Disclosing to informed traders*

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Abstract

We develop a model of voluntary disclosure in the presence of diversely-informed investors. The manager's disclosure strategy influences trading by investors, which in turn affects the manager's incentives to disclose. We document conditions under which there exists a unique equilibrium where the manager discloses only sufficiently favorable news. This equilibrium exhibits two novel features. First, the firm is either over- or under-valued relative to fundamentals, depending on the likelihood the manager is informed and the cost of disclosure. Second, contrary to common intuition, mandatory disclosure can increase managers' incentives to provide voluntary disclosures.

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1 Introduction

Market prices reflect information from many sources. Prices aggregate information that is privately held by and dispersed across investors (Hayek (1945)). In addition, prices reflect information released by firms through both mandatory and voluntary disclosures. Existing analyses have explored how investors' private information interacts with mandatory disclosures (e.g., Diamond (1985); Goldstein and Yang (2017)), but are largely silent on its interaction with voluntary disclosures. Yet, voluntary disclosures play a crucial role in determining prices, generating nearly two-thirds of the return variation created by firm-level public announcements.[1] Thus, it is critical to understand how firms' incentives to provide voluntary disclosures interact with trade on private information, and how this impacts the extent to which prices reflect fundamentals.

The relation between voluntary disclosure and investors' private information is especially important for evaluating financial market regulation. Regulators typically mandate disclosure with the goals of enhancing price efficiency and reducing investor uncertainty. However, disclosure regulation can adversely affect other channels through which investors obtain information. For instance, firms voluntarily provide extensive information regarding sources of uncertainty over which disclosure is not mandatory (e.g., forward-looking and non-financial information), and regulation is often thought to reduce, or "crowd out," such efforts (e.g., Ross (1979)).[2] Similarly, empirical evidence suggests that investors trade more intensely on private information in settings where mandatory disclosure is limited, increasing price informativeness (e.g., Leuz and Verrecchia (2000), Jayaraman and Wu (2019)). This raises the question of how disclosure mandates affect the overall amount of information available to investors, once we account for their impact on both voluntary disclosure and trade on private information.

We develop a model of voluntary disclosure in which a firm's price is determined through trade among privately-informed risk-averse investors and noise traders. Our model integrates two pervasive sources of non-disclosure: proprietary disclosure costs (e.g., Verrecchia (1983))

[1] See Table 1 in Beyer, Cohen, Lys, and Walther (2010), which examines the return variation created by management and analyst forecasts, earnings announcements and pre-announcements, and SEC filings. This table shows that the two voluntary disclosures, management forecasts and earnings pre-announcements, jointly generate ≈66% of the return variation created by these events. Moreover, firms release a number of additional voluntary disclosures not encompassed by this analysis, including press releases, conference calls, and the MD&A.

[2] For instance, despite the absence of regulation requiring ESG reporting, among the S&P 500, 78% provide ESG reports and 36% of these are audited (Kwon, Welsh, Lukomnik, and Young (2018)). More generally, firms provide qualitative discussions, forecasts, and non-GAAP earnings to help investors predict future outcomes that are not captured by mandatory financial reports. In fact, Ross (1979) argues that given firms' incentives to disclose information voluntarily, mandatory disclosure regulation is neither necessary nor desirable.

and uncertainty regarding whether the firm possesses information (e.g., Dye (1985)). In our model, the voluntary nature of the firm's disclosure influences how price aggregates investors' private and public information, which in turn influences the firm's incentives to disclose.

These interactions have important implications for understanding how prices aggregate and reflect information. First, we show that the firm's expected price generically differs from its expected cash flows, and the degree of "mis-valuation" depends on the underlying friction driving non-disclosure.[3] For instance, the firm is under-valued when non-disclosure is driven by disclosure costs, while it can be over-valued when non-disclosure arises because the manager may be uninformed. Second, we find that more ex-ante public information can *increase* voluntary disclosure, in contrast to the common intuition that mandatory disclosure "crowds out" voluntary disclosure. As we discuss below, these results generate a number of testable predictions and highlight the need to take into account the interaction between investors' private information and firms' disclosure incentives when evaluating the impact of mandatory disclosure requirements.

Model Overview and Main Results

Section 3 presents the benchmark model. The firm's future cash flows are normally distributed. With some probability, the manager is privately informed about these cash flows, and can publicly and verifiably disclose this information, potentially at a cost to the firm. Informed investors with CARA utility and noise traders trade in the firm's stock. Each investor observes a conditionally independent, normally-distributed signal about the firm's cash flows, which is noisier than the manager's information. The manager chooses whether to disclose his information before trading begins, in order to maximize the expected stock price.

In Section 4, we derive conditions for the existence of a unique "threshold" equilibrium in which the manager discloses only when his signal exceeds a cutoff. To do so, we first characterize the firm's price in such an equilibrium. When the manager discloses, the firm's price is fully determined by the disclosure, which is a sufficient statistic for investors' private information. When the manager does not disclose, the firm's price is determined by the investors' trading behavior. Since investors infer that either the manager is uninformed or has observed news that is below the threshold for disclosure, their conditional beliefs about the firm's cash flows are no longer normal. While this rules out a "linear" equilibrium as in traditional models (e.g., Hellwig (1980)), we show that, for a given threshold, there exists a

[3] As we discuss further in Section 4.3, this difference does not arise from a traditional risk-premium because we assume the firm's stock is in zero net supply and so cash flows are effectively idiosyncratic.

unique non-linear equilibrium price that reflects the firm's value with noise.[4]

For a threshold equilibrium to exist, the manager must be more inclined to disclose when his signal is higher. This requires that the price when the manager refrains from disclosure does not rise too rapidly in his signal. In classic disclosure models in which investors lack private information, this holds trivially because the non-disclosure price is uninformative. In contrast, in our model, the non-disclosure price partially reflects the manager's signal through investors trading on their private information. We show that when either the manager is known to be informed or investors' private signals are not too informative, the non-disclosure price responds only partially to an increase in the manager's signal, and as a result, there exists a unique threshold equilibrium. However, when investor information is sufficiently precise and there is uncertainty about whether the manager is informed, the threshold equilibrium may break down.[5]

The specific friction driving non-disclosure has important consequences for equilibrium outcomes. Our model nests two natural benchmarks, which are the focus of existing voluntary disclosure literature. The "costly disclosure" benchmark considers the case where the manager is known to be informed but voluntary disclosure is costly, as in Verrecchia (1983). The "probabilistic information" benchmark assumes there are no disclosure costs, but the manager may be uninformed with some probability (e.g., Dye (1985), Jung and Kwon (1988)). As we discuss below, in practice, these benchmarks reflect different economic environments.

Our first main result is that the firm's non-disclosure price deviates, on average, from its expected cash flows, even when the firm's risk is purely idiosyncratic. Moreover, whether the firm is over- or under-valued depends on the underlying friction that drives non-disclosure. This "mis-valution" results from the interaction between strategic disclosure and trade among investors: existing models that feature only one of these dimensions predict that idiosyncratic cash flows are priced at their expected value.

In the costly disclosure benchmark, the manager always discloses when the firm's fundamentals are sufficiently high. Thus, when the manager remains silent, investors infer that the firm's value must be below a threshold, which limits the potential losses from short-selling the stock. Intuitively, long positions are riskier than short positions for investors because

[4] We build on the techniques developed by Breon-Drish (2015) to show that the price given non-disclosure still is instead a concave function of the firm's cash flows and noise trade - see Section 2 for additional discussion.

[5] In particular, we show that when investors' signals are very precise, uncertainty about whether the manager is informed is very sensitive to signals, especially near the disclosure threshold. As an extreme, when signals are infinitely precise, investors know that the manager is uninformed when signals exceed the disclosure threshold. We show that this can imply that the net benefit from disclosure is hump-shaped in firm value near the threshold, which then leads to non-existence of a threshold equilibrium.

the stock's payoff is bounded above, and so they require a larger price adjustment to accommodate aggregate noise-trader sales than noise-trader purchases. This pushes the price down on average, leading to under-valuation.

In contrast, in the probabilistic information benchmark, the manager does not disclose either because he has bad news or because he is uninformed. As a result, the firm's value is not bounded above when the manager is silent. Instead, the firm's value is positively skewed: the possibility that the manager is uninformed and the firm's value is high creates an "upper tail" in the payoff distribution. As a result, taking short positions can expose investors to ***greater*** downside risk than taking long positions. This can push prices upwards on average and lead to over-valuation because investors require a larger price adjustment to accommodate aggregate purchases by noise traders, relative to sales.

Building on these results, we next explore how public information, such as mandatory disclosure, affects voluntary disclosure and investor uncertainty. To do so, we introduce a public signal about cash flows to the benchmark model in Section 5. In standard voluntary disclosure models without privately informed investors, public information "crowds out" voluntary disclosure. Intuitively, public information reduces uncertainty about payoffs, which attenuates the negative inference investors draw when the manager does not disclose. This increases the non-disclosure price on average, which reduces the manager's incentive to disclose. While this force is also present in our setting, we uncover two novel channels through which higher-quality public information may instead increase managers' desire to provide voluntary disclosures.

First, when ex-ante public information improves, the degree of mis-valuation that arises due to non-disclosure is attenuated – we refer to this as the ***valuation channel***. Intuitively, when investors have more precise ex-ante public information, they face less uncertainty about the firm's payoffs. As a result, they demand less compensation to accommodate aggregate noise trading, which shrinks the price distortion due to non-disclosure. This channel increases (decreases) incentives to disclose when, given non-disclosure, the firm is over-valued (under-valued, respectively). Second, an increase in public information leads investors to trade less intensely on their private information – we refer to this as the ***substitution channel***. Holding fixed the public signal, this leads the non-disclosure price to be less informative about fundamentals.[6] Importantly, because the threshold firm's manager knows that expected cash flows weakly exceed investors' expectations given non-disclosure, this lowers the non-disclosure price that he expects.[7] As a result, this channel tends to increase disclosure.

[6]The manager's disclosure decision depends on the informativeness of the non-disclosure price after conditioning on the public signal because the public signal is known at the time of this decision.

[7]The threshold manager's expected value strictly exceeds the investors' expectations given non-disclosure when disclosure is costly, and is equal when disclosure costs are zero.

The overall impact of these channels depends on the friction that drives non-disclosure. In the costly disclosure benchmark, the valuation channel decreases the benefit to voluntary disclosure, as the firm is under-valued given non-disclosure. However, when disclosure costs are high and investors' private information is precise, the substitution channel dominates and causes public information to crowd in voluntary disclosure. Intuitively, when disclosure costs are large, the manager is on the margin between disclosing and not when his signal is very high. Thus, a reduction in the informativeness of the non-disclosure price causes a large drop in the non-disclosure price this manager expects.

On the other hand, in the probabilistic information benchmark, the substitution channel is shut down. In this case, because costs are zero, the manager is on the margin between disclosing and not when his firm's expected cash flows are precisely equal to the expected non-disclosure price. Thus, an increase in the extent to which this price reflects fundamentals has little effect on the price this manager expects. However, the valuation channel mitigates over-valuation, which leads to crowding in of voluntary disclosure, and dominates when the public signal is sufficiently precise.

Empirical and policy implications

An important takeaway of our analysis is that differences in the underlying frictions for non-disclosure can lead to very different implications for how valuations and voluntary disclosure respond to changes in the information environment. This suggests one must be careful in identifying what drives non-disclosure in a given setting when conducting empirical and policy analysis.

One can turn to guidance from standard setters like the Financial Accounting Standards Board (FASB). For instance, FASB (2001) recognizes that voluntary disclosures can impose costs from a variety of sources, including: i) greater competition, ii) higher bargaining costs vis-a-vis customers, suppliers and employees, iii) and litigation risk arising from the disclosures. Such costs may for instance affect a firm's decision to patent an innovation (see e.g., Glaeser (2018); Saidi and Zaldokas (2021)), redact information about its contracts from its financial statements (Verrecchia and Weber (2006); Boone, Floros, and Johnson (2016)), break down segment-level performance (Berger and Hann (2007)), or file for an IPO (Dambra, Field, and Gustafson (2015)). In such settings, our model predicts that non-disclosure is associated with lower stock prices and higher costs of capital, even after controlling for systematic risk-factor loadings. This is broadly consistent with the evidence from Boone et al. (2016), who show that firms that redact information from their IPO filings tend to experience substantial underpricing and higher costs of capital.

FASB (2001) also identifies settings wherein the presence of firm's private information is uncertain to investors. In general, accounting standards prohibit recognition of (internally developed) intangible assets. Therefore, the impact of these assets can only be voluntarily disclosed. For instance, in the pharmaceutical/bio-technology industry, firms can withhold information about business setbacks, such as clinical trial failures on new drugs or vaccines, to avoid or delay negative market reactions (e.g., Dobson (2000)). Indeed, a 2014 analysis found that 4 years after 400 randomly selected trials finished, 30% of them had not disclosed their results (Saito and Gill (2014); Reardon (2016)). Our analysis suggests that non-disclosure in such industries may be associated with over-valuation and lower costs of capital.

Our model also sheds light on the negative, cross-sectional relation between idiosyncratic return skewness and expected returns (e.g., see Jiang, Xu, and Yao (2009); Conrad, Dittmar, and Ghysels (2013); Boyer and Vorkink (2014)). This relation is difficult to reconcile in traditional models, and existing theories often assume that investors have a preference for such skewness.[8] To the extent that voluntary disclosures drive significant variation in skewness across firms and over time, our mis-valuation results provide a novel explanation. Specifically, conditional on non-disclosure in the costly disclosure benchmark, (i) payoffs are bounded above and so returns are negatively skewed, and (ii) the firm is under-valued and so earns positive returns on average. Similarly, in the probabilistic information benchmark, payoffs are positively skewed and the stock can be over-valued on average conditional on non-disclosure. Together, these results imply a robust negative relation between skewness and expected returns in our setting.

