the cash flow volatility. Second, we use the intangible intensity of the firm's assets, measured as the ratio of intangible to total capital as in Peters and Taylor (2017). This calculation employs the most recent observation before year t, looking back no more than 2 years. To be consistent with (20), we scale the intangible intensity by the firm's cash flow volatility.

Measuring Supply Variation. A firm's ESG score is a likely driver of its supply of capital, as we explain in Section 2.2. For instance, sin stocks are known to have higher expected returns than otherwise comparable stocks (Hong and Kacperczyk 2009). We use the MSCI social score of firm n in year t, $Sscore_{n,t}$, which ranges between 1 and 10. Consistent with the approach developed by Pastor, Stambaugh, and Taylor (2021), we define the firm's social shifter by

$$s_{n,t} = -(10 - Sscore_{n,t}) \times Sweight_{n,t}/100, \tag{38}$$

where $Sweight_{n,t}$ denotes the weight of social issues in the ESG score of firm n in year t assigned by MSCI. The quantity $10 - Sscore_{n,t}$ measures how far the firm is from a perfect score of 10. The product $(10 - Sscore_{n,t}) \times Sweight_{n,t}$ measures the interaction of how poorly the firm scores on social issues and how large is the social impact for the industry's typical firm. The minus sign ensures that a high score is desirable for investors.

One limitation of the MSCI dataset is that it is only available after 2007. For this reason we assign a firm in year t's cross-section the social score from the firm's latest MSCI available rating between years t and $t+5$. Using the t to $t+5$ window for the calculation of the social score allows us to maximize the sample size by including three cross-sections of stocks corresponding to years $t = 2004$, $t = 2009$, and $t = 2014$. In the Appendix, we verify that our results are robust to using the two most recent cross-sections.

[12] EBIT corresponds to the sum net income, net interest expense, and net tax expense. We divide EBIT by the book value of total assets from the previous quarter.

[13] We winsorize at 5% from above and 1% from below.

We do not use other ESG metrics as supply instruments for the following reasons. A firm's governance score may have a direct impact on its profitability and demand for capital, and as a result is not a clean supply shifter. A firm's environmental score is in principle a great candidate to measure supply variation, but the period we study coincides with a major shift in global asset allocation toward green investing. As Pastor, Stambaugh, and Taylor (2021) explain, this shift has caused a large increase in the valuation of green firms, implying *lower* future expected returns for these firms. Even though this effect is consistent with the impact of a supply shift predicted by our model, it is problematic for the empirical analysis because we use realized Sharpe ratios as proxies for future Sharpe ratios. During this time period, realized Sharpe ratios were high whereas expected Sharpe ratios were low. For this reason we do not use a firm's environmental score as a supply instrument. However, given the importance of green investing today (Bolton and Kacperczyk 2021), we expect that a firm's environmental score will serve as a good supply instrument in future years.

An alternative supply instrument is the firm's correlation with non-financial wealth. Section 2.1 predicts that high correlation with unlisted business equity is not desirable to investors and therefore reduces a firm's supply of capital. We collect data on the quarterly value of aggregate private business equity held by households from the Financial Accounts database (Z1) at the Board of Governors of the Federal Reserve. For each firm in our dataset, we measure the correlation between quarterly private business equity growth and the firm's quarterly stock return from $t-5$ to t.

Summary Statistics. We obtain an unbalanced panel consisting of three cross-sections of stocks corresponding to December 31 of years 2004, 2009, and 2014. The sample contains 1,243 stocks and 2,255 stock-year observations. With the exception of the ROA metrics which require 10 years of quarterly observations, there is no overlap in the data used for each of the three cross-section.

Table 1 presents summary statistics of the sample. Over the 2005 to 2019 period, the market portfolio delivers an annualized risk premium of 9.7%, a volatility of 18.1%, and a Sharpe ratio of 0.53. During the same period, the average stock in our sample yields a risk premium of 15.5%, a volatility of 35.8%, and a Sharpe ratio of 47.5%. The average firm has a market value of \$17.6 billion, a book value of \$4.5 billion, and a risk capital of \$3.3 billion, which is approximately 14 basis points of the market risk capital of \$2.3 trillion. This ratio corresponds to the firm's internal market correlation. As the statistics indicate, the sample is biased toward large firms, which are more likely to be assigned ESG scores by

		Percentiles			Standard
	Mean	25th	Median	75th	Deviation
General firm characteristics					
Risk Premium	0.155	0.072	0.151	0.230	0.143
Volatility	0.358	0.266	0.332	0.423	0.132
Sharpe Ratio	0.475	0.212	0.463	0.737	0.385
Market Correlation	0.511	0.431	0.523	0.609	0.141
Risk Capital (Billion)	3.314	0.289	0.721	2.162	10.022
Market Equity (Billion)	17.623	1.187	3.414	11.149	58.544
Book Equity (Billion)	4.531	0.340	0.930	2.540	15.958
Supply shifters					
Social Score	−2.399	−3.168	−2.295	−1.500	1.143
Business Equity Correlation	0.028	−0.092	0.028	0.151	0.178
Demand shifters					
Profitability	3.096	1.004	2.570	4.722	2.885
Cash Flow Volatility	0.058	0.025	0.042	0.073	0.049
Intangible Intensity	14.785	5.822	11.260	20.738	11.645
Market portfolio					
Risk Premium $\mu_{\mathrm{M}} - r_f$	0.097				
Volatility σ_{M}	0.181				
Sharpe Ratio λ_{M}	0.535				
Risk Capital Q_{M} (Billion)	2,372.926				

Table 1: **Summary Statistics.** This table reports summary statistics on the panel of U.S. stocks. We construct the panel by merging the CRSP-Compustat database and by applying the set of filters described in Section 3.2 of the main text. The panel consists of three cross-sections corresponding to December 31 of years 2004, 2009, and 2014 and contains 1,243 stocks and 2,255 stock-year observations. We compute firm statistics on December 31 of year t as follows. Market equity and book equity are assessed on the day. Risk capital is the annualized volatility of daily dollar excess returns between $t-5$ and t. Risk premia, volatility, and Sharpe ratio are estimated using daily stock returns between t and $t+5$. Market correlation is estimated using daily returns from $t-5$ to t. We calculate the average profitability per unit of risk from profitability data between $t-10$ and t. Profitability is defined as the firm's return of asset (ROA), which is measured as the ratio of the firm's quarterly income to its lagged book asset value. Cash flow volatility is the volatility of ROA. Intangible intensity is the ratio of intangible to total capital at t. Social score is obtained from MSCI at $t+5$. Business equity correlation is the correlation between the stock's quarterly return and the growth of aggregate business equity held by households from $t-5$ to t. Reported statistics are equal-weighted averages of firm-year variables. Daily data on the market factor are retrieved from Ken French's data library. Estimates of the market premium, volatility, and Sharpe ratio are computed over the same periods as individual stocks.

