Inefficient Automation*

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Abstract

How should the government respond to automation? We study this question in a heterogeneous agent model that takes worker displacement seriously. We recognize that displaced workers face two frictions in practice: reallocation is slow and borrowing is limited. We first show that these frictions result in inefficient automation. Firms are effectively too patient when they automate, and (partly) overlook the time it takes for workers to reallocate and for the benefits of automation to materialize. We then analyze a second best problem where the government can tax automation but lacks redistributive tools to fully overcome borrowing frictions. The equilibrium is (constrained) inefficient — automation and reallocation impose pecuniary externalities on workers. The government finds it optimal to tax automation while labor reallocates, even when it has no preference for redistribution. Using a quantitative version of our model, we find significant welfare gains from slowing down automation.

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1 Introduction

Automation technologies — like AI and robots — raise productivity but disrupt labor markets, displacing workers and lowering their earnings (Graetz and Michaels, 2018; Acemoglu and Restrepo, 2018a). The increasing adoption of automation has fueled an active debate about appropriate policy interventions (Atkinson, 2019; Acemoglu et al., 2020). Despite the growing public interest in this question, the literature has yet to produce optimal policy results that take into account the frictions that workers face in practice when they are displaced by automation.

The existing literature that justifies taxing automation assumes that worker reallocation is frictionless or absent altogether. First, recent work shows that a government that has a preference for redistribution should tax automation to mitigate its distributional consequences (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). This literature assumes that automation and labor reallocation are instrinsically efficient, and that the government is willing to sacrifice efficiency for equity. Second, an extensive literature finds that a government should tax capital — and automation, by extension — to prevent dynamic inefficiency (Diamond, 1965; Aguiar et al., 2021), or address pecuniary externalities when markets are incomplete (Lorenzoni, 2008; Conesa et al., 2009; Dávila et al., 2012; Dávila and Korinek, 2018). This literature abstracts from worker displacement and labor reallocation.

In this paper, we take worker displacement seriously and study how a government should respond to automation. In particular, we recognize that workers face two important frictions when they reallocate or experience earnings losses. First, reallocation is slow: workers face barriers to mobility and may go through unemployment or retraining spells before finding a new job (Davis and Haltiwanger, 1999; Jacobson et al., 2005; Lee and Wolpin, 2006). Second, credit markets are imperfect: workers have a limited ability to borrow against future incomes (Jappelli and Pistaferri, 2017), especially when moving between jobs (Chetty, 2008). We show that these frictions result in *inefficient* automation. A government finds it optimal to tax automation — even if it has no preference for redistribution — when it lacks redistributive instruments to fully alleviate borrowing frictions. Quantitatively, we find large welfare gains from slowing down automation.

We incorporate reallocation and borrowing frictions in a dynamic model with endogenous automation and heterogeneous agents. There is a continuum of occupations, and workers come in overlapping generations. Firms invest in automation to expand their productive capacity. Automated occupations become less labor intensive, which displaces workers. These workers face reallocation frictions: they receive random opportunites to

move between occupations, experience a temporary period of unemployment or retraining when they do so (Alvarez and Shimer, 2011), and incur a permanent productivity loss when reallocating due to the specificity of their skills (Violante, 2002; Adão et al., 2020). Workers also face financial frictions: they are not insured against the risk that their occupation is automated and face borrowing constraints (Huggett, 1993; Aiyagari, 1994) which can prevent consumption smoothing.

We have two main theoretical results. Our first result shows that the interaction between slow reallocation and borrowing constraints results in inefficient automation. Displaced workers experience earnings losses when their occupation is automated, but expect their income to increase as they slowly reallocate. This creates a motive for borrowing to smooth consumption during this transition. When borrowing and reallocation frictions are sufficiently severe, displaced workers are pushed against their borrowing constraints.[1] This drives a wedge between the (intertemporal) marginal rate of substitution of displaced workers and the equilibrium interest rate that firms face when they automate. Effectively, firms are excessively patient when they automate. They (partly) overlook the time it takes for labor to reallocate and for the benefits of automation to materialize.

Our second result characterizes optimal policy. In principle, the government could restore efficiency without taxing automation if it was able to fully relax borrowing constraints using redistributive taxes and transfers.[2] In practice, the government is unlikely to have access to this rich set of instruments.[3] This motivates us to study second best interventions, where the government can tax automation and (potentially) implement active labor market interventions but is unable to fully alleviate the borrowing constraints of displaced workers by redistributing income directly.

We find that the equilibrium is generically (constrained) inefficient, as defined by Geanakoplos and Polemarchakis (1985) and Farhi and Werning (2016). Automation and reallocation choices impose *pecuniary externalities* on workers. Firms do not internalize that automation displaces workers and lowers their earnings, and workers do not internalize how their reallocation affects the wage of their peers. These pecuniary externalities do not net out when displaced workers are pushed against their borrowing constraints.

We then show that the government should tax automation on efficiency grounds —

[1] This is consistent with the empirical evidence. Displaced workers increase their borrowing to smooth consumption (Sullivan, 2008; Collins et al., 2015) when they are able to. Many workers are constrained and are either unable to borrow or forced to delever their existing debt (Bethune, 2017; Braxton et al., 2020). Job displacement also increases defaults (Gerardi et al., 2018; Keys, 2018), which further limits access to credit.

[2] In fact, the government can decentralize any first best allocation using targeted lump sum transfers — a version of the Second Welfare Theorem holds in our model.

[3] The absence of (targeted) lump sum transfers is precisely what motivates the existing literature on the taxation of automation. We allow for various sources of social insurance in our quantitative model.

even when it has no preference for redistribution. The reason is that the output gains from automation are *back-loaded*, since they materialize slowly as more workers reallocate. The government values *less* future gains than firms do as it recognizes that the equilibrium interest rate is lower than the (intertemporal) marginal rate of substitution of the average worker. In other words, automation imposes adverse pecuniary externalities on displaced workers early on in the adjustment process, precisely when these workers are borrowing-constrained. As a result, the government should *slow down* automation while labor reallocation takes place. The optimal policy improves *aggregate efficiency* – as defined by Bhandari et al. (2021) — by preventing excessive investment in automation and raising consumption when displaced workers value it more. However, the government should not intervene in the long-run once labor reallocation is complete and workers no longer have a motive to borrow.[4]

We then consider an intermediate case where the government is able to tax automation (ex ante) but is unable to intervene in the labor market (ex post). This extension is motivated by an extensive empirical literature that finds that these active labor market interventions have mixed results or unintended consequences for untargeted workers (Heckman et al., 1999; Card et al., 2010; Doerr and Novella, 2020; Crépon and van den Berg, 2016). In this case, the government also uses its tax on automation as a proxy for the labor market interventions it cannot implement directly. We show that this can reinforce or dampen the government's desire to tax automation, depending on the average duration of unemployment / retraining spells. When these spells are short, workers rely excessively on labor reallocation as a source of insurance. As a result, the government taxes automation more than it would have with active labor market interventions. The opposite occurs when the spells are long, and the government taxes automation less.

We conclude the paper with a quantitative exploration. We extend our baseline model along various dimensions that are important for welfare analysis. In particular, we introduce gross flows across occupations (Kambourov and Manovskii, 2008; Moscarini and Vella, 2008) and uninsured earnings risk (Floden and Lindé, 2001). These two features are important in the data and affect the ability of displaced workers to self-insurance against automation through mobility and savings. We also introduce progressive income taxation (Heathcote et al., 2017) and unemployment benefits (Krueger et al., 2016) to account for existing sources of insurance that can benefit displaced workers. Slowing down automation generates large welfare gains in our numerical simulations — even absent any equity considerations.

[4] We allow for uninsured idiosyncratic risk in our quantitative analysis. In this case, borrowing constraints bind in the long-run as well.

Our paper relates to several strands of the literature. Our results show that optimal policies can improve *both* efficiency and equity — while there is necessarily a trade-off in the efficient economies studied in the literature on the taxation of automation (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). In this literature, taxing automation results in production inefficiency (Diamond and Mirrlees, 1971). On the contrary, optimal policy preserves (or restores) production efficiency in our model.[5]

The rationale we propose for taxing automation also complements a large literature on capital taxation due to equity considerations (Judd, 1985; Chamley, 1986), dynamic inefficiency (Phelps, 1965; Diamond, 1965; Aguiar et al., 2021), or pecuniary externalities when markets are incomplete (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012; Dávila and Korinek, 2018). Optimal policies in our model also address pecuniary externalities. However, these externalities are distinct from the type encountered in the incomplete markets literature. In particular, the externalities in our economy rely neither on the presence of uninsured idiosyncratic risk, nor on endogenous borrowing constraints. Instead, they occur when firms and workers make *technological choices* — such as automation or reallocation — and borrowing constraints distort the (shadow) prices that these agents face.[6] In addition, the literature on pecuniary externalities has almost exclusively studied static (or two-period) models or long-run stationary equilibria. Therefore, the *timing* of pecuniary externalities (i.e., how front- or back-loaded they are) plays no role for optimal policy. In contrast, the rationale for intervention that we propose applies during the *transition* to the long run, and the timing of pecuniary externalities is central for optimal policy.

Methodologically, our quantitative model borrows from three literatures: (i) the one studying reallocation and wage dispersion using island models in the tradition of Lucas and Prescott (1974) and Alvarez and Shimer (2011); (ii) the one concerned with the impact of technological innovations and trade using dynamic discrete choice models with mobility shocks (Heckman et al., 1998; Artuç et al., 2010; Caliendo et al., 2019); and (iii) the one interested in consumption and insurance using heterogeneous-agent models with idiosyncratic income shocks, life-cyle features, and borrowing constraints (Heathcote et al., 2010; Low et al., 2010).

Finally, our policy analysis contributes to the public finance literature studying opti-

[5] The laissez-faire in our model can be inefficient while the economy still achieves production efficiency. In particular, production inefficiency arises only when the frictions are sufficiently severe. Whether this is the case or not, optimal policy results in production efficiency when the government can also implement active labor market interventions.

[6] Labor reallocation — just like automation — is isomorphic to a technological choice in the Arrow-Debreu construct. Each worker owns a firm that chooses the type of labor services to provide. We show that this choice is efficient *when* workers' borrowing constraints do not bind.

mal taxation (Conesa and Krueger, 2006; Ales et al., 2015; Heathcote et al., 2017) and social insurance (Imrohoroglu et al., 1995; Golosov and Tsyvinski, 2006; Braxton and Taska, 2020) in dynamic models with heterogeneous agents.

Layout. We introduce our baseline environment in Section 2. Section 3 characterizes the set of first best allocations. Section 4 describes the laissez-faire equilibrium. Section 5 presents our inefficiency result. Section 6 characterizes optimal policy. Section 7 introduces our quantitative model. Finally, Section 8 describes our calibration strategy and quantitative exercise.

2 Model

Time is continuous, and there is no aggregate uncertainty. Periods are indexed by $t \geq 0$. The economy consists of a continuum of workers, a continuum of occupations indexed by $h \equiv [0,1]$, and a final goods producer. In this section, we specify the technologies, preferences, reallocation frictions, and resource constraints of this economy. We will describe assets, incomes, and borrowing frictions in Section 4 when discussing the decentralized equilibrium.

2.1 Technology

Occupations use labor as an input and another factor that we assume is fixed.[7] Final goods are produced by aggregating the output of all occupations.

Occupations. Occupations are indexed by $h \equiv [0,1]$. They use a decreasing returns to scale technology

$$y_t^h = F^h\left(\mu_t^h\right), \tag{2.1}$$

where μ_t^h denotes effective labor.

Technology adoption. Each occupation becomes partially automated in period $t = 0$ with probability ϕ. The degree of automation is α. The occupations' technology is given by

$$F^h(\cdot) = \begin{cases} \hat{F}(\cdot;\alpha) & \text{if automated} \\ F(\cdot) & \text{otherwise} \end{cases}, \tag{2.2}$$

[7] The fixed factor plays the same role as capital in our quantitative model (Section 7). In this richer model, capital reallocation is slow (Ramey and Shapiro (2001)) due to adjustment costs at the occupation-level.

where $F(\cdot)$ and $\hat{F}(\cdot)$ are neoclassical technologies. By definition, $\hat{F}(\cdot;0) \equiv F(\cdot)$ absent automation. Automated occupations are less labor intensive than non-automated occupations — i.e., $\hat{F}_\mu(1;\alpha)$ decreases with α. Automation can raise output directly, but it also comes at a cost. The technology has to be maintained: it requires some continued investment which diverts resources away from production. The technology $\hat{F}(\cdot;\alpha)$ implicitly captures these two effects. We will impose some regularity assumptions in the next section.

Final good. Aggregate output is produced by combining the output y^h of all occupations with a neoclassical technology

$$Y_t = G\left(\left\{y_t^h\right\}\right), \tag{2.3}$$

In the following, we suppose that these inputs are complements. Moreover, we impose some *symmetry* across occupations.

Assumption 1 (Symmetry)**.** *The technology of the final good producer $G(\cdot)$ is continuously differentiable, additively separable and symmetric in its arguments.*

This assumption ensures that economy behaves is *as if* there were only automated ($h = A$) and non-automated ($h = N$) occupations.[8] This allows us to define the aggregate production function

$$G^\star(\boldsymbol{\mu};\alpha) \equiv G\left(\left\{F^h\left(\mu^h\right)\right\}\right) \tag{2.4}$$

where $\boldsymbol{\mu} \equiv \left(\mu^A, \mu^N\right)$ are the flow of workers employed in each automated and non-automated occupations, with the degree of automation α being implicit in $\left\{F^h\left(\mu^h\right)\right\}$.[9] The technology $G^\star(\cdot)$ is total production net of automation costs. For concision, we refer to it as *output* in the following.

2.2 Workers

There are overlapping generations of workers who are born and die at a constant rate $\chi \in [0, +\infty)$.[10] A worker is indexed by four idiosyncratic states: their initial occupation of

[8] Formally, this follows from Assumption 1 together with the (strict) concavity of $G(\cdot)$.

[9] For illustration, suppose that the technology belongs to the Kimball (1995) class. Then, the aggregate production function solves

$$1 = \phi\Gamma\left(\frac{\hat{F}\left(\mu^A;\alpha\right)}{G^\star\left(\mu^A,\mu^N;\alpha\right)}\right) + (1-\phi)\,\Gamma\left(\frac{F\left(\mu^N\right)}{G^\star\left(\mu^A,\mu^N;\alpha\right)}\right),$$

for some $\Gamma(\cdot)$ increasing and concave.

[10] We suppose throughout the paper that $1/\chi$ is bounded away from 0 — each generation has a positive (expected) life span.

employment (h); their age (s); their productivity (ξ); and their employment status (e). In the following, we let $\mathbf{x} \equiv (h, s, \xi, e)$ denote the workers' idiosyncratic states and π denote the associated measure.

Preferences. Workers consume, and supply inelastically one unit of labor when employed. Workers' preferences are represented by

$$U_0 = \mathbb{E}_0 \left[\int \exp(-\rho t)\, u(c_t)\, dt \right] \tag{2.5}$$

for some discount rate $\rho > 0$ and some isoelastic utility $u(c) \equiv \frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$.

Reallocation frictions. We assume that the process of labor reallocation is *slow* and *costly*. Reallocation is slow for three reasons. First, existing generations of workers are given the opportunity to reallocate to another occupation with intensity λ.[11] Second, workers who reallocate across occupations enter a temporary state of non-employment which they exit at rate $\kappa > 0$.[12] This state can be interpreted either as involuntary unemployment due to search frictions or a temporary exit from the labor force during which workers retrain for their new occupation. Third, new generations of workers enter the labor market gradually at rate $\chi < \lambda$ — at which point they can choose any occupation.[13] Finally, reallocation is costly for two reasons. First, workers do not produce while not employed. Second, they incur a permanent productivity loss $\theta \in (0, 1]$ after they have reallocated to a new occupation, i.e. $h'_t \neq h$. This productivity loss captures the workers' skill specificity, i.e. the lack of transferability of their skills across occupations. Thus, workers' productivity evolves as

$$\xi_t = \mathbf{1}_{\{e_t=1\}} \zeta_t \quad \text{with} \quad \zeta_t = \begin{cases} \lim_{\tau \uparrow t} \zeta_\tau & \text{if } h'_t(\mathbf{x}) = h \\ (1-\theta) \times \lim_{\tau \uparrow t} \zeta_\tau & \text{otherwise} \end{cases} \tag{2.6}$$

with $\zeta_t \equiv e_t \equiv 1$ at birth. In turn, the employment status is $e_t = 1$ at birth, switches to $e_t = 0$ upon reallocation, and reverts $e_t = 1$ upon exiting unemployment at rate κ.

[11] That is, workers' mobility decision is purely time-dependent, which delivers tractable expressions. We allow for state-dependent mobility in our quantitative model (Section 7).

[12] This assumption follows Alvarez and Shimer (2011). Empirically, reallocation across occupations/sectors raises unemployment (Lilien, 1982; Chodorow-Reich and Wieland, 2020).

[13] We introduce overlapping generations because new cohorts achieve a substantial share of labor reallocation across occupations (Adão et al., 2020; Hobijn et al., 2020; Porzio et al., 2020). We assume that new generations enter at a sufficiently low rate for existing workers to reallocate in equilibrium.

Occupational choices. Occupational decisions for existing or new generations consist of choosing the occupation with the highest value

$$\max_{h' \in \{A,N\}} V_t^{h'} \left(\mathbf{x}' \left(h'; \mathbf{x} \right) \right) \tag{2.7}$$

where $V_t^h(\cdot)$ denotes the continuation value associated to automated and non-automated occupations. For existing generations, the state $\mathbf{x}'(h'; \mathbf{x})$ captures the unemployment or retraining spells that displaced workers go through and the permanent productivity loss they experience. Newborns are subject to neither of those.

2.3 Resource Constraint

The occupation outputs are given by

$$y_t^h = F^h \left(\int \mathbf{1}_{\{h(\mathbf{x})=h\}} \xi d\pi_t \right) \tag{2.8}$$

for each occupation h. Finally, the aggregate resource constraint is

$$G^* \left(\left\{ \mu_t^h \right\} \right) = \int c_t(\mathbf{x}) d\pi_t \tag{2.9}$$

where $c_t(\mathbf{x})$ is the consumption of a worker with idiosyncratic state $\mathbf{x}$.

3 Efficient Allocation

We now characterize the set of efficient allocations. We state the planner's problem in Section 3.1 and characterize its solution in Section 3.2.

3.1 First Best Problem

The planner faces two choices: how to reallocate labor after automation has taken place (ex post); and the degree of automation (ex ante). We start by discussing our choice of Pareto weights. We then state the ex post problem, before turning to the ex ante counterpart. In the following, we let $\phi^A \equiv \phi$ and $\phi^N \equiv 1 - \phi$ denote the mass of automated and non-automated occupations, respectively.

Pareto weights. We allow the planner to place different weights $\boldsymbol{\eta} \equiv \left\{ \eta_s^h \right\}$ on workers

based on their birth period and their initial occupation of employment.[14] The dispersion of these weights *within* a generation determines the strength of the equity motive. Without loss of generality, we suppose that these weights add up to 1. In the following, we let $\bar{\eta}_s > 0$ denote the *average* weight within a given generation with $\bar{\eta}_s^{\frac{1}{\sigma}} \equiv \sum_h \phi^h \left(\eta_s^h\right)^{\frac{1}{\sigma}}$.

Ex post. We start with the efficient reallocation of labor after automation has occurred. The planner solves

$$V^{FB}(\alpha;\boldsymbol{\eta}) \equiv \max_{\{\mathbf{c}_t, m_t, \hat{m}_t, \boldsymbol{\mu}_t\}_{t\geq 0}} \sum_h \phi^h \int_{-\infty}^{+\infty} \eta_s^h \int_0^{+\infty} \exp\left(-(\rho+\chi)\,t\right) u\left(c_{s,t}^h\right) dtds \tag{3.1}$$

subject to the constraints (3.2)–(3.7) below. Here, $c_{s,t}^h$ denotes the consumption in period t of the generation born in period s and initially located in an occupation $h \in \{A, N\}$. First, an allocation must satisfy the resource constraint

$$C_t = G^\star\left(\boldsymbol{\mu}_t; \alpha\right) \tag{3.2}$$

where

$$C_t \equiv \sum_h \phi^h \int_0^{+\infty} \chi \exp\left(-\chi s\right) c_{s,t}^h ds \tag{3.3}$$

denotes aggregate consumption. The laws of motion for *effective* labor supplies $\boldsymbol{\mu}_t \equiv \left\{\mu_t^A, \mu_t^N\right\}$ — accounting for productivity losses upon reallocation — are

$$d\mu_t^A = -\left(\lambda m_t + \chi \hat{m}_t\right) \mu_t^A dt \quad \text{with} \quad \mu_0^A = 1 \tag{3.4}$$

in automated occupations $h = A$, for some $m_t, \hat{m}_t \in [0,1]$, and

$$d\mu_t^N \equiv -\frac{\phi}{1-\phi} d\mu_t^A - d\tilde{\mu}_t - \theta d\hat{\mu}_t \quad \text{with} \quad \mu_0^N \equiv 1 \tag{3.5}$$

in the non-automated occupations $h = N$, with

$$d\tilde{\mu}_t = \left(\lambda \frac{\phi}{1-\phi} m_t \mu_t^A - (\kappa + \chi)\,\tilde{\mu}_t\right) dt \quad \text{with} \quad \tilde{\mu}_0 = 0 \tag{3.6}$$

$$d\hat{\mu}_t = \left(\kappa \tilde{\mu}_t - \chi \hat{\mu}_t\right) dt \quad \text{with} \quad \hat{\mu}_0 = 0 \tag{3.7}$$

for each $t \geq 0$. From (3.4), we see that labor reallocation happens through two margins:

[14] By convention, we treat symmetrically all workers who are not born yet in period $t = 0$, i.e. $\eta_s^A = \eta_s^N = \eta_s$ for all $s > 0$. Thus, we can treat the mass of workers indexed by (h, s) as constant, conditional on survival.

the reallocation of existing generations (at rate λ) or the entry of new generations (at rate χ). The planner chooses the shares m_t and $\hat{m}_t$ of each of these workers to reallocate. From (3.6), we see that existing generations who reallocate enter a temporary pool of unemployed which they leave gradually – either their unemployment ends (at rate κ) or they are replaced by a new generation (at rate χ). At that point, they become active in their new occupation (3.7) but incur a productivity loss ($\theta \leq 1$) and are replaced gradually (at rate χ). From (3.5), we see how the effective labor supply in non-automated occupations evolves. The term Θ_t captures the productivity distorsion due to unemployment and productivity losses.

The planner increases output by closing the wedge between the marginal productivities of labor across occupations $\left(\frac{\partial}{\partial \mu^A} G^\star(\cdot) \text{ and } \frac{\phi}{1-\phi}\frac{\partial}{\partial \mu^N} G^\star(\cdot)\right)$. It does so by reallocating workers. But using each margin of labor reallocation has a different cost. When reallocating existing generations, there is a productivity loss ($\theta > 0$), the foregone production while unemployed ($1/\kappa > 0$), and the delay in closing the wedge by waiting for workers to slowly reallocate ($1/\lambda > 0$) and exit unemployment ($1/\kappa > 0$). When reallocating new generations, there is the delay in closing the wedge by waiting for them to slowly enter the labor market ($1/\chi > 0$). The condition in Assumption 2 guarantees that these costs are such that reallocation happens through both margins at the first best.

Ex ante. We now turn to the efficient automation decision. The planner solves

$$\max_{\alpha \geq 0} V^{FB}(\alpha; \boldsymbol{\eta}) \tag{3.8}$$

Automation creates a wedge between the marginal productivities of labor across occupations. The production possibility frontier expands as labor gets reallocated between those. The efficient level of automation maximizes the present discounted value of the additional output that this reallocation allows, given reallocation frictions $(\lambda, \kappa, \chi, \theta)$ and the cost of automation.

Finally, we impose regularity assumptions to rule out corner solutions. These are needed for a meaningful discussion of automation and labor reallocation. First, we assume that the cost of automation is such that there is positive but partial automation at the first best.[15] Second, we suppose that parameters governing reallocation costs — i.e. the average unemployment or retraining duration $(1/\kappa)$ and the productivity loss (θ) — are small

[15] The first part of Assumption 2 is satisfied whenever the cost of automation $F(1) - \hat{F}(1; \alpha)$ is sufficiently convex — i.e. more and more workers are diverted from production as the level of automation increases.

enough that reallocation takes places at the first best.

