First-price auctions are a common auction format, but rarely occur in isolation:
• Multiple objects are often auctioned simultaneously
• Auctions are repeated regularly whenever new lots become available.

This is a problem for standard empirical methods when pay-offs are non-additive across auctions. Consider a bidder with diminishing marginal returns:
1. If they previously won many lots, they will bid less aggressively in future
2. If many auctions occur simultaneously, they may only bid on a small subset

It is difficult to model both repeated and simultaneous auctions. Previous work, including Gentry et al. (2018) and Jofre-Bonet and Pesendorfer (2003), focuses on either a static multi-object model, or a dynamic single-object model.

→ It is an open research question what effect these simplifications will have, and the magnitude of the inaccuracy created on estimates and counterfactuals.

This paper develops an empirical model of bidding in this environment, combining the models of Gentry et al. (2018) and Jofre-Bonet and Pesendorfer (2003)

Research Questions:
1. Is the model non-parametrically identified?
2. Is it computationally feasible to estimate such a model?
3. What is the inaccuracy of using a simpler single object or static model?

I build an infinite horizon model of risk neutral bidders bidding in repeated simultaneous first-price auctions. The model has the following features:
• L lots are auctioned each period. At time t bidder i places bid $b_{it}$ on lot l.
• There are 2^L possible outcomes for bidder $i$ at time $t$, denoted $s_t \in \{1, \ldots, 2^L\}$. Combination outcomes include winning none, 1 lot, a lot, a lot, 2 lots, etc.
• The state of the world at the beginning of period $i$ is denoted $\sigma_i$. The ex-post state at the end of period $i$ is denoted $\sigma_i$. This depends on the outcome $s_i$.
• If $i$ wins lot l, they receive pay-off $\tau_{s_i(l)}$. This value is known privately, and drawn independently from distribution $\mathcal{F}_l$.
• If $i$ ends the period in state $\sigma_l$, they receive deterministic pay-off $\chi_{s_i}(\cdot)$.
• Players believe, given the state $\sigma_t$, their bids $b_{it}$, and others’ strategies, that:
  1. They will win lot $l$ with (marginal) probability $\mathcal{F}_l(\sigma_t,\sigma_{-t})$. 
  2. The combination outcome will be $\sigma_l$ with probability $\mathcal{F}_l\mathcal{F}_l(\sigma_t,\sigma_{-t})$.

The Bellman equation for this problem is given by:

$$W(s,\sigma,\ldots) = \max_i \left\{ \sum_l \frac{\tau_l}{2} \mathcal{F}_l(\sigma_t,\sigma_{-t}) + \mathcal{F}_l(\sigma_t,\sigma_{-t}) \right\}$$

→ Where the Continuation Value $V'(s') = E_{\sigma}[W(s',\sigma,\ldots)]

This optimisation problem yields $V$. First Order Conditions for optimal bidding:

$$0 = \frac{\partial V(s)}{\partial b_i} - \tau_i \mathcal{F}_l(\sigma_t,\sigma_{-t}) + \mathcal{F}_l(\sigma_t,\sigma_{-t}) \left( \frac{\partial \mathcal{F}_l(\sigma_t,\sigma_{-t})}{\partial b_i} \right) + \mathcal{F}_l(\sigma_t,\sigma_{-t}) \mathcal{F}_l(\sigma_t,\sigma_{-t})$$

Following Gentry et al. invert the FOCS (i.e. $\frac{\partial}{\partial b_i}$) for the inverse Bid Function $\zeta_i$:

$$\zeta_i(b) = b_i + \frac{\tau_i}{2} \mathcal{F}_l(\sigma_t,\sigma_{-t}) + \mathcal{F}_l(\sigma_t,\sigma_{-t}) \left( \frac{\partial \mathcal{F}_l(\sigma_t,\sigma_{-t})}{\partial b_i} \right) + \mathcal{F}_l(\sigma_t,\sigma_{-t}) \mathcal{F}_l(\sigma_t,\sigma_{-t})$$

I investigate the extent of the inaccuracies from incorrectly estimating either a static or single-object model. I apply the model to data from Michigan Department of Transport’s procurement auctions for highway maintenance contracts:
• 10 years of data, with auction rounds taking place every 2-4 weeks.
• 45 auctions take place each round, with roughly 200 (potential) bidders.

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I compare estimates of the marginal cost function across three models:

We observe that costs are decreasing in the backlog.

→ This suggests firms take advantage of increasing returns to scale

When we compare the mis-specified models the costs are under-estimated.