MSCI. In Section 4, we show that we obtain similar results when we re-estimate the supply and demand system on a larger sample that includes smaller firms.

The table also reports summary statistics of the supply and demand instruments. We observe substantial levels of cross-sectional variation in these variables, which is important for the identification of supply and demand schedules. These statistics play a useful role in Section 4.2 when we study how supply and demand shifts impact a firm's cost and quantity of capital.

3.3 Estimates of Supply and Demand Schedules

Ordinary Least Squares. Columns 1 and 2 of Table 2 report pooled OLS estimates of the supply and demand schedules. In column 1, we estimate the supply equation (36) by regressing a stock's Sharpe ratio on its market correlation, $\rho_{\mathrm{M},nt}$, and social score, $s_{n,t}$. Market correlation has a positive and highly significant coefficient. This result indicates that there is a positive cross-sectional relationship between a stock's market correlation and its Sharpe ratio, consistent with the extant literature documenting a positive link between risk and return. The OLS estimate of the supply slope, Δ_{S}, is 0.37. Under the CAPM, this coefficient would coincide with the Sharpe ratio of the market portfolio, which is 0.53 over the period (Table 1). The OLS regression shows that the cross-sectional relation between a stock's compensation and its market risk exposure is weaker than the CAPM implies, which confirms the findings an extensive literature (see, e.g., Asness, Moskowitz, and Pedersen 2013, and Fama and MacBeth 1973).

In column 2 of Table 2, we report OLS estimates of the demand schedule (36). We regress a stock's Sharpe ratio on internal market correlation and profitability and obtain a coefficient on internal market correlation that is negative, although small and weakly significant. The OLS results are of course driven by a combination of supply and demand effects.

Two-Stage Least Squares. In columns 3 to 6 of Table 2, we report the two-stage least squares (2SLS) estimates of the supply and demand schedules. Market correlation is instrumented by profitability, $a_{n,t}$, in the supply equation, and internal market correlation is instrumented by the social score, $s_{n,t}$, in the demand equation.

Supply Schedule. In the first stage of 2SLS supply estimation, we regress a firm's market

	OLS		2SLS			
	Supply	Demand	Supply		Demand	
	Sharpe Ratio	Sharpe Ratio	Market Corr.	Sharpe Ratio	Risk Capital	Sharpe Ratio
	(1)	(2)	(3)	(4)	(5)	(6)
Supply						
Market Correlation	0.371***					
	(0.057)					
Fitted Market Correlation				0.474***		
				(0.181)		
Social Score	−0.050***		0.015***	−0.051***	0.000***	
	(0.007)		(0.002)	(0.008)	(0.000)	
Demand						
Internal Market Correlation		−3.317*				
		(1.763)				
Fitted Internal Market Correlation						−151.791***
						(24.114)
Profitability		0.007**	0.015***		0.000***	0.043***
		(0.003)	(0.001)		(0.000)	(0.006)
Year FE	no	no	no	no	no	no
Observations	2255	2255	2255	2255	2255	2255
F Statistic	40.332***	4.471**	150.196***	22.555***	31.492***	22.555***

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

Table 2: **OLS and 2SLS Estimates of Supply and Demand Schedules** This table reports estimates of supply and demand schedules obtained on an unbalanced panel of 1,243 U.S. firms observed on December 31 of years 2004, 2009, and 2014. Columns 1 and 2 report OLS estimates, columns 3 and 4 report 2SLS estimates of the supply schedule, and columns 5 and 6 report 2SLS estimates of the demand schedule. Statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels.

correlation, $\rho_{\mathrm{M},n,t}$ on its social score, $s_{n,t}$, profitability per unit of risk, $a_{n,t}$, and an intercept. The results, reported in column 3 of Table 2, show that the market correlation loads positively on social score and profitability, consistent with our estimation methodology. The F statistic is 150, which is substantially higher than the threshold of 10 advocated by Stock and Yogo (2005). The corresponding p value is substantially lower than 0.01. Hence profitability is a strong instrument.

Column 4 of Table 2 reports estimates of the second stage. We regress the firm's Sharpe ratio on the fitted market correlation from the first stage, the social score, and an intercept. The coefficient on $\widehat{\rho}_{\mathrm{M},n,t}$ is our estimate of the supply slope Δ_{S}. It is significantly positive and equal to 0.47. The second stage also reveals that social score is significantly and negatively tied to the stock's Sharpe ratio, consistent with an outward shift in the supply schedule. Hence social score is a plausible proxy for supply variation, which motivates its use as an instrument for estimating the demand schedule.

Demand Schedule. In the first stage of 2SLS demand estimation, we regress a firm's internal market correlation, $\rho_{\mathrm{M},n,t} - \rho^{ext}_{\mathrm{M},n,t}$, on profitability, social score, and an intercept. Column 5 of Table 2 shows that the coefficients on social score and profitability are both positive and highly significant, as theory predicts. The F statistic is high at 31 and has a p value substantially lower than 0.01. Social score is therefore a strong instrument.

Column 6 of Table 2 reports the second stage. We regress a firm's Sharpe ratio on its fitted market correlation from the first stage, profitability, and an intercept. The 2SLS estimate of the demand slope Δ_{D} is equal to -152. The demand slope is strongly significant, which showcases the strength of the instrumental variable approach. Furthermore, the demand schedule is much steeper than the supply schedule.

