# Monetary Policy in a Small Open Economy with Multiple Monetary Assets

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#### Abstract

To examine the impact of openness on the volatility of macroeconomic variables in a small and open economy, we revisit the issues of money measurement. I compare the behavior of different money measures in the context of the New Keynesian framework with sticky price. I introduce the banking sector into the model, which allows the accommodation of multiple monetary assets like currency and interest-bearing-deposits. The central bank conducts its monetary policy via a simple interest rate rule. I explore the responses of different money measures, namely simple-sum, monetary base, and Divisia quantity aggregate with respect to domestic and foreign shocks and compare these responses with those from a theoretical benchmark. I find that Divisia tracks the movement of money most closely to the benchmark, followed by monetary base, while simple sum often does not match the correct trend. I analyze the impact of openness, which has an inverse relation with home-bias in consumption, on the volatility of macroeconomic variables. I find that as a small economy becomes more open, domestic inflation and nominal interest rate are more volatile while term of trade and exchange rate become more stable. Among the different money measures, monetary base and Divisia follow the monetary aggregate benchmark to become less volatile as consumption becomes less home-biased, while simple-sum, again, does not.

Keywords: Divisia monetary aggregates; open-economy macroeconomics; monetary policy; New Keynesian model; small open economy

JEL Classification: E31; E32; E41; E47; E51; E52

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## 1 Introduction

As the world is getting more and more integrated, all economies are open and most of them are small. Macroeconomic research on small open economy has caught more and more attention lately. The so-called New Open Economy Macroeconomics literature has grown rapidly in recent years. The discussion of New Keynesian literature on optimal monetary policy in an open economy focuses on whether exchange rate stability should be part of a central bank's strategy, for example, see McCallum (2007). Though the role of nominal variables like interest rate, money supply and exchange rate are emphasized in recent New Keynesian literature to have impact on real economic variables like output and growth in short-run due to the stickiness of price and wage. However, recent DSGE macroeconomic models often ignore aggregate quantity of money as an instrument of monetary policy. They tend to rely on (nominal) interest rate, such as the federal fed fund rate in modeling the thrust of monetary policy. In case of a small open economy, exchange rate is considered. The entire intervention of the central bank is expressed via some kind of interest rate rule, like the Taylor rule (1993). The support for this view lies in the empirical evidence in 1980s by Bernanke and Blinder (1988) that demand for money is more unstable than the demand for credit, therefore, monetary policy would have better success in stabilizing output if it stabilized interest rate rather than money supply. Moreover, Sims (1980) argues that money loses its predictive power on output when interest rate is included in the regression.

In response to the lacking attention on money quantities in the New Keynesian literature due to some empirical evidence that the demand for money is unstable and money has low power in explaining output and other macroeconomic variables, other economists have brought up the issue of money measurement. Up until the 1980s, economists throughout the world measured different levels of monetary aggregation, such as M0/MB (monetary base), M1 (narrow money), M2 (broad money), and M3 and M4 (financial liquidity), by simply adding up the quantities of component assets. This simple sum assigns the same weight to every monetary asset and implicitly assumes all monetary assets are perfect substitutes. As the financial system becomes more and more sophisticated with different type of monetary assets, which produce different interest rates, possess different liquidity and are at different levels of risk, this simple sum measure is clearly not a proper way to aggregate money. Chrystal and MacDonald (1994) used the term 'Barnett critique' to refer to the misleading and potential distortion of economic inferences if the simple-sum measure of money keeps being used.

Barnett (1978, 1980) derived the formula to measure the user cost price of monetary services and proposed a method to aggregate money based on solid microeconomic theory

foundation and index number theory. Accordingly, to properly aggregate components in monetary service aggregation, we need both their quantities and prices. Monetary asset services are not analogous to perishable consumer good services, such as apples, but to capital goods or durable goods, such as houses or automobiles. Hence, their prices are measured in term of user cost prices. The Divisia measure of money that Barnett proposed is a weighted index whose weights are based on the expenditure shares of component assets. Since the theory of monetary aggregation became available, central banks such as the Federal Reserve (FED) in the US, the Bank of England (BOE) in the UK, the European Central Bank (ECB), the Bank of Japan (BoJ), the National Bank of Poland, and the Bank of Israel, among others, have, at various times and in diverse ways, produced and maintained Divisia indexes for monetary aggregation. Simply replacing the traditional simple-sum measure of money by Divisia measures, Belongia (1996), Barnett and Chauvet (2010), Belongia and Ireland (2015), among many other researchers have shown that money still shares a strong relationship with aggregate economic activity, and the demand for money function still exhibits stability.

Though the New Open Macroeconomics has been getting more attention during the past two decades, the discussion on monetary policy in an open economy centers around the matter of stabilizing the exchange rate and money supply is completely out of sight. In this paper, we would like to bring attention to the measurement of money in a context of small open economy with home-bias in consumption. We used the recent developed microfounded, dynamic and stochastic New Keynesian model to examine the responses of different measures of money supply, including the official simple-sum measure, monetary base and Divisia measure to various macroeconomic shocks. To do so, we extended the New Keynesian model for a small open economy from Faia and Monacelli (2008) in a similar manner to Belongia and Ireland (2014) by introducing private financial institutions, who create deposit as imperfect substitute for government-issued currency. This framework allows us to construct and compare the behavior of different measures of monetary aggregates. It also allows us to look at the impact of openness to the stability of macroeconomic variables. In such an environment, we showed that Divisia measure is strictly better than simple-sum measure and monetary base in tracking the movement of money. Our findings, consistent with many others in the literature, reemphasize the Barnett critique that simple-sum is misleading and using it can distort inference about the economy. We advocate the use of Divisia index in measuring money supply. In the future work, we plan to further examine the role of monetary aggregate as a potential intermediate monetary policy target and compare it with interest rate target and exchange rate target in the context of a small open economy.

The rest of the paper is organized as the following. Section 2 provides details of the theoretical model. Section 3 presents the parameters calibration for numerical solution of

the model and discusses the results. Section 4 concludes. The full set of the equilibrium system can be found in the appendix.

## 2 Model

The world consists of two countries: home country (Home) and the rest of the world (Foreign). Home country is of sized n, and relatively small compared to Foreign whose size is 1 - n. Final goods are traded among two countries. The international financial market is accessible to residents in both economies. In addition to the interaction among traditional sectors in a small open economy, households, firms, and foreign sector, we introduce banking sector who receive deposits from and make loans to households. There also exists a central bank who conducts monetary policy. We character behaviors of each sector as below

#### 2.1 Households sector

Households consume a composite final goods, which is a combination of domestic goods and imported goods. These goods can be substituted with the elasticity of substitution  $\eta$ . The first optimization problem of households is to allocate their expenditure among domestic and foreign goods, taking the prices as given.

#### 2.1.1 First (intra-temporal) optimization problem

The composite bundle for consumption (CES) in home country

$$C_{t} = \left[ (1 - \gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$
(1)

where  $0 \le \gamma \le 1$  is the weight of domestic goods in the consumption bundle,  $\gamma = (1 - n)\alpha$ with 1 - n is the relative size of the Foreign and  $\alpha$  is the openness of the Home economy. The elasticity of substitution between domestic good and imported goods is  $\eta > 0$ . In a similar manner, the composite consumption bundle in Foreign is

$$C_t^* = \left[ (1 - \gamma^*)^{\frac{1}{\eta}} C_{H,t}^{*\frac{\eta-1}{\eta}} + \gamma^{*\frac{1}{\eta}} C_{F,t}^{*\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$
(2)

Home bias in consumption requires  $\alpha < 1$ , see Faia and Monacelli (2008) for more details.

$$1 - \gamma = 1 - (1 - n)\alpha > \gamma^* = n\alpha^* \tag{3}$$

In final consumption stage, each consumption bundle  $C_H, C_F$  itself is composed of imperfect substitutable varieties with elasticity of substitution  $\varepsilon$ 

$$C_{H,t} = \left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \left(\int_0^n C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
(4)

$$C_{F,t} = \left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \left(\int_{1}^{n} C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
(5)

The optimal allocation of expenditure between domestic and foreign goods depends on the ralative price of domestic and foreign goods,

$$C_{H,t} = (1 - \gamma) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \text{ and } C_{F,t} = \gamma \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{6}$$

and the optimal allocation within each variety of goods depends on the relative price of goods for each variety

$$C_{H,t}(j) = \frac{1}{n} \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \text{ and } C_{F,t}(j) = \left(\frac{1}{1-n}\right) \left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}$$
(7)