Assumption 2 (Interior solutions). *The direct effect of automation $G^{\star}(\mu, \mu'; \alpha)$ is concave in α and satisfies $\partial_{\alpha} G^{\star}(\mu, \mu'; \alpha)|_{\alpha=0} > 0$ and $\lim_{\alpha \to +\infty} \partial_{\alpha} G^{\star}(\mu, \mu'; \alpha) = -\infty$ for any $0 \leq \mu < 1$ and $\mu' > 1$. Finally, the average unemployment duration $(1/\kappa)$ and the productivity loss (θ) are sufficiently small*

$$\theta \leq 1 - \frac{Z^A}{\int_0^{+\infty} (1 - \exp(-\kappa t)) Z_t^N dt}$$

that labor reallocation takes place at the first best. The coefficients Z_A and $\{Z_{N,t}\}$ are defined in Appendix A.1, and are positive, exogenous, and independent of (θ, κ).

3.2 Efficient Automation and Reallocation

We now characterize labor reallocation and automation at the first best. We define the following felicity function

$$U_t(C_t; \boldsymbol{\eta}) \equiv \beta_t(\boldsymbol{\eta}) u(C_t) \tag{3.9}$$

with

$$\beta_t(\boldsymbol{\eta}) \equiv \chi \int_0^{+\infty} \bar{\eta}_{t-s} \exp(-\rho(s-t) - \chi s) u \left(\frac{(\bar{\eta}_{t-s} \exp(-\rho s))^{\frac{1}{\sigma}}}{\chi \int_0^{+\infty} \exp(-\chi\tau) (\bar{\eta}_{t-\tau} \exp(-\rho\tau))^{\frac{1}{\sigma}} d\tau} \right)^{1-\sigma} ds$$

We focus on aggregate, first best allocations $X_t \equiv \{C_t, \mu_t, \Theta_t\}$. It is understood that the planner can choose any set of individual allocations that satisfy (3.3).

Reallocation. We begin by characterizing the solution to the ex post problem — the efficient reallocation of labor *fixing* the degree of automation α.

Proposition 1 (Efficient labor reallocation). *An aggregate allocation $\{X_t\}$ is part of an efficient allocation if and only if there exist two stopping times (T_0^{FB}, T_1^{FB}) with $0 < T_0^{FB} < T_1^{FB} < +\infty$ such that: (i) existing generations reallocate to non-automated occupations until T_0^{FB} and new generations until T_1^{FB}; (ii) after T_1^{FB}, new generations are allocated so that marginal productivities are equalized across occupations; and (iii) these stopping times satisfy the smooth pasting conditions*

$$\int_{T_0^{FB}}^{+\infty} \exp(-\rho t) U_t'(C_t; \boldsymbol{\eta}) \Delta_t dt = 0 \quad \text{and} \quad \mathcal{Y}^N_{T_1^{FB}} = \mathcal{Y}^A_{T_1^{FB}} \tag{3.10}$$

where

$$\Delta_t \equiv \underbrace{\exp\left(-\chi t\right)}_{OLG}\left\{\underbrace{\left(1-\theta\right)}_{\text{Productivity cost}}\underbrace{\left(1-\exp\left(-\kappa\left(t-T_0\right)\right)\right)}_{\text{Unemployment spell}}\mathcal{Y}_t^N-\mathcal{Y}_t^A\right\} \tag{3.11}$$

for all $t \geq T_0^{FB}$ *denotes the response of output to labor reallocation,* $C_t = G^\star\left(\boldsymbol{\mu}_t;\alpha\right)$ *denotes aggregate consumption, and* $\mathcal{Y}_t^h \equiv 1/\phi^h \partial_h G^\star\left(\boldsymbol{\mu}_t;\alpha\right)$ *denotes the marginal productivity of labor.*

Proof. See Appendix A.1. □

Automation drives a wedge between the marginal productivities of labor in the two occupations, i.e. $\mathcal{Y}_0^A < \mathcal{Y}_0^N$. The planner reallocates workers between those to increase output. Effective labor supplies evolve as

$$\mu_t^A = \exp\left(-\lambda \min\left\{t, T_0^{\text{FB}}\right\} - \chi t\right) \tag{3.12}$$

$$\mu_t^N = 1 - \mu_t^A + \left(\theta - 1\right)\frac{\phi}{1-\phi}\exp\left(-\chi t\right)\left\{1-\exp\left(-\lambda \min\left\{t, T_0^{\text{FB}}\right\}\right)\right\} \tag{3.13}$$

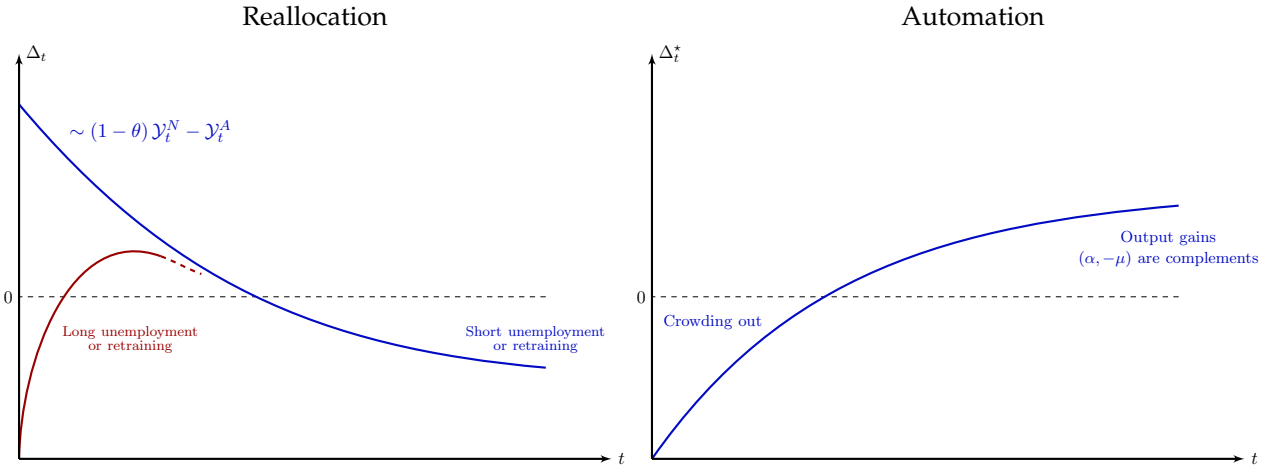
for all $t \in \left[0, T_1^{\text{FB}}\right)$ with no unemployment or retraining, or as (A.7)–(A.10) in the general case. Initially, both existing generations (at rate λ) and new generations (at rate χ) are assigned to non-automated occupations. This adjustment process is slow and labor misallocation declines gradually, i.e. $\mathcal{Y}_0^A/\mathcal{Y}_0^N < \mathcal{Y}_t^A/\mathcal{Y}_t^N$ for $t > 0$. In period $t = T_0^{FB}$, the planner finds its optimal to stop reallocating existing generations since they experience a productivity loss when moving. New generations continue to be assigned to non-automated occupations until period $t = T_1^{FB}$. After that, the planner allocates new generations across occupations so as to keep the marginal productivities equal across those.

The left panel of Figure 3.1 illustrates the dynamics of the output response to reallocation Δ_t for the case of no overlapping generations $(\chi \to 0)$. This response governs the incentives to reallocate workers displaced by automation, as seen in equation (3.10). When unemployment / retraining spells are short $(1/\kappa \to 0)$, the flows Δ_t are *front-loaded.* The response is initially positive and then gradually declines as more workers enter non-automated occupations. At longer horizons the flows become negative $(\lim_{t\to+\infty}\Delta_t < 0)$, as the productivity of non-automated occupations is depressed by the productivity loss due to skill specificity $(\theta > 0)$.[16] On the contrary, the flows Δ_t are *back-loaded* when unemployment spells are sufficiently long. The response is negative at short horizons since displaced workers who reallocate do not produce while unemployed. It becomes positive

[16] In the general case with overlapping generations, this distorsion decreases and vanishes asymptotically as these workers are replaced by new, more productive generations.

later on, as workers exit unemployment and produce in non-automated occupations, and then negative again at long horizons due to the productivity loss.

Figure 3.1: Impulse responses of output to reallocation and automation



Automation. We now turn to the ex ante problem — the efficient degree of automation α^{FB}. The next proposition characterizes this choice.

Proposition 2 (Efficient automation)**.** *The degree of automation α^{FB} is unique and interior. A necessary and sufficient condition is*

$$\int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t; \boldsymbol{\eta})\, \Delta_t^\star dt = 0 \tag{3.14}$$

where

$$\Delta_t^\star \equiv \frac{\partial}{\partial \alpha} G^\star\left(\boldsymbol{\mu}_t; \alpha^{FB}\right) \tag{3.15}$$

for all $t \geq 0$ denotes the response of output to automation, with consumption $\{C_t\}$ and the labor supplies $\{\boldsymbol{\mu}_t\}$ are given by Proposition 1 when evaluated at α^{FB}.

Proof. See Appendix A.2 □

An increase in the degree of automation α has two effects on output (net of automation costs). First, automation increases output as labor gradually reallocates to non-automated occupations with a higher labor productivity.[17] Second, automation comes at a cost that diverts resources away from consumption. The exact time profile of these benefits and costs depends on the degree of complementarity between automation and reallocation. We maintain the following assumption throughout.

[17] The formulation (2.4) also allows for direct productivity gains through automation.

Assumption 3 (Complementarity)**.** *Automation and labor reallocation are complements. That is,*

$$\hat{G}(\mu, \alpha) \equiv G^{\star}(\mu, 1 + \Theta(1 - \mu); \alpha)$$

has decreasing differences in (μ, α) *for all* $\mu \in (0, 1)$ *and* $\Theta \in (0, 1]$.

This assumption ensures that the gains from automation are realized gradually, as more workers reallocate to occupations where the marginal productivity of labor is higher. This assumption is satisfied in our quantitative model (Section 7) where we adopt standard functional forms. Figure 3.1 depicts the returns on automation Δ_t^* in this case. Automation crowds out consumption early on, but eventually expands the production possibility frontier as labor reallocates. In other words, the returns on automation are *backloaded*.

4 Decentralized Equilibrium

We now turn to the decentralized equilibrium. We first describe the problem of a representative firm which chooses automation and labor demands. We next describe the workers' problem, including the assets they trade, the frictions they face and their sources of incomes. Finally, we define a competitive equilibrium.

4.1 Firms

The representative firm chooses the degree of automation α and labor demands $\boldsymbol{\mu}$ to maximize the value of its equity

$$\max_{\alpha \geq 0} \hat{V}(\alpha) \quad \text{with} \quad \hat{V}(\alpha) \equiv \int_0^{+\infty} Q_t \hat{\Pi}_t(\alpha)\, dt \tag{4.1}$$

where $\{Q_t\}$ is the appropriate stochastic discount factor,[18] and

$$\hat{\Pi}_t(\alpha) \equiv \max_{\boldsymbol{\mu} \geq 0} G^{\star}(\boldsymbol{\mu}; \alpha) - \phi \mu^A w_t^A - (1 - \phi)\, \mu^N w_t^N \tag{4.2}$$

are optimal profits given equilibrium wages $\{w_t^h\}$.

[18] Equity is implicitly priced by workers who are marginally unconstrained.

4.2 Workers

We now specify the assets that workers trade, and the constraints they face beyond reallocation frictions.

Assets and states. Workers save in bonds available in zero net supply. We suppose that financial markets are incomplete: workers are unable to trade contingent securities against the risk that their occupation becomes automated.[19] We suppose that workers initially employed in automated occupations form a large household. This allows them to achieve full risk sharing against the risk of being allowed to move across occupations (at rate λ) or not.[20] Workers trade annuities (Blanchard, 1985; Yaari, 1965) against the risk of their death. Workers are now indexed by five idiosyncratic states: their holdings of bonds (a); their initial occupation of employment (h); their age (s); their idiosyncratic productivity (ξ); and their employment status (e). We let $\mathbf{x} \equiv (a, h, s, \xi, e)$ denote the vector of idiosyncratic states and π denote the associated measure.

Budget constraint. Worker's flow budget constraint is

$$da_t(\mathbf{x}) = \left[\mathcal{Y}_t^\star(\mathbf{x}) + (r_t + \chi)\, a_t(\mathbf{x}) - c_t(\mathbf{x})\right] dt \tag{4.3}$$

where $\mathcal{Y}_t^\star$ denotes total income consisting of labor income, profits and taxes, $r_t \geq 0$ denotes the return on savings, and c_t denotes individual consumption. The initial condition is $a_s(\mathbf{x}) = a^{\text{birth}}(\mathbf{x})$ at birth, where $a^{\text{birth}}(\mathbf{x})$ is the stock of assets inherited by a given generation.

Borrowing frictions. Workers are subject to borrowing constraints

$$a_t(\mathbf{x}) \geq \underline{a} \tag{4.4}$$

where the borrowing limit is $\underline{a} \leq 0$.

[19] We rule out complete markets for two reasons: financial markets participations is limited in practice (Mankiw and Zeldes, 1991; Heaton and Lucas, 2000); and workers' equity holdings are typically not hedged against their employment risk (Poterba, 2003; Massa and Simonov, 2006; Huberman, 2001). The absence of contingent securities (or lump sum tranfers) is precisely what motivates the literature on the regulation of automation (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018). The equilibrium would be efficient if workers could trade contingent securities before automation risk is realized.

[20] This assumption prevents an artificial dispersion in the distribution of asset holdings. It allows us to retain tractability and abstract from insurance considerations at this point. We relax this assumption in our quantitative model (Section 7).

Income and occupational choice. Total income consists of effective labor income $\hat{\mathcal{Y}}_{s,t}^h$, profits Π_t and lump sum taxes $T_t(\mathbf{x})$.[21] That is,

$$\mathcal{Y}_t^\star(\mathbf{x}) = \hat{\mathcal{Y}}_{s,t}^h + \Pi_t - T_t(\mathbf{x}) \tag{4.5}$$

Profits are

$$\Pi_t \equiv G^*(\boldsymbol{\mu}_t;\alpha) - \int \mathbf{1}_{\{h(\mathbf{x})=h\}} \xi w_t^h d\pi_t \quad \text{with} \quad \mu_t^{h'} = \frac{\int \mathbf{1}_{\{h=h'\}} \xi d\pi_t}{\phi^h} \tag{4.6}$$

For simplicity, we suppose that profits are claimed symmetrically — all our results carry through if we assume that workers displaced by automation claim no profits.[22] To retain tractability and prevent a source of dispersion in the wealth distribution, we suppose that the returns on annuities are taxed lump sum by the government $T_t(\mathbf{x}) \equiv \chi a_t(\mathbf{x})$ and rebated to new generations.[23] Finally, workers still face the occupational choice (2.7) with the values now indexed by the new idiosyncratic state $\mathbf{x}$.

4.3 Equilibrium

The rest of the model is unchanged. The resource constraint is still given by (2.8)–(2.9). The wages that ensure labor market clearing in each occupation are

$$w_t^h \equiv 1/\phi^h G_h^\star(\boldsymbol{\mu}_t;\alpha) \tag{4.7}$$

All agents act competitively. We choose the price of the final good as *numéraire*. A competitive equilibrium is defined as follows.

Definition 1 (Competitive equilibrium)**.** A competitive equilibrium consists of a degree of automation α, and sequences for effective labor supplies $\{\mu_t^h\}$, consumption and savings functions $\{c_t(\mathbf{x}), a_t(\mathbf{x})\}$, interest rate, wages, profits and incomes $\{r_t, \{w_t^h\}, \Pi_t, \{\mathcal{Y}^\star(\mathbf{x})\}\}$, and distributions of states $\{\pi_t(\mathbf{x})\}$ such that: (i) automation and labor demands are consistent with the firm's optimization (4.1)–(4.2); (ii) consumption and savings are consistent

[21] A worker's effective labor income is not indexed by their reallocation history because they are insured against idiosyncratic reallocation risk.

[22] We implicitly assume that all workers are endowed with one unit of equity, which is fully illiquid. That is, we effectively impose two restrictions. First, workers are unable to hedge ex ante against the risk that their occcupation becomes automated. Second, workers are limited in their ability to self-insure ex post by selling their equity after being affected by automation. These two restrictions are in-line with the evidence discussed in footnote 19.

[23] We relax this last assumption in our quantitative model (Section 7).

with workers' optimization; (iii) interest rates ensure that the resource constraint is satisfied

$$\int c_t(\mathbf{x})\, d\pi_t = G^*(\boldsymbol{\mu}_t; \alpha); \tag{4.8}$$

(iv) wages, profits and incomes are given by (4.5)–(4.7); and (v) the distribution of states evolves consistently with workers' choices and the law of motion of productivity (2.6).

5 Inefficient Automation

We now show that automation is inefficient when reallocation and borrowing frictions are sufficiently severe. Section 5.1 proves that the equilibrium is inefficient and discusses the role of the frictions. Section 5.2 explains why automation and labor reallocation are inefficient. Section 5.3 contrasts our results to the existing literature.

5.1 Inefficient Equilibrium

We now state the first main result of this paper. The decentralized equilibrium is inefficient when reallocation is slow and borrowing constraints are tight.[24]

Proposition 3 (Failure of First Welfare Theorem)**.** *The laissez-faire equilibrium is inefficient if and only if reallocation frictions* (λ, κ, χ) *and borrowing frictions* $(\underline{a})$ *are such that* $a^\star(\lambda, \kappa, \chi) < \underline{a} \leq 0$ *for some threshold* $a^\star(\cdot)$ *defined in Appendix A.4. The threshold satisfies* $a^\star(\lambda, \kappa, \chi) < 0$ *— i.e., inefficiency can occur — if and only if labor reallocation is slow* $1/\lambda > 0$ *or* $1/\kappa > 0$ *or* $1/\chi < +\infty$*.*

Proof. See Appendix A.4. □

Figure 5.1a illustrates this result in the space of reallocation frictions $(1/\lambda)$ and borrowing frictions $(\underline{a})$.[25] This space is partitioned in two main regions. The equilibrium is efficient as long as the frictions fall in the white region — that is $\underline{a} \leq a^\star(\cdot)$. This occurs when either reallocation is sufficiently fast or borrowing constraints are sufficiently loose. In constrast, the equilibrium is inefficient when the frictions fall into either one of the colored regions — that is $\underline{a} > a^\star(\cdot)$.[26]

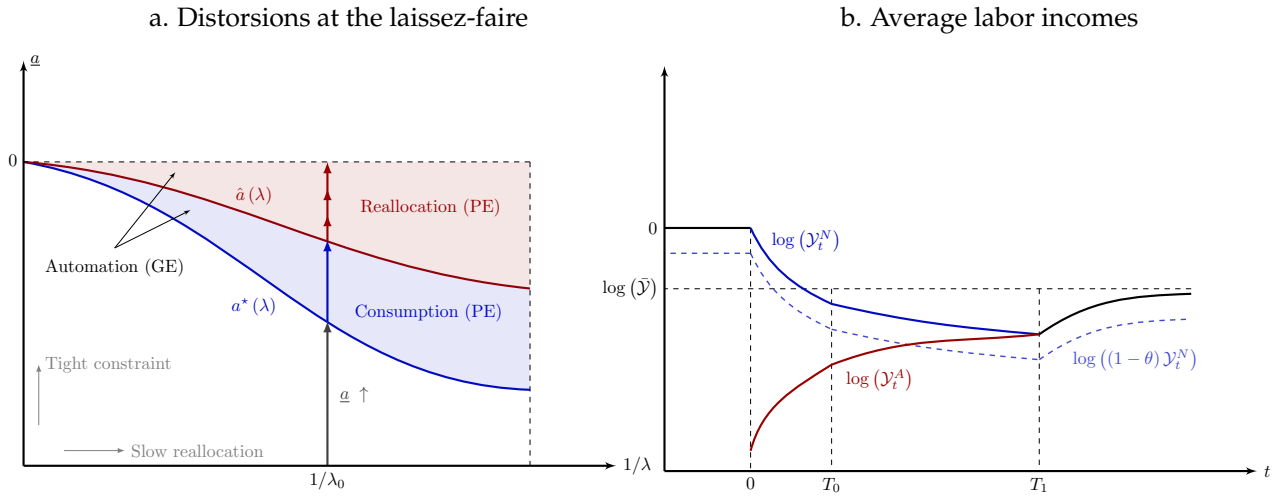
[24] Throughout, we continue to assume that workers incur a productivity loss $\theta > 0$ when moving across occupations (Section 2.2).

[25] For exposition, we focus on the effect of the slow arrival of mobility opportunities $(1/\lambda)$ and abstract from the other forms of slow reallocation $1/\kappa \to 0$ and $1/\chi \to +\infty$.

[26] It should be noted that the threshold $a^\star(\lambda)$ is non-monotonic in its arguments. In particular, $\lim_{1/\lambda \to +\infty} a^\star(\lambda) = 0$ when existing workers never reallocate (as in Guerreiro et al., 2017).

To understand the nature of this inefficiency, we momentarily adopt a partial equilibrium (PE) approach and fix the sequence of prices that prevail in an efficient economy. This allows us to focus on how reallocation and borrowing frictions direcly affect workers' consumption, savings and reallocation decisions. The discussion remains informal at this point. We formalize these insights in Appendix A.10.

Figure 5.1: Laissez-faire: distorsions and labor incomes



Labor incomes for workers born in period s are[27]

$$\hat{\mathcal{Y}}_{s,t}^h = w_t^A + \overbrace{(1-\exp(-\lambda \min\{t, T_0\}))}^{\text{Mass of workers who reallocate}} \overbrace{\left[\Theta_t(\lambda,\kappa) \times (1-\theta) w_t^N - w_t^A\right]}^{\text{Self-insurance through reallocation}} \tag{5.1}$$

when these workers are initially employed in automated occupations, i.e. $h = A, s < 0$, and $\hat{\mathcal{Y}}_{s,t}^h = w_t^N$ otherwise. Here, the term $\hat{\Theta}_t(\lambda,\kappa)$ captures the share of workers who exited unemployment or retraining among those who reallocated (Appendix A.3). Figure 5.1b depicts the paths of the average labor incomes in each of the occupations. When reallocation is *slow*, automation decreases the income of workers initially employed in automated occupations. This decrease is not fully persistent though. Their income slowly rises after they reallocate — it increases from $\mathcal{Y}_t^A$ to $(1-\theta)\mathcal{Y}_t^N$ — or their peers do —$\mathcal{Y}_t^A$ increases over time. This makes workers displaced by automation want to borrow while they reallocate. The following remark states this insight.

[27] The large family of workers initially employed in automated occupations earns w_t^A for each worker who has not relocated, and $(1-\theta) w_t^A$ for each worker who reallocated *and* exited unemployment at rate κ — which is captured by the term $\Theta_t(\lambda,\kappa)$ (Appendix A.3).

Remark 1. *Workers displaced by automation expect their income to increase as they slowly reallocate. This creates a motive for borrowing.*

When reallocation and borrowing frictions are sufficiently mild, workers are never borrowing constrained and the equilibrium is efficient — the white region in Figure 5.1a. As the frictions become more severe, borrowing constraints eventually bind $\underline{a} > a^{\star}(\cdot)$. In this case, workers initially employed in automated occupations are unable to achieve consumption smoothing over $t \geq 0$. So consumption choices are distorted — the blue region in Figure 5.1a. When borrowing constraints are still relatively loose, workers stop being constrained *before* they would have stopped reallocating absent borrowing frictions — see the blue curve in Appendix Figure A.1. In this case, reallocation decisions remain undistorted since they are forward-looking. As the frictions become even more severe, workers remain constrained in the period when they would have stopped reallocating absent borrowing frictions — see the red curve in Appendix Figure A.1. In this case, the reallocation decision becomes distorted too — the red region in Figure 5.1a.

Turning to the general equilibrium (GE), the distortions in consumption and labor allocations cause automation choices to become distorted too, adding to the partial equilibrium distortions of labor reallocation – the colored regions in Figure 5.1a. We elaborate on this point in the next section.