The second stage also reveals that profitability has a significantly positive effect on the stock's Sharpe ratio, which corresponds to an outward shift in the demand schedule. We note that the magnitude of the profitability coefficient (0.04) is smaller than that the unit value predicted by our static model. In the Appendix, we show that the prediction of unit slope is a byproduct of the static setting. In a multi-period setting, discount rate shocks attenuate the effect of profitability on the Sharpe ratio, consistent with the empirical evidence. An attenuation bias from noisy profitability estimates or underestimated cash flow volatilities may also drive the empirical estimate reported in Table 2.

The system of simultaneous equations (36)–(37) provides an empirically tractable characterization of capital supply and demand at the firm level. We next examine the sensitivity of our results to alternative estimation techniques and sets of instruments.

3.4 Robustness Checks and Extensions

Controlling for the Correlation of Residuals. In Table 3, we estimate the supply and demand system (36)–(37) by three-stage least squares ("3SLS," Zellner and Theil 1962). Unlike 2SLS, the 3SLS procedure takes into account the correlation of the equation residuals $u_{\mathrm{S},n,t}$ and $u_{\mathrm{D},n,t}$. In column 1, we use the profitability and social score instruments as in Table 2. The point estimate of the supply slope Δ_{S} is 0.47, with a high level of significance, and the demand slope Δ_{D} is negative at -152 and is very strongly significant. Thus our results are robust to controlling for the correlation of the residuals.

Alternative Instruments. One may be concerned about the reliability of social score and profitability as supply and demand instruments. If a firm's social score has a direct impact on its profitability and its demand for capital, then the social score may not satisfy Identifying Restriction 1 and may therefore not be a valid supply shifter. Similarly, if investors have a behavioral preference for the stocks of profitable companies, then profitability may drive supply and may therefore not be a valid demand shifter.

We address these threats to identification by verifying that our estimated supply and demand slopes are robust to alternative sets of instruments. In column 2 of Table 3, we use a firm's correlation with private business equity (instead of the social score) as the supply shifter. Consistent with theory, a high correlation with private business equity is significantly and positively linked to the stock's Sharpe ratio, which suggests that investors require higher compensation for holding stocks that co-move positively with private equity (an inward supply shift). Under this alternative instrument, the supply and demand slope estimates, which are respectively equal to 0.4 and -165, are remarkably similar to the baseline coefficients in column 1 and are also strongly significant.

In column 3 of Table 3, we include a firm's intangible intensity as an additional demand instrument. A high intangible intensity is linked with a lower Sharpe ratio, which is consistent with an inward shift in the demand schedule as our model predicts. Once again, the point coefficients of supply and demand slopes are similar to the estimates in column 1.

In column 4 of Table 3, we use cash flow volatility, $\sigma_{\mathrm{CF},n}$ as an instrument and we also

	Dependent Variable: Sharpe Ratio			
	(1)	(2)	(3)	(4)
Supply				
Market Correlation	0.474***	0.396**	0.598***	0.394**
	(0.180)	(0.182)	(0.180)	(0.157)
Business Equity Correlation		0.224***		
		(0.047)		
Social Score	−0.051***		−0.053***	−0.050***
	(0.008)		(0.008)	(0.007)
Demand				
Internal Market Correlation	−151.791***	−165.171**	−154.087***	−136.455***
	(49.516)	(66.611)	(49.216)	(40.145)
Profitability	0.043***	0.046***	0.058***	0.050***
	(0.013)	(0.017)	(0.018)	(0.015)
Cash Flow Volatility				1.122**
				(0.517)
Intangible Intensity			−0.005*	
			(0.002)	
Year FE	no	no	no	no
Observations	2255	2250	2223	2255
F Statistic	593.351***	584.938***	468.377***	504.78***

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

Table 3: **3SLS Estimates of Supply and Demand Schedules with Alternative Instruments** This table reports estimates of supply and demand schedules obtained by three-stage least squares. We conduct the estimation on an unbalanced panel of 1,243 U.S. firms observed on December 31 of years 2004, 2009, and 2014. We report estimates using alternative sets of supply and demand instruments. Statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels.

control for the ROA mean-to-volatility ratio, a_n. A high cash flow volatility is associated with a high Sharpe ratio, in line with the model's predictions. The point coefficients of supply and demand slopes again remain similar.

Full Set of Supply and Demand Shifters. We have hitherto shown that when we consider small sets of supply and demand shifters, the measured impact of each shifter has the sign predicted by the general equilibrium model in Section 2. While informative, these exploratory steps do not represent a full-fledged test because our model also predicts the impact of each shifter when *all* shifters are used simultaneously. In particular, the micro-founded supply schedule in equation (23) implies that profitability, cash flow volatility, and intangible intensity should all play a role. Similarly, the supply schedule (27) allows for multiple drivers of the aggregate deviation portfolio.

In column 1 of Table 4, we combine all the instruments from Table 3 into a single estimation. We find that the effect of each instrument is unchanged and remains statistically significant. The evidence thus confirms that each instrument captures a distinct supply and demand effect. The supply and demand slopes are also unchanged. The results of 3SLS with all instruments therefore provide a remarkable confirmation of the empirical validity of our supply and demand approach to capital markets.

4 Empirical Applications and Robustness Tests

This section explores equilibrium implications of our estimated supply and demand system and reports additional robustness checks. Section 4.1 plots the cross-section of firm Sharpe ratios and market correlations and documents the heterogeneity of estimated supply and demand schedules. Section 4.2 measures the equilibrium impact of each supply or demand shifter on a firm's cost and quantity of capital. Section 4.3 reports robustness checks.