The general price level in the home country can be expressed as an index of domestic produced and imported prices (Consumer Price Index (CPI)).

$$P_t = \left[ (1 - \gamma) P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
(8)

#### 2.1.2 Second (inter-temporal) maximization problem

In the second optimization problem, households choose to allocate their resources to maximize the lifetime expected discounted utility.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t h_t U(C_t, N_t, \frac{M_t}{P_t})$$
(9)

where  $0 < \beta < 1$  is the discount factor and  $h_t$  is a preference shock, which is assumed to follow an AR(1) process

$$\ln(h_t) = \rho_h \ln(h_{t-1}) + \varepsilon_{h,t} \tag{10}$$

with  $0 \leq \rho_h < 1$  and  $\varepsilon_{h,t} \sim i.i.dN(0, \sigma_h^2)$ . Utility function is assumed to be increasing in (log) consumption and decreasing in labor.

$$U_t = \ln C_t - \psi_N \frac{N_t^{1+\xi}}{1+\xi} + \psi_M \ln\left(\frac{M_t}{P_t}\right)$$
(11)

with the inverse elasticity of labor  $\xi > 0$ , and coefficients  $\psi_M, \psi_N > 0$ . We also have money (real balance) entered in the utility function in log form. In this model, with the present of private financial institutions like commercial banks, households allocate their monetary assets between cash  $Ca_t$  and deposit  $D_t$  in commercial banks. Money in the utility function depends on a (true/theoretical) monetary aggregate relation

$$M_t = \left[\nu^{\frac{1}{\omega}} C a_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} D_t^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$
(12)

where  $0 \leq \nu \leq 1$  is the weight of cash among monetary assets, and  $\omega$  is the elasticity of substitution between cash and deposit. The above true monetary aggregate can not be observed in real data. For the purpose of comparing the behavior of different money measures in the model, we assume that we know the true monetary aggregation's functional form.

Households enter each period with a portfolio of maturing bonds  $B_{t-1}$  and monetary assets  $A_{t-1}$ . In the first sub-period, they receive lump-sum transfer from the central bank and allocate their monetary assets between cash and deposit. They can also take loan from commercial banks to finance these transactions.

$$\frac{B_t}{1+r_t} + Ca_t + D_t = A_{t-1} + B_{t-1} + T_t + L_t$$
(13)

In the second sub-period, households get wage from working. They also collect interest from their deposit  $r_t^D$  and pay interest on their loan  $r_t^L$ . At the end of each period, households receive nominal dividends from holding shares of intermediate firms. After all the payments are made, households carry  $A_t$  into the next period.

$$P_t C_t + A_t + (1 + r_t^L) L_t = C a_t + W_t N_t + F_t + (1 + r_t^D) D_t$$
(14)

Households choose the optimal sequences  $\{C_t, N_t, B_t, A_t, M_t, Ca_t, D_t, L_t\}_{t=0}^{\infty}$  to maximize the expected discounted utility subject to constraints (12),(13),(14). Let  $\Lambda_t^1, \Lambda_t^2, \Lambda_t^3$  be the Lagrangian multiplier on each constraint, we solved for the FOCs and obtained the Euler equation

$$\Lambda_t^3 = \beta \mathbb{E}_t \Lambda_{t+1}^3 (1 + r_{t+1}) \frac{P_t}{P_{t+1}}$$
(15)

where

$$\Lambda_t^3 = \frac{h_t}{C_t} \tag{16}$$

## 2.2 Foreign sector

Consumption goods is being traded among Home and Foreign. Export depends on the demand for domestic goods from the rest of the world. Assume that the structure of the Foreign economy is similar to the Home economy (except the size), hence, we can derive the Foreign demand for Home products as

$$C_{H,t}^{*}(j) = \frac{1}{n} \left( \frac{P_{H,t}^{*}(j)}{P_{H,t}^{*}} \right)^{-\varepsilon} C_{H,t}^{*}$$

$$\implies C_{H,t}^{*}(j) = \frac{1}{n} \left( \frac{P_{H,t}^{*}(j)}{P_{H,t}^{*}} \right)^{-\varepsilon} \gamma^{*} \left( \frac{P_{H,t}^{*}}{P_{t}^{*}} \right)^{-\eta} C_{t}^{*}$$
(17)

The general price level in Foreign (CPI) becomes

$$P_t^* = \left[ (1 - \gamma^*) (P_{F,t}^*)^{1-\eta} + \gamma^* (P_{H,t}^*)^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
(18)

Define term of trade as the relative price of imported goods

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} \tag{19}$$

Compute the CPI-PPI ratios for Home and Foreign, we see that they depend on the term of trade

$$\frac{P_t}{P_{H,t}} = [(1-\gamma) + \gamma S_t^{1-\eta}]^{\frac{1}{1-\eta}} \equiv q(S_t)$$
(20)

$$\frac{P_t^*}{P_{F,t}^*} = \left[ (1 - \gamma^*) + \gamma^* S_t^{\eta - 1} \right]^{\frac{1}{1 - \eta}} \equiv q^*(S_t)$$
(21)

The *law of one price* tells us that goods must be sold at the same price everywhere, after being converted into the same unit of currency either in Home or Foreign.

$$P_{H,t}(j) = NE_t P_{H,t}^*(j) \text{ and } P_{F,t}(j) = NE_t P_{F,t}^*(j) \text{ for all } j \in [0,1]$$
 (22)

where  $NE_t$  is the nominal exchange rate measured by the number of units of home currency per one unit of foreign currency. Real exchange rate is the relative price of one unit of domestic goods in term of imported goods.

$$Q_t \equiv N E_t \frac{P_t^*}{P_t} \tag{23}$$

where for eign price level  $P_t^\ast$  is exogenous. We see that term of trade and real exchange rate are linked through

$$Q_t = S_t \frac{P_t^*}{P_{F,t}^*} \left(\frac{P_t}{P_{H,t}}\right)^{-1}$$
(24)

We see that real exchange rate depends on the term of trade and other parameters that characterize the open economy.

$$Q_t = S_t \frac{q^*(S_t)}{q(S_t)} = S_t \frac{[(1 - \gamma^*) + \gamma^* S_t^{\eta - 1}]^{\frac{1}{1 - \eta}}}{[(1 - \gamma) + \gamma S_t^{1 - \eta}]^{\frac{1}{1 - \eta}}}$$
(25)

Notice that

$$\frac{\partial q(S_t)}{\partial S_t} > 0, \frac{\partial q^*(S_t)}{\partial S_t} < 0 \text{ and } \frac{\partial Q_t}{\partial S_t} > 0$$

We are looking at a small economy  $n \to 0$ , which implies  $P_{F,t}^* = P_t^*$  and  $q^*(S_t) = 1$ . Therefore,

$$Q_t = \frac{S_t}{q(S_t)} \tag{26}$$

and

$$S_{t} = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{t}^{*} N E_{t}}{P_{H,t}}$$
(27)

*Risk sharing/ Interest rate parity*: From the inter-temporal maximization problem of households, recall the optimal condition for bonds' holdings in Home is the Euler equation in (15). The analogue optimal condition for bond holding in Foreign is

$$\Lambda_t^{3*} = \beta \mathbb{E}_t \Lambda_{t+1}^{3*} (1 + r_{t+1}^*) \left( \frac{P_t^*}{P_{t+1}^*} \right)$$
(28)

Perfect capital mobility means both domestic residents and foreigners can invest in the bond market, hence their expected return from this bond must be the same after being converted into domestic currency.

$$\beta \mathbb{E}_t \left( \frac{\Lambda_{t+1}^3}{\Lambda_t^3} \frac{P_t}{P_{t+1}} \right) = \beta \mathbb{E}_t \left( \frac{\Lambda_{t+1}^{3*}}{\Lambda_t^{3*}} \frac{P_t^* N E_t}{P_{t+1}^* N E_{t+1}} \right)$$
(29)

Interest rate parity implies no arbitrage opportunity on bond market

$$1 + r_{t+1} = (1 + r_{t+1}^*) \left(\frac{NE_{t+1}}{NE_t}\right)$$
(30)

Rearrange Equation (29), we get

$$NE_{t}\frac{P_{t}^{*}}{P_{t}} = \frac{\Lambda_{t}^{3*}}{\Lambda_{t}^{3}}\mathbb{E}_{t}\left(\frac{\Lambda_{t+1}^{3}}{\Lambda_{t+1}^{3*}}\frac{P_{t+1}^{*}}{P_{t+1}}NE_{t+1}\right)$$
(31)

From households' FOCs, we know that  $\Lambda_t^3 = h_t/C_t$ , we further assume  $\Lambda_t^{3*} = 1/C_t^*$  and other initial conditions for the two economies so that we can iterate the expectation in (31) and rewrite the interest parity as

$$Q_t = \kappa \frac{h_t C_t}{C_t^*} \tag{32}$$

where Foreign demand is exogenous given. In our model, it follows an exogenous process,

$$\ln(C_t^*) = \rho_c^* \ln(C_{t-1}^*) + \varepsilon_{c,t}^*$$
(33)

with  $0 < \rho_c^* < 1$  and  $\varepsilon_{c,t}^* \sim i.i.dN(0, \sigma_c^{*2})$ .