Finally, our results have important implications for empirical analysis and regulatory policy. Regulators often motivate disclosure requirements as means to mitigate adverse selection across investors and "level the playing field." While a standard critique of such policies is that they "crowd out" voluntary disclosure by firms (e.g., Verrecchia (1990)), the empirical evidence is mixed. Some papers suggest that firms increase voluntary disclosures to mitigate reductions in external information quality (e.g., Balakrishnan, Billings, Kelly, and Ljungqvist (2014), Guay, Samuels, and Taylor (2016), and Barth, Landsman, and Taylor (2017)), but others argue that public information and voluntary disclosures are positively correlated (e.g., Francis, Nanda, and Olsson (2008); Ball, Jayaraman, and Shivakumar (2012); Bischof and Daske (2013); Kim and Ljungqvist (2021)).

Our analysis helps reconcile this evidence, and clarifies how the impact of regulatory changes varies across firms and depends crucially on the underlying drivers of non-disclosure. Our model implies that, when investors are informed, increased mandatory disclosure com-

[8]In standard settings, idiosyncratic skewness should be diversifiable, and so irrelevant for expected returns. As we discuss in Section 4.3, existing explanations for the negative relation (e.g., Mitton and Vorkink (2007); Barberis and Huang (2008)) assume that (some) investors have a preference for positive skewness and so over-value "lottery-like" stocks.

plements voluntary disclosure and tends to improve the overall public information environment when disclosure costs are high or investors face uncertainty about whether the manager is informed.[9] This includes mandatory disclosure concerning R&D outcomes, for which information arrives sporadically and disclosure can enable follow-on innovations by competitors. On the other hand, mandatory disclosure substitutes voluntary disclosure and can increase residual uncertainty when disclosure costs are low and managers are very likely to be informed. These settings can be readily identified: they correspond to situations in which managers are very likely to issue informative voluntary disclosures in the absence of regulation (as appears to be the present state of ESG reporting; see Kwon et al. (2018)).

The rest of the paper is as follows. Section 2 reviews the related literature. Section 3 presents the benchmark model and discusses our assumptions. Section 4 presents the equilibrium characterization and discusses features of our model, including the implications for firm valuation. Section 5 introduces an ex-ante public signal to the benchmark model, and characterizes when public information can "crowd in" voluntary disclosure. Section 6 concludes, and proofs and extensions can be found in Appendix A and B, respectively.

2 Related Literature

Our paper contributes to two strands of literature: models of voluntary disclosure and models of heterogeneously-informed investors. The literature on voluntary disclosure, starting with Jovanovic (1982), Verrecchia (1983), and Dye (1985), typically models financial markets in a stylized manner, assuming that investors are uninformed, risk neutral, or both.[10] Notable exceptions are Bertomeu, Beyer, and Dye (2011), Petrov (2016) and Einhorn (2018) who also study the interaction between informed trade and disclosure. Bertomeu et al. (2011) and Petrov (2016) analyze settings in which there is a single risk-neutral informed trader, while Einhorn (2018) considers trade based on private information only when non-disclosure is completely uninformative.[11]

In contrast, analysis of disclosure in the context of heterogeneously-informed investors has largely focused on either non-strategic disclosure or settings in which the manager can commit, ex-ante, to a public signal with chosen precision (see Goldstein and Yang (2017) for a recent survey). For example, Diamond (1985), Kurlat and Veldkamp (2015), and

[9]Our measure of overall informativeness captures the residual uncertainty about payoffs given all public information. Specifically, we measure the average variance in payoffs, conditional on prices, mandatory disclosures and voluntary disclosures.

[10]Examples of voluntary disclosure models with risk-averse, but uninformed traders include Verrecchia (1983), Cheynel (2013), Jorgensen and Kirschenheiter (2015), and Dye and Hughes (2018).

[11]Almazan, Banerji, and MOTTA (2008) endogenize the manager's incentives to use cheap talk communication when facing a market with risk-neutral, informed investors.

Banerjee, Davis, and Gondhi (2018) study how public signals influence the acquisition of private information, and Goldstein and Yang (2019) analyzes how disclosure on different components of firm value influences real efficiency. Gao and Liang (2013) studies a setting where the firm can commit to disclose its information and show how disclosure crowds out learning by investors and thus can distort real decisions. Yang (2020) considers a setting where firms choose whether to disclose information before learning information from asset prices that inform their production decisions. Schneemeier (2019) studies how firms can use ex-ante disclosure policies to direct investor attention towards their firm. Cianciaruso, Marinovic, and Smith (2020), following the Bayesian persuasion tradition, study a setting where the firm maximizes its expected price by designing ex-ante a signal that investors will publicly observe prior to trading the stock.

To the best of our knowledge, our paper is the first to study voluntary disclosure to a market of heterogeneously-informed, risk-averse investors when the manager cannot commit to a disclosure strategy ex-ante. A key step is to allow investors to learn from prices in an environment where the price does not have a standard "linear-normal" form. We build on the insights of Breon-Drish (2015) to overcome this challenge: as in his paper, we show that there exists a unique equilibrium in which the price is a generalized linear function of a noisy signal about fundamentals.[12]

Our analysis relates to the theoretical literature on the relationship between prior public information (e.g., mandatory disclosure or analyst reports) and voluntary disclosure. The common intuition in the existing literature (e.g., Verrecchia (1990); Bertomeu, Vaysman, and Xue (2019)) is that these two types of information are substitutes, especially when they are concerned with the same underlying fundamental shocks.[13] Our analysis suggests that these two types of information may instead be complementary when investors are privately informed. The existing literature has documented economically distinct channels through which prior public information and voluntary disclosure may be complements. Friedman, Hughes, and Michaeli (2020a,b) show that these information sources may be complements when firms experience a discrete gain should investors' expectations exceed a cutoff. Einhorn (2005) find that certain correlation structures between public information and voluntary disclosure lead them to be complements. Frenkel, Guttman, and Kremer (2020) find that disclosure by an external party may crowd in firm disclosure when the external party and

[12] Other papers that have considered rational expectations equilibria with non-linear prices include Albagli, Hellwig, and Tsyvinski (2021), Banerjee and Green (2015), Glebkin (2015), Albagli, Hellwig, and Tsyvinski (2020), Chabakauri, Yuan, and Zachariadis (2021), Smith (2019), Lenkey (2020), and Glebkin, Malamud, and Teguia (2020).

[13] As Goldstein and Yang (2017) and Goldstein and Yang (2019) point out, this may not be the case if the two sources of information are about different components of payoffs.

the firm possess information with correlated probabilities. Lassak (2020) shows that voluntary disclosures can encourage more information acquisition by investors when they increase public uncertainty.

Our finding that the firm may be over- or under-valued appears similar to existing results in the literature, but the underlying economic mechanism is distinct. For instance, Albagli et al. (2021) shows that a wedge between expected price and expected dividends can arise when cash flows are ***exogenously*** specified, concave / convex functions of an underlying fundamental about which investors are informed.[14] In contrast, in our model, cash flows are not a non-linear function of some other fundamental, and investors are directly informed about the cash flows themselves. However, voluntary non-disclosure endogenously leads to conditional cash flow distributions that are skewed, and the over-/under-valuation in our model results from risk compensation demanded by investors for this endogenous skewness. Moreover, in contrast to this earlier work, the price in the probabilistic information benchmark we consider is neither a globally concave or convex function of noise trade and fundamentals, and as such, we develop novel approaches to establish these results.

3 Model Setup

Our model features verifiable disclosure (e.g., Jovanovic (1982); Verrecchia (1983); Dye (1985)) in a market with privately-informed investors (e.g., Hellwig (1980)).

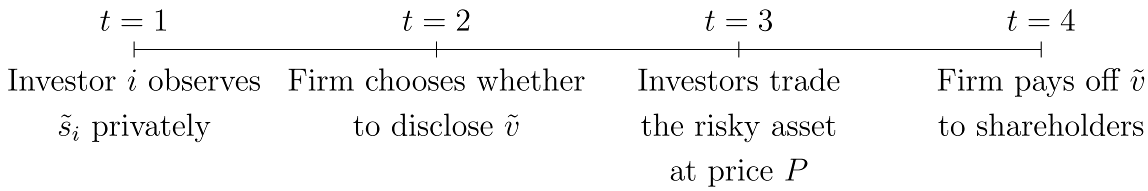
Payoffs. Investors trade in both a risky and a risk-free security. The gross return on the risk-free security is normalized to 1. The risky security is the stock of a firm, which pays of a terminal dividend $\tilde{v}$ that is normally distributed with mean m and variance σ_v^2, i.e., $\tilde{v} \sim N\left(m, \sigma_v^2\right)$. We normalize the mean to zero (i.e., $m = 0$) without loss of generality. We assume that there are noise/liquidity traders who submit demands of $\tilde{z} \sim N\left(0, \sigma_z^2\right)$. The aggregate supply of the risky asset is $\kappa \geq 0$.

Preferences and Information. There is a continuum of investors indexed by $i \in [0, 1]$. Each investor i is endowed with initial wealth W_0 and zero shares of the stock, and exhibits CARA utility with risk-tolerance τ over terminal wealth W_i, where:

$$W_i = W_0 + D_i(\tilde{v} - P),$$

[14] In Albagli et al. (2021), the wedge results from limits to arbitrage with risk-neutral investors. They show that payoff convexity/concavity generates price convexity/concavity. As a result, Jensen's inequality implies the expected price is higher/lower than the expected value. In a model with privately-informed risk-averse investors, Chabakauri et al. (2021) derive an approximate relationship between skewness and prices using a polynomial expansion.

Figure 1: Timeline of events



and D_i denotes his demand for the stock. Investor i observes a private signal $\tilde{s}_i$ of the form:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i. \tag{1}$$

The error terms follow the distributions $\tilde{\varepsilon}_i \sim N\left(0, \sigma_\varepsilon^2\right)$ and are independent of all other random variables. In what follows, $\mathbb{E}[\cdot]$ and $\mathrm{var}[\cdot]$ denote the expectation and variance operators, respectively.

Disclosure Decision. Prior to trade, the firm's manager privately observes $\tilde{v}$ with probability $p \in [0, 1]$. Thus, as in Dye (1985), the manager's information endowment is probabilitistic. Conditional on being informed, the manager can verifiably disclose this information to the market, subject to a disclosure cost of $c \geq 0$ borne by the firm. The manager aims to maximize his expectation of the equilibrium price. Moreover, if the manager does not learn $\tilde{v}$, he is unable to credibly convey this lack of information to the market.

Note that our model allows for either or both standard disclosure frictions: a disclosure cost and a random information endowment. To prevent "unravelling", we assume that at least one of these two frictions is present, i.e., that at least one of $c > 0$ and $p < 1$ holds. For specific results, we will focus on the two benchmarks from the literature: (i) $c > 0$ and $p = 1$, which we refer to as the costly disclosure benchmark, and (ii) $c = 0$ and $p < 1$, which we refer to as the probabilistic information benchmark.

The timing of events is summarized in Figure 1. At date $t = 1$, investor i observes his private signal $\tilde{s}_i$. At $t = 2$, if informed, the manager chooses whether to disclose $\tilde{v}$. Conditional on disclosure, the price at date $t = 3$ is completely determined by the disclosed information. Conditional on non-disclosure, investors use their private signals and the information in prices to submit demands and the price is determined by market clearing. Finally, the firm pays off $\tilde{v}$ to shareholders at $t = 4$.

3.1 Discussion of assumptions

Our benchmark analysis makes a number of simplifying assumptions for analytical tractability.

Perfect verifiable disclosure. The assumption that disclosure is verifiable, as opposed to manipulable, is common in the literature. Einhorn and Ziv (2012) shows that the possibility of costly manipulation does not qualitatively affect the analysis; hence we rule it out for parsimony. Note we also assume the manager observes the value of the firm perfectly. The essential assumption that lends tractability to our analysis is that the manager has superior information to the market; qualitatively similar results hold when the firm's value also includes a component that is unknown to all agents.

Noise trade. As usual in models of informed trade, noise trade ensures that the equilibrium price does not fully reveal the firm's value. In our model, it also causes the manager to face uncertainty regarding the market reaction should they ***refrain from disclosure***, and thus the relative payoffs to disclosing and not disclosing. This complements related work that considers situations in which managers face uncertainty about the market reaction ***should they disclose*** (Suijs (2007)), or audience preferences (Bond and Zeng (2021)). Note further that our model also accommodates the case in which the firm's disclosure decision influences extent of noise trade. This might occur if, say, a fraction of liquidity traders times their trades until after uncertainty is resolved and thus avoids trading when the firm withholds its news. As we will see, noise trade only plays a pertinent role when the firm does not disclose. Thus, our model can capture this case if we interpret the level of noise trade as its level conditional on non-disclosure.

Public Information. Our benchmark analysis focuses on how the presence of investors' private information affects voluntary disclosure. In Section 5, we extend the model to allow for an ex-ante public signal and study how the precision of public information affects the probability of voluntary disclosure in our setting. As we shall see, the presence of privately-informed investors can have important implications for whether public information crowds out or crowds in voluntary disclosure.[15]

4 Equilibrium

We focus on equilibria in which the manager discloses $\tilde{v}$ if and only if it exceeds a threshold T, which we refer to as threshold equilibria. In classical disclosure models, any equilibrium must take this form, as the manager's payoff to non-disclosure is constant and his payoff to

[15] In Appendix B, we further extend the benchmark model to allow for both an ex-ante public signal and an ex-post public signal, and show that our equilibrium characterization remains qualitatively the same.

disclosure increases in $\tilde{v}$. However, it is less clear that all equilibria must take this form in our model: not only does the manager's payoff given disclosure depend upon $\tilde{v}$, but so too does his payoff conditional on non-disclosure (through investors' trading behavior).