4.1 Heterogeneity of Supply and Demand Schedules

Figure 4 plots supply and demand schedules and the cross-section of stocks in the Sharpe ratio-market correlation plane. The supply schedules are predicted values from equation (36) for firms in the 10th, 30th, 50th, 70th, and 90th percentiles of the supply intercept, $\boldsymbol{\beta}_{\mathrm{S}}'\boldsymbol{x}_{\mathrm{S},n,t}$. The demand schedules plot predicted values from equation (37) for firms in the 10th, 30th,

	Dependent Variable: Sharpe Ratio			
	(1)	(2)	(3)	(4)
Supply				
Market Correlation	0.399**	0.658***	0.657***	0.670***
	(0.164)	(0.147)	(0.077)	(0.069)
Business Equity Correlation	0.225***	0.095***	0.108***	0.097***
	(0.042)	(0.036)	(0.028)	(0.025)
Social Score	−0.050***	−0.031***		
	(0.007)	(0.007)		
Demand				
Internal Market Correlation	−144.464***	−86.455**	−280.264**	−136.279**
	(32.565)	(36.530)	(109.735)	(64.851)
Profitability	0.063***	0.040***	0.095***	0.054***
	(0.015)	(0.014)	(0.035)	(0.020)
Cash Flow Volatility	1.287***	0.516	0.939	0.348
	(0.490)	(0.409)	(0.582)	(0.327)
Intangible Intensity	−0.003*	−0.001	−0.001	−0.000
	(0.002)	(0.001)	(0.002)	(0.001)
Year FE	no	yes	no	yes
Observations	2218	2218	5659	5659
F Statistic	372.274***	254.69***	715.11***	526.403***

***$p < 0.01$; **$p < 0.05$; *$p < 0.1$

Table 4: **3SLS Estimates with Multiple Instruments, Year Fixed Effects, and Alternative Samples.** This table reports estimates of supply and demand schedules obtained by three-stage least squares. In columns 1 and 2, we conduct the estimation on an unbalanced panel of 1,243 U.S. firms observed on December 31 of years 2004, 2009, and 2014 and report estimates computed by using the full set of instruments (1) without fixed effects, (2) with year fixed effects. In colunms 3 and 4, we consider a larger panel of 2,425 U.S. firms observed on December 31 of years 1999, 2004, 2009, and 2014 and report estimates computed (3) without fixed effects, (4) with year fixed effects. Statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels.

50th, 70th, and 90th percentiles of $\boldsymbol{\beta}'_{\text{D}}\boldsymbol{x}_{\text{D},n,t} - \Delta_{\text{D}}\rho^{\text{ext}}_{M,n,t}$, which corresponds to the demand intercept. All coefficients are computed using estimates from column 1 of Table 4.

The figure reveals substantial heterogeneity in the capital supply schedules of U.S. firms. The Sharpe ratio differential between the 10th and 90th percentiles amounts to 0.18, or about one-third of the market portfolios's Sharpe ratio. Heterogeneity in demand schedules appear even more substantial, with a Sharpe ratio differential between the 10th and 90th percentiles close to 0.5. This suggests that heterogeneity in demand dominates heterogeneity in supply.

The figure also reports the scatter plot of the fitted Sharpe ratio and market correlation of individual firms, which we obtain by solving the system (36)–(37). This approach is akin to estimating the risk and return characteristics of securities via an asset pricing model, as is routinely implemented in finance. Our supply and demand system is able to produce substantial dispersion in fitted equilibrium values of market correlation and Sharpe ratio. Most firms are located between the most extreme supply and demand curves. The scatter plot also confirms that the dispersion of firm outcomes is driven primarily by cross-sectional variation in demand shifters, but the dispersion of supply shifters also plays a role.

The dominance of demand heterogeneity explains why the relation between market correlation and Sharpe ratio is generally positive. The black dotted line in Figure 4 is obtained by running a univariate regression of equilibrium Sharpe ratios on equilibrium market correlations. By construction, this line picks up a mix of supply and demand effects and is therefore less steep than the estimates of the supply schedules.

4.2 Price and Quantity Impact of Supply and Demand Shifts

We now investigate how supply and demand shifts impact a firm's cost and quantity of capital. We take shifters one at time and measure the equilibrium outcome of a one standard-deviation change in the shifter. Throughout the section, the coefficients of the supply and demand schedules are from column 1 of Table 4.

Supply Shifters. A firm's social score, $s_{n,t}$, has a mean of -2.4 and a standard deviation of 1.14 (Table 1). In the Appendix, we report that the social score exhibits substantial heterogeneity across industries. For instance, the tobacco industry has an average social score of -4.12, while the food products industry has an average social score of -2.50. Within