## 2.3 Production sector and price setting

Monopolistic intermediate production firms use labor to produce homogeneous goods under a constant return to scale technology. These outputs are used to assemble final goods for domestic consumption and export to foreign consumers. Each monopolistic firm j use labor to produce homogeneous output with linear technology to meet the total demand for their product from the whole world

$$Y_t(j) = Z_t N_t(j) \tag{34}$$

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \tag{35}$$

The level of technology  $Z_t$  follows a random walk process

$$\ln(Z_t) = \ln z + \ln(Z_{t-1}) + \varepsilon_{z,t}$$
(36)

with  $\varepsilon_{z,t} \sim i.i.dN(0, \sigma_z^2)$ . Intermediate firms choose labor input to minimize their production cost,

$$\mathcal{L} = -W_t N_t(j) + \varphi_t(j) \left[ Z_t N_t(j) - \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \right]$$
(37)

FOC for cost minimization problem

$$\varphi_t(j) = \frac{W_t}{Z_t} \text{ for all } j \in [0, 1]$$
(38)

where the Lagrangian multiplier can be interpreted as nominal marginal cost. Hence, real marginal cost is expressed as

$$MC_t(j) = \frac{W_t}{Z_t P_{H,t}} \text{ for all } j \in [0,1]$$
(39)

Monopolistic intermediate firms have the power to set their price to maximize profit. In a sticky price environment, we set an adjustment cost for price setting according to Rotemberg (1982). Accordingly, all production firms face the same quadratic cost of adjusting their nominal prices. This cost is measured in term of unit of domestic final goods

$$\frac{\phi}{2} \left( \frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)} - 1 \right)^2 Y_t$$

and depends on the steady state of gross Home producer's inflation  $1 + \pi_H$ . If  $\phi = 0$ , prices are flexible. Each firm chooses to set its price  $P_{H,t}(j)$  to maximize its expected discounted profit

$$\max_{P_{H,t}(j)} \mathbb{E}_t \sum_{i=0}^{\infty} \beta^j \Lambda_{t+i}^3 F_{t+i}$$

where nominal profit is

$$F_t(j) = \left[ P_{H,t}(j)Y_t(j) - W_t N_t(j) - \frac{\phi}{2} \left( \frac{P_{H,t}(j)}{(1+\pi_H)P_{H,t-1}(j)} - 1 \right)^2 P_{H,t} Y_t \right]$$
(40)

subject to

$$Y_t(j) \le \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\varepsilon} Y_t \text{ and } Y_t(j) = Z_t N_t(j)$$
(41)

#### 2.4 Financial sector and central bank

Private financial firms like commercial banks accept households' deposit and make loan. They follow the required reserve set by the central bank. During each period, bank receives deposit  $D_t$ , and pays interest rate  $r_t^D$  on that deposit. They also give out loan  $L_t$  to households and charge interest rate  $r_t^L$  on the loans. The relation of loan and deposit depends on the actual reserve ratio  $0 \le \tau \le 1$ ,

$$L_t = (1 - \tau)D_t \tag{42}$$

Commercial banks' operating cost depends on their real revenue linearly  $x_t = vD_t/P_t$ . Com-

mercial banks' nominal profits during period t are

$$F_t^b = r_t^L L_t - r_t^D D_t - P_t \upsilon D_t / P_t \tag{43}$$

Competition among commercial banks drive their profits to zero. Profit maximization condition (w.r.t  $D_t$ ) requires that

$$r_t^D = r_t^L (1 - \tau_t) - \upsilon \tag{44}$$

The central bank injects/withdraws money in the economy via lumpsum transfer to households. Central bank's budget constraint

$$T_t = A_t - A_{t-1} \tag{45}$$

Suppose the ultimate goal of the central bank is to stabilize price level, and it conducts monetary policy via targeting nominal interest rate. In this model, we follow the literature to employ a simple version of Taylor rule for monetary policy

$$r_t = (1 - \rho_r)r + \rho_r r_{t-1} + (1 - \rho_r)\rho_\pi (\pi_{H,t} - \pi_H) + \varepsilon_{r,t}$$
(46)

with  $\varepsilon_{r,t} \sim i.i.dN(0, \sigma_r^2)$ .  $\rho_r > 0$  and  $\rho_{\pi}$  large enough to avoid indeterminacy. The Home producer's inflation target  $\pi_H$  is chosen by the central bank (exogenous).

## 2.5 Market clearing condition

Beside the assumption for a small economy  $n \to 0$ , we further assume symmetric openness among Home and Foreign  $\alpha = \alpha^*$ , so that  $\gamma = \alpha$ . Notice that the level of home-bias in consumption  $1 - \gamma$  is of inverse degree with the level of openness  $\alpha$ .

Symmetric equilibrium implies that all intermediate firms end up setting the same price, produce the same level of output using the same amount of labor in equilibrium,  $P_{H,t}(j) = P_{H,t}, N_t(j) = N_t$ , and  $Y_t(j) = Y_t$  for all j and t. Define  $1 + \pi_{H,t} \equiv (P_{H,t}/P_{H,t-1})$  and use it to rewrite the nominal profit of intermediate firms in equilibrium

$$F_t = P_{H,t}Y_t - W_t N_t - \frac{\phi}{2} \left(\frac{1 + \pi_{H,t}}{1 + \pi_H} - 1\right)^2 P_{H,t}Y_t$$
(47)

and derive the FOC for optimal price setting

$$(1-\varepsilon)\Lambda_t^3 Y_t + \varepsilon \Lambda_t^3 \frac{W_t Y_t}{Z_t P_{H,t}} - \phi \Lambda_t^3 \left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right) \left(\frac{1+\pi_{H,t}}{1+\pi_H}\right) Y_t$$

$$+\beta\phi\mathbb{E}_{t}\Lambda_{t+1}^{3}\left(\frac{1+\pi_{H,t+1}}{1+\pi_{H}}-1\right)\frac{1+\pi_{H,t+1}}{1+\pi_{H}}Y_{t+1}=0$$
(48)

Market clearing condition for bonds requires  $B_t = 0$  for all t. Since the economy is open with trade, aggregate resource constraint meets the domestic and foreign demand and covers the cost from production sector and private financial sector.

$$Y_t = nC_{H,t} + (1-n)C_{H,t}^* + \frac{\phi}{2} \left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right)^2 Y_t + \upsilon \frac{D_t}{P_{H,t}}$$
(49)

The above condition can be written in term of  $S_t$  and  $q(S_t)$  as

$$Y_t = (1 - \alpha)q(S_t)^{\eta}C_t + \alpha(S_t)^{\eta}C_t^* + \frac{\phi}{2}\left(\frac{1 + \pi_{H,t}}{1 + \pi_H} - 1\right)^2 Y_t + \upsilon\frac{D_t}{P_{H,t}}$$
(50)

#### 2.6 Monetary aggregation

In this economy, we have multiple monetary assets, cash and deposit, which produce different interest rates, possess different levels of liquidity and are at different levels of risk. Therefore, we have multiple ways to measure money supply. The traditional way is the official simplesum measure that is often reported by central banks all over the world.

$$SM_t = Ca_t + D_t \tag{51}$$

Growth rate of simple-sum measure

$$1 + g_t^S = SM_t / SM_{t-1} (52)$$

The second measure is monetary base, which is also found in the central banks' reports. In our model, it is easy to see that monetary base is equivalent to monetary asset  $A_t$  by market clearing condition and bank's balance sheet.