We can show that in any equilibrium, the manager discloses sufficiently large realizations and withholds sufficiently low realizations of $\tilde{v}$.[16] This rules out equilibria such as those in Clinch and Verrecchia (1997) and Kim and Verrecchia (2001), in which the manager discloses exclusively extreme or moderate values. However, we have not been able to rule out the existence of equilibria consisting of disjoint disclosure sets that are bounded from below.

To characterize a threshold equilibrium, our initial focus is on deriving the firm's price when the manager does not disclose; denote this event by ND. In contrast to standard models without private information, this price depends upon the firm's value through investors' private signals. As we will see, only the average investor signal, $\int s_i di$ influences price, which, given that there is a continuum of investors, simply equals v. Thus, let $P_{ND}(v, z; T)$ denote equilibrium price given non-disclosure when the firm's value is $\tilde{v} = v$, noise trade is $\tilde{z} = z$, and the threshold above which the manager discloses is T.

4.1 Market pricing

Given the threshold nature of the manager's disclosure behavior in a threshold equilibrium, the absence of a disclosure leaves investors with a non-normal posterior. This implies that there does not exist an equilibrium in which $P_{ND}(v, z; T)$ is a linear function of v and z. We solve for the equilibrium by applying and extending the techniques developed in Breon-Drish (2015).[17] In particular, we conjecture and verify the existence of a "generalized" linear equilibrium in which, rather than a linear function, the price is a continuous, monotonic

[16] Equilibria in which the manager discloses upon observing $\tilde{v}$ *below* some threshold T are easily ruled out: if the manager followed such a strategy, the firm's price when the manager does not disclose would be no less than T, for otherwise there would exist an arbitrage opportunity. Moreover, in any equilibrium, the firm's price conditional on disclosure is simply $\tilde{v} - c$. Thus, the manager would prefer to deviate, refraining from disclosure when they observe $\tilde{v} < T + c$. Likewise, in any equilibrium, the manager always discloses upon observing sufficiently high $\tilde{v}$. Intuitively, if the manager did not disclose upon observing $\tilde{v} > T$, then the firm's price conditional on non-disclosure would be bounded above by T. However, this implies that when the manager observes $\tilde{v} > T + c$, he would prefer to deviate to disclosing.

[17] In particular, we apply the results in Proposition 2.1 of the Online Appendix of Breon-Drish (2015).

transformation of a linear function of the firm's value v and noise trade z:[18]

$$P_{ND}(v, z; T) = G\left(v + \beta z; T\right), \tag{2}$$

where $G\left(x; T\right)$ is a strictly increasing, smooth function of x.

The key feature of such an equilibrium is that, just as in a linear equilibrium, investor i can infer a "truth-plus-noise" signal from the price:

$$\tilde{s}_p \equiv \tilde{v} + \beta \tilde{z} \sim N(\tilde{v}, \sigma_p^2), \quad \text{where} \quad \sigma_p^2 = \beta^2 \sigma_z^2. \tag{3}$$

This characterization allows for a tractable calculation of investors' posterior beliefs given their private signals and the information in price. In particular, investors' updated beliefs $\tilde{v}$ given their price and private signals are again normal with mean and variance:

$$\tilde{\mu}_i \equiv \mathbb{E}[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{1} \left(\frac{\tilde{s}_i}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}\right), \tag{4}$$

$$\sigma_s^2 \equiv \mathrm{var}[\tilde{v}|\tilde{s}_i, \tilde{s}_p] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{1}. \tag{5}$$

To complete the derivation of the equilibrium price, we follow a series of steps, which are outlined in detail in the appendix. First, taking as given the form of price in expression (2), we derive each investor's demand as a function of their information set, which includes not only their private signal $\tilde{s}_i$, and the signal contained in the price, $\tilde{s}_p$, but also the knowledge that the firm has not disclosed. Note that the manager does not disclose both when they are uninformed and when they observe $\tilde{v} < T$. Thus, investors' beliefs given non-disclosure are the mixture of a normal distribution and truncated normal distribution, with mean and variance parameters given in expressions (4) and (5). Next, we apply the market-clearing condition to solve for the equilibrium price as a function of β. Finally, we solve for β to ensure the price is consistent with the conjecture in (2), and verify that the price is monotonic in $\tilde{s}_p$.

The following proposition characterizes the resulting equilibrium. In stating this result, we let $\phi\left(x\right)$ and $\Phi\left(x\right)$ denote the density and distribution function of a standard normal distribution, and we let $h\left(x\right) \equiv \frac{\phi(x)}{(x)}$ denote the inverse-Mills ratio.

Proposition 1. *Suppose there exists a $T \in \mathcal{R}$, such that when the manager observes $\tilde{v}$, they*

[18] Note our framework fits into the exponential family of distributions that is necessary to apply the methodology in Breon-Drish (2015). Breon-Drish (2015) also demonstrates that the generalized linear equilibria we consider here are unique among the class of equilibria in which price is a continuous function. We abstract from equilibria with discontinuous prices as those considered by Pálvölgyi and Venter (2015).

disclose if and only if $\tilde{v} > T$. Then, when the manager refrains from disclosure, there exists a unique equilibrium in the financial market. In this equilibrium, the firm's price equals:

$$P_{ND}(v,z;T) = \frac{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) P_I\left(v,z;T\right) + \left(1-p\right) P_U\left(v,z\right)}{p\Phi\left(\frac{T - P_U(v,z)}{\sigma_s}\right) + 1 - p}, \tag{6}$$

where:

$$P_U(v,z) \equiv \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z-\kappa), \tag{7}$$

$$P_I(v,z;T) \equiv P_U(v,z) - \sigma_s h\left(\frac{T - P_U(v,z)}{\sigma_s}\right), \tag{8}$$

$\beta = \frac{\sigma_\varepsilon^2}{\tau}$*, and* $\sigma_p^2 = \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2}$*. Moreover,* $P_{ND}(v,z;T)$ *is strictly increasing in* v*,* z*, and* T*.*

To develop intuition for the equilibrium non-disclosure price, it is helpful to first consider the components P_U in equation (7) and P_I in equation (8) separately.[19] First, note that P_U captures the non-disclosure price when the manager is known to be ***uninformed*** – that is, when $p = 0$. In this case, the absence of disclosure is entirely uninformative and the equilibrium price may be derived as in standard models of trade in which the firm's value is normally distributed and investors possess CARA utility (e.g., Hellwig (1980)). Specifically, the optimal demand for investor i is given by:

$$D_i(\mu_i, P) = \tau\,\frac{\mu_i - P}{\sigma_s^2}, \tag{9}$$

where P is the equilibrium price. Applying market clearing, the price then equals the average investor's posterior mean plus a risk-adjustment term that is proportional to noise traders' excess demand $z - \kappa$:

$$P = \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z-\kappa) \equiv P_U. \tag{10}$$

By substituting the expressions for investor beliefs in (4)-(5), one can easily verify that P_U is a linear function of the price signal $s_p \equiv v + \frac{\sigma_\varepsilon^2}{\tau} z$. Importantly, note that the price P_{ND} depends upon investors' private information signals only through P_U. As a result, the price aggregates these signals in precisely the same manner as in traditional noisy rational expectations models without voluntary disclosure. Thus, the signal investors glean from the non-disclosure price is identical to the one that arises in Hellwig (1980).

[19] We suppress the dependence of P_{ND}, P_U and P_I on (v,z,T) in what follows.

On the other hand, for a fixed T, P_I denotes the non-disclosure price when the manager is known to be ***informed*** – that is, when $p = 1$. A familiar special case is one in which the manager is informed and investors are uninformed (as captured by letting $p = 1$ and $\sigma_\varepsilon \to \infty$). In this case, the non-disclosure price is simply equal to the firm's expected cash flows given that $\tilde{v} < T$, which reduces to (e.g., Verrecchia (1990)):

$$P_I = \mathbb{E}[\tilde{v}|\tilde{v} < T] = \mathbb{E}[\tilde{v}] - \sigma_v h \left(\frac{T - \mathbb{E}[\tilde{v}]}{\sigma_v} \right). \tag{11}$$

Equation (8) illustrates that the non-disclosure price when both investors and the manager have information P_I combines features of expressions (10) and (11). Specifically, this price equals the firm's expected cash flows given $\tilde{v} < T$, where the mean parameter of the payoff reflects the price that would arise were the manager uninformed and the variance parameter reflects investors' variance parameter given their signals.

Finally, expression (6) shows that in the general case, the firm's price is a weighted average of the price if the manager was known to be uninformed (i.e., P_U) and the price if the manager was known to be informed but did not disclose their information (i.e., P_I). The weights reflect the perceived likelihood that the manager is informed, presuming again that the prior mean over firm value is P_U. Thus, in contrast to the Dye (1985) - Jung and Kwon (1988) model, these weights depend upon the noise trader demand z and the investors' private signals: a more optimistic signal indicates that the absence of a disclosure more likely resulted from an uninformed manager, as opposed to an informed manager who observed negative news.

4.2 Disclosure decision

We next analyze the manager's disclosure choice. The manager who observes $\tilde{v} = v$ discloses if and only if his payoff given disclosure exceeds the expected non-disclosure price conditional on $\tilde{v} = v$, i.e.,

$$v - c - \mathbb{E}[P_{ND}|\tilde{v} = v] \geq 0. \tag{12}$$

A disclosure threshold is incentive compatible if the manager is more inclined towards disclosure when his observed signal $\tilde{v} = v$ is greater. This would clearly be the case if the non-disclosure price were independent of the firm's value, as in voluntary disclosure models without informed trade. However, in our setting the non-disclosure price reflects the firm's value through investors' trading behavior, which may cause this condition to be violated.

An intuitive sufficient condition for there to exist a threshold equilibrium is that the non-disclosure price reacts to a marginal change in the firm's value only partially, i.e., $\frac{\partial P_{ND}}{\partial v} < 1$.

This ensures that the manager is more inclined towards disclosure as his signal rises, which implies the left-hand side of (12) increases in v. While this condition may seem natural given that investors observe noisy signals, it is in fact possible that the price responds more than one-for-one with a change in the value of the firm. To determine when this is the case, we next characterize $\frac{\partial P_{ND}}{\partial v}$.

Lemma 1. *Suppose there exists a threshold $T \in \mathcal{R}$ such that the manager discloses if and only if $\tilde{v}$ exceeds T. Then, when the manager does not disclose, the price response to a marginal change in the firm's value satisfies:*

$$\frac{\partial P_{ND}}{\partial v} = \mathit{var}\left[\tilde{v}\,|ND, \tilde{\mu}_j = P_U\right]\left(\mathit{var}^{-1}\left[\tilde{s}_j|\tilde{v}\right] + \mathit{var}^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{13}$$

The price response to a shift in $\tilde{v}$ is equal to the posterior variance perceived by an investor whose posterior mean parameter $\tilde{\mu}_j$ is equal to P_U, multiplied by the combined precision of their private signal and the signal they receive from price. To gain intuition, consider the case when the manager is known to be uninformed ($p = 0$), as in standard models of trade with normal distributions. In this case,

$$\frac{\partial P_U}{\partial v} = \frac{\partial}{\partial v}\left[\int_0^1 \mu_i di + \frac{\sigma_s^2}{\tau} z\right].$$

Upon substituting for μ_i and applying Bayes' rule for normal distributions, this reduces to:

$$\text{var}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]\left(\text{var}^{-1}\left[\tilde{s}_j|\tilde{v}\right] + \text{var}^{-1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{14}$$

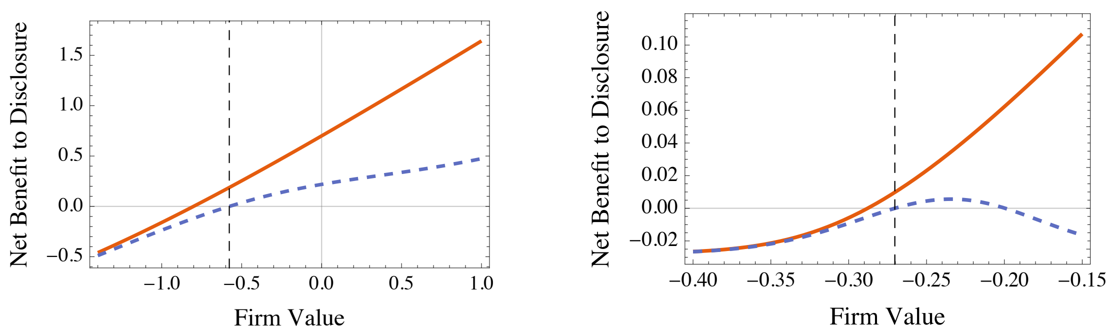
One can verify that this is always less than one, and so the price responds only partially to an increase in firm value. Intuitively, the price response is driven by the product of investors' posterior uncertainty and the total precision of their information signals.

When the manager may be informed with some probability, the posterior variance that appears in expression (14), $\text{var}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right]$, is replaced by $\text{var}\left[\tilde{v}\,|ND, \tilde{\mu}_j = P_U\right]$, which conditions on the event of non-disclosure (for a "representative" investor whose signals $\tilde{s}_j$ and $\tilde{s}_p$ lead them to have the posterior belief $\tilde{\mu}_j = P_U$). Therefore, when the manager may be informed, the event of non-disclosure changes the marginal reaction to the firm's information by adjusting investors' posterior variance. When the manager is *always* informed (i.e., $p = 1$), observing non-disclosure reveals that $\tilde{v} < T$. Because this strictly reduces the possible outcomes for the firm's value, investors' posterior variances fall short of the prior variance, and thus the marginal price response $\frac{\partial P_{ND}}{\partial v}$ falls short of the response when the manager is uninformed. Thus, this response is less than 1.