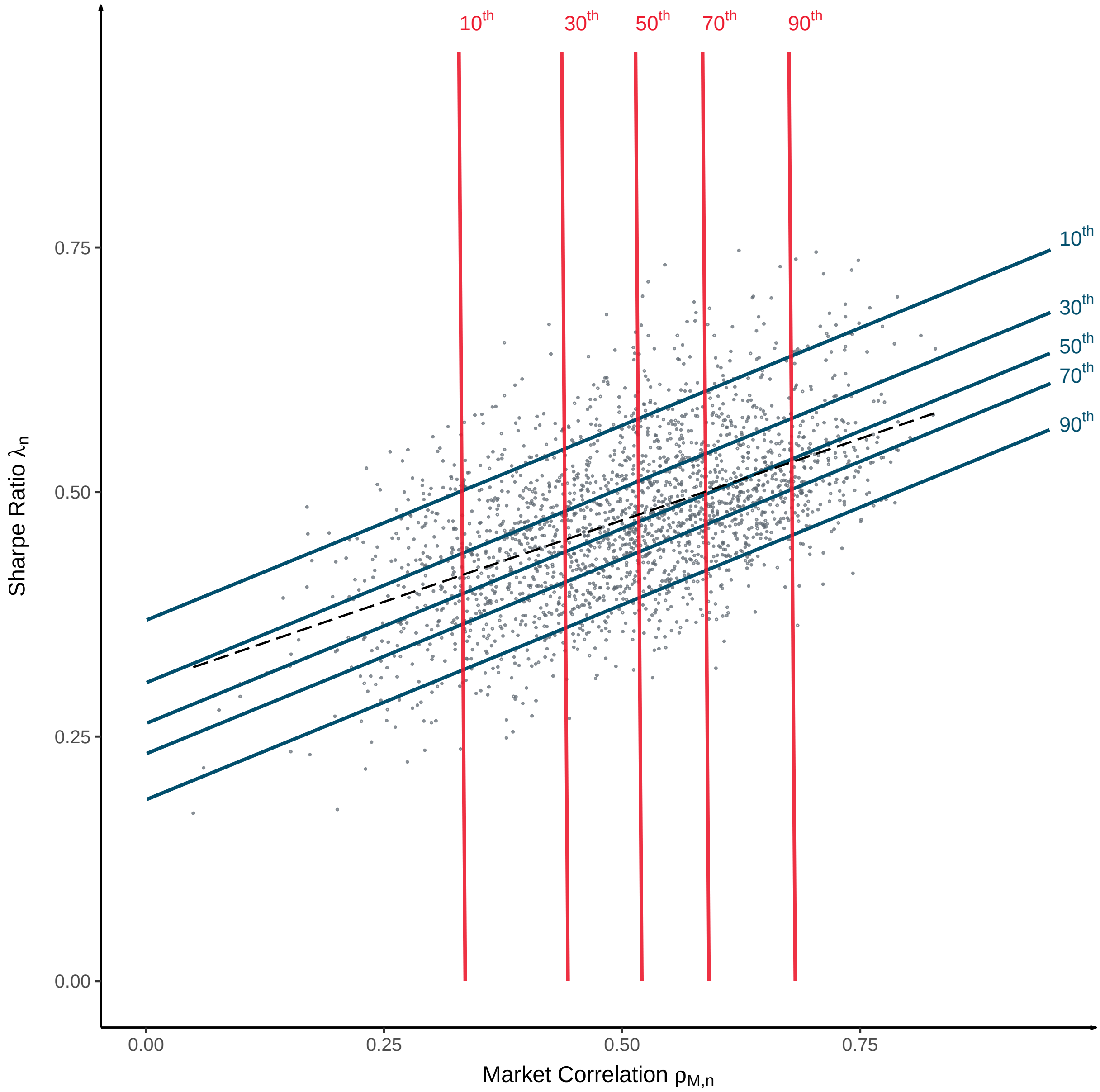

Figure 4: **Equilibrium Outcomes and Supply and Demand Schedules.** This figure plots equilibrium outcomes and the supply and demand schedules in the Sharpe ratio-market correlation plane. It includes 1,243 U.S. firms observed on December 31 of years 2004, 2009, and 2014. We plot supply schedules (blue) for firms in the 10th, 30th, 50th, 70th, and 90th percentiles of the supply intercept, $\boldsymbol{\beta}_{\mathrm{S}}'\boldsymbol{x}_{\mathrm{S},n,t}$. We plot demand schedules (red) for firms in the 10th, 30th, 50th, 70th, and 90th percentiles of the demand intercept, $\boldsymbol{\beta}_{\mathrm{D}}'\boldsymbol{x}_{\mathrm{D},n,t} - \Delta_{\mathrm{D}}\rho_{M,n,t}^{\mathrm{ext}}$. The coefficients are from column 1 of Table 4. We also plot the equilibrium values of market correlation and Sharpe ratio fitted to the model. The dotted line is estimated from a univariate regression of fitted Sharpe ratios on fitted market correlations.

an industry, firms can also have very different social scores. For example, the standard deviation of social score inside the tobacco industry is 0.9.

In Figure 5, we plot the supply and demand schedules of two firms that differ only in their social scores (Panels A and B). In each panel, the baseline firm represented by the solid lines is assigned the average characteristics in the data. The second firm represented by the dashed lines has a social score coefficient that is one standard deviation higher than the social score of the baseline firm. Panel A plots the supply and demand schedules in the Sharpe ratio-correlation plane, while Panel B plots them in the Sharpe ratio-risk capital plane.

A one-standard deviation improvement in a firm's social score reduces its equilibrium Sharpe ratio from 0.48 to 0.42, a substantial decrease in the price of risk. For a firm with average return volatility 0.36 (Table 1), such a decrease corresponds to a 2.16 percentage point drop in expected return. It also triggers a significant increase in risk capital from \$3.5 billion to \$4.5 billion, and slightly increases by 4 basis points the firm's correlation to the market (Panels A and B of Figure 5). Hence, the firm with high social score is able to decrease its cost of capital and increase the level of investment without significantly impacting its market correlation.

Exposure to private business equity also impacts the firm's quantity and price of risk. In the Appendix, we show that a one-standard deviation reduction in a firm's correlation with private business equity reduces its equilibrium Sharpe ratio from 0.48 to 0.44 and increases its risk capital from \$3.5 billion to \$4.2 billion. As with social score, the impact on correlation with private business equity on the firm's market correlation is economically insignificant.

Demand Shifters. In Panels C and D of Figure 5, we plot the supply and demand schedules of two firms that differ only in profitability. As previously, the baseline firm (solid lines) is assigned the average characteristics in the data, while the other firm (dashed lines) has a profitability coefficient that is one standard deviation higher than the baseline.

Panel C of Figure 5 shows that a one standard deviation-increase in profitability has little effect on the firm's Sharpe ratio. The reason is that the supply curve does not change and the outward push in the demand schedule only increases the firm's market correlation by 12 basis points. Consequently, investors with well diversified portfolios do not require a significant change in compensation for supplying more capital to the more profitable firm. Nevertheless, Panel D shows that the increase in profitability leads to a large increase in the