$$MB_t = A_t = Ca_t + \tau D_t \tag{53}$$

Growth rate of monetary base

$$1 + g_t^B = MB_t / MB_{t-1} (54)$$

Another way to aggregate money, the Divisia measure, requires data not only on quantities but also on interest rates of each monetary asset. In this model, we have enough information to construct the Divisia measure. First, we compute the user cost price of currency and deposit

$$u_t^{Ca} = (r_t - 0)/(1 + r_t)$$
(55)

$$u_t^D = (r_t - r_t^D)/(1 + r_t)$$
(56)

then we compute the expenditure shares of currency and deposit

$$s_t^{Ca} = \frac{u_t^{Ca}Ca_t}{u_t^{Ca}Ca_t + u_t^D D_t}$$
(57)

$$s_t^D = \frac{u_t^D D_t}{u_t^{Ca} Ca_t + u_t^D D_t}$$
(58)

The growth rate of Divisia quantity index is the weighted average growth rate of all components

$$1 + g_t^Q = (Ca_t/Ca_{t-1})^{(s_t^{Ca} + s_{t-1}^{Ca})/2} (D_t/D_{t-1})^{(s_t^D + s_{t-1}^D)/2}$$
(59)

Growth rate of true monetary aggregate

$$1 + g_t^M = M_t / M_{t-1} (60)$$

## 3 Calibration and results

## 3.1 Calibration

In order to solve the model numerically, we first choose reasonable values for parameters. The households' discount factor  $\beta = 0.98525$  indicates one period in the model as a month in real time, and an annual interest rate of 2% in equilibrium. The inverse elasticity of labor,  $\xi = 1$  implies that labor enters utility function in a quadratic form, and the coefficient  $\psi_N = 3.5$  orients the steady state value of labor to be in the range from 0.3 to 0.5, which can be understood as 8 hours to 12 hours per day. The elasticity of substitution among different variety of domestic goods reflexes the power of firms. Notice that the steady state of real marginal cost is equal to the inverse markup,  $MC = (\varepsilon - 1)/\varepsilon$ . We choose  $\varepsilon = 10$ , which implies a steady state markup of 11%. The degree of price stickiness  $\phi$  is calibrated to match the slope of the New Keynesian Phillips Curve in another sticky price manner using Calvo (1983) approach. We set  $\phi = 105$  which is equivalent to a probability of not resetting prices in a given period  $\theta = 0.75$ . The literature is quite varied in the value of  $\eta$ , the elasticity of substitution between domestic and foreign goods. In a special case,  $\eta = 1$ , domestic and foreign goods are perfect substitutes. Other than that, most of the papers in

the literature adopts a value of  $\eta$  above unity. As a benchmark case, we choose  $\eta = 2$ , but we also do parameter variation for sensitivity analysis. In our model, the size of the small economy is assumed to be very small compared to the rest of the world  $n \to 0$ , hence the share of imported goods on composite consumption bundle becomes  $\gamma = (1 - n)\alpha \to \alpha$ . If  $\alpha = 0$ , the economy is closed. The level of home bias in consumption  $1 - \gamma$  has an inverse relation with the level of openness  $\alpha$  and requires that  $\alpha < 1$ . We choose the benchmark value of  $\alpha = 0.4$  and we vary it from 0 to 1 to see the impact of openness in our model. For the elasticity of substitution between cash and deposit,  $\omega$ , we choose the benchmark value of 2, and set the weight of cash on monetary assets  $\nu = 0.625$  to match the ratio of currency in circulation (Ca/SM) about 10%. Other values of  $\omega$ , 1.16 and 3 are considered to resemble other scenarios in the economy with 25% and 3% currency in circulation. The financial sector cost  $\nu = 0.005$  is measured in unit of final goods. It implies that banking activity accounts for about of total output in the steady state. The reserve ratio is set at  $\tau = 0.02$  based on the average reserve ratio in a small economy such as Singapore. Other parameters,  $\kappa = 1, \psi_M = 0.01$  are set for simplicity.

Table 1: Calibrated values of selected parameters

Case	ω	$\eta$	$\alpha$	ca/sm	Ν
1	2	2	0.4	0.11	0.4305
2	1.16	2	0.4	0.27	0.4299
3	3	5	0.4	0.03	0.3438
4	2	5	0.1	0.11	0.4022
5	3	2	0.8	0.03	0.4089

Column 5 and 6 show the steady state values of the ratio of currency in circulation and labor w.r.t to each set of parameters.

We assume the central bank's ultimate goal is to stabilize price level, so exogenous inflation target in Home country is set at  $\pi_H = 0$ , and similarly, we assume the central bank in Foreign succeeds in control inflation so that exogenous Foreign inflation  $\pi^* = 0$ . The calibrated value z = 1.005 implies a growth of productivity (technology) of 6% per year in the model. The relative productivity of Home versus Foreign  $z_t^f = Z_{t-1}/Z_{t-1}^*$  is assumed to be  $z^f = 1$ , implying the same level of productivity all over the world in steady state.

For the coefficients of monetary policy rule (a simple version of Taylor rule), we followed Belongia and Ireland (2014) to set the coefficient on reaction to interest rate  $\rho_r = 0.75$ . We need to set the reaction to inflation  $\rho_{\pi}$  large enough to avoid indeterminacy, i.e, to meet Blanchard and Kahn (1980) condition for unique equilibrium solution of the rational expected model. Given  $\rho_r = 0.75$ , we set  $\rho_{\pi} = 1.5$  to meet this condition. Other coefficients,  $\rho_h = 0.9, \rho_c^f = 0.95, \rho_z^f = 0.95, \rho_{\pi}^f = 0.95$ . All standard deviations are set at 0.01, which means 1% shock.

#### 3.2 Results and discussion

As the benchmark case, we choose the elasticity of substitution between cash and deposit,  $\omega =$ 2, the elasticity of substitution between domestic and foreign goods,  $\eta = 2$ , and the openness,  $\alpha = 0.4$ , which implies a weight of 0.6 on domestic goods in consumption. With this setting of parameters, we are looking at a small economy with 10% currency in circulation in steady state. We shock this economy with various shocks including a preference shock. a monetary policy shock, a home-productivy shock, a shock to foreign demand, a shock to foreign productivity, and a shock to foreign inflation. The impulse responses of growth rate of different money measures, including the true monetary aggregate, Divisia quantity aggregate, monetary base and simple-sum measure are reported in Figure 1 and Figure 2. For the impulse responses of other macroeconomic variables, see Figure 12, Figure 13, Figure 14, Figure 15, Figure 16, and Figure 17. Notice that shock in foreign inflation is entirely absorbed by the exchange rate and all other macroeconomic variables do not response to this shock. This is due to the assumption of perfect capital mobility. The small economy chooses to let capital flow in and out freely, so that the domestic interest rate will be determined by the interest rate in the world. To ensure such an environment, the economy must float its exchange rate. As a consequence, the shock to foreign inflation does not impact other macroeconomic variables, except the exchange rate. For the left 5 stochastic shocks, while Divisa quantity aggregate tracks alomst perfectly the movement of the true monetary aggregate, monetary base fails in the home-productivity shock and over-reacts with respect to the monetary policy shock. More seriously, simple-sum behaves differently with that of the true money in 3 out of 5 shocks, and for the other 2 shocks, its responses are not quite close to the movement of the true money.

In order to check for the robustness of our results, we vary parameters  $\omega$ ,  $\eta$  and  $\alpha$ . Figure 3 and Figure 4 show the responses under the variation of parameter  $\omega$ . A higher value of the elasticity of substitution between cash and deposit, a smaller  $\omega$ , implies an economy with less cash in circulation and vice versa. See Table 1 for a numerical image. The behavior of Divisia quantity aggregate and monetary base does not change when we vary  $\omega$ , however, simple-sum does. With a smaller value of  $\omega$ , simple-sum is less likely to misbehave, but with bigger  $\omega$ , the misbehavior becomes more and more serious. The intuition behind this result is that as cash and deposit becomes far away from perfect substitues, people are willing to

hold less cash. The simple-sum measure which implicity assumes perfect substitution among components and assigns the same weight to different monetary assets gets more and more distorted.

In a similar manner, we vary the elasticity of substitution between domestic and foreign goods in Home's consumption, see Figure 5 and Figure 6. We later vary the openness (or the inverse degree of home-bias in consumption), see Figure 7 and Figure 8, for sensitivity analysis. Our conclusion is the same, in every case, Divisia index can track the movement of the true money very closely, monetary base is good in some cases but fails in some others, and simple-sum often behaves very differently compared with the theoretical monetary aggregate.

We further look at the volatility of macroeconomic variables under different values of  $\alpha$  from 0 to 1. See Figure 9 and Figure 10 for a sample of simulated data from the benchmark model with  $\alpha = 0.4$ . As  $\alpha$  goes from 0 to 1, the economy becomes more open (it is a closed economy with  $\alpha = 0$ ) and consumption becomes less home-biased. We find that nominal interest rate and domestic inflation fluctuate more as the economy is more open while the growth rate of exchange rate and true money become more stable, see Figure 11. Once again, Divisa index follows the correct trend of the theoretical money aggregate, while simple-sum behaves totally different. This pattern does not change under different financial structure (different value of  $\omega$ ).