In contrast, when $p < 1$, the posterior variance following the observation of non-disclosure

Figure 2: Existence and non-existence of a threshold equilibrium.

The plot shows the net benefit to disclosure $v - \mathbb{E}[P_{ND}|\tilde{v} = v] - c$ as a function of the observed value v for $p = 1$ (solid) and $p = 0.95$ (dashed) respectively. The left panel illustrates an example of low investor information precision ($\sigma_\varepsilon = 0.75$) while the right panel illustrates the case of high information precision ($\sigma_\varepsilon = 0.2$). The remaining parameters are $c = 0.025$ and $\sigma_z = 1$.



(a) Low Private Information ($\sigma_\varepsilon = 0.75$) (b) High Private Information ($\sigma_\varepsilon = 0.2$)

may *increase* relative to the prior variance (Dye and Hughes (2018)). Intuitively, in this case, investors face an additional source of uncertainty given non-disclosure: they do not know whether the manager was informed with bad news or uninformed, outcomes that have very different implications for firm value. Nevertheless, we show in the next proposition that if the combined precision of investors' private and price signals are not excessively large, the sensitivity of the non-disclosure price to v is bounded above by 1, which, as previously mentioned, ensures the existence of a threshold equilibrium. Furthermore, we show that, when a threshold equilibrium exists, it is unique.

Proposition 2. *Suppose that either $p = 1$ or $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}$ is su ciently small. Then, there exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T$. The equilibrium threshold satis es:*

$$T - c = \mathbb{E}[P_{ND}(T, \tilde{z})]. \tag{15}$$

When the condition does not hold – i.e., when both $p \neq 1$ and investors' private information is highly precise – a threshold equilibrium may break down. Figure 2 shows an example of this case. The left panel illustrates the benefit to a manager of disclosing relative to not disclosing, $v - \mathbb{E}[P_{ND}|\tilde{v} = v]$, as a function of their observed signal, for low private information (high σ_ε), while the right panel depicts the analogous case for high private information (low σ_ε). To be precise, it plots this benefit when investors believe the manager discloses when $\tilde{v} > T$, where T is the minimum solution to $T - \mathbb{E}[P_{ND}|\tilde{v} = T] = c$, i.e., the threshold that would arise as an equilibrium if it were the case that $\frac{\partial P_{ND}}{\partial v} < 1$.

When investors' signals are sufficiently noisy (left panel), the benefit to disclosure is always increasing, which implies the existence of a threshold equilibrium. However, when investors' signals are sufficiently precise (right panel), the benefit to disclosure can decline for $v > T$, returning to negative values, which implies that the manager prefers not to disclose for some values $v > T$. This, in turn, rules out the existence of a threshold equilibrium.

4.3 Implications for firm valuation

We now characterize how the interaction of privately-informed investors and voluntary disclosure affects the firm's valuation, i.e., its expected price. Relative to a setting in which investors are not privately informed, we find that the expected price may be higher or lower, and that the nature of the "mis-valuation" depends on the underlying friction for non-disclosure. We then explore the implications of these results for the managers' incentives to acquire information and for the relation between skewness and expected returns.

For expositional clarity, we normalize the aggregate supply of the risky asset to zero (i.e., $\kappa = 0$).[20] This lets us abstract from standard risk-premia effects. Specifically, when investors have no private information, the non-disclosure price is simply the firm's expected value conditional on non-disclosure (e.g., Verrecchia (1990)):

$$P_{ND} \equiv \frac{p\Phi\left(\frac{T}{\sigma_v}\right)\mathbb{E}[\tilde{v}|\tilde{v} < T]}{p\Phi\left(\frac{T}{\sigma_v}\right) + (1-p)} = \mathbb{E}[\tilde{v}|ND]. \tag{16}$$

In the following result, we show that the expected non-disclosure price may be higher or lower than the firm's expected value when investors are privately informed. This result is particularly striking because investors do not bear aggregate risk when holding the stock (since $\kappa = 0$). Thus, this result suggests that, even when disclosure is idiosyncratic, i.e., it has no impact on risk premia, it may still influence a firm's expected price and returns.

Proposition 3. *Suppose $\kappa = 0$. Conditional on non-disclosure, the rm's expected value (generically) di ers from its expected price.*
(i) In the costly disclosure benchmark (i.e., $p = 1$, $c > 0$), the rm's expected value exceeds its expected price i.e.,

$$\mathbb{E}\left[P_{ND}|ND\right] < \mathbb{E}\left[\tilde{v}|ND\right].$$

[20]When $\kappa > 0$, the equilibrium price reflects a risk-premium that depends on investors' posterior variance about asset payoffs. To isolate the impact of private information on valuations after adjusting for the risk-premium, one needs to compare the equilibrium P_{ND} to the price that would arise in equilibrium if (i) investors had no private signals, but (ii) observed a public signal that leads to the same posterior variance as in equilibrium with private information. While our results obtain after making these adjustments, this would obscure the underlying economic channel we highlight in our analysis.

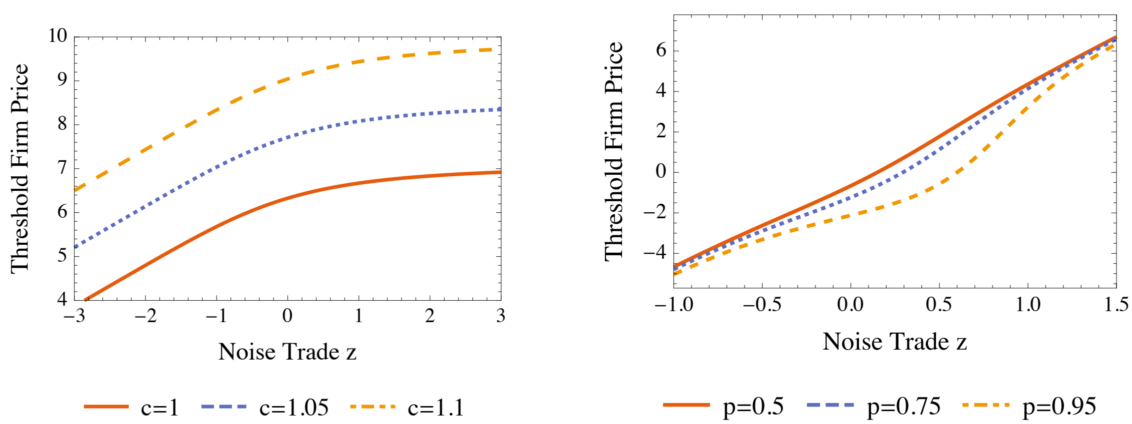
(ii) In the probabilistic information endowment benchmark (i.e., $p < 1$, $c = 0$), when investors' private signal precision $1/\sigma_\varepsilon$ is su ciently low, the rm's expected price exceeds its expected value i.e.,

$$\mathbb{E}\left[P_{ND}|ND\right] > \mathbb{E}\left[\tilde{v}|ND\right].$$

To gain intuition for the above result, first consider the costly disclosure benchmark (i.e., $p = 1$). Panel (a) of Figure 3 illustrates that the non-disclosure price in this case is concave in the underlying price signal, and specifically, in noise trading demand. The concavity of the firm's price and resulting under-valuation are rooted in the investors' risk preferences. When noise traders purchase the firm's shares, investors short the stock and demand a boost in price to do so. But because the firm's value is truncated from above, the downside from shorting is limited, and the price compensation they require is relatively small. In contrast, when noise traders sell, investors must bear the risk of being long. In this case, their downside is unlimited and so investors charge a larger compensation (price discount) for bearing the risk. On average, this pushes prices down.[21]

Figure 3: Curvature of the Price

The figure plots the price of the firm when when its value is equal to the disclosure threshold, i.e., $\tilde{v} = T$, as a function of noise trade z. The left (right) plot depicts the case in which $p = 1$ and $c > 0$ ($p < 1$ and $c = 0$). The parameters in the left (right) plot are set to $\sigma_v = 3$, $\sigma_\varepsilon = 1$, and $\sigma_z = 1.25$ ($\sigma_v = \sigma_\varepsilon = \sigma_z = 3$).



(a) Costly Disclosure Benchmark (b) Probabilistic Endowment Benchmark

Next, consider the probabilistic information benchmark, where $c = 0$ and $p < 1$. Impor-

[21]This asymmetric risk-compensation effect is absent in traditional models with linear prices because the value is symmetric and unbounded (usually normal). However, it is analogous to the "skewness effect" discussed in Albagli et al. (2021), Chabakauri et al. (2021) and Cianciaruso et al. (2020). It can be formally shown that the concavity/convexity of price is driven by the conditional skewness of cash flows given the price signal; this proof is available upon request.

tantly, as panel (b) of Figure 3 illustrates, when $p < 1$, the non-disclosure price is neither globally concave nor convex in the underlying noise trader demand. As a result, the overall impact of noise trade on the firm's valuation is less straightforward than in the costly disclosure benchmark. We apply an argument based upon the "minimum principle" of Guttman, Kremer, and Skrzypacz (2014) to demonstrate that noise trade tends to raise valuations.[22] Intuitively, this principle implies that the equilibrium disclosure threshold minimizes the non-disclosure price over all potential thresholds. Moreover, the price expression (6) reveals that, on average, noise trade has the same effect on price as creating random variation in the disclosure threshold.

This result can also be traced back to investor preferences for skewness. In this case, the firm's cash flows are not bounded above: non-disclosure can arise either because the manager is informed but the cash flows are low, or because the manager is uninformed and the cash flows are unbounded. As a result, payoffs can exhibit positive skewness – even though the price, conditional on non-disclosure, is low, there is a possibility that the payoff is extremely high. This implies that investors demand a large price compensation (increase) for short positions when noise traders buy, especially when they face high uncertainty about the payoffs (e.g., when their private information is noisy). This can lead to higher prices on average and, consequently, over-valuation. Figure 4 illustrates these results, showing that the extent of mis-valuation depends upon investors' prior uncertainty.

Next, we discuss implications of Proposition 3 for the firm's incentives to acquire information and for the empirical relation between skewness and expected returns.

Value of idiosyncratic information

In traditional models of voluntary disclosure, the ability to disclose idiosyncratic information does not benefit a firm and is detrimental when disclosure is costly. Intuitively, since the firm is correctly valued on average in these models, (i.e., $\mathbb{E}[P] = \mathbb{E}[\tilde{v}]$), idiosyncratic disclosure has no expected impact on the firm's price except through its cost. As a result, a firm would be weakly better off if it could commit to not acquiring information (Shavell (1994)).[23]

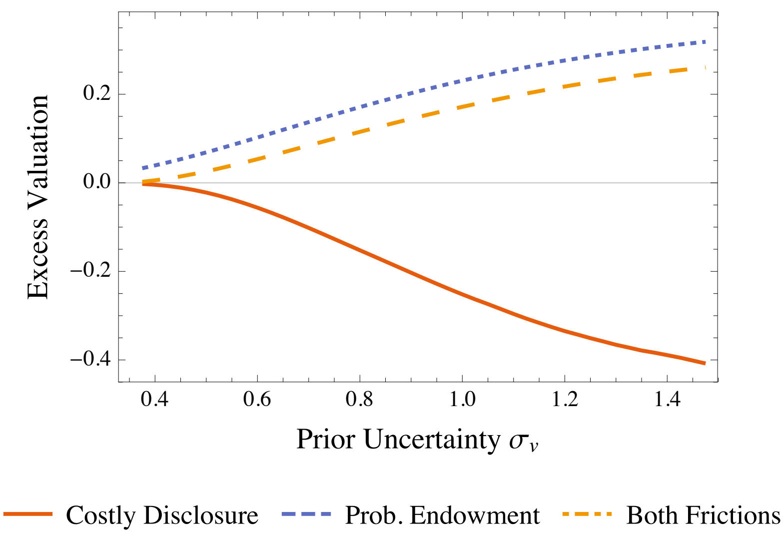
However, in our setting, this is no longer the case. Specifically, Proposition 3 (ii) and Figure 4 suggest that if disclosure costs are low, the firm may be better off by acquiring information with some probability than committing not to do so, because it can exploit the over-valuation that results from non-disclosure. In this case, the firm may optimally engage

[22] While we can only demonstrate analytically that the firm is over-valued when private information is imprecise, we have not been able to find a set of numerical parameters for which the firm is not over-valued in this benchmark.

[23] When the quality of the firm's information is an unobservable choice, DeMarzo, Kremer, and Skrzypacz (2019) demonstrate that in equilibrium the firm chooses imprecise information.

Figure 4: Excess valuation for different p and c

The figure plots the firm's expected price less its expected cash flows conditional on non-disclosure, $\mathbb{E}\left[P_{ND} - \tilde{v} | \tilde{v} < T\right]$ as a function of prior uncertainty σ_v. Parameters are set to $\sigma_\varepsilon = 2$, $\sigma_z = 2$, $c = 1; p = 1$ in the costly disclosure benchmark, $c = 0; p = 0.85$ in the probabilistic endowment benchmark, and $c = 0.25; p = 0.85$ in the case with both frictions.



in information acquisition and disclosure even though it is socially inefficient to do so.

An additional implication of Proposition 3 (ii) is that a firm can have a *lower* cost of capital, on average, when it chooses not to disclose information. This is in contrast to the common intuition from existing models that suggests more disclosure leads to a lower cost of capital (e.g., Dye and Hughes (2018)). Our model predicts that the relation between voluntary disclosure and cost of capital depends on the underlying friction that leads to non-disclosure. Thus, conditioning on the disclosure friction in a given empirical setting is important when studying the impact of disclosure on cost of capital.