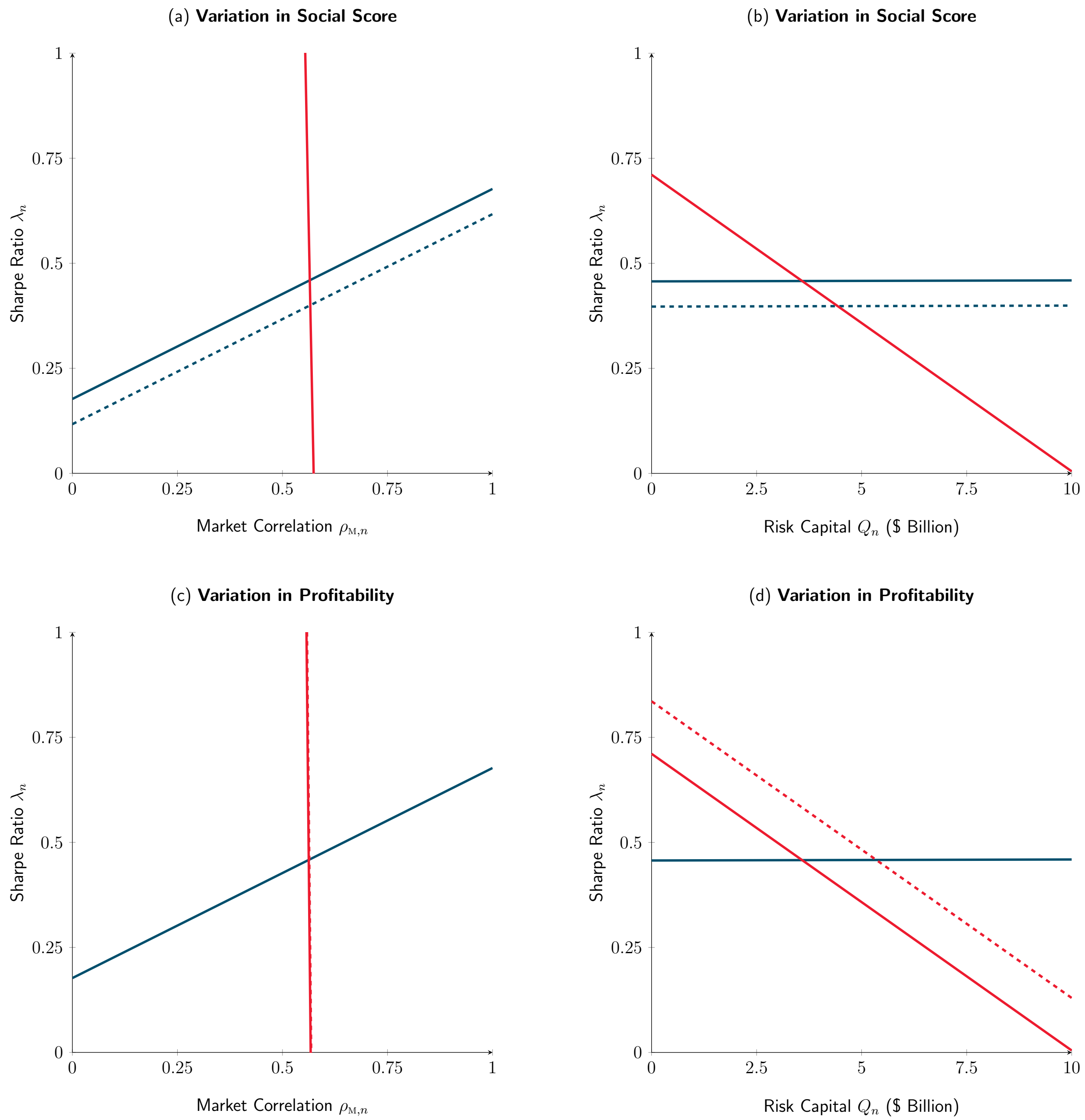

Figure 5: **Impact of Supply and Demand Shifters** This figure illustrates the supply and demand schedules for two firms that differ either in social score (Panels A and B) or in profitability per unit of risk (Panels C and D) by one standard deviation. The firm whose schedules are represented by the solid lines is assigned the average firm characteristics in the data. The firm whose schedules are represented by the dashed lines differs only in social score or profitability. The coefficients of the supply and demand schedules are from column 1 of Table 4. The left panels plot supply and demand schedules in the $\lambda_n - \rho_{M,n}$ plane, while the right panels plot the schedules in the $\lambda_n - Q_n$ plane.

firm's risk capital, from \$3.5 billion to \$6.5 billion.

By the same logic, variation in cash flow volatility and variation in intangible intensity do not impact a firm's market correlation and Sharpe ratio but substantially change its risk capital. A one-standard deviation increase in cash flow volatility increases the firm's risk capital from \$3.5 billion to \$4.5 billion. Likewise, a one-standard deviation *decrease* in intangible intensity boosts risk capital from \$3.5 billion to \$4.1 billion. We note that these effects, although large, are not as economically important as the impact of a one-standard deviation change in profitability.

A key insight from Figure 5 is that the slopes of supply and demand schedules vary significantly depending on the plane of analysis. In the $\lambda_n - \rho_{\mathrm{M},n}$ plane, which characterizes the risk-return trade-off of investors, demand schedules are almost vertical since firm decisions have little impact on their level of correlation to the stock market. By contrast, in the $\lambda_n - Q_n$ plane characterizing the capital budgeting decisions of individual firms, the supply schedule is almost horizontal and the cost of capital appears nearly constant. To a first approximation, firms operate in an "small open economy" context with an exogenous price of risk. These contrasted perspectives help to understand why most studies in asset pricing tend to focus on a firm's price of risk and take its market correlation as given, while studies in corporate finance focus on quantities and take a firm's cost of capital as given.

The small impact of demand shifters on a firm's Sharpe ratio should be interpreted with caution because the figure compares individual firms that only differ along one dimension. If we instead compare a group of firms with high profitability to a group of firms with low profitability, then firms in the more profitable cluster will also have high external market correlation. As we show in the next section, an increase in external market correlation generates a significant increase in a firm's Sharpe ratio, so a portfolio of high-profitability firms would have a higher Sharpe ratio than a portfolio of low-profitability firms.

External Market Correlation. While the external correlation is not an exogenous demand shifter in the strict sense, it is informative to study its impact on a firm's equilibrium outcome. This exercise roughly corresponds to comparing firms that only differ in their exogenous correlations to the EBIT of other firms.

Figure 6 considers two firms that differ by a unit standard deviation in external market correlation, which is about 0.14 in the data. All other parameters are assigned their average values. The firm with lower external market correlation (dashed lines) has a significantly