## 4 Conclusion

Since Divisia monetary aggregate and the monetary aggregation theory became available from the 1980s, hundreds of theoretical and empirical work have been repeatedly showing that Divisa measure is strictly preferable to its official simple-sum counterpart, however, the availability of the simple-sum aggregates has continued. This paper revisits the issue of money measurement in a context of small open economy using the recent highly microfounded DSGE model. Our results are consistent with others such as Barnett and Chauvet (2010), Keating et al (2019). This paper is among the first work of Divisia measure in a small open economy. It introduced banking sector and Divisia measure of money in a New Keynesian framework for a small open economy. It is also the first paper to analyze the effect of openness (home bias in consumption) to the volatility of macroeconomc variables in a such an economy.

In the future work, we plan to continue our analysis on Divisia monetary aggregate in a small open economy. We want to check if money is more informative than interest rate and exchange rate in explaining/predicting output and inflation to see whether the relation of quantity of money and macroeconomic variables is still stable. Furthermore, to find out the optimal monetary policy rule, we plan to evaluate the representative household welfare under different targets of monetary policy, i.e, an interest rate rule, a rule for fixed growth rate for monetary base, fixed growth rate for simple sum, fixed growth rate for Divisia monetary aggregate, and fixed exchange rate.

## References

- [1] Barnett, W.A., 1978. The user cost of money. Economics Letters 1, 145-149.
- [2] Barnett, W.A., 1980. Economic monetary aggregates: an application of aggregation and index number theory. Journal of Econometrics 14, 11-48.
- [3] Barnett, W.A., Chauvet, M., 2010. How better monetary statistics could have signaled the financial crisis. Journal of Econometrics, 36, 2301-2310.
- [4] Belongia, M.T., 1996. Measurement matters: recent results from monetary economics reexamined. Journal of Political Economy 104: 1065–1083.
- [5] Belongia, M.T., Ireland, P.N., 2014. The Barnett critique after three decades: a New Keynesian analysis. Journal of Econometrics 183, 5-21.
- [6] Belongia, M.T., Ireland, P.N., 2015. Interest rates and money in the measurement of monetary policy. Journal of Business & Economic Statistics 33: 255–269.
- [7] Bernanke, B.S., Blinder, A.S. 1988. Credit, money, and aggregate demand. American Economic Review 78: 435–39.
- [8] Blanchard. O.J., Kahn, C.M., 1980. The solution of linear difference models under rational expectations. Econometrica 48, 1305-1311.
- [9] Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12, 383-398
- [10] Chow, H., Lim, G., McNelis, P., 2013. Monetary regime choice in Singapore: Would a Taylor rule outperform exchange-rate management? Journal of Asian Economics 30: 63–81.
- [11] Chrystal, K.A., MacDonald, R., 1994. Empirical evidence on the recent behavior and use-fulness of simple-sum and weighted measures of the money stock? Proceedings of the 18-th Annual Economic Policy Conference of the Federal Reserve Bank of St. Louis, Mar/Apr 1994, Vol 76, No 2, 73–109.

- [12] Faia, E., Monacelli, T., 2008. Optimal monetary policy in a small open economy with home bias. Journal of Money, Credit and Banking 40(4), 721-750.
- [13] Keating, J.W., Kelly, L.J., Smith A.L., Varcarcel V.J., 2019. A model of monetary policy shocks for financial crises and normal conditions. Journal of Money, Credit and Banking, vol 51(1), 227-259.
- [14] Keating, J.W., Smith A.L., 2019. The optimal monetary instrument and the (mis)use of causality tests. Journal of Financial Stability, 42(2019), 90-99.
- [15] McCallum, B.T., 2007. Monetary Policy in East Asia: The Case of Singapore. IMES Discussion Paper Series 07-E-10. Tokyo: Institute for Monetary and Economic Studies, Bank of Japan.
- [16] Rotemberg, J.J., 1982. Sticky prices in the United States. Journal of Political Economy 90, 1187–1211.
- [17] Sims, C.A., 1980. Comparison of interwar and postwar business cycles. American Economic Review 70(2), 250-257.
- [18] Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy. Pittsburgh: Carnegie Mellon University, vol. 39, pp. 195-214.

# A Appendix

#### A.1 Full set of equilibrium system

We have 31 endogenous variables in our original system:  $A_t, C_t, Ca_t, D_t, L_t, N_t, NE_t, M_t, P_{H,t}, P_t, Q_t, r_t, r_t^D, r_t^L, S_t, q(S_t), W_t, Y_t, \Lambda_t^1, \Lambda_t^2, \Lambda_t^3, \pi_{H,t}, SM_t, g_t^S, g_t^B, g_t^M, g_t^Q, s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D$  and 4 exogenous variables:  $h_t, Z_t, C_t^*, P_t^*$ .

We transformed this system into a stationary system with real effective variables. We now have 30 endogenous variables:  $c_t = C_t/Z_{t-1}, y_t = Y_t/Z_{t-1}, a_t = (A_t/P_t)/Z_{t-1},$ 

 $\begin{aligned} ca_t &= (Ca_t/P_t)/Z_{t-1}, d_t = (D_t/P_t)/Z_{t-1}, l_t = (L_t/P_t)/Z_{t-1}, m_t = (M_t/P_t)/Z_{t-1}, \\ w_t &= (W_t/P_t)/Z_{t-1}, \lambda_t^1 = Z_{t-1}\Lambda_t^1, \lambda_t^2 = Z_{t-1}\Lambda_t^2, \lambda_t^3 = Z_{t-1}\Lambda_t^3, sm_t = (SM_t/P_t)/Z_{t-1}, Q_t, N_t, \\ r_t, r_t^D, r_t^L, S_t, q(S_t), \pi_t, \pi_{H,t}, g_t^S, g_t^B, g_t^M, g_t^{NE}, g_t^Q, s_t^{Ca}, s_t^D, u_t^{Ca}, u_t^D; \text{and 5 exogenous variables:} \\ h_t, z_t &= Z_t/Z_{t-1}, c_t^f = C_t^*/Z_{t-1}^*, \pi_t^f \text{ and } z_t^f = Z_{t-1}/Z_{t-1}^*. \end{aligned}$ 

(1) Theoretical/True monetary aggregation

$$m_t = \left[\nu^{\frac{1}{\omega}} ca_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

(2) Households first sub-period budget constraint

$$ca_t + d_t = a_t + l_t$$

(3) FOC for households

$$\frac{h_t}{c_t} = \lambda_t^3$$

(4) Labor supply

$$\lambda_t^3 w_t = \psi_N h_t N_t^{\xi}$$

(5) Euler equation

$$\lambda_t^3 = \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}^3 (1+r_{t+1})}{1+\pi_{H,t+1}} \right) \frac{q(S_t)}{q(S_{t+1})z_t}$$

(6)

 $r_t^L = r_t$ 

(7) Money demand

$$\lambda_t^1 m_t = \psi_M h_t$$

(8)  
$$\lambda_t^2 - \lambda_t^3 = \lambda_t^1 \left[ \nu^{\frac{1}{\omega}} c a_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} \nu^{\frac{1}{\omega}} c a_t^{-\frac{1}{\omega}}$$
(9)

(10)  
$$\lambda_t^2 - (1+r_t^D)\lambda_t^3 = \lambda_t^1 \left[ \nu^{\frac{1}{\omega}} c a_t^{\frac{\omega-1}{\omega}} + (1-\nu)^{\frac{1}{\omega}} d_t^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} (1-\nu)^{\frac{1}{\omega}} d_t^{-\frac{1}{\omega}}$$
(10)

$$\lambda_t^2 = \lambda_t^3 (1 + r_t^L)$$

(11) Term of trade

$$\frac{S_t}{S_{t-1}} = \frac{1 + \pi_t^*}{1 + \pi_{H,t}} (1 + g_t^{NE})$$

(12)  
$$\frac{q(S_t)}{q(S_{t-1})} = \frac{1 + \pi_t}{1 + \pi_{H,t}}$$

(13) CPI-PPI ratio

$$q(S_t) = [(1 - \gamma) + \gamma S_t^{1 - \eta}]^{\frac{1}{1 - \eta}}$$

(14) Real exchange rate

$$Q_t = \frac{S_t}{q(S_t)}$$

(15) Interest rate parity

$$\frac{S_t}{q(S_t)} = \kappa \frac{h_t c_t}{c_t^f} z_t^f$$

(16) Production function

 $y_t = z_t N_t$ 

(17) FOC for optimal price setting

$$(1-\varepsilon) + \varepsilon \frac{w_t q(S_t)}{z_t} - \phi \left(\frac{1+\pi_{H,t}}{1+\pi_H} - 1\right) \left(\frac{1+\pi_{H,t}}{1+\pi_H}\right) + \beta \phi \mathbb{E}_t \frac{\lambda_{t+1}^3 y_{t+1}}{\lambda_t^3 y_t} \left(\frac{1+\pi_{H,t+1}}{1+\pi_H} - 1\right) \frac{1+\pi_{H,t+1}}{1+\pi_H} = 0$$