5.2 Why Is Automation Inefficient?

To understand why automation is inefficient, we compare the private and social incentives to automate[28]

$$\text{(LF)} \qquad \int_0^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_{0,t}^N\right)}{u'\left(c_{0,0}^N\right)} \Delta_t^{\star} dt \;\; = 0 \tag{5.2}$$

$$\text{(FB)} \qquad \int_0^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_{0,t}^A\right)}{u'\left(c_{0,0}^A\right)} \Delta_t^{\star} dt \;\; = 0 \tag{5.3}$$

respectively, where

$$\Delta_t^{\star} \equiv \frac{\partial}{\partial \alpha} G^{\star}\left(\boldsymbol{\mu}_t; \alpha\right) \tag{5.4}$$

[28] To obtain (5.3), we use Proposition 2, and the envelope condition (A.39) in Appendix A.4. This expression holds for *any* weights $\boldsymbol{\eta}$ that the planner assigns to workers.

is the response of output to automation, as depicted in Figure 3.1. Firms — just like the government — increase automation until the returns $\Delta_t^\star$ are zero in present discounted value. The (intertemporal) *marginal rate of substitution* (MRS) that they internalize are potentially different, however. Absent borrowing constraints, all workers share the same MRS — which is inversely related to the gross interest rate. In this case, the private and social incentives to automate coincide, and the equilibrium is efficient. When workers in automated occupations become borrowing constrained, their planning horizon is effectively shorter (Woodford, 1990) than their peers' in non-automated occupations. Thus, binding borrowing constraints drive a wedge between the interest rate that firms face when they automate and the MRS of workers displaced by automation.[29] In this case, private and social incentives to automate differ. The following remark states this insight. In Section 6, we further show that automation is *excessive* when automation and labor reallocation are *complements* — the gains from automation are realized gradually as more workers reallocate.

Remark 2. *The degree of automation is inefficient. Firms are effectively too patient and (partly) overlook the time it takes for labor to reallocate and for the benefits of automation to materialize.*

It is worth noting that the economy can be *inefficient* while still achieving *production efficiency* (Diamond and Mirrlees, 1971). This is the case when borrowing and reallocation frictions are sufficiently severe to prevent consumption smoothing, but not sufficiently so to distort the reallocation choices — i.e., the blue region in Figure 5.1a. In this case, displaced workers are still pushed against their borrowing constraints, and the private (5.2) and social (5.3) incentives to automate still differ. However, reallocation is efficient *conditional* on the equilibrium degree of automation. The distorsion in automation simply affects the timing of the output and consumption stream $\{C_t\}$ and the economy moves *along* its production possibility frontier (as opposed to inside). Production efficiency also fails whenever the borrowing and reallocation frictions are particularly severe — i.e., the red region in Figure 5.1a.

5.3 Relation to the Literature

To conclude this section, we draw a connection to two strands of the literature.

[29] This wedge would occur even if the sequence of interest rates is fixed (as in a small open economy). Beyond this wedge, automation is distorted for two additional reasons. We find them less interesting and realistic, but mention them here for completeness. First, when consumption profiles are distorted, so is the MRS of unconstrained non-automated workers and, therefore, the interest rate that firms face. Second, when reallocation is distorted, so are the returns on automation Δ_t^* since automation and reallocation are complements.

Regulation of automation. A burgeoning literature argues that a government should tax automation when it has a preference for redistribution. This literature has worked with *efficient* economies. These efficient benchmarks obtain in our economy in two limit cases. Suppose first that the process of labor reallocation is *instantaneous* — as in Costinot and Werning (2018).[30] In our model, this occurs when workers receive immediately reallocation opportunities $1/\lambda \to 0$, do not go through unemployment or retraining spells when moving between occupations $1/\kappa \to 0$, and no reallocation takes place through new generations $1/\chi \to +\infty$. In this case, the model is static $T_0, T_1 \to 0$ — the economy jumps to its new steady state. To understand why the equilibrium is efficient in this case, it is useful to inspect workers' labor incomes (5.1). When reallocation is instantaneous, workers initially employed in automated occupations flow immediately across occupations so as to ensure $(1-\theta)\, w_t^N = w_t^A$. Workers initially employed in non-automated occupations earn w_t^A. In other words, automation has *distributional* effects. However, there is no motive for borrowing since income changes are fully permanent. As a result, borrowing frictions are irrelevant and the equilibrium is efficient. This explain why slow reallocation is *necessary* for inefficiency.

In turn, suppose that reallocation is slow but there are no borrowing frictions — as in Guerreiro et al. (2017).[31] In our model, this occurs when the borrowing constraints are sufficiently loose $\underline{a} \to -\infty$. In this case, wages remain persistently higher in non-automated occupations, until the gap closes in period $t = T_1$. Therefore, automation has distributional effects and creates a motive, but there is no wedge between the MRS of automated and non-automated workers.

Dynamic inefficiency. An extensive literature argues that capital investment can be dynamically inefficient. This can occur in economies with overlapping generations (Samuelson, 1958; Phelps, 1965; Diamond, 1965), or when precautionary saving depresses the interest rate (Aiyagari, 1995; Aguiar et al., 2021). In these environments, the stock of capital is excessively high in the *long-run* and a planner can achieve a Pareto improvement by redistributing resources across generations. The source of inefficiency is different in our model. In Section 6.4, we extend our baseline environment to allow for gradual investment in automation (as opposed to a one-time choice). We find that the equilibrium is inefficient

[30] The general production technology in Costinot and Werning (2018) effectively allows for labor reallocation between occupations.

[31] In Guerreiro et al. (2017), reallocation taking place (entirely) through new generations $1/\chi < +\infty$. That is, existing generations are not allowed to move in their model. In our model, this corresponds to the case where workers never receive reallocation opportunities $1/\lambda \to +\infty$ or unemployment spells $1/\kappa$ are prohibitively long (Assumption 2).

during the *transition* — while labor reallocates and displaced workers are borrowing constrained — but converges to an efficient allocation in the long-run. The inefficiency that we document relies on the presence of multiple occupations and slow reallocation — two features that the literature on dynamic inefficiency and capital taxation abstracts from.

6 Optimal Policy Interventions

We now discuss optimal policy. We state the Ramsey problem and discuss our choice of policy instruments in Sections 6.1. In Section 6.2, we show that the equilibrium is generically *constrained inefficient* due to pecuniary externalities. In Section 6.3, we show that the government should tax automation on efficiency grounds — even when it does not value equity directly. Section 6.4 draws connections to the literature.

6.1 Ramsey Problem

We start by stating the government's problem in its general form, allowing for a rich set of policy instruments. We then discuss the types of interventions that would implement first best outcomes without a tax on automation. Even if these tools are unrealistic in practice, this discussion clarifies and motivates the restrictions that we impose on the set of policy instruments. Finally, we state the constrained Ramsey problem that we work with in the remaining of this section. For tractability and to obtain more compact expressions, we assume in the following that workers cannot borrow $\underline{a} \to 0$ and abstract from overlapping generations $1/\chi \to +\infty$.[32]

6.1.1 General Problem

We suppose at this point that the government has access to a set of taxes $\{\boldsymbol{\tau}_t\}$. This set includes a distorsionary tax on automation $\{\tau^\alpha\}$, and arbitrary taxes and transfers to redistribute income such as complex lump sum tranfers $\{T_t^h\}$, non-linear income taxes $\{\mathcal{T}_t(\cdot)\}$, severance payments $\{\varsigma_t\}$, etc. The government chooses these taxes to solve

$$\max \sum_h \phi^h \eta^h \int_0^{+\infty} \exp(-\rho t)\, u\left(c_t^h\right) dt \tag{6.1}$$

for a given set of Pareto weights $\boldsymbol{\eta}$, subject to the following implementability constraints. First, consumption and reallocation choices are consistent with workers' optimization,

[32] All the insights carry through in the general case with $1/\chi < +\infty$.

i.e., equations (A.18)–(A.22), (A.26) and (A.28) in Appendix A.3 augmented with taxes. Second, effective labor supplies are given by equations (A.7)–(A.10). Third, automation is consistent with firms' optimality condition (A.36) given taxes. Finally, wages and profits are given by (A.29)–(A.30) and labor incomes are given by (5.1).

6.1.2 Implementing a First Best

The type of inefficiency that we document operates when displaced workers are pushed against their borrowing constraints. If the government has access to a sufficiently rich set of redistributive tools to fully alleviate these borrowing constraints, efficiency can be restored without taxing automation directly. To see this, consider the *wedge* between the optimality conditions for automation at the laissez-faire (5.3) and the first best (5.2), namely[33]

$$\tau^{\alpha} \equiv \int_0^{+\infty} \exp\left(-\rho t\right) \left(\frac{u'\left(c_{0,t}^N\right)}{u'\left(c_{0,0}^N\right)} - \frac{u'\left(c_{0,t}^A\right)}{u'\left(c_{0,0}^A\right)} \right) \Delta_t^{\star} dt \tag{6.2}$$

Suppose for the moment that the government is able to effectively relax workers' borrowing constraints by redistributing income directly. In this case, the MRS of automated and non-automated workers coincide: a first best can be implemented without taxing automation ($\tau^{\alpha} = 0$). Three interventions could in principle achieve this outcome.

First, *targeted* lump sum tranfers $\left\{T_t^h\right\}$ (indexed by worker and time) could implement any efficient allocation — a version of the Second Welfare Theorem holds in our economy (Proposition 10 in Appendix B.5). This type of transfers are unlikely to be feasible in practice, however. They would require the government to know which occupations are automated and discriminate between workers who are displaced and those who are temporarily unemployed. These informational constraints motivate a large literature on optimal income taxation (Piketty and Saez, 2013 for a review) and the existing literature on the regulation of automation (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020).

Second, the government could undo workers' borrowing constraints via *symmetric* transfers $\{T_t\}$ (Araujo et al., 2015). Effectively, the government would borrow on behalf of workers in the short-term and repay its debt later on by taxing them. This intervention could be implemented with a temporary form of negative income tax or Universal Basic Income (Friedman, 1962; Moffitt, 2003). In practice, the fiscal cost is likely to be prohibitive. The payments would have to be given to *all* workers and be generous enough

[33] This wedge corresponds to the linear tax on automation that would implement a particular first best.

to ensure that no worker is constrained — a scenario that the literature on heterogeneous agents has not seriously considered. These programs would also have to be financed with distorsionary taxation with potentially large welfare costs (Daruich and Fernández, 2020; Conesa et al., 2021; Guner et al., 2021; Luduvice, 2021). Future higher taxes could also push some workers into default if they face uninsured idiosyncratic risk (as in our quantitative model), limiting the size of early transfers. The government's ability to relax borrowing constraints could also be limited when the future tax burden tightens these constraints (Aiyagari and McGrattan, 1998).

Finally, it is worth noting that a non-linear income tax $\mathcal{T}_t(\cdot)$ (Mirrlees, 1971; Atkinson and Stiglitz, 1976) or unemployment insurance could benefit displaced workers and help relax their borrowing constraints. However, these interventions would typically not implement first best allocations in practice — e.g., they would reduce labor supply and distort the incentives to reallocate between occupations. In addition, non-linear income taxation is a particularly blunt and costly tool to redistribute resources *across* occupations when there is a relatively large dispersion in incomes *within* occupations due to idiosyncratic shocks (as in our quantitative model).

6.1.3 Constrained Ramsey Problem

In the following, we assume that the government is not able to *fully* alleviate the borrowing constraints of displaced workers. In this case, the equilibrium is inefficient (Section 5.1). For simplicity, we abstract from social insurance programs altogether at this point and re-introduce them later in our quantitative analysis. Instead, we assume that the government has access to a simple set of taxes that depend on calendar time alone: a linear tax on automation τ^{α}; and active labor market interventions (Card et al., 2010 for a survey) that subsidize labor reallocation $\{\varsigma_t\}$.[34] It is understood that these taxes and subsidies can be positive or negative. These instruments are already used in the U.S. and other advanced economies, and do not require the government to know which occupations become automated or which workers are displaced.[35]

The government effectively controls two choices with its instruments: the degree of automation α; and the reallocation of displaced workers T_0. All other choices must be

[34] Because we abstract from social insurance at this point, we suppose that the government requires the large family (Section 4.2) to reimburse any reallocation subsidies received by its members. These subsidies can take the form of credits for retraining programs or unemployment insurance (when positive), or penalties such as imperfect vesting of retirement funds (when negative).

[35] The tax code already treats various forms of capital differently in many countries. In particular, Acemoglu et al. (2020) show that the U.S. tax code already favors investment in automation over investment in labor-augmenting capital.

consistent with workers' and firms' optimality. The government's constrained Ramsey problem reduces to the following primal problem (Lucas and Stokey, 1983).

Lemma 1 (Primal problem). *The government's problem reduces to*

$$\max_{\{\alpha, T_0, \boldsymbol{\mu}_t, \mathbf{c}_t\}} \sum_h \phi^h \eta^h \int_0^{+\infty} \exp\left(-\rho t\right) u\left(c_t^h\right) dt$$

subject to the laws of motion for effective labor

$$\begin{aligned}
\mu_t^A &= \exp\left(-\lambda \min\left\{t, T_0\right\} - \chi t\right) \\
\mu_t^N &= \left\{1 + \left(\theta - 1\right) \frac{\phi}{1-\phi} \exp\left(-\chi t\right) \frac{1 - \exp\left(-\lambda \min\left\{t, T_0\right\}\right)}{1 - \mu_t}\right\} \left(1 - \mu_t^A\right)
\end{aligned}$$

and the consumption allocations

$$c_t^h = \underbrace{1/\phi^h \partial_h G^\star\left(\boldsymbol{\mu}_t; \alpha\right)}_{\text{Initial wage}} + \underbrace{\left(1 - \exp\left(-\lambda \min\left\{t, T_0\right\}\right)\right) \Gamma_t^h}_{\text{Reallocation gains}} + \underbrace{G^\star\left(\boldsymbol{\mu}_t; \alpha\right) - \sum_h \mu_t^A \partial_h G^\star\left(\boldsymbol{\mu}_t; \alpha\right)}_{\text{Profits}},$$

where reallocation gains are

$$\Gamma_t^h \equiv \left(1 - \theta\right) 1/\phi^h \partial_h G^\star\left(\boldsymbol{\mu}_t; \alpha\right) - 1/\phi^A \partial_A G^\star\left(\boldsymbol{\mu}_t; \alpha\right)$$

for each occupation $h \in \{A, N\}$, in the particular case without unemployment / retraining spells $(1/\kappa \to 0)$. The general case is similar but involves the richer laws of motion for effective labor (A.7)–(A.10) and reallocation gains (A.24)–(A.25) in Appendices A.1 and A.3.

6.2 Constrained Inefficiency

We now show that the government should intervene even when its instruments are limited — the equilibrium is *constrained inefficient*.[36] To understand why this is the case, it is useful to compare the private and social inventives to automate and reallocate

$$\begin{aligned}
\int_0^{+\infty} \exp\left(-\rho t\right) u'\left(c_{0,t}^N\right) \Delta_t^\star dt &= -\Phi^\star\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \\
\underbrace{\int_{T_0^{\text{SB}}}^{+\infty} \exp\left(-\rho t\right) u'\left(c_{0,t}^A\right) \Delta_t dt}_{\text{laissez-faire}} &= \underbrace{-\Phi\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right)}_{\text{pecuniary externalities}}
\end{aligned}$$

[36] By definition, an allocation is constrained efficient whenever there exists *some* set of Pareto weights $\boldsymbol{\eta}$ such that the allocation coincides with the solution to (6.1) given these weights $\boldsymbol{\eta}$.

where the terms $\Phi^{\star}(\cdot)$ and $\Phi(\cdot)$ capture the *pecuniary externalities* that automation and reallocation impose on workers — which we define in Appendix A.5. The government takes into account that an increase in automation (α) reduces wages in automated occupations, but increases profits that benefit all workers (or *some* workers when profits are not claimed symmetrically).[37] Similarly, the government takes into account that an increase in reallocation (T_0) reduces wages in non-automated occupations, but lifts wages in automated occupations. Firms and workers do not internalize these effects.

We show in the following that these pecuniary externalities typically do not net out in presence of reallocation and borrowing frictions. Formally, we establish that the laissez-faire is *generically* constrained inefficient in the sense of Geanakoplos and Polemarchakis (1985) and Farhi and Werning (2016).

Proposition 4 (Constrained inefficiency)**.** *Fix the production function $G^{\star}$. Suppose that the* laissez-faire *is constrained efficient for some Pareto weights $\boldsymbol{\eta}$. Then, there exists a perturbation of the production function $G^{\star,\prime} = \mathcal{G}(G^{\star}, \epsilon)$ (with $G^{\star,\prime}(G^{\star}, \epsilon) \to G^{\star}$ uniformly as $\epsilon \to 0$) and a threshold $\bar{\epsilon} > 0$ such that the resulting second best and* laissez-faire *do not coincide for all $0 < \epsilon \leq \bar{\epsilon}$.*

Proof. See Appendix A.5. □

In other words, the government should intervene *regardless* of its preference for redistribution $\boldsymbol{\eta}$. This finding echoes the constrained-inefficiency results in the incomplete markets literature (Lorenzoni, 2008; Farhi and Werning, 2016; Dávila and Korinek, 2018). The nature of the inefficiency is different, however. Constrained inefficiency occurs in our economy despite the absence of uncertainty and incomplete markets, or endogenous borrowing constraints. Instead, it occurs when firms and workers make *technological choices*, and borrowing constraints distort the (shadow) prices that these agents face.[38] It is well-known that technological choices can result in inefficiencies by *themselves* (Acemoglu and Zilibotti, 1997; Acemoglu, 2009). However, our model is set up so that they are a source of inefficiency *only* when borrowing constraints bind, as we have shown in Section 5.

6.3 Taxing Automation on Efficiency Grounds

We now present the second main set of results of this paper, which characterizes and signs optimal policy interventions. We show that the government should tax automation on ef-

[37] Again, all our results carry through in the case where displaced workers do not claim profits. Assuming that profits are claimed symmetrically is conservative, if anything, since the increase in profits partly compensates for the decline in labor income experienced by displaced workers.

[38] Labor reallocation — just like automation — is isomorphic to a technological choice in the Arrow-Debreu construct. Each worker owns a firm that chooses the type of labor services to provide.

ficiency grounds — even when it does not have a preference for redistribution. We start by discussing our choice of Pareto weights.

Pareto weights. Taxing automation has two effects. The first effect is *aggregate*: it generates an intertemporal substitution between current resources (the automation cost, or investments more generally) and future output. The importance of this first effect for welfare (and hence for the government) depends on the distribution of marginal utilities over time — the intertemporal marginal rate of substitution. The second effect is *distributional*: some workers benefit more from this intervention than others through the pecuniary externalities we discussed above. The importance of this second effect depends on the distribution of marginal utilities across workers and the Pareto weights that the government places on each worker. The two effects correspond to the *aggregate efficiency* and *redistribution* components of the welfare decomposition proposed by Bhandari et al. (2021).[39] To highlight the new rationale for policy intervention that we propose, we initially abstract from equity altogether — the second effect. We suppose that the government intervenes *exclusively* to improve aggregate efficiency — the first effect. This is achieved by choosing Pareto weights $\boldsymbol{\eta}^{\text{effic}}$ such that the distributional effects net out when taking into account the worker's marginal utilities and the weigths $\boldsymbol{\eta}^{\text{effic}}$.[40] In particular, these weights are such that the government values constrained workers relatively *less* compared to a utilitarian government that values equity. We reintroduce equity considerations in Section 6.4.

6.3.1 With Active Labor Market Interventions

We are now ready to sign optimal policy interventions. At this point, we continue to assume that the government has the necessary tools to intervene ex post in the labor reallocation process. The following result shows that the government should *curb* (or *tax*) automation on efficiency grounds.

Proposition 5 (Second best)**.** *Suppose that the government controls automation, as well as labor reallocation. Then, curbing automation is optimal.*

Proof. See Appendix A.6. □

[39] Bhandari et al. (2021) decompose the welfare effects of policy changes into: (i) gains in aggregate efficiency from changes in total resources, (ii) gains in redistribution from changes in consumption shares of ex-ante heterogeneous agents, and (iii) gains in insurance from changes in idyosincratic consumption risk. In our baseline model, taxing automation affects welfare via (i) and (ii) alone. In our quantitative model, (iii) is also present.

[40] The details are provided in Appendices A.6–A.7. These weights are inversely related to the workers' marginal utilities. Absent borrowing constraints, these weights take the familiar form $1/\eta^{\text{effic},h} \propto u'\left(c_0^h\right)$.

To understand this result, it is useful to inspect the private and social benefits to automate

$$\text{(LF)} \quad \int_0^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_{0,t}^N\right)}{u'\left(c_{0,0}^N\right)} \Delta_t^\star dt = 0 \tag{6.3}$$

$$\text{(SB)} \quad \int_0^{+\infty} \exp\left(-\rho t\right) \left\{ \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} \right\} \Delta_t^\star dt = 0 \tag{6.4}$$

where $\Delta_t^\star$ is the response of output to automation and is given by (5.4). Firms — just like the government — increase automation until the returns $\Delta_t^\star$ are zero in present discounted value. Their effective (intertemporal) *marginal rate of substitution* (MRS) are different, however. The reason is that the government takes into account the welfare of *all* workers. In contrast, the firm's decisions are based on the equilibrium interest rate which reflects the welfare of unconstrained workers who were initially employed in non-automated occupations. In other words, firms are excessively patient compared to the government.

The direction of the intervention depends on the time profile of $\{\Delta_t^\star\}$. By assumption, automation and reallocation are complements (Assumption 3). Therefore, the flows $\Delta_t^\star$ are *back-loaded* — as depicted in the left panel of Figure 3.1. The firm initially incurs a productivity cost when adopting automation technologies ($\Delta_t^\star < 0$ for small t), and the benefits get realized gradually as labor reallocates between occupations ($\Delta_t^\star > 0$ for large t). Comparing (6.3) and (6.4), it follows that the government prefers a flatter time profile of $\{\Delta_t^\star\}$. Curbing automation achieves so by reducing the cost of automation in the short-run at the cost of smaller productivity gains in the long-run. The following remark states this insight.

Remark 3. *Taxing automation prevents excessive investment and raises consumption early on in the transition, precisely when displaced workers are borrowing-constrained.*

6.3.2 Without Active Labor Market Interventions

In practice, ex post policies can be difficult to implement. Active labor market interventions often produce mixed results (Heckman et al., 1999; Card et al., 2010; Doerr and Novella, 2020), or have unintended consequences for untargeted workers (Crépon and van den Berg, 2016).[41] For this reason, we now suppose that the government controls automation (ex ante) but is unable to control directly labor reallocation (ex post).

[41] This would be the case with *gross* labor flows between occupations — as in our quantitative model.

Proposition 6 (Second best — ex ante only)**.** *Suppose that the government only controls automation — but labor reallocation T_0 must be consistent with workers' optimization. This reinforces the government's desire to curb automation when unemployment / retraining spells are short $(1/\kappa \to 0)$. On the contrary, this reduces the government's desire to curb automation unemployment / retraining spells are long $(1/\kappa > 1/\kappa^\star)$ for some threshold $1/\kappa^\star > 0$.*[42]

Again, it is useful to inspect the social incentives to automate

$$\text{(SB)} \quad \int_0^{+\infty} \exp\left(-\rho t\right) \times \left\{ \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} \right\} \left\{ \Delta_t^\star + T_0'\left(\alpha^{\text{SB}}\right) \Delta_t \right\} dt \ = 0, \tag{6.5}$$

and compare them to the private incentives (6.3). Here, Δ_t denotes the response of output to reallocation, and is defined by (3.11). Missing active labor market interventions provide an additional motive for policy intervention. The government internalizes the indirect effect of automation on output Δ_t due to the reallocation it induces $T_0'(\cdot) > 0$, in addition to the direct effect. Workers's reallocation at the laissez-faire satisfies

$$\text{(LF)} \quad \int_{T_0^{\text{LF}}}^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_{0,t}^A\right)}{u'\left(c_{0,0}^A\right)} \Delta_t dt = 0 \tag{6.6}$$

Absent borrowing constraints, all workers share the same MRS. In this case, the indirect effect of automation Δ_t is no cause for intervention either, given (6.6). When borrowing constraints bind, the private and social incentives to automate differ due to *both* the direct effect $\Delta_t^\star$ and the indirect effect Δ_t. The government should curb automation based on the direct effect (Section 6.3.1). The sign of the indirect effect depends on the duration of unemployment / retraining spells.