The relation between skewness and expected returns

The negative relation between idiosyncratic skewness and average returns (e.g, Jiang et al. (2009); Conrad et al. (2013); Boyer and Vorkink (2014)) is difficult to reconcile in a model with rational investors and standard utility functions. In traditional representative agent models, coskewness with the market may be priced (e.g., Kraus and Litzenberger (1976)), but idiosyncratic skewness is diversified away. As such, earlier explanations of the negative relation have relied on investors exhibiting a preference for firm-specific skewness (e.g., Mitton and Vorkink (2007); Barberis and Huang (2008)).[24]

Our model provides a complementary, rational explanation for the negative relation. Note that the firm is correctly priced when it discloses. However, Proposition 3 (i) suggests that in the costly disclosure benchmark, payoffs are negatively skewed and the stock is under-valued conditional on non-disclosure. On the other hand, Proposition 3 (ii) implies that in the probabilistic information benchmark, payoffs are positively skewed and the stock is over-valued. To the extent that variation in skewness across firm and over time depends sufficiently on variation in voluntary disclosures, this implies a negative relation between skewness and expected returns unconditionally.[25]

5 Public information and the probability of disclosure

An extensive empirical literature has studied whether public information crowds in or crowds out voluntary disclosure, documenting mixed evidence. Using management forecasts as the proxy for voluntary disclosure and earnings volatility as the proxy for information quality, Imhoff Jr (1978), Cox (1985), and Waymire (1985) find that firms' forecast frequency is negatively related with their earnings volatility, consistent with a complementary relation, or crowding in. Francis et al. (2008) find that firms with more informative earnings have more expansive (i.e., detailed) voluntary disclosures. More recently, Kim and Ljungqvist (2021) show that firms increase voluntary disclosures in response to increased mandatory disclosures by peer firms.

Other work suggests that public information and voluntary disclosure are substitutes. Balakrishnan et al. (2014) find that firms respond to a loss of public information by providing more earnings guidance. Barth et al. (2017) find that proprietary cost concerns that eliminate previously mandatory disclosures lead firms to provide additional disclosures. Finally, Billings, Jennings, and Lev (2015) find evidence consistent with the notion that managers react to rising market volatility by providing guidance.

The ambiguous nature of this evidence is at odds with traditional models of disclosure, which suggest that public information either crowds out disclosure (e.g., Verrecchia (1990)) or leaves it unchanged (e.g., Dye and Hughes (2018)). Moreover, in standard models with informed investors, disclosure is usually modeled as a non-discretionary commitment to

[24] Mitton and Vorkink (2007) show that a negative relation can arise in an economy where investors have heterogeneous preferences for skewness, while Barberis and Huang (2008) rely on investors having cumulative prospect theory preferences.

[25] Since Beyer et al. (2010) find that 66% of return variation attributed to accounting information is driven by voluntary disclosures, this is plausible. Moreover, McNichols (1988) and Kothari, Shu, and Wysocki (2009) document a direct link between disclosure and return skewness.

release a public signal to the market. In these settings, better external information also tends to crowd out disclosure when both types of information are about the same dimension of fundamentals.[26] Furthermore, the direction of this relationship is critical to assessing the efficacy of disclosure regulations, as it determines their impact on the overall level of information available to market participants.

To study how public information affects voluntary disclosure when investors also have access to private information, we extend our benchmark model to incorporate public information. Specifically, suppose that a mandatory (non-strategic), ex-ante public signal $\tilde{y}$ is publicly disclosed in date $t = 1$:

$$\tilde{y} = \tilde{v} + \tilde{\eta}, \tag{17}$$

where $\tilde{\eta} \sim N(0, \sigma_\eta^2)$ is independent of all other random variables. Note because the disclosure arrives at date $t = 1$, the manager observes the outcome of the public signal before making his disclosure decision.[27] While this assumption is made primarily for tractability, empirical evidence suggests that this is a realistic feature of prominent voluntary disclosures. For instance, Beyer et al. (2010) find that, on average, management earnings forecasts generate significantly larger price reactions than earnings. This suggests that managers are aware of much of the information in forthcoming earnings when deciding whether to provide a forecast.

In Appendix B, we consider how the introduction of a public signal that arrives after the voluntary disclosure decision is made affects our results. We show that our results for equilibrium characterization and existence extend naturally to this case. While we are unable to characterize the impact of ex-post signal precision on voluntary disclosure analytically, numerically we find that such information can crowd in voluntary disclosure, especially when investors are uncertain about whether the manager is informed.

We begin by generalizing our equilibrium characterization to this case.

Proposition 4. *Suppose that either $p = 1$ and/or $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}$ is su ciently small, and x a realization of $\tilde{y} = y$. Then, there exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T(y)$. The equilibrium threshold satis es:*

$$T(y) - c = \mathbb{E}\left[P_{ND} | \tilde{v} = T(y), \tilde{y} = y\right], \tag{18}$$

where:

[26] See, e.g., Diamond (1985). See also Goldstein and Yang (2017), which discusses when this finding might not hold in such models.

[27] Our results would not change if the manager knows the public signal's outcome when disclosing, but this signal arrives after the voluntary disclosure.

$$P_{ND}(v,z,y) = \frac{p\Phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right) P_I(v,z,y) + (1-p) P_U(v,z,y)}{p\Phi\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right) + 1 - p}, \tag{19}$$

$$P_U(v,z,y) \equiv \int_i \mu_i di + \frac{\sigma_s^2}{\tau}(z-\kappa), \tag{20}$$

$$P_I(v,z,y) \equiv P_U(v,z,y) - \sigma_s h\left(\frac{T(y)-P_U(v,z,y)}{\sigma_s}\right), \tag{21}$$

and investor beliefs are given by:

$$\tilde{\mu}_i \equiv \mathbb{E}[\tilde{v}|\tilde{y},\tilde{s}_i,\tilde{s}_p] = \left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}\left(\frac{\tilde{y}}{\sigma_\eta^2}+\frac{\tilde{s}_i}{\sigma_\varepsilon^2}+\frac{\tilde{s}_p}{\sigma_p^2}\right), \tag{22}$$

$$\sigma_s^2 \equiv var[\tilde{v}|\tilde{y},\tilde{s}_i,\tilde{s}_p] = \left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)^{-1}, \tag{23}$$

where $\sigma_p^2 = \frac{\sigma_\varepsilon^4\sigma_z^2}{\tau^2}$. *Moreover, the equilibrium threshold* $T(y)$ *satisfies:*

$$T(y) = T(0) + \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right], \tag{24}$$

and is increasing in y.

This proposition clarifies the public signal's impact on the equilibrium outcomes. In particular, equation (24) shows that the equilibrium threshold increases with expected cash flows given the public signal, and thus rises in the signal. Intuitively, when expected cash flows are greater, the price given non-disclosure rises, which discourages disclosure. However, we next show that, as in standard models (e.g., Einhorn (2005)), the realization of such a signal has no impact on the *probability of disclosure*. An increase in the signal not only raises the threshold, but also increases the likelihood that the firm's value exceeds a given threshold; these two forces have precisely offsetting impacts on the probability of disclosure.

Lemma 2. *Fix a realization of the public signal* $\tilde{y}=y$. *Then, the probability of disclosure in equilibrium* $\Pr(\tilde{v}<T(y)|y)$ *does not depend on the realization* y *of the public signal.*

Given the above observation, the following result characterizes the impact of public information on the probability of voluntary disclosure in our setting.

Proposition 5. *More public information can **crowd-in** voluntary disclosure:*

(i) In the costly disclosure benchmark (i.e., $p=1$, $c>0$*), an increase in the precision of the public signal increases the probability of disclosure when* $\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2} > \frac{1}{var[\tilde{v}|\tilde{y}]}$ *and disclosure is sufficiently expensive.*

(ii) In the probabilistic information benchmark (i.e., $p < 1$, $c = 0$), when investors' private information is not too precise, there exists a range of values of public information precision such that an increase in the precision of the public signal increases the probability of disclosure.

To gain intuition, it is helpful to first express the non-disclosure price as follows:

$$P_{ND}(v,z,y) = P_U(v,z,y) - \frac{p\sigma_s \phi\left(\frac{T(y)\quad P_U(v,z,y)}{\sigma_s}\right)}{p\Phi\left(\frac{T(y)\quad P_U(v,z,y)}{\sigma_s}\right) + 1 - p}. \tag{25}$$

That is, the non-disclosure price can be written as the price were the manager uninformed, $P_U(v,z,y)$, less a "discount" that reflects investors' inference from non-disclosure. Since the probability of disclosure is independent of $\tilde{y}$, we can focus on the case in which $\tilde{y} = 0$.

The effect of public information on voluntary disclosure is primarily determined by how it impacts the threshold firm's expected price when it does not disclose, $\mathbb{E}[P_{ND}(T(0), \tilde{z}, 0)]$.[28] Equation (25) demonstrates that this expected price depends on σ_η through its impact on σ_s and P_U. Let $\Pi(\sigma_s, P_U)$ denote the non-disclosure price $P_{ND}(T(0), \tilde{z}, 0)$ as a function of these two components. Moreover, let $\Pi_{z=0}(\sigma_s, P_U) = P_{ND}(T(0), 0, 0)$ denote the price in the hypothetical alternative in which $\tilde{z}$ were fixed at zero. Then, differentiating and adding and subtracting terms, we arrive at:

$$\begin{aligned} \frac{\partial \mathbb{E}[P_{ND}(T(0), \tilde{z}, 0)]}{\partial \sigma_\eta} &= \frac{d\mathbb{E}[\Pi(\sigma_s, P_U)]}{d\sigma_\eta} \\ &= \underbrace{\frac{\partial \Pi_{z=0}}{\partial \sigma_s}\frac{\partial \sigma_s}{\partial \sigma_\eta}}_{\text{Standard channel}} + \underbrace{\frac{\partial \mathbb{E}[\Pi - \Pi_{z=0}]}{\partial \sigma_s}\frac{\partial \sigma_s}{\partial \sigma_\eta}}_{\text{Valuation channel}} + \underbrace{\mathbb{E}\left[\frac{\partial \Pi}{\partial P_U}\frac{\partial P_U(T(0), \tilde{z}, 0)}{\partial \sigma_\eta}\right]}_{\text{Substitution channel}}. \end{aligned} \tag{26}$$

This equation reveals that better public information affects the firm's incentives to disclose via three channels. The first channel, which is captured by $\frac{\partial \;\;_{\tilde{z}=0}}{\partial \sigma_s}\frac{\partial \sigma_s}{\partial \sigma_\eta}$, is directly analogous to standard disclosure models: greater public information reduces investor uncertainty, which attenuates the negative inference investors draw from non-disclosure (e.g., Verrecchia (1990)). This raises the non-disclosure price, discouraging disclosure. Equation (25) shows that, holding fixed P_U, σ_s affects P_{ND} purely through the non-disclosure "discount." Thus, $\frac{\partial \;\;_{\tilde{z}=0}}{\partial \sigma_s}\frac{\partial \sigma_s}{\partial \sigma_\eta}$ captures the impact of σ_η on this discount.

Second, better public information reduces the degree of mis-valuation in equilibrium – we

[28] Because $\mathbb{E}[P_{ND}(T, \tilde{z}, 0)]$ increases in the disclosure threshold, the implicit function theorem implies that -1 times this derivative determines how the equilibrium disclosure threshold changes with respect to σ_η. However, the complete argument in the appendix also accounts for the fact that a change in public information quality also changes the likelihood the firm's value falls below a given threshold.

refer to this as the ***valuation channel***. Recall that mis-valuation is driven by the asymmetric risk borne by investors when taking the other side of noise-trader purchases versus sales. Thus, we can think of the "mis-valuation" expected by the threshold firm as its expected non-disclosure price less the non-disclosure price it would receive in a hypothetical alternative where noise trade were instead fixed at zero, i.e., $\mathbb{E}\left[\Pi - \Pi_{z=0}\right]$.[29] Figure 4 shows that mis-valuation can increase with investor uncertainty, and as a result, it can decrease in the precision of public information.

Importantly, the effect of this channel on the likelihood of voluntary disclosure depends on whether the non-disclosure price exhibits over-valuation or under-valuation. In the costly disclosure benchmark, the valuation channel ***reduces under-valuation***, which reduces the benefit from disclosure for the firm. In contrast, the valuation channel can ***reduce over-valuation*** in the probabilistic information benchmark, and thus increase the firm's incentive to disclose information.

Finally, as public information improves, investors place relatively less weight on their private signals. We refer to this as the ***substitution channel***. This is reflected in the model via $\mathbb{E}\left[\frac{\partial}{\partial P_U}\frac{\partial P_U(T(0),z,0)}{\partial \sigma_\eta}\right]$. Intuitively, $P_U\left(T\left(0\right), \tilde{z}, 0\right)$ captures the aggregation of investors' beliefs that is reflected in the non-disclosure price, and $\frac{\partial}{\partial P_U}$ reflects how strongly the non-disclosure price varies in this statistic. Note that this substitution from private to public information makes the non-disclosure price P_{ND} less informative about the firm's value, which weakly increases the firm's incentive to disclose.

The overall impact of public information depends on the interaction of these channels. In the costly disclosure benchmark, the substitution channel dominates and thus public information crowds in voluntary disclosure when investor information is precise and disclosure costs are high. The condition on signal precisions is intuitive: investors' private and price information must be sufficiently precise (relative to the uncertainty given public information) to ensure that their signals play a significant role in determining the equilibrium price. Moreover, when disclosure costs are high, the disclosure threshold is high. As a result, the non-disclosure discount (i.e., the second term in equation (25)) approaches zero, and the non-disclosure price approaches the standard, linear price P_U. This causes both the standard non-disclosure discount channel (which is driven by $P_{ND} - P_U$) and the valuation channel (which is driven by the non-linearity of P_{ND}) to approach zero.[30] In turn, this implies that the "crowding out" effect of these two channels is attenuated.