(18) Loans

$$l_t = (1 - \tau)d_t$$

(19) FOC for profit maximization of commercial banks

$$r^D_t = r^L_t (1-\tau) - \upsilon$$

(20) Aggregate resource constraint

$$y_t = (1 - \alpha)q(S_t)^{\eta}c_t + \alpha(S_t)^{\eta}\frac{c_t^f}{z_t^f} + \frac{\phi}{2}\left(\frac{1 + \pi_{H,t}}{1 + \pi_H} - 1\right)^2 y_t + \upsilon d_t q(S_t)$$

(21) Simple-sum measure

$$sm_t = ca_t + d_t$$

(22) Growth rate of simple-sum measure

$$1 + g_t^S = sm_t / sm_{t-1}$$

Monetary base (equivalent to a by market clearing condition)

$$mb_t = a_t = ca_t + \tau d_t$$

(23) Growth rate of monetary base

$$1 + g_t^B = mb_t/mb_{t-1}$$

(24) User cost price of currency

$$u_t^{Ca} = (r_t - 0)/(1 + r_t)$$

(25) User cost price of deposit

$$\boldsymbol{u}_t^D = (\boldsymbol{r}_t - \boldsymbol{r}_t^D)/(1 + \boldsymbol{r}_t)$$

(26) Expenditure share of currency

$$s_t^{Ca} = \frac{u_t^{Ca} ca_t}{u_t^{Ca} ca_t + u_t^D d_t}$$

(27) Expenditure share of deposit

$$s^D_t = \frac{u^D_t d_t}{u^{Ca}_t ca_t + u^D_t d_t}$$

(28) Growth rate of Divisia quantity index

$$1 + g_t^Q = (ca_t/ca_{t-1})^{(s_t^{Ca} + s_{t-1}^{Ca})/2} (d_t/d_{t-1})^{(s_t^D + s_{t-1}^D)/2}$$

(29) Growth rate of true monetary aggregate

$$1 + g_t^M = m_t / m_{t-1}$$

(30) Monetary policy rule (a simple version of Taylor rule)

$$r_t = (1 - \rho_r)r + \rho_r r_{t-1} + (1 - \rho_r)\rho_\pi(\pi_{H,t} - \pi_H) + \varepsilon_{r,t}$$

(31) Preference shock

$$\ln(h_t) = \rho_h \ln(h_{t-1}) + \varepsilon_{h,t}$$

(32) Home technology/productivity shock

$$\ln(z_t) = \ln(z) + \varepsilon_{z,t}$$

(33) Shock to Foreign productivity

$$\ln(z_t^f) = (1 - \rho_z^f) \ln(z^f) + \rho_z^f \ln(z_{t-1}^f) + \varepsilon_{z,t}^f$$

(34) Shock to Foreign demand

$$\ln(c_t^f) = \rho_c^f \ln(c_{t-1}^f) + \varepsilon_{c,t}^f$$

(35) Foreign inflation is exogenous. Assume the goal for central bank in Foreign is to stabilize the price level.

$$\pi_t^f = (1 - \rho_p^f)\pi^f + \rho_p^f \pi_{t-1}^f + \varepsilon_{p,t}^f$$

## A.2 Figures

Figure 1: Impulse responses of growth rate of different money measures w.r.t domestic shocks, benchmark  $\omega = 2, \eta = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

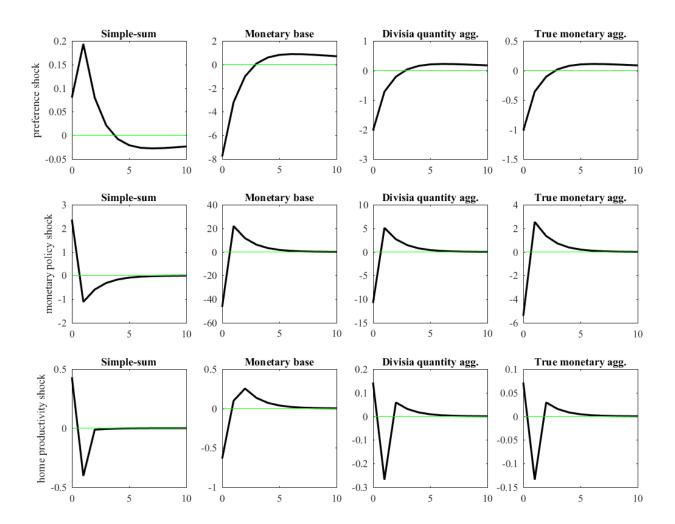


Figure 2: Impulse responses of growth rate of different money measures w.r.t foreign shocks, benchmark  $\omega = 2, \eta = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

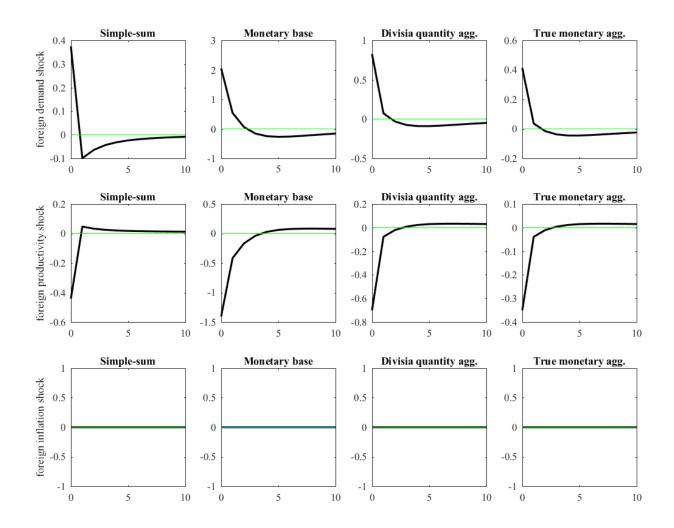


Figure 3: Impulse responses of growth rate of different money measures w.r.t domestic shocks under variation of parameter  $\omega$ , pink for  $\omega = 1.16$ , black for  $\omega = 2$ , blue for  $\omega = 3$ , benchmark  $\eta = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

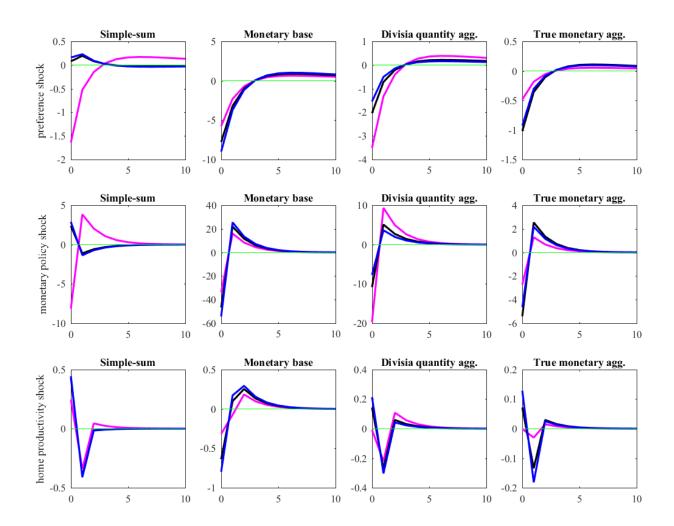


Figure 4: Impulse responses of growth rate of different money measures w.r.t foreign shocks under variation of parameter  $\omega$ , pink for  $\omega = 1.16$ , black for  $\omega = 2$ , blue for  $\omega = 3$ , benchmark  $\eta = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