When unemployment spells are short $1/\kappa \to 0$, the flows Δ_t are *front-loaded* (see Figure 3.1). Workers enjoy a higher wage after they reallocate $\Delta_0 > 0$, but their new wage declines gradually as more workers enter non-automated occupations $\lim_{t\to+\infty} \Delta_t < 0$. Constrained workers put an excessive weight on early, positive payoffs: binding borrowing constraints incentivize them to rely excessively on mobility as a source of self-insurance. This indirect effect reinforces the government's desire to *curb* automation.

When unemployment spells are sufficiently long, the flows Δ_t are *back-loaded* instead.

[42] The average duration of unemployment spells $1/\kappa$ is bounded above by Assumption 2. In theory, the case where the government curbs automation *less* might not present itself. This is an empirical question that we address with our quantitative model (Section 7).

Workers' earnings decrease during unemployment $\Delta_0 < 0$, before they are paid the wage in their new occupation.[43] Constrained workers put an excessive weight on early, negative payoffs: binding borrowing constraints limit their ability to to use mobility as a source of self-insurance. The indirect effect dampens the government's desire to curb automation, and could in principle lead the government to *stimulate* automation.

6.4 Extensions

To conclude this section, we consider a number of extensions of our baseline model. This allows us to clarify the economic mechanisms at play as well as our contribution relative to the literatures on the taxation of automation on equity grounds and on long-run capital taxation.

6.4.1 Equity Concerns

A growing literature argues that a government should curb automation when it has a preference for redistribution (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). To draw a connection to this literature, we now introduce equity concerns in our model. We denote by $\boldsymbol{\eta}^\star$ the weights that support the decentralized allocation with no borrowing frictions.[44] In turn, we denote by $\left(\eta^{\text{utilit}}\right)_s^{\frac{1}{\sigma}} \equiv \sum_h \left(\eta_s^{h,\star}\right)^{\frac{1}{\sigma}}$ the symmetric weights that a *utilitarian* government would use *within* each generation s.[45] We show below that the government curbs automation in an efficient economy with no borrowing frictions when it has a preference for redistribution.

Proposition 7 (Second best with equity concerns)**.** *Consider the special case of our model with no borrowing frictions — so that the laissez-faire is efficient. Suppose that the government is utilitarian, i.e., uses symmetric weights η^{utilit} within generations. Suppose that the government can either control automation and reallocation, or only automation (with reallocation consistent with workers' optimization). Then, the optimal policy curbs automation.*

Proof. See Appendix A.8. □

Figure 6.1 illustrates this result schematically. Automation has distributional effects: it reduces equity at the laissez-faire (LF) compared to the first best of a utilitarian planner ($\text{FB}^{\text{utilit}}$). Displaced workers are worse off and their marginal utility is (persistently)

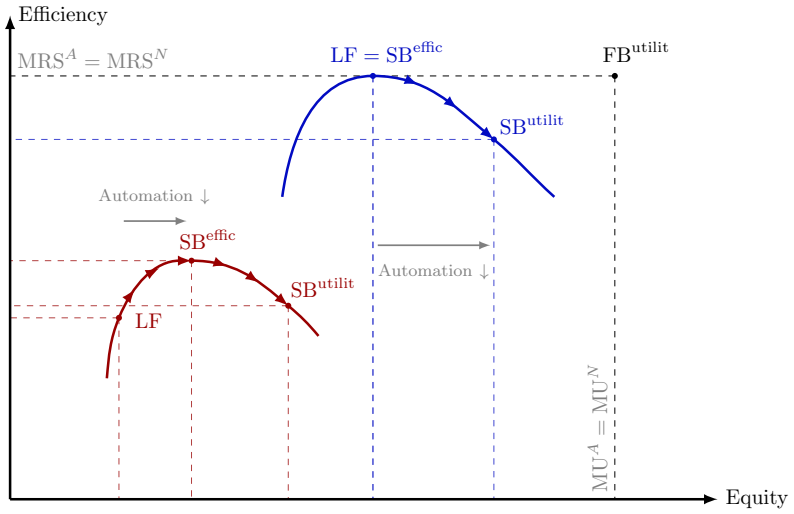
[43] See footnote 42. In the medium term, $\Delta_t > 0$. Eventually, $\lim_{t\to+\infty} \Delta_t < 0$ since workers experience a permanent productivity loss.

[44] These weights exist since the equilibrium is efficient in this case (Proposition 3).

[45] This particular averaging was introduced in Section 3.1, and ensures that the equity motives does not affect the aggregate allocation at the first best — i.e. (3.9).

higher than other workers' $\text{MU}^A > \text{MU}^N$. We consider two economies. The first one is efficient (in blue), which occurs when borrowing constraints are sufficiently loose (Section 5.1). In this case, the (intertemporal) marginal rates of substitutions of displaced workers coincides with the equilibrium interest rate faced by firms who automate $\text{MRS}^A = \text{MRS}^N$. The government does not intervene ($\text{LF} = \text{SB}^{\text{effic}}$) unless it has a preference for redistribution ($\text{SB}^{\text{utilit}}$), in which case it taxes automation and sacrifices efficiency to improve equity. Equity gains can be achieved at a relatively small efficiency cost in this case — an envelope condition applies. This is the canonical trade-off emphasized in the existing literature on the taxation of automation. The second economy is inefficient (in red), which occurs when borrowing constraints are relatively tight. In this case, displaced workers are pushed against their borrowing constraints. This drives a wedge between the (intertemporal) marginal rate of substitution of displaced workers and the equilibrium interest rate faced by firms $\text{MRS}^A > \text{MRS}^N$. Firms are effectively too patient: automation is inefficient. The government can improve *both* efficient and equity by taxing automation — there is no trade-off.

Figure 6.1: Second best with efficiency ($\underline{a} \to -\infty$) and inefficiency ($\underline{a} \to 0$)



6.4.2 Gradual Investment in Automation

An extensive literature on capital taxation with incomplete markets argues that capital should be taxed in the long-run. This literature has proposed two main arguments for taxing capital. First, it can improve insurance against earnings risk by affecting the relative price between labor and capital services (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012). Second, it can improve dynamic efficiency by reducing capital accumulation in economies where the interest rate is reduced by precautionary savings (Chamley, 2001;

Aguiar et al., 2021). These two rationales share two features: they rely on the presence on uninsured idiosyncratic risk; and optimal policies affect investment in the *long-run*.

The rationale that we propose is conceptually distinct. First, we find that taxing automation is optimal even absent idiosyncratic uncertainty. Second, the government should tax automation during the *transition* to the long-run, but has no reason to intervene once labor reallocation is complete. To clarify this point, we now extend our model and allow automation to take place gradually due to convex adjustment costs. Output (net of investment costs) is

$$Y_t = G^* \left(\boldsymbol{\mu}_t; \alpha_t \right) - x_t \alpha_t - \Omega \left(x_t / \alpha_t \right) \alpha_t, \tag{6.7}$$

where x_t is the gross investment rate in automation and $\Omega(\cdot)$ is a convex function. The law of motion of automation is $d\alpha_t = (x_t - \delta \alpha_t)\, dt$ for some depreciation rate $\delta > 0$. Automation is costly as installing (x_t) or maintaining (δ) these technologies diverts resources away from production activities.[46] To obtain realistic long-run dynamics, we reintroduce overlapping generations $1/\chi < +\infty$.

Proposition 8 (No intervention in the long-run)**.** *A (utilitarian) government does not intervene in the long-run* $\alpha_t^{LF} / \alpha_t^{FB} \to 1$ *as* $t \to +\infty$.

Proof. See Appendix A.9. □

The government should *slow down* automation while labor reallocation takes place and workers are borrowing constrained, but has no reason to tax automation in the long-run. This rationale for intervention is specific to automation — as opposed to capital accumulation — since it displaces workers and pushes them against their borrowing constraints.[47,48] It should be noted that a positive long-run tax on automation could still be optimal for reasons that are not captured in this baseline model — e.g., for insurance purposes or to prevent dynamic inefficiency. This parly motivates our quantitative analysis, where we allow for uninsured idiosyncratic risk.

[46] This specification provides a micro-foundation for the cost of automation in our baseline model. In this baseline model, automation takes place once and for all and entails a productivity cost as labor is diverted away from production. In our quantitative model, automation requires maintenance to offset depreciation, which effectively diverts labor away from production. The effective production function for automated occupations — net of investment — is $\hat{F}(\mu; \alpha) \equiv A (\alpha + \mu)^{1-\eta} - (\delta + \Omega(\delta)) \alpha$ at the new steady state.

[47] Empirically, displaced workers have a higher risk of being borrowing constrained (footnote 1).

[48] In theory, workers could also become borrowing constrained after an occupation-specific TFP shock — without ever experiencing an earnings loss. The reason is that expect their earnings to go up in the future after they reallocate to this occupation. In practice, this anticipatory effect is likely to be quantitatively small (Poterba, 1988). The effect might actually be the opposite: workers borrowing constraints are relaxed as their future earnings increase (Jappelli, 1990; Carman et al., 2003).

6.4.3 The Direction of Investments

Firms in our baseline model can only invest in automation technologies. As a result, the optimal tax on automation unequivocally reduces the *level* of investment in new technologies. Consider an extension where firms can also choose to invest in a technology that is Hicks-neutral. In particular, assume that aggregate output is now given by

$$G^{\star}\left(\boldsymbol{\mu};\alpha,A\right)=A\hat{G}\left(\boldsymbol{\mu};\alpha\right)-\psi(\alpha)-\Phi(A),$$

and that firms choose both the degree of automation α and the Hicks-neutral productivity A in order to maximize the present discounted value of profits.

Investments that improve productivity in a Hicks-neutral way do not cause worker displacement or labor reallocation. In turn, the economy's adjustment is instantaneous. The optimal policy for the government may now involve changing the *direction* of investments in new technologies: taxing automation but subdizing Hicks-neutral technological improvements. In practice, this policy can be implemented by discriminating between expenditures associated with automation and more traditional types of capital, similar to how the tax code in many countries already treats various forms of capital differently.

7 Quantitative Model

In the previous sections, we established that the government should slow down automation when reallocation and borrowing frictions are sufficiently severe. In the remaining of this paper, we quantify the welfare gains from optimal policy interventions. To this end, we enrich our baseline model along several dimensions that are potentially important for optimal policy. First, workers are subject to uninsured idiosyncratic earnings risk (Aiyagari, 1994; Huggett, 1993). It is well-known that incomplete markets play an important role for capital taxation, since they create an insurance motive and can contribute to dynamic inefficiency (Section 5.3). Second, workers now face idiosyncratic preference shocks for mobility across occupations (Artuç et al., 2010; Caliendo et al., 2019). This generates gross flows between occupations, which is an important feature of the data (Kambourov and Manovskii, 2008; Moscarini and Vella, 2008) and can create an additional insurance motive to stimulate wages in automated occupations. We also introduce progressive income taxation (Heathcote et al., 2017) and unemployment benefits (Krueger et al., 2016) to account for existing sources of insurance that can benefit displaced workers (Section 6.1.2). Finally, automation now takes place gradually due to convex adjustment costs. As in our baseline model, time is continuous, and there is no aggregate uncertainty. Periods

are still indexed by $t \geq 0$.

7.1 Firms

Production. Occupations are still indexed by $h \in \{A, N\}$. These occupations produce intermediate goods by combining labor, automation and a fixed factor

$$y_t^h = F\left(\mu_t^h; \alpha_t^h\right) = A^h \left(\varphi^h \alpha_t^h + \mu_t^h\right)^{1-\eta} \tag{7.1}$$

for some elasticity $\eta \in (0,1)$ and productivities $A^h, \varphi^h > 0$, with μ_t^h denoting effective labor and $\{\alpha_t^h\}$ denoting automation.[49] We set $\varphi^A \equiv 1$ in automated occupations — so that automation displaces labor *gradually* as investment takes place— and $\varphi^N \equiv 0$ in non-automated occupations. The aggregate technology has a constant elasticity of substitution

$$G\left(\left\{y_t^h\right\}\right) \equiv \left(\sum_h \phi^h \left(y_t^h\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \tag{7.2}$$

for some elasticity $\nu > 1$, where $\phi^A \equiv \phi$ is the mass of automated occupations and $\phi^N \equiv 1 - \phi$ is the mass of non-automated occupations. Automated occupations rent the stock of automation on spot markets (Guerreiro et al., 2017; Costinot and Werning, 2018) at rate $\{r_t^\star\}$ from a mutual fund.

Investment. A mutual fund invests workers' savings in government's bonds and automation. The initial stock of automation is $\alpha_0 = 0$. The law of motion of automation is

$$d\alpha_t = (x_t - \delta\alpha_t)\, dt, \tag{7.3}$$

where δ is the rate at which the automation depreciates, and x_t is the investment rate. Investment is subject to quadratic adjustment cost $\Omega(x_t; \alpha_t) = \omega (x_t/\alpha_t)^2 \alpha_t$ as in (6.7). The government taxes automation linearly at rate $\{\tau_t^x\}$ and rebates the proceedings to the mutual fund. The mutual fund is competitive so that its return is the return on bonds $\{r_t\}$.

[49] That is, labor and automation are perfect substitutes. Each occupation can be interpreted as a task in the frameworks of Acemoglu and Autor (2011) and Acemoglu and Restrepo (2018b). The only difference is that automation is *as* productive as labor in our model, so that labor is not displaced immediately. Instead, labor is crowded out progressively as the stock of automation increases over time.

7.2 Workers

There are still overlapping generations generations of workers that are are replaced at rate $\chi \in [0, +\infty)$. A worker is indexed by five idiosyncratic states: their holdings of liquid assets (a); their current occupation of employment (h); their employment status (e); a binary variable that indicates whether they ever switched between occupations (ξ); and the mean-reverting component of their productivity (z). In the following, we let $\mathbf{x} \equiv (a, h, e, \xi, z)$ denote the workers' idiosyncratic states, and π denote the corresponding measure.

Assets and constraints. Workers now save in two financial assets: a liquid asset (mutual fund) with return $\{r_t\}$, and an illiquid asset (equity). Following Auclert et al. (2018), we suppose that each worker is endowed with one unit of the illiquid asset $\hat{a} \equiv 1$ and is unable to trade it away. This asset pays a random dividend, which is proportional to a worker's mean-reverting productivity (z). In addition, workers have access to annuities (Blanchard, 1985; Yaari, 1965) which allows them to self-insure against survival risk. Financial markets are otherwise incomplete: workers are unable to trade contingent securities against the risk that their occupation becomes automated, against the risk that they are not able to relocate across occupations, against unemployment risk, and against their idiosyncratic productivity risk. Workers now face the flow budget constraint

$$da_t(\mathbf{x}) = \left[\mathcal{Y}_t^{\text{net}}(\mathbf{x}) + (r_t + \chi)\, a_t(\mathbf{x}) - c_t(\mathbf{x})\right] dt \tag{7.4}$$

where $\mathcal{Y}_t^{\text{net}}(\mathbf{x})$ denotes net income and r_t is the return on the liquid asset.[50] Workers still face the borrowing constraint (4.4). They hold $a^{\text{birth}}(\mathbf{x}) = 0$ assets at birth.

Occupational choice. Workers choose their first occupation of employment at birth. During their life, workers supply labor and are given the opportunity to move between automated and non-automated occupations with intensity λ.[51] Workers are subject to linearly additive taste shocks when choosing between occupations. These taste shocks are independent over time and distributed according to an Extreme Value Type-I distribution with mean 0 and variance $\gamma > 0$.[52] In particular, workers choose a *non-automated* occupation

[50] For simplicity, we suppose that the capital income earned through bonds is not taxed.

[51] Our model reduces to a standard income fluctuations problem when $\lambda = 0$ and all workers are employed initially.

[52] This specification is standard in the literature — e.g. Artuç et al. (2010) and Caliendo et al. (2019).

with hazard[53]

$$\mathcal{S}_t(\mathbf{x}) = \frac{(1-\phi)\exp\left(\frac{V_t^N(\mathbf{x}'(N;\mathbf{x}))}{\gamma}\right)}{\sum_{h'}\phi^{h'}\exp\left(\frac{V_t^{h'}(\mathbf{x}'(h';\mathbf{x}))}{\gamma}\right)}, \tag{7.5}$$

where $V_t^h(\cdot)$ denotes the continuation value associated to automated $(h = A)$ and non-automated $(h = N)$ occupations, and the parameter $\gamma > 0$ governs the elasticity of labor supply between those. If workers reallocate between occupations, they enter a state of unemployment or retraining which they exit at rate $\kappa > 0$. Upon exiting unemployment, workers enter their new occupation and experience a permanent productivity loss $\theta \in (0,1)$.[54] Newborns are subject to neither unemployment nor a productivity loss when they choose their first occupation.

Income. Employed workers $(e = E)$ earn a gross labor income

$$\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) = \xi \exp(z)\, w_t^h, \tag{7.6}$$

with the productivity consisting of a permanent component (ξ) and a mean-reverting component (z). The permanent component captures the productivity cost (2.6) that workers incur when reallocating between occupations. The mean-reverting component evolves as

$$dz_t = -\rho_z z_t dt + \sigma_z dW_t \tag{7.7}$$

with persistence $\rho_z^{-1} > 0$ and volatility $\sigma_z > 0$. Following Krueger et al. (2016), we suppose that unemployed workers $(e = U)$ receive benefits that are proportional to the gross labor income they would have earned if they had remained employed in their previous occupation. The replacement rate is $b \in [0,1]$, and we assume that these earnings take the form of home production (Alvarez and Shimer (2011)).[55] Workers net income is

$$\mathcal{Y}_t^{\text{net}}(\mathbf{x}) = \mathcal{T}_t\left(\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) + \exp(z)\,\Pi_t\right)$$

where $\mathcal{T}_t(y) = y - \psi_{0,t} y^{1-\psi_1}$ where $\psi_{0,t}, \psi_1 > 0$ captures progressive taxation (Heathcote et al., 2017).

[53] Our focus is on the reallocation process *between* automated occupations or non-automated occupations. Therefore, we abstract from gross flows *within* those.

[54] Workers experience this productivity loss at most once during their lifetime.

[55] This last assumption is mostly innocuous. Its only purpose is to avoid introducing an additional motive for distorsionary taxation to finance unemployment insurance.

7.3 Policy and Equilibrium

The government's flow budget constraint is[56]

$$dB_t = (T_t + r_t B_t - G_t)\, dt \tag{7.8}$$

where B_t is the government's asset holdings, T_t is total tax revenues and G_t is government spending. The government maintains a constant ratio of spending to GDP G_t/Y_t. It adjusts its stock of assets B_t to maintain a constant ratio of liquidity to GDP $(-B_t + \alpha_t)/Y_t$. The government adjusts the intercept of the linear tax schedule $\psi_{0,t}$ so that its budget clears (7.8). The rest of the model is unchanged. The resource constraint is now

$$\int a_t(\mathbf{x})\, d\pi_t = -B_t + M_t \tag{7.9}$$

where M_t is the equity of the mutual fund. The wages that ensure labor market clearing in each occupation are still given by (4.7). The rental rate of the stock of automation adjusts so that $\alpha_t^A = \alpha_t$. All agents act competitively. We normalize the price of the final good to 1 (*numéraire*). A competitive equilibrium is defined as before.

8 Quantitative Evaluation

We now calibrate the quantitative model and we use it to evaluate the quantitative importance of our mechanisms and perform various optimal policy experiments. Section 8.1 discusses the calibration. Section 8.3 describes our numerical experiments. Finally, Appendix B provides details about our numerical implementation.

8.1 Calibration

We parameterize the model using a mix of external and internal calibration. We first describe the external calibration, before discussing the targeted moments and the parameters we calibrate internally. Table 8.1 shows the parametrization.

External calibration. External parameters are set to standard values in the literature. We interpret our initial stationary equilibrium, i.e. before automation, as the year 1970. We set the initial labor share η to 0.64 based on BLS data. We choose a depreciation rate of

[56] The government also taxes investment in automation. The proceedings are rebated lump sum to occupations.

automation δ of 10%, as in Graetz and Michaels (2018) and Artuç et al. (2020). We set the elasticity of substitution across occupations ν to 0.75, in between the values in Buera and Kaboski (2009) and Buera et al. (2011).[57] We choose an inverse elasticity of intertemporal substitution parameter σ to 2. We set the replacement rate χ to obtain an average active life of 45 years. We pick the unemployment exit hazard parameter κ to match the average unemployment duration in the U.S., as measured by Alvarez and Shimer (2011). The productivity loss θ when moving between occupations is set to match the earnings losses estimated by Kambourov and Manovskii (2009). As in Auclert et al. (2018), we rule out borrowing $\underline{a} = 0$. We use the annual income process estimated by Floden and Lindé (2001) using PSID data and choose the mean reversion ρ_z and volatility σ_z in our continuous time model accordingly. We choose a replacement rate when unemployed b of 0.4, following Shimer (2005) and Ganong et al., 2020. We suppose that the profits are rebated to workers in proportion to their idiosyncratic productivity z, following Auclert et al. (2018).[58] The parameter that governs the elasticity of the progressive tax schedule ψ_1 is 0.181 as in Heathcote et al. (2017). Government spending to private consumption G_t/C_t is 50%. Finally, we choose the ratio of liquidity to GDP $\left(-B_t + M_t\right)/Y_t$ to one-fourth, following Kaplan et al. (2018).

Internal calibration. We calibrate seven parameters internally: the discount rate (ρ); the mobility hazard (λ); the Fréchet parameter (γ); the occupations' productivities $\left(A^h\right)$; the adjustment cost for automation (ω), and the share of automated occupations (ϕ). We pick these parameters to jointly match seven moments. We use the discount rate to target an annualized real interest rate of 2 percent. We adjust the mobility hazard to match an annual occupational mobility rate of 10% per year, which corresponds to the U.S. level in 1970 in Kambourov and Manovskii (2008). We pick the Fréchet parameter so as to obtain an elasticity of labor supply of 0.5 for the stock of workers (i.e. all generations). This is between the 0.26 Hicksian labor supply elasticity in Chetty et al. (2011) and the 0.79 occupational choice elasticity to expected earning changes in Arcidiacono et al. (2020).[59] We obtain a Fréchet parameter of roughly 0.06, which is an order of magnitude smaller

[57] We interpret automated occupations as routine-intensive occupations which are well represented in manufacturing. So we set the elasticity of substitution between the groups of automated and non-automated occupations as broadly corresponding to that between manufacturing and other sectors. Evidence from the structural change literature strongly suggest gross complementarity as the empirically relevant case.

[58] In a one-sector model, this assumption implies that workers claim *both* the wage bill and profits in proportion to their productivity. This assumption is the most neutral possible, in that it ensures that the government has no incentives to regulate automation to improve insurance by affecting the profit share of aggregate output.

[59] We compute this elasticity in our model by simulating a 10% wage increase in one of the occupations and leaving the others unchanged.

than conventional estimates in the literature (Artuç et al., 2010; Hsieh et al., 2019). The reason is twofold: our model features gross flows, so that wage changes are perceived as non-permanent; and old generations have accumulated financial wealth and therefore respond less to perceived changes in human wealth. We choose the occupations' productivity $\{A^h\}$ to normalize output in the initial stationary equilibrium to 1 and ensure that wages at the initial stationary equilibrium are symmetric across automated and non-automated occupations. We set the mass of automated occupations ϕ to target an employment share of 56% in routine occupations in 1970 (Bharadwaj and Dvorkin, 2019). Finally, we choose the adjustment cost for automation ω to match an employment share of 41% in routine occupations in 2015 (Bharadwaj and Dvorkin, 2019), along the laissez-faire transition.