In contrast, the substitution channel does not operate when $c = 0$, i.e., in the probabilistic

[29] This is only an approximate means of isolating the "valuation" channel in our model that is useful for conveying intuition. To fully remove noise trade from the model, we would let $\sigma_z^2 \to 0$. However, this would not only remove any mis-valuation, but would also render the price perfectly informative.

[30] Intuitively, for T large, investors' beliefs given non-disclosure, $\tilde{v}|\tilde{v} < T$, are approximately normal.

information benchmark. In this benchmark, the firm is approximately indifferent between disclosing and not when its value is precisely equal to the price it expects given non-disclosure i.e.,

$$T(y) = \mathbb{E}[P_{ND}(T(y), \tilde{z}, y)].$$

As a result, for a given realization of the public signal, the firm is indifferent over the amount of weight that investors place on their private signals.[31] However, we show that "crowding in" can arise due to the valuation channel. In particular, recall from Proposition 3 that the presence of noise trade causes the firm to be over-valued, i.e., $\mathbb{E}[\Pi - \Pi_{z=0}] > 0$. Thus, better public information reduces over-valuation, which in turn increases the marginal firm's incentive to disclose.

Figure 5 provides a numerical illustration of our results by plotting the probability of voluntary disclosure as a function of public information quality. Our measure of public information quality, $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$ measures the "signal to noise" ratio of the public signal $\tilde{y}$ with respect to the fundamental $\tilde{v}$ i.e., it measures $\frac{\text{cov}(v,y)}{\text{var}(y)}$. Panel (a) illustrates that in the costly disclosure benchmark, public information crowds in voluntary disclosure when disclosure costs are sufficiently high and public information quality is relatively low, or equivalently, private information precision is relatively high. Panel (b) illustrates that, in the probabilistic information benchmark, the crowding in effect is quite robust, but strongest when public information quality is high. Finally, panel (c) suggests that when both frictions are in effect, crowding in can arise for a wide range of parameters. Together these plots suggests that public information "crowding in" voluntary disclosure is a robust feature of our setting, in contrast to traditional models of voluntary disclosure without privately-informed investors.

Ex-ante public information and overall market informativeness

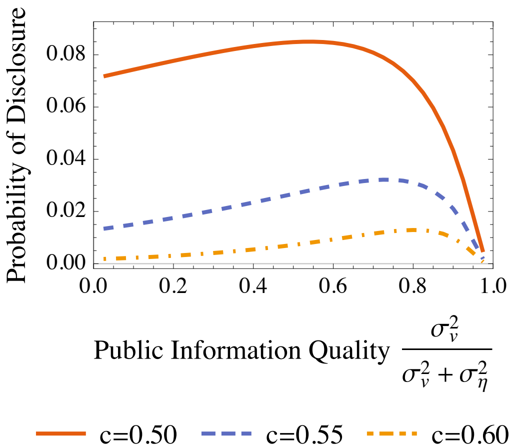
We next consider how a change in ex-ante information quality influences overall market informativeness. We measure overall market informativeness as the posterior variance of payoffs conditional on the publicly available information, i.e., $\mathbb{E}[\text{var}[\tilde{v}|\tilde{y}, \tilde{P}, \tilde{\Lambda}]]$, where $\tilde{\Lambda} \in \{D, ND\}$ denotes whether or not there is voluntary disclosure, and $\tilde{P} = P_{ND}$ when $\tilde{\Lambda} = ND$ and $\tilde{P} = v$ when $\tilde{\Lambda} = D$. This analysis is particularly useful from a policy perspective, because it speaks to how a change in mandatory disclosure affects the average amount of information available to an uninformed, rational investor.[32]

[31]Technically, this manifests as $\mathbb{E}\left[\frac{\partial \Pi}{\partial P_U}\right] \approx 0$ in the probabilistic information benchmark.

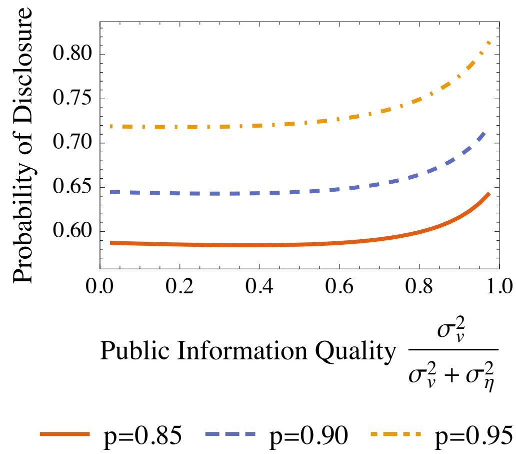
[32]We find similar results to those in this section when examining the information available to an *informed* investor, $\mathbb{E}[\text{var}[\tilde{v}|\tilde{s}_i, \tilde{y}, \tilde{P}, \tilde{\Lambda}]]$, as well as the *relative* uncertainty faced by an uninformed and informed investor, i.e., $\mathbb{E}[\text{var}[\tilde{v}|\tilde{y}, \tilde{P}, \tilde{\Lambda}]] - \mathbb{E}[\text{var}[\tilde{v}|\tilde{s}_i, \tilde{y}, \tilde{P}, \tilde{\Lambda}]]$. Thus, our results speak to both regulatory objectives to reduce the uncertainty faced by investors at large, as well as to "level the playing field" among investors.

Figure 5: Probability of Disclosure vs. Information Quality

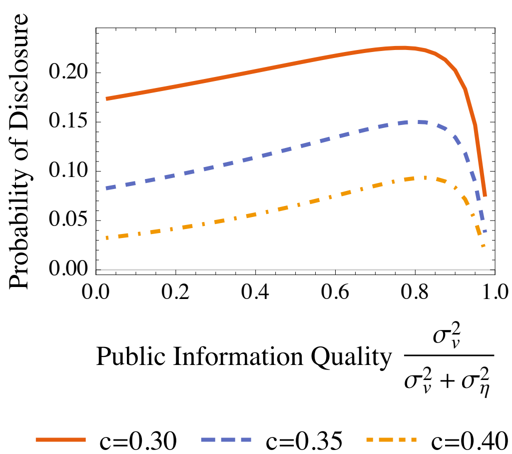
The figure plots the probability that the manager discloses in the costly disclosure and probabilistic endowment benchmarks as a function of public information quality, defined as the "signal-to-noise" ratio of the public signal $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$. Other parameters are $\tau = 1$, $\kappa = 0.1$ and $\sigma_v = 1.5$.



(a) Costly Disclosure Benchmark
($p = 1$, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)



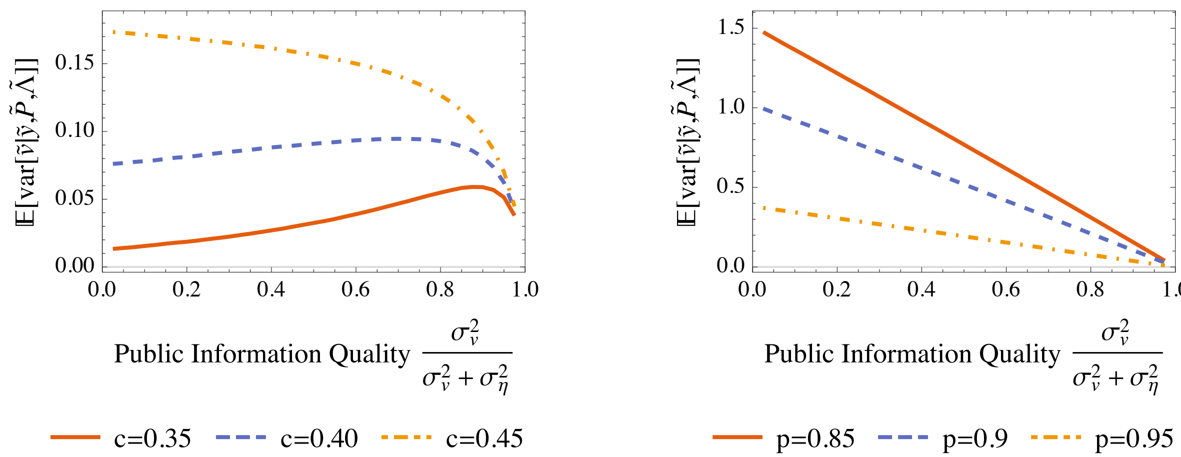
(b) Probabilistic Endowment Benchmark
($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)



(c) Both Frictions
(p=0.85, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)

Figure 6: Overall Informativeness vs. Public Information

The figure plots the expected posterior variance $\mathbb{E}\left[\text{var}\left[\tilde{v}|\tilde{y},\tilde{P},\tilde{\Lambda}\right]\right]$ as a function of the amount of public information, defined as the "signal-to-noise" ratio of the public signal $\frac{\sigma_v^2}{\sigma_v^2+\sigma_\eta^2}$. Other parameters are set to $\kappa = 0.1$, $\tau = 1$, and $\sigma_v = 1.5$.



(a) Costly Disclosure Benchmark ($p = 1$, $\sigma_\varepsilon = 0.75$, $\sigma_z = 1$)

(b) Probabilistic Endowment Benchmark ($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)

Figure 6 illustrates how overall market informativeness changes with public information quality. Panel (a) shows that better ex-ante public information ***reduces*** overall informativeness when disclosure costs are low and the public signal is not very precise. This implies that the crowding out effect of mandatory on voluntary disclosure can be sufficiently potent to cause disclosure mandates to be counterproductive. However, overall informativeness ***increases*** with public information quality in the costly disclosure benchmark when disclosure costs are sufficiently high or when public information quality is high. This demonstrates that mandatory disclosure may be more effective in settings where investors have significant private information.

In contrast, panel (b) shows that public information has a robust positive impact on overall informativeness in the probabilistic endowment benchmark, and has the largest impact when there is more uncertainty about whether the manager is informed (i.e., p is low). This is intuitive – the public signal is not only informative about the fundamental payoff v, but also helps reduce uncertainty about whether the manager is informed when there is no disclosure.[33] Taken together, these results highlight how changes in mandatory disclosure can have different effects on overall informativeness across firms, depending on the interaction between private information of investors and the firm's incentives to disclose.

[33] Recall that an informed manager discloses their information for sufficiently good news, and so a high realization of y together with non-disclosure is indicative of the manager not being informed.

6 Conclusions

Standard voluntary disclosure models assume that investors do not have access to private information. We show that this assumption is an economically important restriction, and relaxing it has qualitatively novel implications. First, in contrast to traditional models, prices generically exhibit over-valuation or under-valuation relative to expected cash flows. Moreover, the direction of mis-valuation depends on the underlying friction that generates non-disclosure. Second, we show that ex-ante public information can "crowd in" more voluntary disclosure, especially when firms face high disclosure costs or when investors face substantial uncertainty about firm payoffs.

Our analysis suggests that it is important to account for voluntary disclosure, and the underlying disclosure frictions, when interpreting empirical evidence and evaluating regulatory policy about the information environment of financial markets. Contrary to common wisdom, non-disclosure may be associated higher valuations and lower costs of capital when investors face uncertainty about the information endowment of managers. Similarly, mandatory disclosures may actually increase voluntary disclosure by firms and improve overall informativeness, in contrast to the standard criticism against such regulations.

Our model is stylized and may be extended along several dimensions. For instance, investors and the manager are endowed with information in our model. It would be interesting to study how the interaction between disclosure and trade affects both parties' incentives to acquire information. In traditional models of costly disclosure, the manager usually prefers to commit not to acquire information (ex-ante) because disclosure is costly but has no real effects. However, as we discuss in Section 4.3, our analysis implies that managers may find it valuable to acquire information with some probability, since the possibility of voluntary disclosure can lead to over-valuation on average. Similarly, while a model of endogenous information acquisition by investors is not immediately tractable, we expect some of our results to extend to this setting. For instance, to the extent that more public information crowds out private information acquisition, it is likely to crowd in voluntary disclosure as in our current model.

Our model assumes that the manager cannot commit to a disclosure policy ex-ante. In a complementary paper, Cianciaruso et al. (2020) study the optimal disclosure policy with commitment. In the class of threshold strategies, they show that the firm prefers to commit to a "recognition" policy which involves disclosing bad news (below a threshold), but withholding good news. We expect the optimal disclosure policy to have a similar form in our setting with privately-informed investors so long as disclosure is not too costly.[34]

[34]When disclosure is sufficiently costly, we expect the firm would commit to no disclosure, and the equi-

Finally, we consider a model without real and feedback effects. As an interesting extension, one could consider the possibility that managers use their disclosure policy to elicit information from the market and inform their investment choices. Alternatively, one might consider how voluntary disclosure influences the incentives of managers to invest, as in Ben-Porath, Dekel, and Lipman (2018), when investors possess private information.

librium would revert to a traditional "linear" equilibrium (e.g., Hellwig (1980)).

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A Proofs

Proof of Proposition 1. To begin, as in the text, let $\tilde{\mu}_j \equiv \mathbb{E}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right] = \sigma_s^2\left(\frac{\tilde{s}_j}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2}\right)$. Moreover, let $g\left(x\right) \equiv \mathbb{E}\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]$ denote investor j's conditional expectation of firm value given non-disclosure when $\tilde{s}_j$ and $\tilde{s}_p$ are such that $\tilde{\mu}_j = x$. This function plays a central role in the analysis and thus we begin by characterizing its properties. As in the text, let $\sigma_p^2 \equiv \text{var}\left[\tilde{s}_p|\tilde{v}\right] = \beta^2\sigma_z^2$ and $\sigma_s^2 \equiv \text{var}\left[\tilde{v}|\tilde{s}_j, \tilde{s}_p\right] = \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}$.