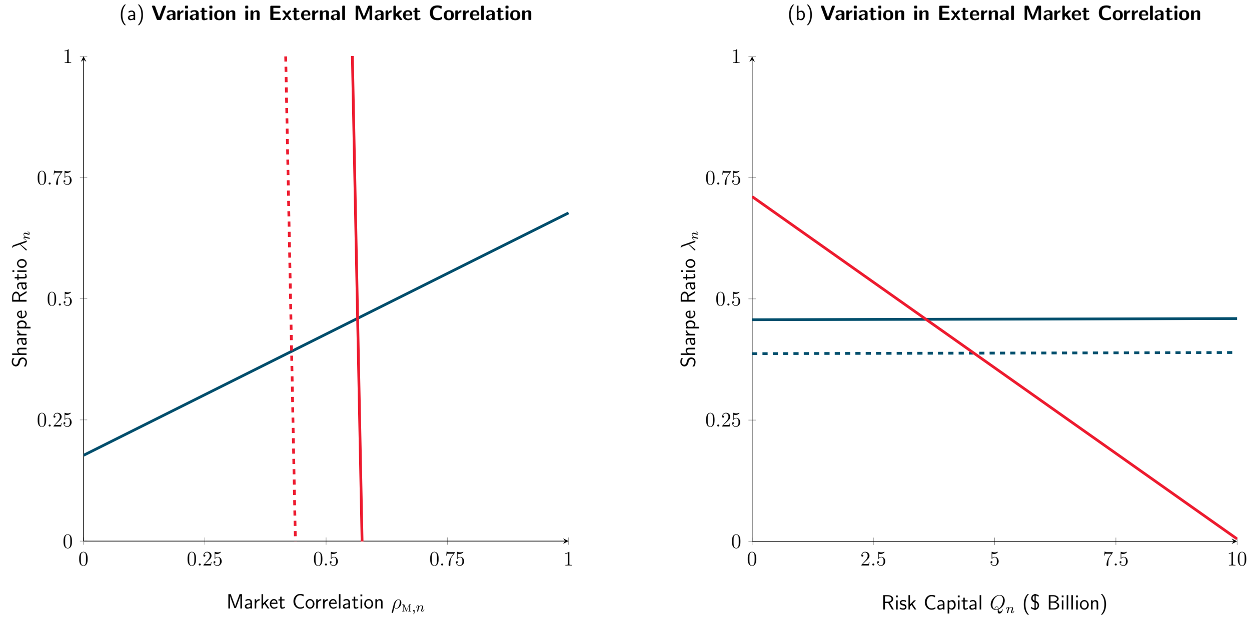

Figure 6: **Variation in external market correlation** This figure illustrates the supply and demand schedules for two firms that differ in external market correlation by one standard deviation. The firm whose schedules are represented by the solid lines is assigned the average firm characteristics in the data. The firm whose schedules are represented by the dashed lines has similar firm characteristics but an external market correlation reduced by one standard deviation. The coefficients of the supply and demand schedules are from column 1 of Table 4. The left panel plots supply and demand schedules in the $\lambda_n - \rho_{\mathrm{M},n}$ plane, while the right panel plots the schedules in the $\lambda_n - Q_n$ plane.

lower Sharpe ratio (0.42 vs. 0.48) and higher risk capital of ($4.5 billion vs. $3.5 billion) than the firm with high external market correlation. A one standard deviation decrease in the external market correlation, $\rho^{ext}_{\mathrm{M},n,t}$,, has a similar impact as a one standard deviation *increase* in the social score, $s_{n,t}$.

A comparison of Panel C of Figure 5 and Panel A of Figure 6 shows that the external market correlation channel dominates the variation in demand schedules in the $\lambda_n - \rho_{\mathrm{M},n}$ plane observed in Figure 4. This source of heterogeneity is therefore responsible for the generally positive relation between market correlation and Sharpe ratio in the cross-section.

A distinct property of the external market correlation effect is that it impacts differently the supply and demand schedules. A fall in external market correlation triggers an inward *demand* shift along the firm's supply schedule (17) in the Sharpe ratio-market correlation plane, consistent with the fact that the firm is now less exposed to systematic risk. In the Sharpe ratio-risk capital plane, however, the reduction in external market correlation triggers an outward shift in the firm's *supply* of capital (15). Intuitively, when the firm's free cash flow becomes less systematic, the firm is no longer crowded out as much by other firms competing for access to capital. The dual interpretation of external market correlation as a supply or demand shifter depending on the plane of analysis stems from the equivalence between risk capital and market correlation in (12).

4.3 Robustness Tests

We next perform additional tests of our supply and demand system to verify that the baseline estimates are robust to alternative specifications.

Sample Selection. A possible concern is that the baseline results in Table 2 are specific to the chosen time-period and pool of stocks. We run two tests that verify that this is not the case.

In column 2 of Table 4, we add year fixed effects that pick up changes in general financial market conditions. Even though some of the instruments lose significance, their sign is unchanged and the positive supply and negative demand slopes remain strongly significant. The specification with time fixed-effects produces a steeper supply schedule and flatter demand schedule.

In columns 3 and 4 of Table 4, we expand the sample size by removing the requirement that firms have a social score. The sample now includes an additional cross-section of stocks corresponding to $t = 1999$ as well as more stocks in each cross-section. In the Appendix we report summary statistics of this larger sample, which contains 5,659 stock-year observations and 2,425 firms, including smaller-size firms. We do not consider time fixed effects in column 3 but include them in column 4. The results are again similar to the baseline results and confirm that our supply and demand estimation holds across multiple samples.

Measuring Market Correlation from Accounting Data. Until now, we have used stock returns to calculate a firm's internal and total market correlations. A possible concern is that stock returns are largely driven by discount rate shocks (Campbell and Shiller 1988) that are absent from our two-period general equilibrium model. Hence, a firm's estimated market correlation may not necessarily correspond to what the model predicts.

Since a firm's risk capital is related to its underlying cash flow volatility by (11), it is

	Dependent Variable: Sharpe Ratio		
	(1)	(2)	(3)
Supply			
Market Correlation	0.474^{***}	0.474^{***}	
	(0.180)	(0.180)	
Market Correlation (EBIT)			0.385^{**}
			(0.151)
Social Score	-0.051^{***}	-0.051^{***}	-0.053^{***}
	(0.008)	(0.008)	(0.008)
Demand			
Internal Market Correlation	-151.791^{***}		
	(49.516)		
Internal Market Correlation (EBIT)		-82.772^{***}	-82.772^{***}
		(26.796)	(26.796)
Profitability	0.043^{***}	0.010^{*}	0.010^{*}
	(0.013)	(0.006)	(0.006)
Year FE	no	no	no
Observations	2255	2255	2255
F Statistic	593.351^{***}	594.05^{***}	566.465^{***}

$^{***}p < 0.01$; $^{**}p < 0.05$; $^{*}p < 0.1$

Table 5: **3SLS Estimates of Supply and Demand Schedules with EBIT-based measures of Market Correlation** This table reports estimates of supply and demand schedules obtained by three-stage least squares, where a firm's market correlation and internal market correlation metrics are calculated from a firm's EBIT instead of its stock return. We conduct the estimation on an unbalanced panel of 1,243 U.S. firms observed on December 31 of years 2004, 2009, and 2014. Statistical significance is indicated by ***, **, and * for the 0.01, 0.05, and 0.10 levels.