8.2 Automation, Reallocation and Inequality

We start by simulating the transition of our economy to its new stationary equilibrium with automation. The economy is initially at its (unstable) steady state without automation. At that point, investment is prohibitive and no automation takes place.[60] In period $t = 0$, an exogenous increase in automation $(\alpha_0 > 0)$ initiates a convergence to the final (stable) steady state with positive automation. We interpret the first period in our simulation as the year 1990 — when automation was still in its infancy. We choose the initial stock α_0 so that it is roughly $1/4$ of its level in 2015 (Acemoglu and Restrepo, 2020) along the laissez-faire transition.[61]

Aggregate dynamics. Figures 8.1–8.2 illustrate the transition dynamics for key aggregate variables. Automation converges to its steady state with a half-life of roughly 40 years. The rise in automation expands the economy's productive capacity over time, but displaces labor since automation and labor are substitutes. The mass of workers employed in automated occupations declines from 56% to 42% over 25 years — as targeted by our calibration — and to 32% in the long-run. The profit share (the complement of the labor share) initially falls below its steady state value (36%) as the firm invests in automation. It then rises rises gradually to 39% as labor gets reallocated to non-automated occupations where it is more productive.[62]

[60] The reason is that we have assumed quadratic adjustment costs of the form $\Omega(x_t; \alpha_t) = \omega (x_t/\alpha_t)^2 \alpha_t$. This specification is standard in the investment literature (Cooper and Haltiwanger, 2006).

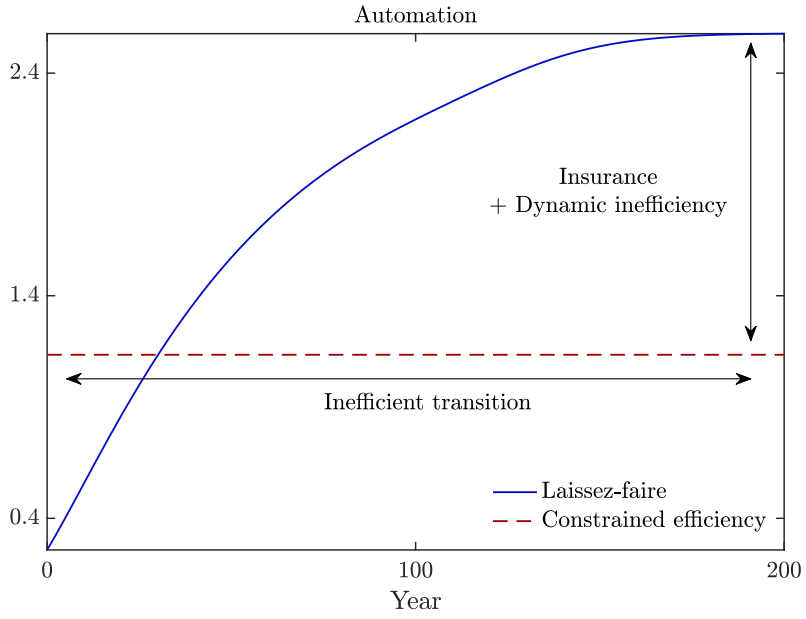
[61] We solve the model non-linearly, so that the rate of convergence over the first decades depends on the initial condition.

[62] In comparison, the labor share (BLS) decreased from 64% in 1970 to 58% in 2019 in the U.S.

Table 8.1: Calibration

Parameter	Description	Calibration	Target / Source
Workers			
ρ	Discount rate	0.13	2% real interest rate
σ	EIS (inverse)	2	-
χ	Death rate	$1/45$	Average working life of 45 years
$\underline{a}$	Borrowing limit	0	Auclert et al. (2018)
Technology			
A^A, A^N	Productivities	$(0.89, 1.26)$	Initial output (1)
η	Initial labor share	0.36	1970 labor share (BLS)
δ	Depreciation rate	0.1	Graetz and Michaels (2018)
ϕ	Fraction of automated occupations	0.53	Routine occs. share in 1970
ω	Adjustment cost	16	Routine occs. share in 2015
ν	Elasticity of subst. across occs.	0.75	(Buera and Kaboski, 2009; Buera et al., 2011)
Mobility frictions			
λ	Mobility hazard	0.49	Occupational mobility rate in 1970
$1/\kappa$	Average unemployment duration	$1/3.2$	Alvarez and Shimer (2011)
$1-\kappa^*$	Probability of return move	0.44	Carrillo-Tudela and Visschers (2020)
θ	Productivity loss from relocation	0.18	Kambourov and Manovskii (2009)
γ	Fréchet parameter	0.06	Elasticity of labor supply
Government			
G/C	Gvt spending / consumption	0.50	BEA
ψ_1	Progressive taxation	0.181	Heathcote et al. (2017)
$(-B+M)/Y$	Liquidity / GDP	0.26	Liquid assets / GDP (Kaplan et al., 2018)
Income process			
ρ_z	Mean reversion	0.0228	Floden and Lindé (2001)
σ_z	Volatility	0.1025	Floden and Lindé (2001)
b	Replacement rate	0.4	(Shimer, 2005; Ganong et al., 2020)

Figure 8.1: Automation



Individual allocations. Figure 8.3 describes the distributional effects of automation for workers initially employed in automated and non-automated occupations. Investment in automation crowds out consumption initially. The decline in consumption is larger for workers initially employed in automated occupations. The reason is that automation decreases the marginal productivity of labor in these occupations, increases the supply of these goods — and hence depresses their price — and forces workers into unemployment. As time passes, workers reallocate across occupations and average consumption converges across these two groups. Importantly, the consumption profile is steeper for workers initially employed in automated occupations. This reflects the higher share of hand-to-mouth (HtM) workers in these occupations, as they borrow to self-insure against their income drop and anticipate that their future reallocation across occupations will improve their earnings in the medium-run.[63] As a result, workers initially employed in automated occupations are more *impatient* than their peers in non-automated occupations. This wedge in intertemporal marginal rates of substitution (MRS) is a source of inefficiency that distorts automation choices.

Long run taxation. The rationale for intervention that we propose in this paper concerns the *transition* to the long-run steady state, while labor reallocates gradually and automated

[63] The share of HtM workers is roughly 31% at the initial steady state. This figure is in line with the estimated found in the literature on heteregeneous agent models (Kaplan et al., 2018; Aguiar et al., 2020).

Figure 8.2: Aggregate dynamics

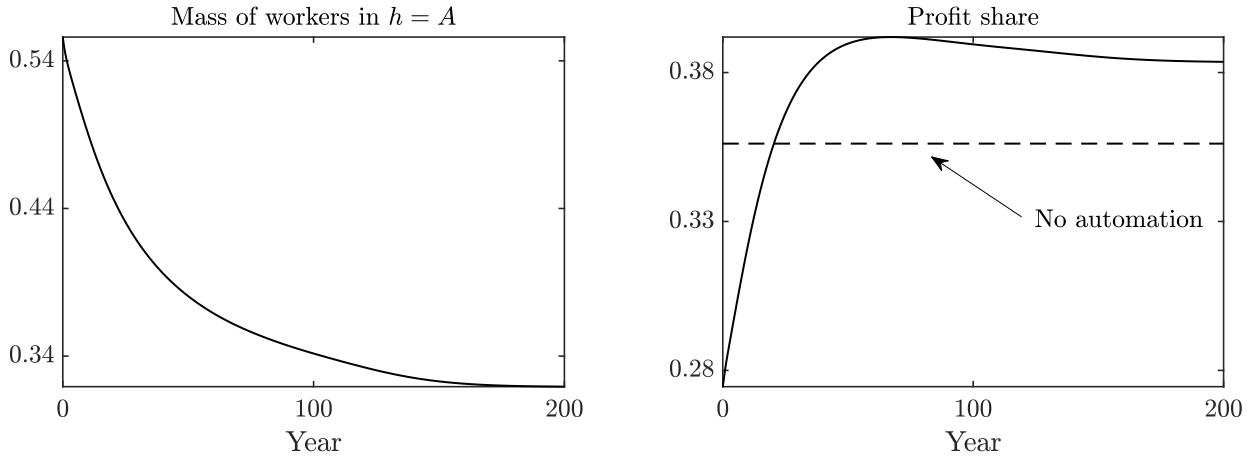
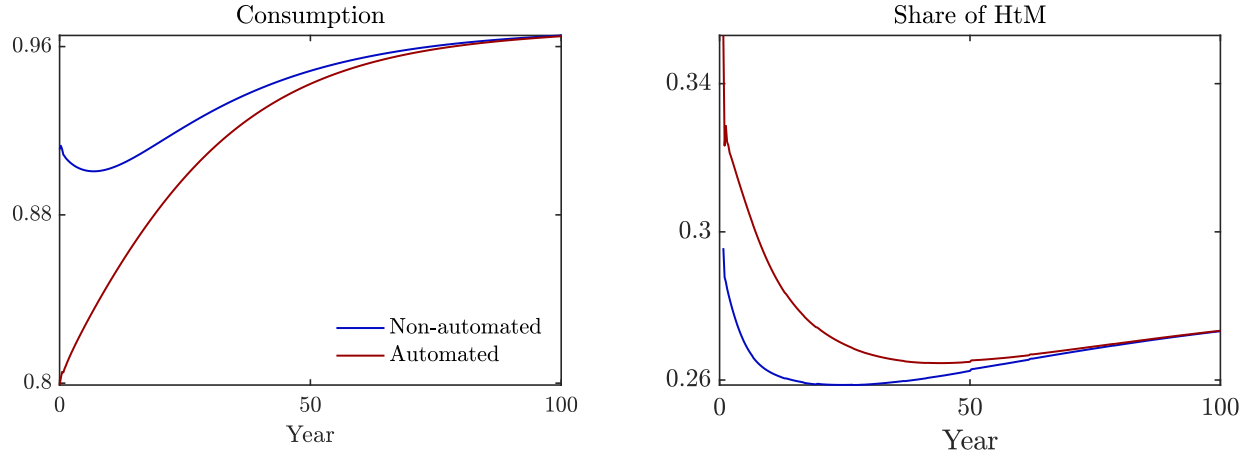


Figure 8.3: Individual allocations



workers are borrowing constrained. In particular, this motive does not justify by *itself* to tax automation in the long run (Section 6.4). However, our quantitative model also features uninsured idiosyncratic risk which introduces an additional motive for intervention. It is well-known that a long-run tax (or subsidy) on *capital* can be optimal when markets are incomplete — it can improve insurance and / or prevent dynamic inefficiency (Section 5.3). We find that this is the case too in the context of automation. Figure 8.1 plots the constrained efficient level of automation that maximizes long-run utilitarian welfare (Figure B.1 in Appendix B.5). The optimal level of automation is roughly 45% that at the laissez-faire.

8.3 Slowing Down Automation [In Progress]

We showed in Section 5 that the government should *slow down* automation, even when it has no preference for redistribution. We now quantify the welfare gains from taxing automation. The government maximizes

$$\mathcal{W}(\boldsymbol{\eta}) \equiv \int_{-\infty}^{+\infty} \int \eta_t(\mathbf{x}) V_t^{\text{birth}}(\mathbf{x}) \, d\pi_t(\mathbf{x}) \, dt \tag{8.1}$$

by choosing the sequence of taxes on investment $\{\tau_t^x\}$ and rebating the proceedings lump sum to the mutual fund. Here, $V_t^{\text{birth}}(\mathbf{x})$ denotes the value function of a worker born in period t that draws a state $\mathbf{x}$, and $\eta_t(\mathbf{x})$ denotes some Pareto weights.[64]

As in our benchmark model, we work with the primal problem since the government can implement any sequence of automation $\{\alpha_t\}$ by choosing taxes appropriately. We focus on relatively simple (or arguably more realistic) interventions, where the government chooses the *speed* at which automation converges to its constrained-efficient level in the long-run (Section 8.2). That is,

$$\alpha_t = \alpha_0 + \left(1 - e^{-\varphi t}\right)\left(\alpha^{\text{CE}} - \alpha_0\right)$$

where $\varphi > 0$ is the speed of convergence. We consider two half-lives that are longer than the one at laissez-faire (40 years). For each of those, we compute the transition dynamics, evaluate welfare (8.1) and express the gains in consumption-equivalent terms.

Table 8.2: Welfare Gains

Half-life (years)	40	70	100
$\mathcal{W}\left(\boldsymbol{\eta}^{\text{effic}}; r_t\right)$	0%	2.3%	3.2%
$\mathcal{W}\left(\boldsymbol{\eta}^{\text{utilit}}; r_t\right)$	0%	2.8%	3.9%

Table 8.2 reports the welfare gains from slowing down automation. We use two sets of Pareto weights $\{\eta_t(\mathbf{x})\}$, which we describe in details in Appendix B.3. The first set of weights $\boldsymbol{\eta}^{\text{effic}}$ focuses on *efficiency*. In this case, the government has no preference for redistribution. The welfare gains are substantial. In other words, slowing down automation improves efficiency. The second set of weights $\boldsymbol{\eta}^{\text{utilit}}$ is *utilitarian*. In this case, the

[64] By assumption, the government treats all *existing* generations (in period $t = 0$) symmetrically. That is, $\eta_t(\mathbf{x}) = \eta_s(\mathbf{x})$ for all $s, t < 0$.

government does has a preference for redistribution. We find even larger welfare gains, since slowing down automation improves not only efficiency but also equity.

9 Conclusion

We presented two novel results in economies where workers displaced by automation face reallocation and borrowing frictions. First, automation is inefficient when these frictions are severe. Second, absent redistributive tools to fully alleviate borrowing frictions, the government should slow down automation while displaced workers reallocate. The optimal policy addresses pecuniary externalities that firms impose on borrowing-constrained workers when they automate and workers impose on their peers when they reallocate. This efficiency rationale for slowing down automation adds to (but is distinct from) the redistributive rationale for taxing automation in efficient economies (Guerreiro et al., 2017; Costinot and Werning, 2018) and the arguments for taxing capital in inefficient economies (Dávila et al., 2012; Dávila and Korinek, 2018; Aguiar et al., 2021). Quantitatively, we found that policies that combine a tax on automation with some redistribution can achieve substantial welfare gains.

To derive sharp results and clarify the mechanisms at play, our model necessarily abstracted from many features. Some of these are worth discussing now. Tax-codes often subsidize capital and R&D expenditures on the grounds that firms face credit constraints or that there are externalities involved — features that our analysis has ignored. Thus, our results *should not* be interpreted as saying that automation ought to be taxed *on net*. Instead, they imply that subsidies on investment in automation should be lowered *temporarily* while the economy adjusts and displaced workers reallocate. In addition, we have abstracted from permanent differences in earnings potential across workers. Certain automation technologies (like industrial robots) displace low-to-middle income workers in routine-intensive occupations, whereas others (like AI software for natural language processing) might impact high-income skilled workers (like lawyers). If low-to-middle income workers have less liquidity or face more severe frictions, this strengthens the rationale for policy intervention when automation affects them. Similarly, inefficiencies caused by automation may be worse in developing countries where borrowing is more limited and social insurance is less generous.

Finally, our quantitative model points to two directions for future work. First, we found that optimal policy is crucially determined by the speed of worker reallocation and the duration of unemployment and retraining spells. It would be interesting to measure these for workers displaced by automation, and compare them to the estimates for the

average U.S. worker used in our quantitative exercises. For instance, reallocation could be slower for workers in automated occupations if their skills differ more from those used in other occupations. Or, it could be faster if it is easier for automated workers to find a new job within the same firm. Second, the quantitative model is rich enough to tackle other optimal policy questions where the dynamics of labor reallocation and asset markets imperfections are relevant, such as how governments should manage declining cities and regions.

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A Proofs and Derivations

A.1 Proof of Proposition 1

The proof consists of two steps. In the first step, we decompose the problem (3.1)–(3.7) into a dynamic problem and a sequence of statics ones. In the second step, we characterize the efficient allocation of labor across occupations and the associated level of consumption.

1. **Objective.** The planner's problem is equivalent to

$$\max_{\{C_t, Q_t, m_t, \hat{m}_t, \mu_t, \Theta_t\}_{t\geq 0}} \int_0^{+\infty} \exp(-\rho t) U_t(C_t) dt \tag{A.1}$$
$$\text{s.t. } (3.2) - (3.7)$$

with the felicity function[65]

$$\exp(-\rho t) U_t(C_t) \equiv \max_{\{c_s^h\}} \chi \sum_h \phi^h \int_0^{+\infty} \eta_{t-s}^h \exp(-(\rho+\chi)s) u\left(c_s^h\right) ds \tag{A.2}$$
$$\text{s.t. } C_t = \chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi s) c_s^h ds$$

Here, $c_s^h \equiv c_{t-s,t}^h$ denotes the consumption in period t of the generation born in period $t-s$ and initially located in occupation h. Solving the static problem,

$$c_s^h = \frac{\left(\eta_{t-s}^h \exp(-\rho s)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp(-\chi\tau) \left(\eta_{t-\tau}^h \exp(-\rho\tau)\right)^{\frac{1}{\sigma}} d\tau} C_t \tag{A.3}$$

and the felicity function $U_t(\cdot)$ is given by (3.9) in the text.

2. **Labor allocation.** Fix some period $T \geq 0$. Consider the planner's decision to reallocate workers employed in automated occupations, i.e. the choice of $\{m_t\}$ and $\{\hat{m}_t\}$. Using a standard variational argument, it is optimal for the planner to reallocate all members of existing generations ($m_t = 1$) if and only if the present discounted value of the marginal labor productivities is higher in non-automated occupations

$$\int_T^{+\infty} \exp(-\rho(t-T)) U_t'(C_t) \Delta_t dt > 0, \tag{A.4}$$

[65] The normalizing factor χ in (A.2) is for convenience. It ensures that $U_t(C) = u(C)$ when the planner discounts each generation with its own discount factor (ρ), i.e. $\eta_{t-s} = \exp(-\rho(t-s))$.

where $\Delta_t \equiv \exp(-\chi t)\left((1-\theta)\left(1-\exp\left(-\kappa\left(t-T\right)\right)\right)\mathcal{Y}_t^N - \mathcal{Y}_t^A\right)$ captures the marginal increase in output from reallocating an additional worker. This term reflects the difference in marginal productivities $\{\mathcal{Y}_t^h\}$ (Proposition 3.1), the productivity loss θ, the average duration of unemployment spells $1/\kappa$, and the share $\exp(-\chi t)$ of the marginal workers that survive. The planner reallocates none of these workers $(m_t = 0)$ if and only if the inequality (A.4) is reversed.[66] Here, similarly, the planner reallocates all members of entering generations $(\hat{m}_t = 1)$ if and only if

$$\int_T^{+\infty} \exp\left(-\rho\left(t-T\right)\right) U_t'\left(C_t\right) \exp\left(-\chi\left(t-T\right)\right)\left[\mathcal{Y}_t^N - \mathcal{Y}_t^A\right] dt > 0, \tag{A.5}$$

reallocates none of them $(\hat{m}_t = 0)$ if and only if the inequality is reversed. The planner chooses an interior solution $(\hat{m}_t \in (0,1))$ otherwise. By Assumptions 1–2, there exists some $T_0^{\text{FB}} > 0$ such that the planner reallocates all members of existing generations $(m_t = 1)$ for all $t \in \left[0, T_0^{\text{FB}}\right)$. In period $T = T_0^{\text{FB}}$, the left-hand side of (A.4) is zero. Inspecting (A.4)–(A.5), the planner continues to reallocate entering generations. That is, there exists some T_1^{FB} with $0 < T_0^{\text{FB}} < T_1^{\text{FB}}$ such that the planner reallocates all members of new generations $(\hat{m}_t = 1)$ for all $t \in \left[0, T_1^{\text{FB}}\right)$. Furthermore, $T_1^{\text{FB}} < +\infty$ since the technologies $F(\cdot)$ and $\hat{F}(\cdot)$ satisfy Inada conditions. From $t = T_1^{\text{FB}}$ onward, the left-hand side of (A.5) holds with equality and the planner chooses $\hat{m}_t \in (0,1)$ to ensure that the marginal productivities are equalized $\mathcal{Y}_t^N = \mathcal{Y}_t^A$ for all $t \in \left[T_1^{\text{FB}}, +\infty\right)$. The planner does not reallocate existing generations $(m_t = 0)$ for all $t \geq T_0^{\text{FB}}$. Summing up,

$$m_t = \begin{cases} 1 & \text{if } t \in \left[0, T_0^{\text{FB}}\right) \\ 0 & \text{if } t \in \left[T_0^{\text{FB}}, +\infty\right) \end{cases} \quad \text{and} \quad \hat{m}_t = \begin{cases} 1 & \text{if } t \in \left[0, T_1^{\text{FB}}\right) \\ \in (0,1) & \text{if } t \in \left[T_1^{\text{FB}}, +\infty\right) \end{cases} \tag{A.6}$$

with $\{\hat{m}_t\}$ chosen for $t \geq T_1^{\text{FB}}$ such that the effective labor supplies in the two occupations remain constant over time. The two stopping times satisfy (3.10) in the text. Solving the differential equation (3.4) and evaluating using (A.6) gives

$$\mu_t^A = \exp\left(-\lambda \min\left\{t, T_0\right\} - \chi t\right) \tag{A.7}$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. Solving (3.5)–(3.7) gives

$$\mu_t^N \equiv 1 - \mu_t^A - \tilde{\mu}_t - \theta\hat{\mu}_t \tag{A.8}$$

[66] As shown below, $\hat{m}_t \geq m_t$ for all $t \geq 0$ so that the left-hand side of (A.4) holds with equality in a single period.

and

$$\tilde{\mu}_t = \frac{\lambda}{\lambda-\kappa}\frac{\phi}{1-\phi}\exp\left(-\left(\kappa+\chi\right)t\right)\left(1-\exp\left(-\left(\lambda-\kappa\right)\min\left\{t,T_0\right\}\right)\right) \tag{A.9}$$

$$\begin{aligned}\hat{\mu}_t = &\frac{\lambda}{\lambda-\kappa}\frac{\phi}{1-\phi}\exp\left(-\chi t\right)\times \\ &\left[\left(1-\exp\left(-\left(\kappa-\chi\right)\min\left\{t,T_0\right\}\right)\right)-\frac{\kappa}{\lambda}\left(1-\exp\left(-\lambda\min\left\{t,T_0\right\}\right)\right)\right] \\ &+\frac{\lambda}{\lambda-\kappa}\frac{\phi}{1-\phi}\left(1-\exp\left(-\left(\lambda-\kappa\right)T_0\right)\right)\exp\left(-\chi t\right)\left[\exp\left(-\kappa T_0\right)-\exp\left(-\kappa\max\left\{t,T_0\right\}\right)\right]ds\end{aligned} \tag{A.10}$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. For $t \geq T_1^{\text{FB}}$, the effective labor supplies $\boldsymbol{\mu}_t$ adjust so that the marginal productivities are equalized across occupations $\mathcal{Y}_t^N = \mathcal{Y}_t^A$. The expression (3.13) in the text is obtained by taking the limit with no unemployment $(1/\kappa \to 0)$. Finally, consumption is given by aggregate output $C_t = G^\star\left(\boldsymbol{\mu}_t; \alpha\right)$.

We have supposed so far that that the average unemployment duration and the productivity loss are sufficiently small that labor mobility takes place at the first best, i.e. $T_0^{\text{FB}} > 0$. This occurs whenever the productivity cost associated to reallocation is sufficiently small

$$\theta \leq 1 - \frac{\int_0^{+\infty}\exp\left(-\left(\rho+\chi\right)t\right)U_t'\left(\tilde{C}_t\right)\tilde{\mathcal{Y}}_t^A dt}{\int_0^{+\infty}\left(1-\exp\left(-\kappa t\right)\right)\exp\left(-\left(\rho+\chi\right)t\right)U_t'\left(\tilde{C}_t\right)\tilde{\mathcal{Y}}_t^N dt} \tag{A.11}$$

where the terms on the right-hand side are defined as above, but evaluated with an alternative technology and a counterfactual sequence of (effective) labor supplies. These labor supplies are still given by (A.7)–(A.10) but are now evaluated at $T_0 \equiv 0$ and T_1 given by (3.10). The technology $G^\star\left(\mu, \mu'; \alpha\right)$ is evaluated at some automation level $\bar{\alpha} > 0$ such that $\partial_\alpha G^\star\left(\mu, \mu'; \bar{\alpha}\right) > 0$.[67,68] By definition, the sequences of consumption and the marginal productivities in (A.11) are not indexed by any of the mobility parameters (θ, κ). Therefore, the restriction (A.11) effectively puts an upper bound (jointly) on the average unemployment duration $1/\kappa$ and the productivity loss θ. The coefficients $\left\{Z^A, Z_t^N\right\}$ in Assumption 2 can be read from the numerator and denominator in (A.11).

[67] Such a threshold $\bar{\alpha} > 0$ exists by Assumptions 1–2.

[68] Whenever the level of automation satisfies $\alpha \geq \bar{\alpha}$ — which turns out to be the case at the first best (Proposition 2) — the right-hand side in (A.11) remains larger than θ so labor reallocation still takes place.