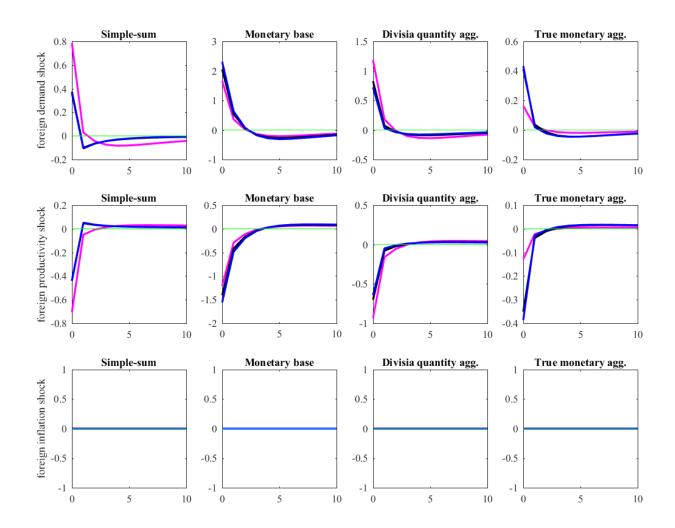


Figure 5: Impulse responses of growth rate of different money measures w.r.t domestic under variation of parameter  $\eta$ , pink for  $\eta = 1.1$ , black for  $\eta = 2$ , blue for  $\eta = 5$ , benchmark  $\omega = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

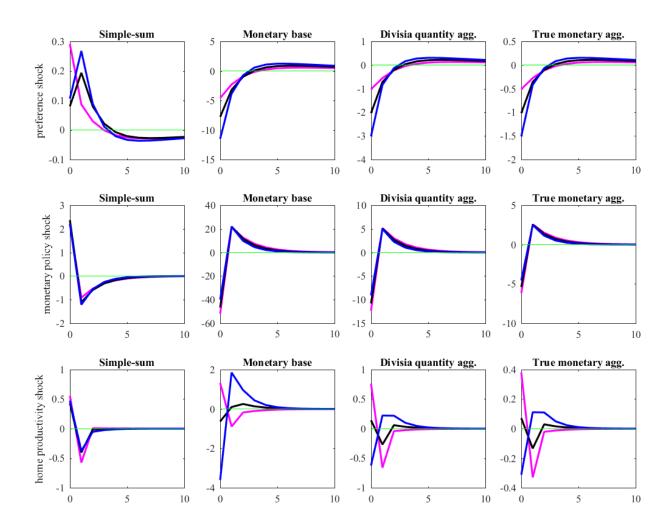


Figure 6: Impulse responses of growth rate of different money measures w.r.t foreign shocks under variation of parameter  $\eta$ , pink for  $\eta = 1.1$ , black for  $\eta = 2$ , blue for  $\eta = 5$ , benchmark  $\omega = 2, \alpha = 0.4$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

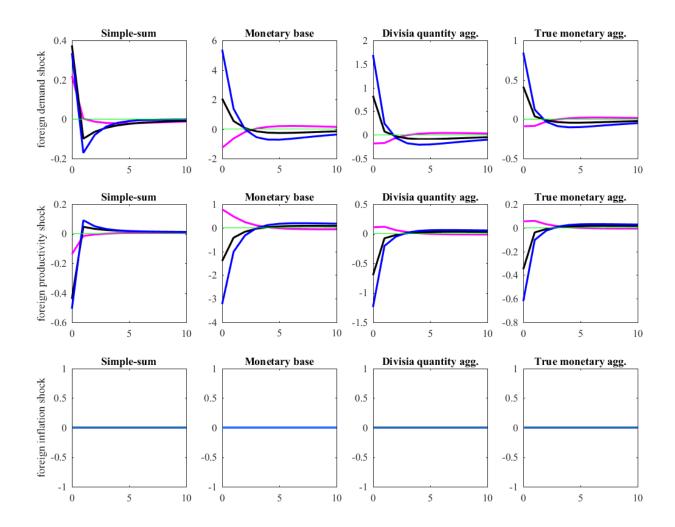


Figure 7: Impulse responses of growth rate of different money measures w.r.t domestic under variation of parameter  $\alpha$ , pink for  $\alpha = 0.1$ , black for  $\alpha = 0.4$ , blue for  $\alpha = 0.8$ , benchmark  $\omega = 2, \eta = 2$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.

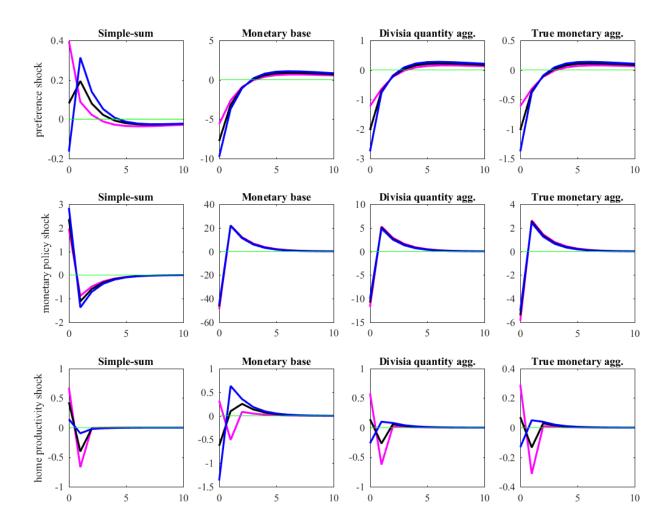
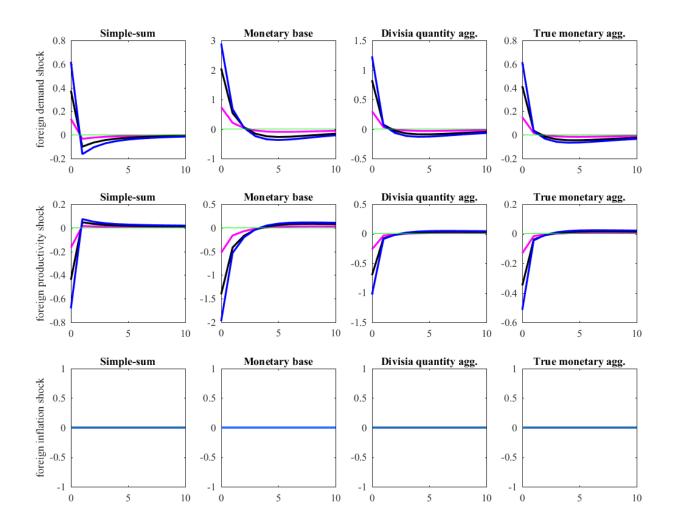


Figure 8: Impulse responses of growth rate of different money measures w.r.t foreign shocks under variation of parameter  $\alpha$ , pink for  $\alpha = 0.1$ , black for  $\alpha = 0.4$ , blue for  $\alpha = 0.8$ , benchmark  $\omega = 2, \eta = 2$ . Each panel shows the percentage-point response to a one-standard deviation innovation in one of the shocks.



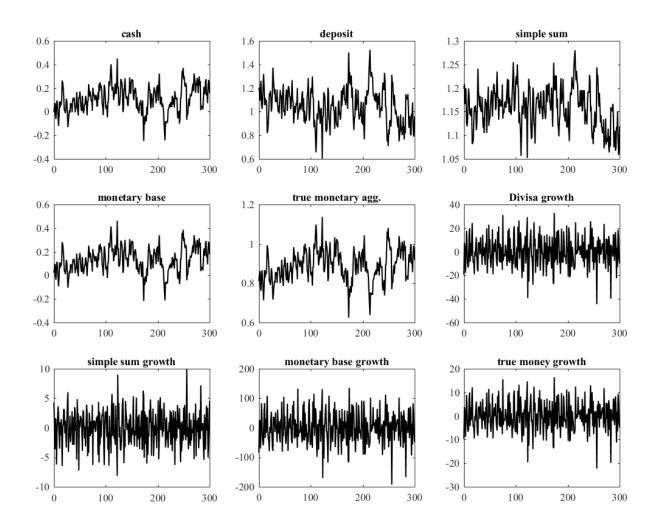


Figure 9: Simulated data for different money measures, benchmark  $\omega = 2, \eta = 2, \alpha = 0.4$ .