Lemma A.1. *The function $g\left(x\right)$ satisfies:*

$$g(x) = \frac{\int_{-\infty}^{\infty} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f\left(v|ND\right) dv}, \tag{27}$$

where $f\left(v|ND\right)$ denotes the density of firm value given non-disclosure. Furthermore,

$$g'\left(x\right) = \frac{var\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]}{\sigma_s^2} > 0. \tag{28}$$

Proof of Lemma A.1. To start, we derive investor j's posterior distribution over $\tilde{v}$ given his signal $\tilde{s}_j = s_j$, the price signal $\tilde{s}_p = s_p$, and the event of non-disclosure ND, whose density we denote by $f\left(v|ND, s_j, s_p\right)$. Note that:

$$\begin{aligned}
f\left(v|ND, s_j, s_p\right) &\propto f\left(v, s_j, s_p|ND\right) \\
&= f\left(s_j, s_p|v\right) f\left(v|ND\right) \\
&\propto \exp\left[-\frac{\left(s_j - v\right)^2}{2\sigma_\varepsilon^2} - \frac{\left(s_p - v\right)^2}{2\sigma_p^2}\right] f\left(v|ND\right) \\
&= \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \left(\frac{s_j}{\sigma_\varepsilon^2} + \frac{s_p}{\sigma_p^2}\right)v - \frac{1}{2}\left(\frac{s_j^2}{\sigma_\varepsilon^2} + \frac{s_p^2}{\sigma_p^2}\right)\right] f\left(v|ND\right),
\end{aligned}$$

where the second line follows from the fact that the event ND is uninformative regarding $\tilde{s}_j$ and $\tilde{s}_p$ conditional on $\tilde{v}$. Now, the above equation implies:

$$\begin{aligned}
f\left(v|ND, s_j, s_p\right) &= \frac{\exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \left(\frac{s_j}{\sigma_\varepsilon^2} + \frac{s_p}{\sigma_p^2}\right)v\right] f\left(v|ND\right)}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \left(\frac{s_j}{\sigma_\varepsilon^2} + \frac{s_p}{\sigma_p^2}\right)v\right] f\left(v|ND\right) dv} \\
&= \frac{\exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{\mu_j}{\sigma_s^2}v\right] f\left(v|ND\right)}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)v^2 + \frac{\mu_j}{\sigma_s^2}v\right] f\left(v|ND\right) dv}.
\end{aligned}$$

Now, let $M\left(t; x\right) = \mathbb{E}\left[\exp\left(t\tilde{v}\right)|ND, \tilde{\mu}_j = x\right]$ denote the moment-generating function of $\tilde{v}$ as perceived by investor j when $\tilde{\mu}_j = x$; see Breon-Drish (2015) for proofs that the integrals under consideration below in fact exist and that derivative-integral interchange is valid. Note that:

$$M(t;x) = \frac{\int_{1}^{1} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \left(t+\frac{x}{\sigma_s^2}\right)v\right] f(v|ND)\, dv}{\int_{1}^{1} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}. \tag{29}$$

Now, observe that, by the properties of the moment-generating function:

$$g(x) = \mathbb{E}\left[\tilde{v}|ND, \tilde{\mu}_j = x\right] = \left[\frac{\partial M(t;x)}{\partial t}\right]_{t=0}. \tag{30}$$

Calculating this derivative, we get:

$$g(x) = \frac{\int_{1}^{1} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}{\int_{1}^{1} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}, \tag{31}$$

which proves the first part of the lemma. To verify the second part of the lemma, differentiating the above equation, we arrive at:

$$\begin{aligned} g^{0}(x) &= \frac{1}{\sigma_s^2}\frac{\int_{1}^{1} v^2 \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}{\int_{1}^{1} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv} \\ &\quad -\frac{1}{\sigma_s^2}\left(\frac{\int_{1}^{1} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}{\int_{1}^{1} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)v^2 + \frac{x}{\sigma_s^2}v\right] f(v|ND)\, dv}\right)^2. \end{aligned}$$

Now, note from equation (29), this implies:

$$\begin{aligned} g^{0}(x) &= \frac{1}{\sigma_s^2}\left\{\left[\frac{\partial^2 M(t;x)}{\partial t^2}\right]_{t=0} - \left\{\left[\frac{\partial M(t;x)}{\partial t}\right]_{t=0}\right\}^2\right\} \\ &= \frac{1}{\sigma_s^2}\left\{\mathbb{E}\left[\tilde{v}^2|ND, \tilde{\mu}_j = x\right] - \mathbb{E}\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]^2\right\} \\ &= \frac{var\left[\tilde{v}|ND, \tilde{\mu}_j = x\right]}{\sigma_s^2}. \end{aligned}$$

□

We next apply this result to derive the investors' demands.

Lemma A.2. *Investor j's demand in the event of non-disclosure given the price P_{ND} equals:*

$$D_j = \frac{\tau}{\sigma_s^2}\left[\mu_j - g^{-1}(P_{ND})\right]. \tag{32}$$

Proof of Lemma A.2. Investor j's demand satisfies:

$$D_j = \arg\max_y - \int_{-\infty}^{\infty} \exp\left\{-\frac{1}{\tau}\left(y\left(\tilde{v} - P_{ND}\right)\right)\right\} f\left(v|ND, s_j, s_p\right) dv.$$

It is easily verified that this function is concave and thus the first-order condition is sufficient for a solution. Applying equation (31), the first-order condition reduces as follows:

$$\begin{aligned} P_{ND} &= \frac{\int_{-\infty}^{\infty} v \exp\left(-\tau^{-1} D_j \tilde{v}\right) f\left(v|ND, s_j, s_p\right) dv}{\int_{-\infty}^{\infty} \exp\left(-\tau^{-1} D_j \tilde{v}\right) f\left(v|ND, s_j, s_p\right) dv} \\ &= \frac{\int_{-\infty}^{\infty} v \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \left(-\frac{D_j}{\tau} + \frac{\mu_j}{\sigma_s^2}\right) v\right] f\left(v|ND\right) dv}{\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v^2 + \left(-\frac{D_j}{\tau} + \frac{\mu_j}{\sigma_s^2}\right) v\right] f\left(v|ND\right) dv} \\ &= g\left(\mu_j - \frac{\sigma_s^2 D_j}{\tau}\right), \end{aligned}$$

where the final line applies Lemma A.1. As Lemma A.1 shows that $g' > 0$, g is invertible, and so we can solve the above equation to arrive at equation (32). □

We may now derive the firm's price by applying the market-clearing condition:

$$\begin{aligned} \kappa = z + \int_0^1 D_i di \iff & \frac{\sigma_s^2}{\tau}\left(\kappa - z\right) = \int \mu_i di - g^{-1}\left(P_{ND}\right) \\ \iff & P_{ND} = g\left(\int \mu_i di + \frac{\sigma_s^2}{\tau}\left(z - \kappa\right)\right). \end{aligned}$$

Substituting for μ_i and σ_s^2 and applying the law of large numbers, we arrive at:

$$\begin{aligned} P_{ND} &= g\left(\int \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}\left[\frac{\tilde{s}_i}{\sigma_\varepsilon^2} + \frac{\tilde{s}_p}{\sigma_p^2} + \frac{z-\kappa}{\tau}\right] di\right) \\ &= g\left(\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}\left[\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) v + \left(\frac{\beta}{\sigma_p^2} + \frac{1}{\tau}\right) z - \frac{\kappa}{\tau}\right]\right). \end{aligned}$$

Observe that, as conjectured, this takes the form of a generalized linear equilibrium, i.e., $P_{ND} = G\left(v + \beta z\right)$, with:

$$G(x) = g\left(\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{-1}\left[\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) x - \frac{\kappa}{\tau}\right]\right);$$

$$\beta = \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)^{1} \left(\frac{\beta}{\sigma_p^2} + \frac{1}{\tau}\right).$$

Solving the second equation for β yields a unique solution $\beta = \frac{\sigma_\varepsilon^2}{\tau}$. The equilibrium solution for β implies that:

$$\sigma_p^2 = \frac{\sigma_\varepsilon^4 \sigma_z^2}{\tau^2},$$

and $s_p = v + \frac{\sigma_\varepsilon^2}{\tau} z$. Note further that, as Lemma A.1 shows that $g^0(x) > 0$, we immediately have that the price is monotonic in s_p. Substituting and simplifying, we have that the unique generalized linear equilibrium price satisfies:

$$P_{ND}(v, z) = g(P_U(v, z)) = \mathbb{E}[\tilde{v} | ND, \tilde{\mu}_j = P_U(v, z)], \tag{33}$$

$$\text{where } P_U(v, z) \equiv \sigma_s^2 \left[\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) \left(v + \frac{\sigma_\varepsilon^2}{\tau} z\right) - \frac{\kappa}{\tau} \right]. \tag{34}$$

To complete the proof, we show that the price expression (33) can be re-expressed as in the proposition. Note that the event of non-disclosure ND results either from an informed manager who observed $\tilde{v} < T$ or an uninformed manager; denote the former event by $\tilde{\Gamma} = 1$ and the latter by $\tilde{\Gamma} = 0$. Then, we have:

$$\mathbb{E}[\tilde{v} | ND, \tilde{\mu}_j = P_U(v, z)] = \Pr\left(\tilde{\Gamma} = 1 | \tilde{\mu}_j = P_U(v, z)\right) \mathbb{E}[\tilde{v} | \tilde{v} < T, \tilde{\mu}_j = P_U(v, z)] + \Pr\left(\tilde{\Gamma} = 0 | \tilde{\mu}_j = P_U(v, z)\right) P_U(v, z). \tag{35}$$

Note that $\Pr(\tilde{v} < T | \tilde{\mu}_j = P_U(v, z)) = \Phi\left(\frac{T \quad P_U(v,z)}{\sigma_s}\right)$. Therefore, we can apply Bayes' rule to arrive at:

$$\Pr\left(\tilde{\Gamma} = 1 | \tilde{\mu}_j = P_U(v, z)\right) = 1 - \Pr\left(\tilde{\Gamma} = 0 | \tilde{\mu}_j = P_U(v, z)\right) = \frac{p\Phi\left(\frac{T \quad P_U(v,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T \quad P_U(v,z)}{\sigma_s}\right) + 1 - p}. \tag{36}$$

To explicitly derive $\mathbb{E}[\tilde{v} | \tilde{v} < T, \tilde{\mu}_j = P_U(v, z)]$, we may apply the formula for the mean of a truncated normal distribution, which yields:

$$\mathbb{E}[\tilde{v} | \tilde{v} < T, \tilde{\mu}_j = P_U(v, z)] = P_U(v, z) - \sigma_s h\left(\frac{T - P_U(v, z)}{\sigma_s}\right) \equiv P_I(v, z). \tag{37}$$

Substituting equations (36) and (37) into equation (35) yields the expression for price defined in the proposition:

$$P_{ND} = \frac{p\Phi\left(\frac{T \;\; P_U(v,z)}{\sigma_s}\right)\left(P_U(v,z) - \sigma_s h\left(\frac{T \;\; P_U(v,z)}{\sigma_s}\right)\right) + (1-p)\,P_U\,(v,z)}{p\Phi\left(\frac{T \;\; P_U(v,z)}{\sigma_s}\right) + 1 - p}. \tag{38}$$

□

Proof of Lemma 1. Observe from equations (33) and (34) that:

$$\frac{\partial P_{ND}(v,z)}{\partial v} \;\; = \;\; g^{0}\left(P_U\left(v,z\right)\right) * \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right). \tag{39}$$

Now, applying equation (28), we have:

$$g^{0}\left(P_U\left(v,z\right)\right) = \frac{1}{\sigma_s^2}\text{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_U\left(v,z\right)\right].$$

Substituting into equation (39) and simplifying, we have:

$$\frac{\partial P_{ND}(v,z)}{\partial v} \;\; = \;\; \text{var}\left[\tilde{v}|ND, \tilde{\mu}_j = P_U\left(v,z\right)\right]\left(\text{var}^{\;\;1}\left[\tilde{s}_j|\tilde{v}\right] + \text{var}^{\;\;1}\left[\tilde{s}_p|\tilde{v}\right]\right). \tag{40}$$

□

Proof of Proposition 2. Let $\Psi(T,\tilde{z}) \equiv T - P_{ND}(T,\tilde{z};T)$. Then, $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ denotes the incremental payoff to the manager who observes $\tilde{v} = T$ from disclosing, relative to not disclosing, when investors conjecture that the manager discloses if and only if $\tilde{v} > T$. We establish the proposition in three lemmas. Lemma A.3 states that, if investors conjecture a threshold disclosure strategy, then the manager's payoff to disclosing is strictly increasing in firm value, i.e., $\frac{\partial}{\partial v}\left(v - c - \mathbb{E}\left[P_{ND}\left(v,\tilde{z};T\right)\right]\right) > 0$. This implies that, if T solves $\mathbb{E}\left[\Psi(T,\tilde{z})\right] = c$, then T corresponds to a threshold equilibrium. Next, Lemma A.4 states that $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ strictly increases in T, and thus, when a threshold equilibrium exists, it is unique. Finally, Lemma A.5 states that there exists a T such that $\mathbb{E}\left[\Psi(T\;,\tilde{z})\right] = c$. Together, these lemmas imply that T corresponds to the unique threshold equilibrium.