A.2 Proof of Proposition 2

We first consider a perturbation of the planner's ex post problem as the level of automation changes and we derive an envelope condition. We then state the optimality condition for the planner's ex ante problem.

1. **Envelope.** By Proposition 1, the planner's ex post problem (3.1)–(3.7) can be equivalently formulated as

$$V^{\text{FB}}(\alpha;\boldsymbol{\eta}) = \max_{\{T_0,T_1\}} \int_0^{+\infty} \exp(-\rho t)\, U_t(C_t)\, dt \tag{A.12}$$

subject to the resource constraint $C_t = G^\star(\boldsymbol{\mu}_t;\alpha)$, the effective labor supplies given by (A.7)–(A.10) and the restriction $0 < T_0 < T_1 < +\infty$. Note that the problem is differentiable in α and is Lipschitz continuous in $\{T_0, T_1\}$. Therefore, the following envelope condition applies

$$\frac{\partial}{\partial\alpha} V^{\text{FB}}(\alpha;\boldsymbol{\eta}) = \int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \underbrace{\frac{\partial}{\partial\alpha} G^\star(\boldsymbol{\mu}_t;\alpha)}_{\equiv \Psi_t(\alpha)}\, dt = 0, \tag{A.13}$$

where consumption $\{C_t\}$, the labor supplies $\{\boldsymbol{\mu}_t\}$ and the terms $\{\tilde{\mu}_t, \hat{\mu}_t\}$ are those characterized in Appendix A.1 when evaluated at α.

2. **Optimality.** The solution to the planner's ex ante problem (3.8) is unique and interior. We first show that the solution is interior. First, note that $\alpha^{\text{FB}} > \bar{\alpha}$ where $\bar{\alpha}$ is the exogeneous level of automation implicit in (A.11) — i.e. Assumption 2. The reason is that $\Psi_t(\alpha) > 0$ for all $\alpha \in [0,\bar{\alpha}]$ and all $t > 0$. This follows by Assumptions 1–2 and the fact that $\mu_t < 1$ and $\Theta_t(1-\mu_t) > 1$ since (at least) some members of existing generations reallocate. Therefore,

$$\int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \Psi_t(\alpha)\, dt > 0 \tag{A.14}$$

for all $\alpha \in [0,\bar{\alpha}]$, so $\alpha^{\text{FB}} > \bar{\alpha}$. Furthermore, $\alpha^{\text{FB}} < 1$ since

$$\lim_{\alpha\to 1} \int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \Psi_t(\alpha)\, dt = -\infty \tag{A.15}$$

by Assumption 2. Therefore, the solution is interior. Uniqueness follows from the concavity of the value (A.12) in α. To see that, consider two automation levels (α_0,α_1) and let $\mu_t(\alpha)$ and $\mu_t'(\alpha)$ denote the associated effective labor supplies in $h \in \{A, N\}$ at

the first best. Now, consider a convex combination $\hat{\alpha} \equiv c\alpha_0 + (1-c)\alpha_1$ of the automation levels, for some $c \in (0,1)$. Note that the effective labor supplies $\hat{\mu}_t^{(\prime)} = c\mu_t^{(\prime)}(\alpha_0) + (1-c)\mu_t^{(\prime)}(\alpha_1)$ are feasible under the laws of motions (3.4)–(3.7). Therefore,

$$\begin{aligned} G^\star\left(\mu_t(\hat{\alpha}), \mu_t'(\hat{\alpha}); \hat{\alpha}\right) \geq & G^\star\left(\hat{\mu}_t, \hat{\mu}_t'; \hat{\alpha}\right) \\ \geq & cG^\star\left(\mu_t(\alpha_0), \mu_t'(\alpha_0); \alpha_0\right) \\ & + (1-c)\, G^\star\left(\mu_t(\alpha_1), \mu_t'(\alpha_1); \alpha_1\right) \end{aligned} \tag{A.16}$$

for all periods $t \geq 0$. The second inequality follows by concavity of the aggregate technology with respect to labor supplies and α (Assumption 2).[69] The concavity of the value (A.12) then follows immediately by concavity of the felicity function (3.9). Therefore, a necessary and sufficient condition for an optimum is

$$\frac{\partial}{\partial \alpha} V^{\text{FB}}\left(\alpha^{\text{FB}}; \boldsymbol{\eta}\right) = 0 \tag{A.17}$$

since $V^{\text{FB}}(\cdot; \boldsymbol{\eta})$ is differentiable everywhere. The result follows immediately from (A.13) and (A.17).

A.3 Characterization of Equilibrium

We now characterize the competitive equilibrium in our baseline model (Section 4). We omit potential distorsionary and lump sum taxes (Section 6) for notational clarity.

Ex post. We start by characterizing the equilibrium conditional on an automation level α. The presence of borrowing frictions implies that some workers are potentially borrowing constrained. As a result, the decentralized equilibrium is characterized by four times: the times until which existing and new generations reallocate to non-automated occupations (T_0, T_1); and the times between which workers initially employed in automated occupations are borrowing constrainted (S_0, S_1).[70] We start by characterizing the latter, before turning to the former and solving for equilibrium prices.

1. **Consumption-savings.** The time at which workers initially employed in automated

[69] The aggregate technology $G^\star$ inherits the concavity of neoclassical technologies used by the final good producer (G) and in each occupation $(F, \hat{F})$.

[70] In principle, workers initially employed in non-automated occupations and new generations could be borrowing constrained *after* wages have converged across occupation (T_1). We abstract from this possibility, since it does not occur for the range of parameters we are interested in.

occupations become borrowing constrained (S_0) is such that workers deplete their savings[71]

$$u'\left(\hat{c}^A_{0,S_0}\right) = \exp\left(\int_0^{S_0} \hat{r}_t dt - (\rho+\chi)\, S_0\right) u'\left(\hat{\mathcal{Y}}^A_{0,S_0} + \Pi_{S_0} + r_{S_0}\underline{a}\right) \tag{A.18}$$

given the budget restriction

$$\hat{c}^A_{0,S_0} = \frac{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right)\left(\hat{\mathcal{Y}}^A_{0,t} + \Pi_t\right) dt + a^A_0 - \exp\left(-\int_0^{S_0} \hat{r}_s ds\right)\underline{a}}{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho t\right)\right)^{-1} dt} \tag{A.19}$$

where $\hat{u} \equiv 1/\left(u'\right)^{-1}$, $\hat{\mathcal{Y}}^A_{0,t}$ is labor income, Π_t are profits, $\hat{r}_t \equiv r_t + \chi$ is the effective return on bonds, and $a^A_0 \equiv 0$. The time at which these workers stop being borrowing constrained (S_1) is the one where their savings flow equals the change in their borrowing constraint[72]

$$\hat{c}^A_{S_1,+\infty} = \hat{\mathcal{Y}}^A_{0,S_1} + \Pi_{S_1} + r_{s_1}\underline{a} \tag{A.20}$$

with $\hat{c}^h_{t,t'}$ defined by analogy with (A.19) and $a^A_{S_1} \equiv \underline{a}$. The same workers are unconstrained for all $t \geq S_1$. That is, the consumption of workers initially employed in automated occupations is given by $c^A_t = \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho s\right)\right)\hat{c}^A_{0,S_0}$ before the borrowing constraint binds $t \in [0, S_0)$, $c^A_t = \hat{\mathcal{Y}}^A_t + \Pi_t$ when the borrowing constraint binds $t \in [S_0, S_1)$ and

$$c^A_t = \hat{u}\left(\exp\left(\int_{S_1}^t r_s ds - \rho\left(t - S_1\right)\right)\right) \times \hat{c}^A_{S_1,+\infty} \tag{A.21}$$

afterwards. In turn, workers initially employed in non-automated occupations and members of generations born at $s > 0$ are unconstrained for all $t \geq 0$. Their consumption is given by

$$\hat{c}^N_t = \hat{u}\left(\exp\left(\int_0^t r_s ds - \rho t\right)\right)\hat{c}^N_{0,+\infty} \tag{A.22}$$

[71] We have $S_0 = S_1 \to +\infty$ when these workers never become borrowing constrained, since all workers effectively become hand-to-mouth as the economy converges to its new stationary equilibrium. Without loss of generality, we can also set $S_0 = S_1 \equiv 0$. For notational convenience, we choose to do so in the following.

[72] In theory, workers could be constrained over multiple, separate intervals of time. We rule this case out since it does not occur for the parametrizations of interest. This explains why (A.20) implicitly assumes that workers are unconstrained for all periods $t \geq S_1$.

Finally, aggregate consumption is given by

$$C_t = \phi \exp(-\chi t)\, c_t^A + (1 - \phi \exp(-\chi t))\, \hat{c}_t^N \tag{A.23}$$

2. **Labor reallocation.** Labor income $\hat{\mathcal{Y}}_{s,t}^h$ in period t for a generation born in s and initially located in occupation h is

$$\hat{\mathcal{Y}}_{s,t}^h = w_t^A + (1 - \exp(-\lambda \min\{t, T_0\})) \left(\Theta_t(\lambda, \kappa)(1-\theta) w_t^N - w_t^A \right) \tag{A.24}$$

if $h = A, s < 0$ and $\hat{\mathcal{Y}}_{s,t}^h = w_t^N$ otherwise, where

$$\Theta_t(\lambda, \kappa) \equiv \frac{1-\phi}{\phi} \frac{\hat{\mu}_t}{1 - \exp(-\lambda \min\{t, T_0\})} \tag{A.25}$$

is the share of workers who exited their unemployment spell after changing occupation, with $\{\hat{\mu}_t\}$ given by (A.10) evaluated at the equilibrium stopping times.

In any period $t = T$, workers initially employed in automated occupations — i.e. $h = A$, $s \leq 0$ — decide as a large household whether to reallocate to non-automated occupations or not. It is never optimal to postpone mobility. Thus, these workers effectively choose a stopping time T_0. When making this choice, they internalize the effect of this stopping time on labor income, taking prices as given — i.e. the direct effect of T_0 in (A.24)–(A.25) as well as the impulse response of $\{\hat{\mu}_t\}$. Therefore, the optimal stopping time satisfies[73]

$$\int_{T_0}^{+\infty} \exp(-\rho t)\, u'\left(\hat{c}_t^A\right) \Delta_t dt = 0 \tag{A.26}$$

where $\Delta_t \equiv \exp(-\chi t) \left\{ (1-\theta)(1 - \exp(-\kappa(t - T_0))) w_t^N - w_t^A \right\}$ captures the marginal increase in labor incomes when the large family reallocates additional workers.[74] This condition becomes

$$\int_{T_0}^{+\infty} \exp\left(- \int_{T_0}^{t} r_\tau d\tau \right) \Delta_t dt = 0 \tag{A.27}$$

in the case where existing workers are unconstrained after they stop reallocating $t \geq T_0$.

[73] Condition (A.26) applies whether workers are constrained or not after they stop reallocating $t \geq T_0$.

[74] A worker who reallocates between occupations internalizes the risk that she will die through her discount factor $\exp(-(\rho + \chi) t)$, not through the flows Δ_t — contrary to the planner. We chose the formulation (A.26) to preserve the symmetry with the first best (3.10)–(3.15).

The second stopping time T_1 is such that wages are equalized across occupations

$$w_{T_1}^A = w_{T_1}^N \tag{A.28}$$

Fixing a sequence of interest rates $\{r_t\}$, the conditions (A.18)–(A.20), (A.24)–(A.25) and (A.27)–(A.28) pin down the equilibrium stopping times (T_0, T_1) and (S_0, S_1).[75] Effective labor supplies $\boldsymbol{\mu}_t$ are given by (A.7)–(A.10) evaluated at the stopping times (T_0, T_1).

3. **Equilibrium prices.** Equilibrium wages and profits are

$$w_t^h = 1/\phi^h \partial_h G^\star\left(\boldsymbol{\mu}_t; \alpha\right) \ \forall h \tag{A.29}$$

$$\Pi_t \equiv Y_t - \phi \mu_t^A w_t^A - (1-\phi)\, \mu_t^N w_t^N \tag{A.30}$$

where $Y_t \equiv G^\star\left(\boldsymbol{\mu}_t; \alpha\right)$ is equilibrium output. Finally, the interest rate that ensures that $C_t = Y_t$ at equilibrium is

$$r_t = \rho + \frac{\sigma}{Y_t}\left(\phi w_t^A \partial_t \mu_t^A + (1-\phi)\, w_t^N \partial_t \mu_t^N\right) \tag{A.31}$$

when the borrowing constraint does not bind $t \in [0, S_0) \cup [S_1, +\infty)$. The expression for the interest rate when the borrowing constraint binds $t \in [S_0, S_1)$ involves additional terms, so we omit it for concision since we do not use it in the following. Finally,

$$\partial_t \mu_t^A = \left(\mathbf{1}_{\{t<T_1\}}\chi + \mathbf{1}_{\{t<T_0\}}\lambda\right)\mu_t^A \tag{A.32}$$

$$\begin{aligned}\partial_t \mu_t^N = &-\frac{\phi}{1-\phi}\partial_t \mu_t^A - \left(\lambda \frac{\phi}{1-\phi}\mathbf{1}_{\{t<T_0\}}\mu_t^A - (\kappa+\chi)\,\tilde{\mu}_t\right) \\ &+ (\theta-1)\left(\kappa\tilde{\mu}_t - \chi\hat{\mu}_t\right)\end{aligned} \tag{A.33}$$

using (3.4)–(3.7) and the definition of the stopping times.

Ex ante. We now characterize the equilibrium choice of automation. A necessary condition for an interior optimum is[76,77]

$$\int_0^{+\infty} \exp\left(-\int_0^t r_s ds\right) \frac{\partial}{\partial \alpha}\Pi_t\left(\alpha\right) dt = 0 \tag{A.34}$$

[75] We can actually show that (T_0, T_1) and (S_0, S_1) are unique, given $\{r_t\}$.

[76] The static profit function (4.2) is differentiable in the level of automation by Assumption 1

[77] We suppose that equity — which is fully illiquid in our model — is priced using the stochastic discount factor of unconstrained workers. By no arbitrage with bonds, the return on equity (pre-annuities) is $\{r_t\}$.

Furthermore, the following envelope condition applies

$$\frac{d}{d\alpha}\Pi_t(\alpha) = \frac{\partial}{\partial\alpha}G^\star(\boldsymbol{\mu}_t;\alpha) \tag{A.35}$$

Therefore,

$$\int_0^{+\infty}\exp\left(-\int_0^t r_s ds\right)\frac{\partial}{\partial\alpha}G^\star(\boldsymbol{\mu}_t;\alpha) = 0 \tag{A.36}$$

This condition is both necessary and sufficient, by Assumption 2.

A.4 Proof of Proposition 3

The result states that laissez-faire equilibrium is inefficient if and only if the borrowing constraints are sufficiently important $\underline{a} > a^\star$ for some $a^\star \leq 1$. For our purpose, it is sufficient to show that the laissez-faire either satisfies all the restrictions that characterizes first best allocations (Section 3) or violates *at least* one of those. At this point, we do not elaborate on the nature of the inefficiency. Throughout, we define the aggregate and individual allocation $\{\bar{X}_t\}$ with $\bar{X}_t \equiv \left(\left\{\bar{c}^h_{s,t}, \bar{a}^h_{s,t}\right\}, \left\{\bar{\mathcal{Y}}^h_t\right\}, \bar{Y}_t\right)$ to be the one that occurs in the laissez-faire equilibrium without borrowing frictions ($\underline{a} \to -\infty$). We let $(\bar{T}_0, \bar{T}_1)$ denote the associated stopping times. Prices are defined similarly. To economize on notation, the dependence on the reallocation parameters $(\lambda, \eta, \theta, \chi)$ is implicit when there is no ambiguity. We show sufficiency first, then necessity.

Sufficiency. Define the threshold $a^\star \equiv \inf_t \bar{a}^0_{s,t}$ for any existing generations $s < 0$. Then, the laissez-faire allocation coincides with $\{\bar{X}_t\}$ whenever $\underline{a} \leq a^\star$.[78] It suffices to show that $\{\bar{X}_t\}$ is efficient — i.e. there exist some weights $\left\{\eta^h_s\right\}$ that implement this allocation as a first best. When workers are unconstrained,

$$\bar{c}^h_{s,\tau}/\bar{c}^h_{s,t} = \hat{u}\left(-\exp\left(\int_t^\tau r_k dk - \rho(\tau - t)\right)\right) \quad \text{for all } (h,s) \text{ and } t,\tau \geq s \tag{A.37}$$

using Assumption 3. This quantity does not depend on the initial occupation of employment (h) nor the birth date (s). Therefore, there exists a set of weights $\left\{\eta^h_s\right\}$ and coefficients $\{b_t\}$ such that $c^h_{s,t} = b_t\left(\eta^h_{t-s}\exp(-\rho s)\right)^{\frac{1}{\sigma}} C_t$ for all initial occupations (h), generations (s) and periods $t \geq s$. The sequence $\{b_t\}$ is chosen to satisfy the definition of aggregate consumption (3.2). As a result, the equilibrium consumption allocation coincides with its first best counterpart (A.3) when the planner uses the weights $\left\{\eta^h_s\right\}$. It remains

[78] All other agents are net savers at equilibrium $\bar{a}^h_{s,t} \geq 0$ for any occupation h and generation $s \geq 0$.

to show that the equilibrium stopping times $(\bar{T}_0, \bar{T}_1)$ also coincide with their first best counterparts. When workers are unconstrained, the first stopping time is characterized by (A.26). Then,

$$\int_{T_0}^{+\infty} \exp(-\rho t)\, u'\left(\bar{c}_{s,t}^{h}\right) \Delta_t dt = 0 \quad \text{for all } (h,s) \tag{A.38}$$

using the workers' optimality conditions (A.27) and (A.37). Furthermore, the following envelope condition applies

$$U_t'(C_t) = \eta_s^h \exp(-\rho s)\, u'\left(\bar{c}_{s,t}^{h}\right) \quad \text{for all } (h,s) \text{ and } t \geq s \tag{A.39}$$

using the planner's intratemporal problem (A.2) with the proportionality factor independent of the period t. It follows that the first stopping time $(\bar{T}_0)$ coincides with its first best counterpart (3.10), using (A.38)–(A.39). Finally, so does the second stopping time $(\bar{T}_1)$ since effective labor supplies still evolve as (A.7)–(A.10) in both cases. To complete the proof of sufficiency, note that $-\infty < a^\star \leq 0$. In the limit where reallocation is fast $1/\lambda, 1/\kappa \to 0$ and $1/\chi \to +\infty$, we have $\bar{\mathcal{Y}}_t^A = \bar{\mathcal{Y}}_t^N$ for all $t \geq 0$ by Proposition 1. Therefore no borrowing takes place and $a^\star \to 0$ in this limit.

Necessity. Define $a^\star$ as above. Let $\underline{a} > a^\star$. Then, there exist some periods $0 \leq t < \tau$ such that

$$c_{s,\tau}^A / c_{s,t}^A > \hat{u}\left(-\exp\left(\int_t^\tau r_k dk - \rho(\tau - t)\right)\right) \quad \text{for all } s < 0 \tag{A.40}$$

at the laissez-faire for workers initially employed in automated occupations, using Assumption 3. In contrast, the relation above holds with equality for all other workers h and s since they are unconstrained at equilibrium. It follows that there exist occupation h, generations $s < 0$ and s' and periods $s' \leq t < \tau$, such that $c_{s,\tau}^A / c_{s,t}^A \neq c_{s',\tau}^h / c_{s',t}^h$. Therefore, the equilibrium allocation does not satisfy the first best restriction (A.3). We conclude that this equilibrium is inefficient.

A.5 Proof of Proposition 4

The government's optimality conditions to reallocate and automate are

$$\int_{T_0^{\text{SB}}}^{+\infty} \exp(-\rho t)\, u'\left(c_{0,t}^A\right) \Delta_t dt = -\exp\left(\lambda T_0^{\text{SB}}\right) \frac{\Phi\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right)}{\phi^A \eta^A} \tag{A.41}$$

and

$$\int_0^{+\infty} \exp(-\chi t)\, u'\left(c_{0,t}^N\right) \Delta_t^\star dt = -\frac{\Phi^\star\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right)}{\phi^N \eta^N} \tag{A.42}$$

respectively. The terms on the left-hand side of (A.41)–(A.42) correspond to the private incentives to automate and reallocate, respectively. The terms on the right-hand capture pecuniary externalities that affect workers through wags and profits — that firms and do not internalize. These pecuniary externalities are given by

$$\Phi\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \equiv \int_{T_0^{\text{LF}}}^{+\infty} \exp\left(-\chi t\right) \hat{\Phi}_t\left(\cdot\right) dt \tag{A.43}$$

$$\Phi^{\star}\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \equiv \int_{0}^{+\infty} \exp\left(-\chi t\right) \hat{\Phi}_t^{\star}\left(\cdot\right) dt \tag{A.44}$$

where

$$\begin{aligned}\hat{\Phi}_t\left(\cdot\right) \equiv & \phi^A \eta^A u'\left(c_{0,t}^A\right)\left[\exp\left(-\lambda T_0^{\text{SB}}\right)\hat{w}_t^A + \left(1-\theta\right)\hat{\mu}_t\left(T_0^{\text{SB}}\right)\hat{w}_t^N - \sum_h \phi^h \mu^h \hat{w}_t^h\right] \\ & + \phi^N \eta^N u'\left(c_{0,t}^N\right)\left[\hat{w}_t^N - \sum_h \phi^h \mu^h \hat{w}_t^h\right]\end{aligned} \tag{A.45}$$

$$\begin{aligned}\hat{\Phi}_t^{\star}\left(\cdot\right) \equiv & \phi^A \eta^A u'\left(c_{0,t}^A\right)\left[\Delta_t^{\star} + \exp\left(-\lambda T_0^{\text{SB}}\right)\hat{w}_t^{A,\star} + \left(1-\theta\right)\hat{\mu}_t\left(T_0^{\text{SB}}\right)\hat{w}_t^{N,\star} - \sum_h \phi^h \mu^h \hat{w}_t^{h,\star}\right] \\ & + \phi^N \eta^N u'\left(c_{0,t}^N\right)\left[\hat{w}_t^{N,\star} - \sum_h \phi^h \mu^h \hat{w}_t^{h,\star}\right]\end{aligned} \tag{A.46}$$

for the reallocation and automation decisions, respectively.[79] In turn, $\hat{\mu}_t\left(T_0^{\text{LF}}\right)$ denotes the mass of workers (A.10) who have reallocated and completed their unemployment spell, while the sequences $\left\{\hat{w}_t^h\right\}$ and $\left\{\hat{w}_t^{h,\star}\right\}$ denote the perturbation of wages $w_t^h \equiv \partial_h G\left(\mu_t, \Theta_t\left(1-\mu_t\right); \alpha\right)$ with respect to a change in T_0 and α, respectively.[80]

The equilibrium is *constrained efficient* if and only if

$$\Phi\left(\alpha^{\text{LF}}, T_0^{\text{LF}}; \boldsymbol{\eta}\right) = \Phi^{\star}\left(\alpha^{\text{LF}}, T_0^{\text{LF}}; \boldsymbol{\eta}\right) = 0 \tag{A.47}$$

for *some* weights $\boldsymbol{\eta}$. We now show that whenever these conditions hold, there exists a small perturbation of the production function such that the resulting second best and laissez-faire do not coincide. To see this, suppose that (A.47) holds for some weights $\boldsymbol{\eta}$. Consider the variation

$$\mathcal{G}\left(G^{\star}, \epsilon\right) = G^{\star} + \epsilon g\left(\boldsymbol{\mu}; \alpha\right) \tag{A.48}$$

[79] The last term in each of the brackets in (A.45)–(A.46) corresponds to the change in profits. This is obtained using the definition of profits $\Pi_t = G^{\star}\left(\cdot\right) - \phi w_t^A \mu_t - \left(1-\phi\right) w_t^N \Theta_t\left(1-\mu_t\right)$ and equilibrium wages $w_t^h = 1/\phi^h G_h^{\star}\left(\cdot\right)$.