possible to re-estimate risk capital as the volatility of a firm's EBIT. Similarly, one can re-estimate market correlation as the correlation between a firm's EBIT and the EBIT of the market portfolio. By construction, these accounting variables only take into account cash flow risk. We measure market $EBIT_{\mathrm{M},t}$ as the sum of $EBIT_{n,t}$ for all firms in our sample. We then calculate Q_n^{EBIT} and Q_{M}^{EBIT} as the annualized standard deviation of each asset's quarterly EBIT from $t-10$ to t. A firm's internal market correlation corresponds to the ratio of Q_n^{EBIT} to Q_{M}^{EBIT}. Similarly, a firm's market correlation $\rho_{\mathrm{M},n,t}^{EBIT}$ is measured as the correlation between $EBIT_{n,t}$ and $EBIT_{\mathrm{M},t}$ from $t-10$ to t. In the Appendix, we find that the return-based and EBIT-based measures of market correlation are strongly correlated in our data.

Table 5 reports the results of the 3SLS estimation that includes the baseline shifters and the EBIT-based market correlation metrics. Column 1 uses return-based market correlations and is identical to column 1 of Table 3. In column 2, we replace a firm's internal market correlation with its EBIT-based analog. The point estimate of the demand schedule is lower in absolute value at -83 but remains highly significant. In column 3, we also replace a firm's market correlation with its EBIT-based analog. The supply slope coefficient reduces to 0.385 but remains strongly significant. Social score and profitability have nearly the same coefficients as in column 1.

This exercise confirms that our empirical results are robust to using an accounting-based measure of correlation. More generally, the analysis suggests that cash flow risk represents an important component of capital supply and demand.

5 Conclusion

This paper develops a tractable and parsimonious empirical model of capital markets. We define general equilibrium by a system of simultaneous linear equations expressing firm-level supply and demand for financial capital. We provide micro-foundations of a specific system using classic assumptions on firm and investor behavior and develop an estimation methodology based on two- and three-stage least squares. We quantify the sensitivity of a firm's size and capital cost to systematic risk and supply and demand shifters, including the social score, correlation with aggregate private business equity risk, profitability, cash flow volatility, and intangible intensity.

This approach produces coherent and reasonable levels of statistical and economic significance when applied to a panel of individual stocks. The estimation method is robust to multiple specifications and is sufficiently accurate to by-pass the construction of well-diversified portfolios that may result in a reduction of cross-sectional variability (Ang, Liu, and Schwarz 2020). Another key benefit is that this methodology naturally resolves the endogeneity issues that arise when using reduced-form models of capital markets.

Our paper highlights the usefulness of the supply and demand approach for applications at the intersection of corporate finance and asset pricing. Our supply and demand system can be used to address key policy questions, such as the impact of investor tilts or tax policies on the pricing of risk and capital budgeting decisions of firms. It can also be used to understand how the risk-return relation can evolve over time and across economies as a function of the underlying supply and demand drivers.

The growing availability of comprehensive investor holdings data provides a promising avenue for the inclusion of additional instruments reflecting the supply of capital of specific institutional investors (Koijen and Yogo 2019) and individual investors (Betermier et al. 2021). The recent developments in ESG reporting standards also provide the opportunity to further investigate how the growth of ESG investing impacts a firm's cost and quantity of capital. We leave these questions for future research.

A Proof of Proposition 1

We infer from equations (23) and (27) that

$$\lambda_\ell = a_\ell - (1+r_f)\,(Q_\ell/\sigma_{\mathrm{CF},\ell}^{\eta+1})^{1/\eta} = \gamma\,\delta_\ell + \frac{\gamma}{W_0}\sum_i \rho_{\ell,i}\,Q_i. \tag{39}$$

We note that

$$\sum_{i=1}^N \rho_{\ell,i}\,Q_i = (1-\rho_{\ell,n})Q_\ell + \rho_{\ell,n}(Q_\ell + Q_n) + \sum_{i\notin\{\ell,n\}} \rho_{\ell,i}\,Q_i. \tag{40}$$

The first term on the right-hand side, $(1-\rho_{\ell,n})Q_\ell$, is specific to firm ℓ, while the other terms on the right-hand side are common to firms ℓ and n.

Assume that $a_\ell > a_n$ and $\delta_\ell = \delta_n$. We infer from equations (39) and (40) that

$$a_\ell - (1+r_f)\,(Q_\ell/\sigma_{\mathrm{CF},\ell}^{\eta+1})^{1/\eta} - (1-\rho_{\ell,n})Q_\ell = \gamma\,\delta_\ell + \frac{\gamma}{W_0}\left[\rho_{\ell,n}(Q_\ell+Q_n) + \sum_{i\notin\{\ell,n\}} \rho_{\ell,i}\,Q_i\right]. \tag{41}$$

The right-hand side is the same for both firms. The left-hand side increases in profitability and decreases in capital risk. Since $a_\ell > a_n$ and all other parmaters are equal, we infer that $Q_\ell > Q_n$. The demand schedule implies that $\lambda_\ell > \lambda_n$. Equation (40) implies that $\sum_{i=1}^N \rho_{\ell,i}\,Q_i > \sum_{i=1}^N \rho_{n,i}\,Q_i$ and therefore $\rho_{\mathrm{M},\ell} > \rho_{\mathrm{M},n}$.

We now assume that $\delta_\ell > \delta_n$ and $a_\ell = a_n$. We infer from equations (39) and (40) that

$$\gamma\,\delta_\ell + (1+r_f)\,(Q_\ell/\sigma_{\mathrm{CF},\ell}^{\eta+1})^{1/\eta} + \frac{\gamma}{W_0}(1-\rho_{\ell,n})Q_\ell = \gamma\,\delta_n + (1+r_f)\,(Q_n/\sigma_{\mathrm{CF},\ell}^{\eta+1})^{1/\eta} + \frac{\gamma}{W_0}(1-\rho_{\ell,n})Q_n.$$

Hence $Q_\ell < Q_n$ and $\rho_{\mathrm{M},\ell} < \rho_{\mathrm{M},n}$. The demand schedule implies that firm ℓ also has a higher Sharpe ratio. ■

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