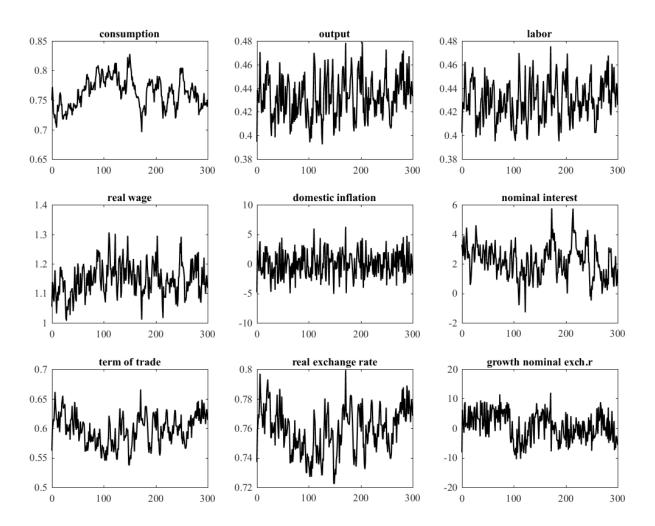


Figure 10: Simulated data for other macroeconomics variables, benchmark  $\omega = 2, \eta = 2, \alpha = 0.4$ .

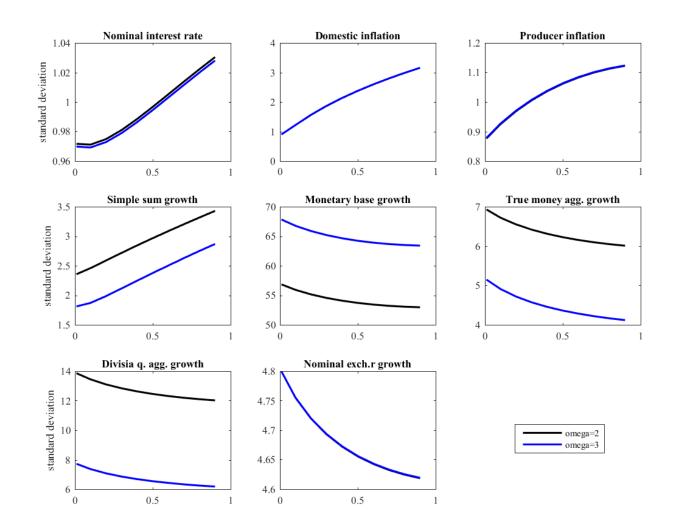


Figure 11: Volatility of macroeconomic variables under variation of parameter  $\alpha$ . Each panel shows the standard deviation in percentage-point under different values of  $\alpha$  from 0 to 1.

Figure 12: Impulse responses of other macroeconomics variables w.r.t preference shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

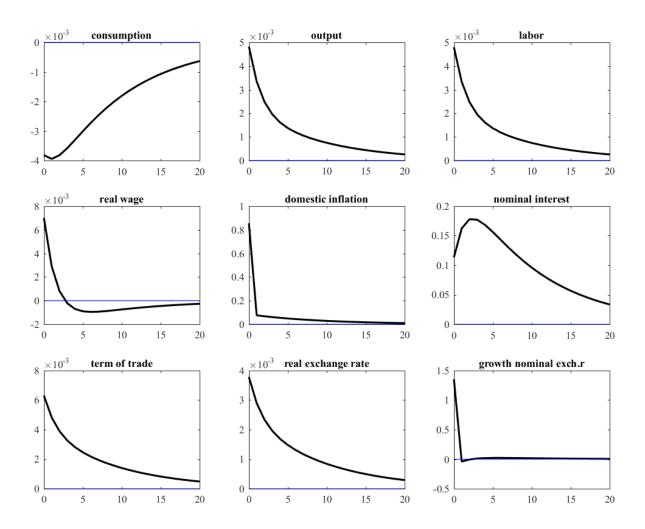


Figure 13: Impulse responses of other macroeconomics variables w.r.t monetary policy shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

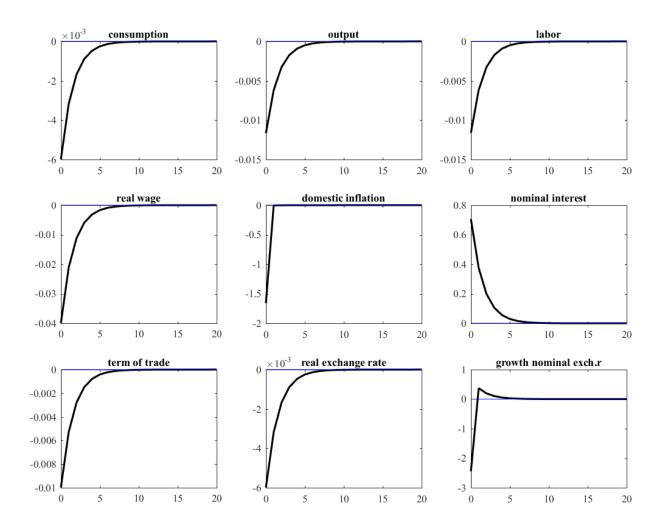


Figure 14: Impulse responses of other macroeconomics variables w.r.t home productivity shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

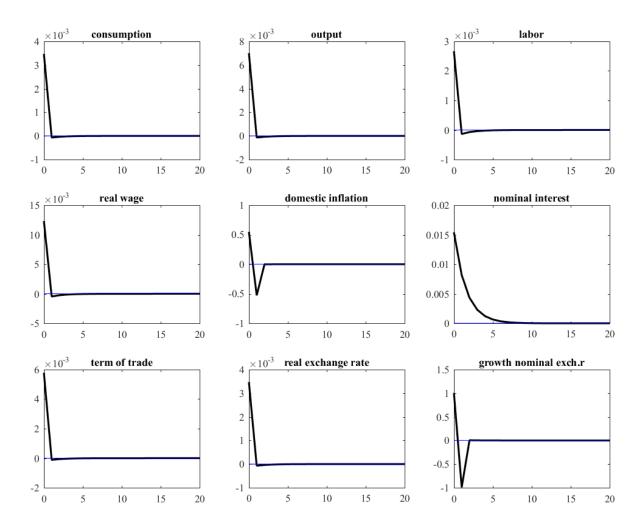


Figure 15: Impulse responses of other macroeconomics variables w.r.t foreign productivity shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

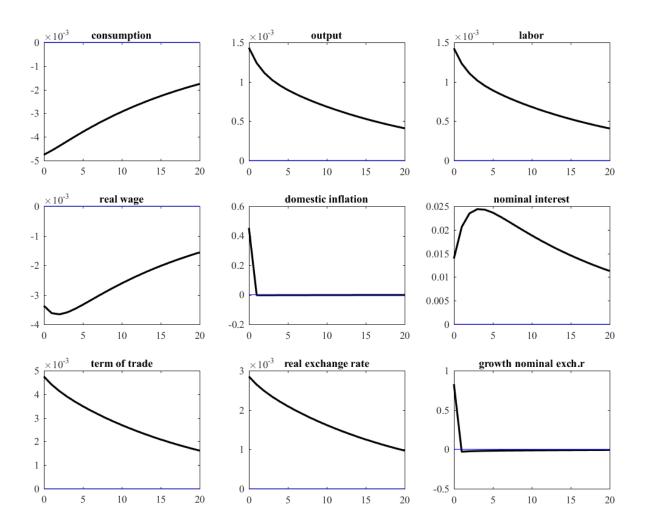


Figure 16: Impulse responses of other macroeconomics variables w.r.t foreign demand shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

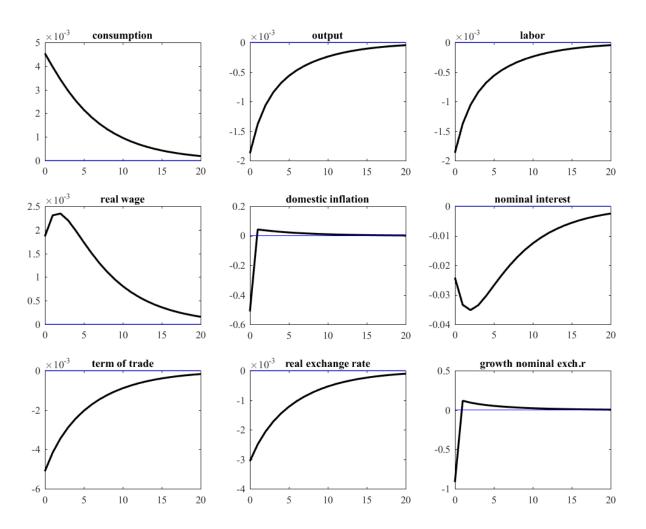


Figure 17: Impulse responses of other macroeconomics variables w.r.t foreign inflation shock. Each panel shows the response to a one-standard deviation innovation in the shock. Responses of domestic inflation, nominal interest rate and growth rate of nominal exchange rate are in percentage-point.