[80] Effective labor supplies $\{\mu_t, \Theta_t\}$ are effectively indexed by T_0, as is apparent from (A.7)–(A.10). These quantities are evaluated at the degree of automation α^{SB} and the stopping time T_0^{SB}.

where g is any function that satisfies

$$g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)=0 \tag{A.49}$$

for all $t\geq 0$,

$$\int_0^{+\infty}\exp\left(-\chi t\right)u'\left(c_{0,t}^{A}\right)\partial_h g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)dt \quad =0 \tag{A.50}$$

for each occupation $h\in\{A,N\}$, and

$$\int_0^{+\infty}\exp\left(-\chi t\right)u'\left(c_{0,t}^{N}\right)\partial_\alpha g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)dt \quad =0 \tag{A.51}$$

along the initial equilibrium. For instance,

$$g\left(\boldsymbol{\mu}_t;\alpha\right)\equiv\left\{\mu_t^{A,\mathrm{LF}}+\varrho\mu_t^{N,\mathrm{LF}}\right\}\left(\alpha^{\mathrm{LF}}-\alpha\right) \tag{A.52}$$

satisfies (A.49)–(A.51) when choosing $\varrho<0$ appropriately.

Then, the allocation $\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)$ still satisfies all equilibrium conditions — workers' reallocation (A.26), firms' automation (5.2), and the resource constraint (4.8) — after a variation $\varepsilon>0$. That is, the laissez-faire is unchanged. It follows that the pecuniary externality that concerns labor reallocation (A.45) still nets out $\Phi\left(\alpha^{\mathrm{LF}},T_0^{\mathrm{LF}};\boldsymbol{\eta}\right)=0$ after this variation. The reason is that this pecuniary externality involves exlusively terms in $D_{\boldsymbol{\mu}}^2G^\star$, while the perturbation (A.52) is linear in $\boldsymbol{\mu}$ and cannot affect these terms.

Now, note that $\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)$ is *increasing* over time when evaluated at $\alpha=\alpha^{\mathrm{LF}}$, and $\partial_\alpha g\left(\boldsymbol{\mu}_0;\alpha\right)<0$ and $\lim_{t\to+\infty}\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)>0$.[81] Furthermore, note that the sequence of relative marginal utilities $\left\{u'\left(\mathcal{Y}_{0,t}^{A}+\Pi_t\right)/u'\left(\mathcal{Y}_{0,t}^{N}+\Pi_t\right)\right\}$ is *decreasing* over time given (5.1). It follows that

$$\int_0^{+\infty}\exp\left(-\chi t\right)\left\{\sum_h\phi^h\eta^{h,\mathrm{effic}}u'\left(\mathcal{Y}_{0,t}^{h}+\Pi_t\right)\right\}\partial_\alpha g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)dt \quad <0 \tag{A.53}$$

given (A.51). Therefore, we have constructed a variation $\mathcal{G}\left(G^\star,\epsilon\right)$ such that $\Phi^\star\left(\alpha^{\mathrm{SB}},T_0^{\mathrm{SB}};\boldsymbol{\eta}\right)\neq 0$. It follows that

$$\int_0^{+\infty}\exp\left(-\left(\chi\right)t\right)u'\left(c_{0,t}^{N}\right)\Delta_t^\star dt\neq 0, \tag{A.54}$$

which is inconsistent with firms' automation (5.2). That is, the second best and the laissez-

[81] The first property follows from the definition (A.52) and the law of motions (A.7)–(A.10) for effective labor supplies $\{\boldsymbol{\mu}_t\}$. The second and third properties follow immediately from (A.51). It is worth noting that the sequence of labor supplies — and hence $\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)$ — could actually be non-monotonic after the second stopping time T_1 when there are overlapping generations — which we abstract from here (Section 6.1).

faire do not coincide after the perturbation $\varepsilon > 0$. Finally, $G^{\star,\prime}(G^\star, \epsilon) \to G^\star$ uniformly as $\epsilon \to 0$ given (A.52), as claimed.

A.6 Proof of Proposition 5

We first derive the optimality conditions associated to the problem (6.1). We then sign the wedge at the laissez-faire.

The equilibrium level of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp(-\rho t) \times \frac{u'\left(c_{0,t}^A\right)}{u'\left(c_{0,0}^A\right)} \Delta_t^\star dt = 0, \tag{A.55}$$

where $\Delta_t^\star$ is defined by (5.4) and denotes the response of aggregate output to automation. In turn, the second-best level of automation $\alpha^{\text{SB}}(\boldsymbol{\eta})$ satisfies

$$\int_{T_0^{\text{SB}}(\boldsymbol{\eta})}^{+\infty} \exp(-\rho t) \times \sum_h \phi^h \eta^h \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} \left(\Delta_t^\star + \Phi_t^{\star,h}\right) dt = 0 \tag{A.56}$$

where $\left\{\Phi_t^{\star,h}\right\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \Phi_t^{h,\star} \equiv 0$ for all periods t. By assumption, the government chooses weights $\boldsymbol{\eta} \equiv \boldsymbol{\eta}^{\text{effic}}$ that ensure that the distributional terms net out. Therefore,

$$\int_{T_0^{\text{SB}}(\boldsymbol{\eta})}^{+\infty} \exp(-\rho t) \times \sum_h \phi^h \eta^h \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} \Delta_t^\star dt = 0 \tag{A.57}$$

In the following, we let $\lambda_t^h \equiv \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)}$. The sequence $\left\{\lambda_t^A\right\}$ is more *front-loaded* than $\left\{\lambda_t^N\right\}$ since labor incomes (and thus consumption) satisfy $\hat{\mathcal{Y}}_{0,t}^A \leq \hat{\mathcal{Y}}_{0,t}^N$ and the two converge eventually (Appendix A.3). For the reasons outlined in Section 6.3.1, the sequence $\{\Delta_t^\star\}$ is itself *back-loaded* by Assumption 3. Thus, the left-hand side of (A.57) is negative at α^{LF} since the government's values relatively less flows which are more distant in the future. Therefore, the government finds it optimal to *curb* automation.

A.7 Proof of Proposition 6

We proceed as in Appendix A.6. The equilibrium stopping time T_0^{LF} satisfies

$$\int_{T_0^{\text{LF}}}^{+\infty} \exp\left(-\rho t\right) \times \frac{u'\left(c_{0,t}^{A}\right)}{u'\left(c_{0,0}^{A}\right)} \Delta_t = 0, \tag{A.58}$$

where Δ_t is defined by (3.11) and denotes the response of aggregate output to labor reallocation. In turn, the second-best level of automation $\alpha^{\text{SB}}\left(\boldsymbol{\eta}\right)$ satisfies

$$\int_{0}^{+\infty} \exp\left(-\rho t\right) \times \left\{\sum_h \phi^h \eta^h \frac{u'\left(c_{0,t}^{h}\right)}{u'\left(c_{0,0}^{h}\right)}\right\}\left\{\Delta_t^{\star} + T_0^{\prime,\text{LF}}\left(\alpha^{\text{SB}}\right)\Delta_t + \hat{\Phi}_t^h\right\} dt \;= 0, \tag{A.59}$$

where $T_0^{\prime,\text{LF}}\left(\cdot\right) > 0$ denotes the response of reallocation at the laissez-faire and $\left\{\hat{\Phi}_t^h\right\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \hat{\Phi}_t^h \equiv 0$ for all periods t. By assumption, the government chooses weights $\boldsymbol{\eta} \equiv \boldsymbol{\eta}^{\text{effic}}$ that ensure that the distributional terms net out. Therefore,

$$\int_{0}^{+\infty} \exp\left(-\rho t\right) \times \left\{\sum_h \phi^h \eta^h \frac{u'\left(c_{0,t}^{h}\right)}{u'\left(c_{0,0}^{h}\right)}\right\}\left\{\Delta_t^{\star} + T_0^{\prime}\left(\alpha^{\text{SB}}\right)\Delta_t\right\} dt \;= 0, \tag{A.60}$$

Again, let $\lambda_t^h \equiv \frac{u'\left(c_{0,t}^{h}\right)}{u'\left(c_{0,0}^{h}\right)}$. Note that the sequence $\left\{\lambda_t^A\right\}$ is more *front-loaded* than $\left\{\lambda_t^N\right\}$. For the reasons outlined in Section 6.3.2, the sequence $\{\Delta_t\}$ can itself be front- or back-loaded depending on the average duration of unemployment / retraining spells. When this reallocation is fast, i.e. $1/\kappa$ small, the sequence $\{\Delta_t\}$ is *front-loaded*.[82] In this case, the term involving $\{\Delta_t\}$ in (A.60) is negative at α_0^{LF} since the government's values relatively more flows which are more distant in the future. This reinforces the government's desire to *curb* automation. When this reallocation is slow, i.e. $1/\kappa > 1/\kappa^{\star}$ large, the sequence $\{\Delta_t\}$ is *back-loaded*.[83] Therefore, term involving $\{\Delta_t\}$ in (A.60) is α_0^{LF} is positive. This reduces the government's desire to curb automation. In theory, this case might not present itself. The reason is that workers might decide not to reallocate altogether if the average

[82] The sequence is initially positive as $(1-\theta)\, w_t^N > w_t^A$ at the equilibrium stopping time (Appendix A.3 and the left panel of Figure 5.1a). It declines over time as wages converge, and eventually becomes negative.

[83] In the limit with infinitely long unemployment spells, $1/\kappa \to +\infty$, the sequence is entirely back-loaded since workers are unemployed for a long-time. The sequence thus increases over time. However, $(1-\theta)\, w_t^N < w_t^A$ when workers exit unemployment (Appendix A.3 and the left panel of Figure 5.1a), so workers would choose not to reallocate in the first place.

duration of unemployment is too long (Assumption 2). In this case, we set $1/\kappa^\star \equiv +\infty$.

A.8 Proof of Proposition 7

We focus on the case with ex-ante interventions only to streamline the exposition. The proof is very similar in the case where the government intervenes ex ante (automation) and ex post (labor reallocation). Suppose that there are no borrowing frictions ($\underline{a} \to -\infty$). Then, the decentralized equilibrium is efficient (Proposition 3). As a result, there exist some weights $\boldsymbol{\eta}^\star$ that support this allocation as a first best (Section 3). This allocation is necessarily second best as well. Abstracting again from overlapping generations ($\chi \to 0$), the equilibrium level of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp\left(-\rho t\right) \times \sum_h \phi^h \eta^{h,\star} \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} \left\{\Delta_t^\star + \Phi_t^{\star,h} + T_0'\left(\alpha^{\text{LF}}\right)\left(\Delta_t + \Phi_t^h\right)\right\} = 0 \tag{A.61}$$

where Δ_t and $\Delta_t^\star$ are defined by (3.15) and (3.15) evaluated at the relevant allocation, and Φ_t^h and $\Phi_t^{\star,h}$ are distributional effects associated to more automation and more reallocation, respectively. By definition, these distributional pecuniary effects satisfy $\sum_h \phi^h \Phi_t^h = \sum_h \phi^h \Phi_t^{h,\star} = 0$ for all periods $t \geq 0$. Now, note that

$$\exp\left(-\rho t\right) \eta^{h,\star} \frac{u'\left(c_{0,t}^h\right)}{u'\left(c_{0,0}^h\right)} = \exp\left(-\rho t\right) \frac{U'\left(C_t^{\text{LF}}\right)}{U'\left(C_0^{\text{LF}}\right)} \tag{A.62}$$

using (A.39), where C_t^{LF} denotes aggregate output at the laissez-faire. Therefore, the pecuniary effects net out

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U'\left(C_t^{\text{LF}}\right)}{U'\left(C_0^{\text{LF}}\right)} \left\{\Delta_t^\star + T_0'\left(\alpha^{\text{LF}}\right)\Delta_t\right\} = 0 \tag{A.63}$$

Now, consider the second best problem for a goverment which values equity and uses symmetric weights $\bar{\eta}^{\frac{1}{\sigma}} \equiv \sum_h \phi^h \left(\eta^{h,\star}\right)^{\frac{1}{\sigma}}$. The second best degree of automation with equity concerns $\alpha^{\text{SB}}\left(\bar{\boldsymbol{\eta}}\right)$ then satisfies (A.61) with these new weights.

Following the same approach as in Appendices A.7–A.6, we evaluate the government's optimality condition at the laissez-faire level of automation. The second best level of au-

tomation remains unchanged if

$$\int_0^{+\infty} \exp\left(-\rho t\right) \times \frac{U'\left(C_t^{\mathrm{LF}}\right)}{U'\left(C_0^{\mathrm{LF}}\right)} \sum_h \left(\frac{\left(\eta^{h,\star}\right)^{\frac{1}{\sigma}}}{\sum_{h'} \phi^{h'} \left(\eta^{h',\star}\right)^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Delta_t^\star + \Phi_t^{\star,h} + T_0' \left(\alpha^{\mathrm{LF}}\right) \left(\Delta_t + \Phi_t^h\right) \right\} = 0 \tag{A.64}$$

Equivalently,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U'\left(C_t^{\mathrm{LF}}\right)}{U'\left(C_0^{\mathrm{LF}}\right)} \sum_h \left(\frac{\left(\eta^{h,\star}\right)^{\frac{1}{\sigma}}}{\sum_{h'} \phi^{h'} \left(\eta^{h',\star}\right)^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Phi_t^{\star,h} + T_0' \left(\alpha^{\mathrm{LF}}\right) \Phi_t^h \right\} = 0, \tag{A.65}$$

using (A.63). Furthermore, note that $\eta^{A,\star} < \eta^{N,\star}$. The reason is that workers initially employed in automated occupations claim a lower human wealth (Appendix A.3), and the weights are inversely proportional to consumption (A.39). In addition, note that

$$\Phi_t^{\star,A} + T_0' \left(\alpha^{\mathrm{LF}}\right) \Phi_t^A < 0 \quad \text{and} \quad \Phi_t^{\star,N} + T_0' \left(\alpha^{\mathrm{LF}}\right) \Phi_t^N < 0, \tag{A.66}$$

since these terms capture the distributional effects of automation in general equilibrium. As automation increases, workers initially employed in these occupations are worse off. They relocate more as a result of this change, but still earn no more than those initially employed in automated occupations (Appendix A.3). Putting this together,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U'\left(C_t^{\mathrm{LF}}\right)}{U'\left(C_0^{\mathrm{LF}}\right)} \sum_h \left(\frac{\left(\eta^{h,\star}\right)^{\frac{1}{\sigma}}}{\sum_{h'} \left(\eta^{h',\star}\right)^{\frac{1}{\sigma}}} \right)^{-\sigma} \left\{ \Phi_t^{\star,h} + T_0' \left(\alpha^{\mathrm{LF}}\right) \Phi_t^h \right\} < 0, \tag{A.67}$$

since the left-hand side of (A.67) puts a higher weight on negative payoffs. Therefore, automation is *excessive*, regardless of the average duration of unemployment / retraining spells.

A.9 Proof of Proposition 8

We show that the laissez-faire allocation converges to its *first best* counterpart in the long-run. It follows that it also converges to its *second best* counterpart, regardless of whether

the government has commitment or not.[84]

We now guess and verify that the laissez-faire converges to the first best with utilitarian weights $\eta_s^h \propto \exp(-\rho s)$. It suffices to verify that the equilibrium sequence of interest rates $r_t \to \rho$ as $t \to +\infty$. The reason is twofold. First, other aggregates allocations are continuous in $\{r_t\}$ (Section A.3) so that the guess that $\{\alpha_t, x_t, \boldsymbol{\mu}_t, \Pi_t\}$ converges to its first best steady states counterparts is verified too — this part is very similar to the proof of Proposition 10 in Appendix A.10 so we omit it. Second, individual allocations $\mathbf{c}$ are symmetric across workers $c_{s,t}^h = C_t$ both at the laissez-faire and the first best with weights $\boldsymbol{\eta}$.[85] As a result, individual allocations necessarily concide too in the long-run.

To show that $r_t \to \rho$ as $t \to +\infty$, note that all workers are unconstrained at equilibrium except for the surviving mass $\exp(-\chi t)$ of workers born in $s < 0$ and initially employed in automated occupations $h = A$. Furthermore, note that all these other workers earn the same income

$$\hat{\mathcal{Y}}_t^{\text{unconstr}} \equiv \frac{1}{1 - \phi \exp(-\chi t)} \left\{ G^\star(\cdot) - x_t \alpha_t - \omega x_t^2 \alpha_t - \phi \exp(-\chi t) \hat{\mathcal{Y}}_t^{\text{constr}} \right\} \tag{A.68}$$

where $\hat{\mathcal{Y}}_t^{\text{constr}} < +\infty$ is the income of constrained workers. Therefore, the income of unconstrained workers converges to the long-run aggregate consumption at the first best $\hat{\mathcal{Y}}_t^{\text{unconstr}} \to C^{\text{FB}}$ as $t \to +\infty$, using the fact that all other aggregates converge to their first best counterpart and the aggregate resource constraint. It follows that individual consumption $c_t^{\text{unconstr}} \to C^{\text{FB}}$ as $t \to +\infty$ by market clearing. As a result, the interest rate converges to the subjective discount factor $r_t \to \rho$ as $t \to +\infty$, using (A.18).

A.10 Additional Results

Proposition 9 (Distorsions in PE and GE)**.** *Fix prices and profits at the level that prevails in an efficiency economy without borrowing constraints $\underline{a} \to -\infty$. Then, the consumption choices are distorted if and only if $\underline{a} > a^\star(\lambda, \kappa, \theta, \chi)$ where $a^\star(\cdot)$ is defined in Proposition 3. Furthermore, the labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^\star(\cdot)$. Turning to general equilibrium, automation α and reallocation $\{\boldsymbol{\mu}_t\}$ are distorted if and only if $\underline{a} > a^\star(\lambda, \kappa, \theta, \chi)$.*

Proof. This result consists of two parts: a partial equilibrium one, and a general equilib-

[84] The reason is that workers are hand-to-mouth $\underline{a} \to 0$ so that all the state variables in the government's problem are under its control.

[85] At the laissez-faire, workers are hand-to-mouth so that $c_{s,t}^h = \hat{\mathcal{Y}}_{s,t}^h$ and $\hat{\mathcal{Y}}_{s,t}^h / \hat{\mathcal{Y}}_{s',t}^{h'} \to 1$ as $t \to +\infty$ for all occupations (h, h') and generations (s, s'), using (5.1) and the fact that wages are equalized in the long-run. At the first best, symmetry follows directly from (A.3).

rium one. We consider the former first, and return to the latter at the end of the proof. We have already shown that consumption choices are distorted if and only if $\underline{a} > a^\star(\lambda, \kappa, \theta)$ as part of Proposition 3. We now show that labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^\star(\cdot)$. Figure A.1 depicts the dynamics of assets and these thresholds graphically. Throughout, we denote by $\bar{T}_0$ the stopping time that prevails at the efficiency equilibrium without borrowing constraints. All prices are understood to be the ones at this particular equilibrium.

Sufficiency. We proceed in three steps. First, we show that the labor supply choices are distorted only if borrowing constraints are binding at equilibrium at $t = \bar{T}_0$, where $\bar{T}_0$ is the stopping time in the frictionless economy ($\underline{a} \to -\infty$). Second, we show that these borrowing constraints are binding in period $t = \bar{T}_0$ if $\underline{a} = 0$. Third, we show that there exists some $\hat{a}$ with $a^\star \leq \hat{a} \leq 0$ such that the borrowing constraints are binding at equilibrium in period $t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. Finally, we show that the borrowing constraints are *not* binding at equilibrium in period $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. The desired result follows immediately.

Step 1. We show that labor supply choices are distorted only if borrowing bind at equilibrium at $t = \bar{T}_0$. To see this, note that the reallocation decisions (A.27) and (A.27) when unconstrained are purely forward-looking. In particular, they are not indexed by workers' asset holdings, and whether they were constrained in any period $t < \bar{T}_0$. Therefore, the labor supply choices are distorted only if borrowing constraints bind in period $t = \bar{T}_0$.[86]

Step 2. We now show that the borrowing constraints are binding in period $t = \bar{T}_0$ if $\underline{a} = 0$. To derive a contradiction, suppose that this is not the case. Then, all workers are hand-to-mouth since none of them can save at equilibrium. Furthermore, their Euler equations hold with equality since they are unconstrained. Therefore,

$$\frac{\hat{\mathcal{Y}}^A_{s,t} + \Pi_t}{\hat{\mathcal{Y}}^A_{s,\bar{T}_0} + \Pi_{\bar{T}_0}} = \frac{\hat{\mathcal{Y}}^N_{s,t} + \Pi_t}{\hat{\mathcal{Y}}^N_{s,\bar{T}_0} + \Pi_{\bar{T}_0}} \tag{A.69}$$

for all $s < 0$ and $t \geq \bar{T}_0$ since preferences are isoelastic. However, this restriction cannot hold since $\hat{\mathcal{Y}}^A_t$ increases over time while $\hat{\mathcal{Y}}^N_t$ decreases, using labor incomes (A.24)–(A.25). This leads to the desired contradiction.

Step 3. By continuity of the equilibrium with respect to $\underline{a}$, there exists some $\hat{a}$ with $a^\star \leq \hat{a} \leq 1$ such that the borrowing constraints are binding at equilibrium in period

[86] We have assumed continue to assume that borrowing constraints either bind in period $t = T_0$ or never bind afterwards (Appendix A.3).

$t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. This threshold satisfies

$$\hat{a} = \begin{cases} a' & \text{if } a' \leq 0 \\ -\infty & \text{otherwise} \end{cases}, \tag{A.70}$$

where a' ensures that workers initially employed in automated occupations do not want to save or dissave in period $t = \bar{T}_0$

$$\frac{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} \hat{r}_s ds\right) \left(\hat{\mathcal{Y}}_t^0 + \Pi_t\right) dt + a'}{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_{\bar{T}_0}^{t} r_s ds - \rho\left(t - \bar{T}_0\right)\right)\right)^{-1} dt} = \hat{\mathcal{Y}}_{\bar{T}_0}^A + \Pi_{\bar{T}_0} + \hat{r}_{\bar{T}_0} a', \tag{A.71}$$

where incomes and prices are those that prevail at the equilibrium with no borrowing frictions.

Step 4. It remains to show that the borrowing constraints are *not* binding at equilibrium in period $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. This is the case when

$$\hat{r}_{\bar{T}_0}^{-1} \geq \int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} \hat{r}_s ds\right) \hat{u}\left(\exp\left(\int_{\bar{T}_0}^{t} r_s ds - \rho\left(t - \bar{T}_0\right)\right)\right) dt \tag{A.72}$$

as total income exceeds consumption when $\underline{a} \leq \hat{a}$. Otherwise, $a' > 0$ using (A.71), since the sequence $\{\hat{\mathcal{Y}}_t^0 + \bar{\Pi}_t\}$ is increasing at the original equilibrium, and $r_t \geq \rho$ for all t if the economy without borrowing constraints grows over time. Therefore, $\hat{a} = -\infty$ — i.e. borrowing constraints are always binding — and $\underline{a} \leq \hat{a}$ is never satisfied.

Necessity. Let $\underline{a} > \hat{a}$ where the threshold is given by (A.70). Then, workers initially employed in automated occupations are constrained in period $t = \bar{T}_0$. We now show that the labor reallocation and automation decisions are distorted in this case. To see this, fix the continuation sequence of interest rates $\{\bar{r}_t\}_{t \geq \bar{T}_0}$ and wages $\{\bar{w}_t^h\}_{t \geq \bar{T}_0}$ that prevail in the frictionless equilibrium. By definition of the stopping time $\bar{T}_0$,

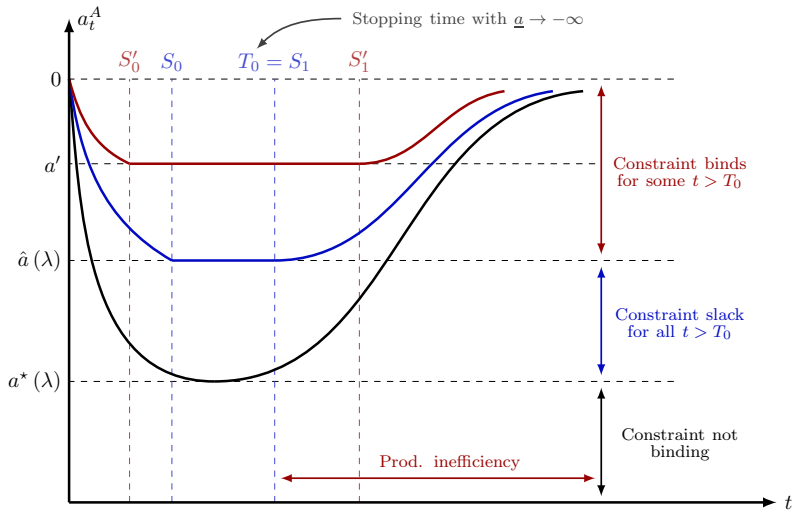
$$\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} r_\tau d\tau\right) \bar{\Delta}_t dt = 0 \tag{A.73}$$

using (A.27), where Δ_t was defined in Appendix A.3 . However, these workers are constrained in period $t = \bar{T}_0$, by assumption. That is, they are hand-to-mouth over a choice-specific interval $t \in [\bar{T}_0, S_1)$. Note that the optimality condition (A.26) is *generically* not satisfied when (A.73) holds.[87] We show in Section 6 that reallocation can be excessive

[87] Whenever this condition happens to be satisfied, there exists a small perturbation of the average duration

$(T_0 > \bar{T}_0)$ or insufficient $(T_0 < \bar{T}_0)$ depending on the average duration of unemployment / retraining spells $1/\kappa$.

Figure A.1: Assets



Turning to the general equilibrium part of the result, automation and reallocation are distorted when $\underline{a} > a^\star(\lambda, \kappa, \theta, \chi)$. The reason is that borrowing constraint bind in this case, by definition of $a^\star(\cdot)$. Therefore, the equilibrium sequence of interest rates $\{r_t\}$ differs from the one $\{\bar{r}_t\}$ in the economy without borrowing constraints. As a result, labor supply choices (A.27) are distorted — even if workers remain unconstrained in period $t = \bar{T}_0$ — and so is the automation choice (A.36). □

Proposition 10 (Second Welfare Theorem)**.** *A first best allocation supported by some Pareto weights $\boldsymbol{\eta}$ can be decentralized with lump sum transfers*

$$\tau_{s,t}^h = \frac{\left(\eta_s^h \exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^h \exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau} C_t - \left\{\hat{\mathcal{Y}}_{s,t}^h + C_t - \sum_k \phi^k \hat{\mathcal{Y}}_{s,t}^k\right\}$$

for each initial occupation h, all ages s and calendar time t, where the quantities on the right-hand side are given by Proposition 1 and (5.1).

Proof. We first show that the first best allocation $\{X_t\}$ associated to the weights $\boldsymbol{\eta}$ is part of an equilibrium (ex post), given the transfers

$$\tau_{s,t}^h = \frac{\left(\eta_s^h \exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^h \exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau} C_t - \left\{\hat{\mathcal{Y}}_{s,t}^h + C_t - \sum_k \phi^k \hat{\mathcal{Y}}_{s,t}^k\right\} \quad \text{(A.74)}$$

of unemployment spells $1/\kappa$ that ensures it does not.

and the level of automation α^{FB}. Then, we show that the equilibrium level of automation is α^{FB} (ex ante) when anticipating $\{X_t\}$.

Ex post. Fix the level A.3 of automation α^{FB} and the transfers (A.74). We conjecture and verify that the following sequence of interest rate $\{r_t\}$, wages $\{w_t^h\}$, profits $\{\Pi_t\}$ are part of an equilibrium[88],

$$\exp\left(-\int_0^t r_s ds\right) = \exp\left(-\rho t\right) \frac{U_t'\left(C_t\right)}{U_0'\left(C_0\right)} \quad \text{for all } t \geq 0, \tag{A.75}$$

and

$$w_t^h = \mathcal{Y}_t^h \quad \text{for each } h \in \{0,1\} \text{ and all } t \geq 0, \tag{A.76}$$

and

$$\Pi_t \equiv C_t - \phi \mu_t^A \mathcal{Y}_t^A - \left(1-\phi\right) \mu_t^N \mathcal{Y}_t^N, \tag{A.77}$$

and that the associated allocation coincides with the first best. The quantities on the right-hand side of (A.75)–(A.77) correspond to the first best. It suffices to show that the planner's allocations of consumption (A.3) and labor (A.7)–(A.10) are consistent with workers' optimality given these prices. By construction, the remaining equilibrium conditions are satisfied: labor markets clear given wages (A.76) and the resource constraint (4.8) is satisfied.

Focusing on the consumption allocation first, we now show that: (i) workers can afford these consumption allocations with a balanced budget given the sequence of wages (A.76), profits (A.77) and the optimal mobility decision implicit in (A.7)–(A.10); and (ii) these consumption allocations ensure that workers' Euler equations hold with equality given the interest rate (A.75). Pre-transfer labor incomes $\hat{\mathcal{Y}}_{s,t}^h$ are given by (A.24)–(A.25). By construction, transfers (A.74) ensure that the first best consumption allocations allocations of consumption (A.3) are affordable for workers and ensure that they have a balanced budget

$$\frac{\left(\eta_s^h \exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi\tau\right) \left(\eta_{t-\tau}^h \exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau} C_t = \hat{\mathcal{Y}}_{s,t}^h + \Pi_t + \tau_{s,t}^h \tag{A.78}$$

since wages $\{w_t^h\}$ and profits $\{\Pi_t\}$ are given by (A.76)–(A.77) and equilibirum output satisfies $Y_t = C_t$. We still have to show that the consumption allocations (A.78) are optimal. Consider the planner's intratemporal problem (A.2). At the optimum of this

[88] The sequence $\{C_t\}$ characterized by (3.10)–(3.13) is continuous but not differentiable at $t \in \{T_0, T_1\}$.

problem, consumption allocations satisfy

$$\eta_{t-s}^{h} \exp\left(-\rho s\right) u'\left(c_{t-s,t}^{h}\right) = \exp\left(-\rho t\right) U_t'\left(C_t\right) \tag{A.79}$$

It follows that workers' Euler equations hold with equality, by definition of the sequence of interest rates (A.75) and using restriction (A.79).

Turning to the labor allocation, we now show that the effective labor supplies coincide with the first best ones (A.7)–(A.10) given the sequence of wages (A.76), profits (4.6) and interest rate (A.75). Occupational choices are still characterized for two equilibrium stopping times $\left(T_0^{LF}, T_1^{LF}\right)$. We now show that those coincide with their first best counterparts $\left(T_0^{\text{FB}}, T_1^{\text{FB}}\right)$. The first stopping time $\left(T_0^{LF}\right)$ is characterized by

$$\begin{aligned}\int_{T_0^{LF}}^{+\infty} \exp\left(-\int_{T_0^{LF}}^{t} r_\tau d\tau\right) \exp\left(-\chi t\right) \times \\ \left(\left(1-\theta\right)\left(1-\exp\left(-\kappa\left(t-T_0^{LF}\right)\right)\right) w_t^N - w_t^A\right) dt = 0\end{aligned} \tag{A.80}$$

since transfers (A.74) ensure that workers are unconstrained, and using (A.27). It follows that the equilibrium stopping time coincides with the first best $T_0^{LF} = T_0^{\text{FB}}$, using the definition of the first best stopping time (3.10), wages (A.76) and the stochastic discount factor (A.75). The proof for the second stopping time $\left(T_1^{LF}\right)$ is very similar, so we omit it for concision.

Ex ante. Finally, we show that the first best degree of automation α^{FB} solves the firm's problem (4.1) when it anticipates the equilibrium sequence $\{X_t\}$. Using (A.36), the interest rates (A.75) and the fact that labor reallocation is unchanged when the degree of automation is α^{FB},

$$\int_0^{+\infty} \exp\left(-\rho t\right) U_t'\left(C_t\right) \frac{\partial}{\partial \alpha} G^\star\left(\boldsymbol{\mu}_t; \alpha^{FB}\right) = 0 \tag{A.81}$$

with $C_t = G^\star\left(\boldsymbol{\mu}_t; \alpha^{FB}\right)$, so the degree of automation is efficient $\alpha^{LF} = \alpha^{\text{FB}}$. □

Lemma 2. *Suppose that either: there are no reallocation frictions $1/\lambda, 1/\kappa, \theta \to 0$; or there are no borrowing frictions $\underline{a} \to -\infty$. Then, $\tau_{s,t}^h = \mathbf{0}$ implements a first best allocation.*

Proof. Consider first the case without mobility frictions $1/\lambda, 1/\kappa, \theta \to 0$. Then, the marginal productivities are equalized across occupations $\mathcal{Y}_t^A = \mathcal{Y}_t^N$. Fix the set of weights $\eta_t^A = \eta_t^N \propto \exp\left(\rho t\right)$. Using (A.74), $\left\{T_{s,t}^h\right\} = \mathbf{0}$ with these particular weights.

Now, consider the case without borrowing frictions $\underline{a} \to -\infty$. Automation has distributional consequences in this case, but the economy is otherwise efficient since borrowing

is frictionless. Fix the set of weights

$$\left(\eta_t^h\right)^{\frac{1}{\sigma}} \propto \int \exp\left(-\left(\rho+\chi\right)t\right) U_\tau'\left(C_\tau\right)\left(1+\sum_k\left(1-\phi^k\right)\hat{\mathcal{Y}}_{t,\tau}^k/C_\tau\right) C_\tau d\tau \tag{A.82}$$

Using (A.74),

$$\int \exp\left(-\left(\rho+\chi\right)\tau\right) U_\tau'\left(C_\tau\right) T_{t,\tau}^h d\tau = 0 \tag{A.83}$$

Therefore, $\left\{T_{s,t}^h\right\} = \mathbf{0}$ implements the same allocation since workers are unconstrained. □

B Quantitative Appendix

In this appendix, we describe our quantitative model in more details and we discuss the approach used to simulate and calibrate the model. Section B.1 provides a recursive formulation of the workers' problem. Section B.2 states and characterizes the solution to the occupations' problem. Section B.3 discusses our choice of Pareto weights for our normative exercise. Finally, Section B.4 provides details about our numerical implementation.

B.1 Workers' Problem

We discretize time into periods of constant length $\Delta \equiv 1/N > 0$, and solve the workers' problem in discrete time.[89] The workers' problem can be formulated recursively

$$\begin{aligned} V_t^h\left(a,e,\xi,z\right) = \max_{c,a'} \; & u\left(c\right)\Delta + \exp\left(-\left(\rho+\chi\right)\Delta\right) V_{t+\Delta}^{h,\star}\left(a',e,\xi,z\right) \\ \text{s.t. } & a' = \left(\mathcal{Y}_t\left(\mathbf{x}\right)-c\right)\Delta + \frac{1}{1-\chi\Delta}\left(1+r_t\Delta\right)a \\ & a' \geq 0 \end{aligned} \tag{B.1}$$

for employed workers $(e = E)$ and unemployed workers $(e = U)$ The continuation value

[89] Alternatively, we could have formulated the workers' problem in continuous time and solved the associated partial differential equation using standard finite difference methods. However, (semi-)implicit schemes are non-linear in our setting due to the discrete occupational choice. This requires iterating on (B.1)–(B.5) to compute policy functions — which limits the efficiency of these schemes. We found that explicit schemes were unstable unless we use a particularly small time step Δ — which again proves relatively inefficient. In constrast, formulating and solving the workers' problem in discrete time proves to be relatively fast.

$V^{\star}$ before workers observe the mean-reverting component of their income is given by

$$V_t^{h,\star}\left(a',e,\xi,z\right)=\int \hat{V}_t^h\left(a',1,\xi,z'\right)P\left(dz',z\right), \tag{B.2}$$

where $\hat{V}_t\left(\cdot\right)$ is the continuation value associated to the discrete occupational choice. The continuation value for employed workers $(e=E)$ associated to this discrete chocie problem is[90]

$$\begin{aligned}\hat{V}_t^h\left(a,e,\xi,z\right)=&\left(1-\lambda\Delta\right)V_t^h\left(a,e,\xi,z\right)+\\&\lambda\Delta\gamma\log\left(\sum_{h'}\phi^{h'}\exp\left(\frac{V_t^{h'}\left(a,e'\left(h',\mathbf{x}\right),\xi,z\right)}{\gamma}\right)\right)\end{aligned} \tag{B.3}$$

with $e'\left(\cdot\right)=E$ if $h'=h$ and $e'\left(\cdot\right)=U$ otherwise. The associated mobility hazard across occupations is

$$\mathcal{S}_t\left(h';\mathbf{x}\right)=\frac{\phi^{h'}\exp\left(\frac{V_t^{h'}(\mathbf{x}'(h';\mathbf{x}))}{\gamma}\right)}{\sum_{h''}\phi^{h''}\exp\left(\frac{V_t^{h''}(\mathbf{x}'(h'';\mathbf{x}))}{\gamma}\right)} \tag{B.4}$$

In turn, the continuation value for unemployed workers $(e=U)$ is ñ

$$\hat{V}^h\left(a,e,\xi,z\right)=\left(1-\kappa\Delta\right)V^h\left(a,e,\xi,z\right)+\kappa\Delta\sum_{h'}\mathcal{S}\left(h',\mathbf{x}\right)V^{h'}\left(a,1,\xi'\left(h',\mathbf{x}\right),z\right) \tag{B.5}$$

where $\mathcal{S}\left(\cdot\right)$ isthe mobility hazard, and $\xi'\left(\cdot\right)=\xi$ if $h'\neq h$ and $\xi'\left(\cdot\right)=\left(1-\theta\right)\xi$ otherwise. The associated mobility hazard across occupations is

$$\mathcal{S}_t\left(h';\mathbf{x}\right)=\begin{cases}1-\kappa^{\star} & \text{if } h'\neq h\\ \kappa^{\star} & \text{otherwise}\end{cases} \tag{B.6}$$

New generations who enter the labor market draw a random productivity z from its stationary distribution and then choose their occupation with a hazard similar to the employed workers'. The only difference is that they experience neither an unemployment spell nor a productivity loss. Worker's labor income is

$$\mathcal{Y}_t\left(\mathbf{x}\right)=\begin{cases}\left(1-\tau_t\right)\xi\exp\left(z\right)w_t^h & \text{if } e=E\\ b\mathcal{Y}_t^{h'}\left(a,E,\xi,z\right)/\left(1-\tau_t\right) & \text{otherwise}\end{cases}, \tag{B.7}$$

[90] See Artuç et al. (2010) or Caliendo et al. (2019) for the derivation.

with $h' \neq h$ denoting the previous occupation of employment. The permanent component of workers' income (ξ) is reduced by a factor $(1-\theta)$ whenever a worker who exits unemployment chooses to enter her new occupation in (B.6). Finally, the mean-reverting component income (z) evolves as

$$z' = (1 + (\rho_z - 1)\Delta) z + \sigma_z \sqrt{\Delta} W' \quad \text{with} \quad W' \sim \text{i.i.d.} \mathcal{N}(0,1) \tag{B.8}$$

B.2 Firms' Problem

The occupations' production problem and the mutual fund's investment problem can be consolidated into a single problem. We solve the consolidated problem in continuous time. This problem can be formulated recursively

$$\begin{aligned} r_t W_t^h(\alpha) = \max_{\{\mu, x, \alpha'\}} \Big\{ & A^h p_t^h (\alpha + \mu)^{1-\eta} - w_t^h \mu - (1 + \tau_t^x) x - \omega \left(\frac{x}{\alpha}\right)^2 \alpha \\ & + (x - \delta\alpha) W_t^{h,\prime}(\alpha) \Big\} + \frac{\partial}{\partial t} W_t^h(\alpha) \\ \text{s.t. } & x \in \Sigma^h \end{aligned} \tag{B.9}$$

for each occupation $h \in \{A, N\}$, where x denotes gross investment, i.e. $d\alpha_t = (x_t - \delta\alpha_t) dt$. The constraint set Σ^h captures the non-negativity of x and the constraint $\alpha^{(N)} = 0$ for non-automated occupations. For notation, we set $\tau_t^x = 0$ in the following. At optimum, the labor demand is characterized by

$$(1-\eta) A^h p_t^h \left(\alpha_t^h + \mu_t^h\right)^{-\eta} = w_t^h, \tag{B.10}$$

where $\{\mu^h\}$ denotes effective labor (demand). In turn, the automation choice for automated occupation $(h = A)$ is characterized by two differential equations. First, investment satisfies

$$(r_t + \delta)(1 + 2\omega x_t^\star) = \left\{ (1-\eta) A p_t^h \left(\mu_t^h + \alpha_t\right)^{-\eta} + \omega (x_t^\star)^2 \right\} + 2\omega \partial_t x_t^\star \tag{B.11}$$

together with a standard transversality condition. Second, automation satisfies the law of motion

$$d\alpha_t = (x_t^\star - \delta) \alpha_t dt \tag{B.12}$$

with the initial condition $\alpha_0 = 0$. The price of the good produced in each occupation is

given by

$$p_t^h = \left(\sum_g \phi^g \left\{ A^g \left(\alpha_t^g + \mu_t^g \right)^{(1-\eta)} \right\}^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}-1} \left\{ A^h \left(\alpha_t^h + \mu_t^h \right)^{(1-\eta)} \right\}^{-\frac{1}{\nu}} \tag{B.13}$$

in the two occupations. The equity of the mutual fund is[91]

$$M_t \equiv -\int_0^{+\infty} \exp\left(-\int_0^s r_{t+s} \right) \left(r_t^\alpha - \left(1 + \tau_t^x\right) x_t^\star - \omega \left(x_t^\star \right)^2 \right) \alpha_t ds \tag{B.14}$$

where $r_t^\alpha = w_t^A$ is the rental rate of automation. Finally, the firms' profits are

$$\Pi_t \equiv Y_t - \sum_h \phi^h w_t^h \mu_t^h - r_t^\alpha \alpha_t \tag{B.15}$$

with $Y_t \equiv G^\star \left(\mu_t^A, \mu_t^N; \alpha_t \right)$ denoting firms' output.

B.3 Pareto Weights

The government's objective is

$$\begin{aligned} \mathcal{W} \equiv & \chi \int_{-\infty}^0 \int \eta_s \left(\mathbf{x} \right) \exp\left(\left(\rho + \chi \right) s \right) V_0 \left(\mathbf{x} \right) \pi_{s,0}^{\text{old}} \left(d\mathbf{x} \right) ds \\ & + \chi \int_0^{+\infty} \eta_s \left(z \right) \int V_s^{\text{new}} \left(z \right) P^\star \left(dz \right) ds, \end{aligned} \tag{B.16}$$

for some Pareto weights $\boldsymbol{\eta}$. The first and second terms capture the contributions of existing $(s < 0)$ and new generations $(s \geq 0)$, respectively. The value $\exp\left(\left(\rho + \chi \right) s \right) V_0$ is the continuation utility of existing generations over periods $t \geq 0$. Following Calvo and Obstfeld (1988) and Itskhoki and Moll (2019), this continuation value is evaluated *at birth*, i.e., in period $t = s < 0$, to ensure time-consistency. The measure $\pi_{s,0}^{\text{old}}$ is the distribution of idiosyncratic states for existing generations in period $t = 0$ (conditional on survival).[92] In turn, the value

$$V_t^{\text{new}} \left(z \right) \equiv \gamma \log \left(\sum_h \phi^h \exp\left(\frac{V_t^h \left(0, 1, 0, z \right)}{\gamma} \right) \right)$$

[91] This equity is a liability for the mutual firm, hence the negative sign. We suppose that the proceedings of the distorsionary tax τ_t^x are rebated lump sum to the mutual fund.

[92] Conditioning on survival (as in Calvo and Obstfeld, 1988) is without loss of generality, since the weights $\eta_s \left(\mathbf{x} \right)$ can be adjusted accordingly.

is the continuation utility for new generations born in period $t = s \geq 0$ reflects their occupational choice.[93] Finally, $P^\star$ denotes the ergodic distribution of the income process $z \sim P$, i.e., the distribution of states at birth.

We choose a set of weights that captures the *efficiency* motive for policy intervention. Our approach is similar to the one we adopted with our benchmark model (Section 6.3). The weights that the government puts on a given worker are inversely related to this worker's marginal utility at birth. This ensures that the government has no incentive to redistribute resources (at birth) to improve equity. In particular, the government weights (constrained) workers with a higher marginal utilitary *less* compared to a utilitarian government that values equity. We also assume the the government discounts generations at rate ρ over time, which ensures that consumption at the first best would be symmetric across generations, by analogy with (A.3). Therefore, we choose

$$\eta_s^h \propto \exp\left(-\rho s\right) \times 1/\partial_a V_s\left(\cdot\right),$$

where $1/\partial_a V_s\left(\cdot\right)$ is marginal utility of wealth at birth. In this case, the weight assigned to new generations satisfies

$$\exp\left(-\rho s\right)/\eta_s\left(z\right) = \sum_h \tilde{\mathcal{S}}_t\left(h,z\right) \partial_a \tilde{V}_t^h\left(a,h,z\right)\Big|_{a=0} \tag{B.17}$$

for all $t \geq 0$, where $\tilde{V}_t^h\left(\cdot\right)$ and $\tilde{S}_t\left(\cdot\right)$ denote the value (B.1) and mobility hazard (B.4) of a worker who is employed $e = 1$ and has never moved between occupations before $\xi = 0$.

Summing up, the government's objective becomes

$$\begin{aligned}\mathcal{W} \equiv & \int \frac{V_0\left(\mathbf{x}\right)}{\partial_a V_0\left(\mathbf{x}\right)} \pi_0\left(d\mathbf{x}\right) ds \\ & + \chi \int_0^{+\infty} \exp\left(-\rho s\right) \int \frac{V_s^{\text{new}}\left(z\right)}{\sum_h \tilde{\mathcal{S}}_t\left(h,z\right) \partial_a \tilde{V}_t^h\left(a,h,z\right)\big|_{a=0}} P^\star\left(dz\right) ds,\end{aligned} \tag{B.18}$$

since

$$\pi_0\left(d\mathbf{x}\right) \equiv \int_{-\infty}^0 \chi \exp\left(\chi s\right) \pi_{s,0}^{\text{old}}\left(d\mathbf{x}\right) ds$$

is the (unconditional) initial distribution of idiosyncratic states.

[93] Members of a new generation are born with no assets $a = 0$, are employed $e = 1$, and have not incurred the productivity cost associated to switching occupations $\xi = 0$.

B.4 Numerical Implementation

We now describe the numerical approach we adopt to solve for the stationary equilibrium and the transition dynamics.

Workers' problem. The problem (B.1) is potentially non-convex since it involves a discrete choice across occupations.[94] To address this issue, we follow an approach similar to Druedahl and Jørgensen (2017) and Iskhakov et al. (2017). Specifically, we recover a map $a' \mapsto a$ using the standard endogenous grid method (Carroll, 2006). This map is not necessarily monotonic since the problem is non-convex. In other words, this map defines a correspondence $a \rightrightarrows a'$ — this map contains the optimal policy $a \mapsto a'$. We recover this optimal function as follows. We first partition the map $a' \mapsto a$ into monotone segments. For each a, we interpolate linearly each of these segments to obtain candidates for a'. We then compare the values (B.1) obtained using these candidates. Optimal policies are those who achieve the highest value. We use a generalization of Young (2010)'s non-stochastic simulation method with multiple assets to iterate on the distribution. Finally, we discretize the income process on a 11-point grid using the method of Rouwenhorst (1995).

Firm's problem. The firm's optimal choice of investment and automation is characterized by the non-linear system of differential equations (B.11)–(B.12). We solve this system as follows when computing the transition paths from the initial to the final stationary equilibrium. First, we fix $\alpha_t = 0$ for all t and $x_T = \delta\alpha^\star$ for some terminal period T, where $\alpha^\star$ denotes the level of automation in $h = A$ at the final stationary equilibrium. We then solve for the sequence $\{x_t\}$ using (B.11) and a standard Runge-Kutta method. Next, we solve for the sequence $\{\alpha_t\}$ using (B.12). We repeat the previous two steps until the sequence $\{\alpha_t\}$ converges.

B.5 Additional Results

In Section 8.3, we discussed why the government finds it optimal to tax automation in the long-run. Figure B.1 plots utilitarian welfare as a function of the steady state level of automation α as a share of its laissez-faire counterpart α^{LF}.

[94] A finite elasticity of labor supply ($\gamma < +\infty$) ensures that $V^\star$ does not feature kinks. However, $V^\star$might remain convex on part of its domain. An Euler equation is still necessary but not sufficient since it can admit multiple candidates for consumption c and savings $s\,(c, \mathbf{x})$.

Figure B.1: Constrained efficiency (steady state)