Lemma A.3. *Suppose that* $p = 1$ *and/or* $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1 + \frac{1}{2}p\left(1-p\right)\right)\right]^{\;\;1}$. *Then,* $\forall v, T \in \mathcal{R}$*:*

$$\frac{\partial}{\partial v}\left(v - c - \mathbb{E}\left[P_{ND}\left(v,\tilde{z};T\right)\right]\right) > 0.$$

Proof of Lemma A.3. We first argue that it is sufficient to show that:

$$\forall v, z, T \in \mathcal{R}, \quad \frac{\partial P_{ND}(v, z; T)}{\partial v} < 1. \tag{41}$$

To see why this is sufficient, note that, because $\frac{\partial P_{ND}(v,z;T)}{\partial v} = \frac{\tau}{\sigma_\varepsilon^2} \frac{\partial P_{ND}(v,z;T)}{\partial z}$, condition (41) implies that $|P_{ND}(v, z; T)|$ is sublinear in z. That is, we have $\left| \frac{1}{\sigma_z} P_{ND}(v, z; T)\, \phi\left(\frac{z}{\sigma_z}\right) \right| < \left| \frac{1}{\sigma_z} A \phi\left(\frac{z}{\sigma_z}\right) \right|$ for some $A \in \mathcal{R}$ that does not depend upon z, and, being the expectation of an absolute normal, $\int_{1}^{1} \left| \frac{1}{\sigma_z} A \phi\left(\frac{z}{\sigma_z}\right) \right| dz$ is finite. Thus, by the dominated convergence theorem, when condition (41) holds,

$$\frac{\partial}{\partial v} \mathbb{E}\left[P_{ND}(v, \tilde{z}; T)\right] = \frac{1}{\sigma_z} \int_{1}^{1} \frac{\partial P_{ND}(v, z; T)}{\partial v} \phi\left(\frac{z}{\sigma_z}\right) dz < 1.$$

We proceed to show that condition (41) holds in each of the two cases stated in the lemma.

Case 1: $\mathbf{p = 1}$. Let:

$$\Delta_v \equiv \frac{\partial}{\partial v} P_U(v, z) = \frac{\sigma_v^2 \left(\sigma_\varepsilon^2 + \sigma_p^2\right)}{\sigma_\varepsilon^2 \sigma_p^2 + \sigma_v^2 \left(\sigma_\varepsilon^2 + \sigma_p^2\right)} \quad \text{and} \tag{42}$$

$$\Delta_z \equiv \frac{\partial}{\partial z} P_U(v, z) = \frac{\sigma_\varepsilon^2}{\tau} \frac{\sigma_v^2 \left(\sigma_\varepsilon^2 + \sigma_p^2\right)}{\sigma_\varepsilon^2 \sigma_p^2 + \sigma_v^2 \left(\sigma_\varepsilon^2 + \sigma_p^2\right)}, \tag{43}$$

and notice that $\Delta_v \in (0, 1)$. Appealing to Proposition 1, we have that, when $p = 1$, $P_{ND}(v, z; T)$ reduces to:

$$P_{ND}(v, z; T) = P_U(v, z) - \sigma_s h\left(\frac{T - P_U(v, z)}{\sigma_s}\right).$$

Differentiating this expression with respect to v yields:

$$\Delta_v \left[1 + h^{0}\left(\frac{T - P_U(v, z)}{\sigma_s}\right)\right].$$

It may be verified that the inverse-mills ratio satisfies $h^{0}(x) \in (-1, 0)$ and thus this belongs to $(0, 1)$.

Case 2: $\frac{1}{\boldsymbol{\sigma_\varepsilon^2}} + \frac{1}{\boldsymbol{\sigma_p^2}} < \left[\boldsymbol{\sigma_v^2}\left(\mathbf{1} + \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{p}\left(\mathbf{1} - \boldsymbol{p}\right)\right)\right]^{-\mathbf{1}}$. Recall from expression (40) that we have:

$$\frac{\partial P_{ND}(v, z; T)}{\partial v} = \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right) \operatorname{var}\left[\tilde{v} | ND, \tilde{\mu}_j = P_U(v, z)\right] > 0.$$

Let $\tilde{\Gamma} = 1$ when the manager is informed and 0 otherwise. Then, applying the law of total variance:

$$\begin{aligned} \text{var}\left[\tilde{v}|ND,\tilde{\mu}_j = P_U(v,z)\right] &= \mathbb{E}\ \left\{\text{var}\ \left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\} \\ &\quad +\text{var}\ \left\{\mathbb{E}\ \left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\}, \end{aligned} \tag{44}$$

where the subscripts on the expectations and variances indicate they are taken over $\tilde{\Gamma}$ only. Now, applying the fact that the variance of a truncated normal always lies below the prior variance, we have:

$$\begin{aligned} &\mathbb{E}\ \left\{\text{var}\ \left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\} \\ = \ &\frac{p\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)\text{var}\left[\tilde{v}|\tilde{v} < T,\tilde{\mu}_j = P_U(v,z)\right] + (1-p)\,\sigma_s^2}{p\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right) + 1 - p} < \sigma_s^2. \end{aligned} \tag{45}$$

Next, applying the variance of a binary distribution, we have:

$$\begin{aligned} &\text{var}\ \left\{\mathbb{E}\ \left[\tilde{v}|\tilde{\Gamma},ND,\tilde{\mu}_j = P_U(v,z)\right]\right\} \\ = \ &\Pr\left(\tilde{\Gamma} = 1|ND,\tilde{\mu}_j = P_U(v,z)\right)\Pr\left(\tilde{\Gamma} = 0|ND,\tilde{\mu}_j = P_U(v,z)\right) * \\ &\left\{\mathbb{E}\left[\tilde{v}|\tilde{\Gamma} = 1,ND,\tilde{\mu}_j = P_U(v,z)\right] - \mathbb{E}\left[\tilde{v}|\tilde{\Gamma} = 0,ND,\tilde{\mu}_j = P_U(v,z)\right]\right\}^2 \\ = \ &\frac{p(1-p)\,\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)\sigma_s^2 h\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)^2}{\left(p\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right) + 1 - p\right)^2} \\ = \ &\frac{p(1-p)\,\sigma_s^2\phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)^2}{\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)\left(p\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right) + 1 - p\right)^2} < p(1-p)\,\sigma_s^2\frac{\phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)^2}{\Phi\left(\frac{T\quad P_U(v,z)}{\sigma_s}\right)}. \end{aligned} \tag{46}$$

It may be verified that $\frac{\phi(x)^2}{(x)}$ is bounded above by $\frac{1}{2}$ and thus the above expression is bounded over all realizations of v and z by $\frac{p(1\quad p)\sigma_s^2}{2}$. Combining (44), (45), and (46), we have:

$$\frac{\partial P_{ND}(v,z;T)}{\partial v} < \sigma_s^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)\left(1 + \frac{p(1-p)}{2}\right) < \sigma_v^2\left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)\left(1 + \frac{p(1-p)}{2}\right),$$

such that, for $\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2} < \left[\sigma_v^2\left(1 + \frac{1}{2}p(1-p)\right)\right]^{\ 1}$, $\frac{\partial P_{ND}(v,z;T)}{\partial v} \in (0,1)$. □

Lemma A.4. $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ *strictly increases in* T*. Thus, if a threshold equilibrium exists, it is unique.*

Proof of Lemma A.4. We first show that, fixing any $T \in \mathcal{R}$, $|\Psi(T,z)\,\frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is bounded above by an integrable function. Note that, because $h^{0}(x) < 0$, when $T - P_U(T, z) > 0$, we have:

$$
\begin{aligned}
|\Psi(T, z)| &= T - P_U(T, z) + \frac{p\sigma_s \phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right) + 1 - p} \\
&< T - P_U(T, z) + \sigma_s h\left(\frac{T - P_U(T, z)}{\sigma_s}\right) < T - P_U(T, z) + \sigma_s h(0),
\end{aligned}
$$

which is linear in z, and hence its product with the PDF a normal distribution is integrable. Hence, $|\Psi(T, z) \frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is integrable on $\{z : T - P_U(T, z) > 0\}$. To see that $|\Psi(T, z) \frac{1}{\sigma_z}\phi\left(\frac{z}{\sigma_z}\right)|$ is also integrable on $\{z : T - P_U(T, z) < 0\}$, note for $T - P_U(T, z) < 0$,

$$
|\Psi(T, z)| = \left| T - P_U(T, z) + \frac{p\sigma_s \phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right)}{p\Phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right) + 1 - p} \right| < |T - P_U(T, z)|,
$$

which is also linear in z. Given these results, we may apply the dominated convergence theorem to arrive at $\frac{\partial}{\partial T}\mathbb{E}[\Psi(T, \tilde{z})] = \mathbb{E}\left[\frac{\partial}{\partial T}\Psi(T, \tilde{z})\right]$. Now, absorbing T into the numerator of $\mathbb{E}[P_{ND}(T, \tilde{z}; T)]$ and expressing $\mathbb{E}[\tilde{v}|\tilde{v} < T, \tilde{\mu}_j = P_U(T, z)]$ in its integral form, we may write $\frac{\partial}{\partial T}\Psi(T, \tilde{z})$ as:

$$
\frac{\partial}{\partial T}\Psi(T, z) = \frac{\partial}{\partial T} \frac{p\left[\Phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right)T - \int_1^T \frac{v}{\sigma_s}\phi\left(\frac{v \;\; P_U(T,z)}{\sigma_s}\right)dv\right] + (1-p)(T - P_U(T, z))}{p\Phi\left(\frac{T \;\; P_U(T,z)}{\sigma_s}\right) + 1 - p}. \tag{47}
$$

Now, integration by parts yields:

$$
\int_1^T \frac{v}{\sigma_s}\phi\left(\frac{v - P_U(T, z)}{\sigma_s}\right)dv = T\Phi\left(\frac{T - P_U(T, z)}{\sigma_s}\right) - \int_1^T \Phi\left(\frac{v - P_U(T, z)}{\sigma_s}\right)dv. \tag{48}
$$

Note further that:

$$
\begin{aligned}
\frac{\partial}{\partial T}\int_1^T \Phi\left(\frac{v - P_U(T, z)}{\sigma_s}\right)dv &= \Phi\left(\frac{T - P_U(T, z)}{\sigma_s}\right) - \int_1^T \frac{\Delta_v}{\sigma_s}\phi\left(\frac{v - P_U(T, z)}{\sigma_s}\right)dv \\
&= (1 - \Delta_v)\Phi\left(\frac{T - P_U(T, z)}{\sigma_s}\right),
\end{aligned} \tag{49}
$$

where Δ_v, as defined in (42), belongs to $(0, 1)$. Applying equations (48) and (49), we may calculate the derivative in expression (47) as follows:

$$
\begin{aligned}
\frac{\partial}{\partial T}\Psi\left(T,z\right) = \frac{\partial}{\partial T}\frac{p\int_{-\infty}^{T}\Phi\left(\frac{v-P_U(T,z)}{\sigma_s}\right)dv+(1-p)\left(T-P_U\left(T,z\right)\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)+1-p}& \\
\propto \quad \left(1-\Delta_v\right)\left[p\Phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)+1-p\right]^2 & \\
-\frac{1-\Delta_v}{\sigma_s}p\phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)\left[p\int_{-\infty}^{T}\Phi\left(\frac{T-P_U\left(v,z\right)}{\sigma_s}\right)dv+(1-p)\left(T-P_U\left(T,z\right)\right)\right] & \\
\propto \quad \frac{p^2}{1-p}\left[\Phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)^2-\frac{1}{\sigma_s}\phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)\int_{-\infty}^{T}\Phi\left(\frac{T-P_U\left(v,z\right)}{\sigma_s}\right)dv\right] & \\
+1-p+2p\Phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)-p\phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)\frac{T-P_U\left(T,z\right)}{\sigma_s}. &
\end{aligned}
$$

Now, note that the normal distribution is log concave, which implies that, $\forall x\in\mathcal{R}$, $\Phi\left(x\right)^2-\frac{1}{\sigma_s}\phi\left(x\right)\int_{-\infty}^{x}\Phi\left(v\right)dv>0$ (Bagnoli and Bergstrom (2005)). Thus, we have:

$$
\begin{aligned}
\frac{\partial}{\partial T}\Psi\left(T,z\right) &> 1-p+2p\Phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)-p\phi\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right)\frac{T-P_U\left(T,z\right)}{\sigma_s} \\
&\propto 1-\frac{T-P_U\left(T,z\right)}{\sigma_s}\frac{p\phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}{1-p+2p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)}.
\end{aligned}
$$

When $\frac{T-P_U(T,z)}{\sigma_s}<0$, this is trivially positive. When $\frac{T-P_U(T,z)}{\sigma_s}>0$, we have that it exceeds:

$$
1-\frac{1}{2}\frac{T-P_U\left(T,z\right)}{\sigma_s}h\left(\frac{T-P_U\left(T,z\right)}{\sigma_s}\right).
$$

Now, this is positive since, $\forall x\in\mathcal{R}$, $xh\left(x\right)<1$. □

Lemma A.5. *There exists a* $T\in\mathcal{R}$ *such that* $\mathbb{E}\left[\Psi(T,\tilde{z})\right]=c$.

Proof of Lemma A.5. It is easily seen that $\mathbb{E}\left[\Psi(T,\tilde{z})\right]$ is a continuous function of T. Thus, to prove the lemma, it is sufficient to show that:

$$
\lim_{T\to-\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right]<c<\lim_{T\to\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right]. \tag{50}
$$

To see that this holds, note that, applying equation (38), we have:

$$
\Psi\left(T,z\right) = T-\frac{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\left(P_U(T,z)-\sigma_s h\left(\frac{T-P_U(T,z)}{\sigma_s}\right)\right)+(1-p)P_U\left(T,z\right)}{p\Phi\left(\frac{T-P_U(T,z)}{\sigma_s}\right)+1-p}
$$

$$= \quad T(1-\Delta_v) - \Delta_z z + \tau^{-1}\sigma_s^2\kappa + \frac{p\sigma_s\phi\left(\frac{T(1-\Delta_v)-\Delta_z z+\tau^{-1}\sigma_s^2\kappa}{\sigma_s}\right)}{p\Phi\left(\frac{T(1-\Delta_v)-\Delta_z z+\tau^{-1}\sigma_s^2\kappa}{\sigma_s}\right)+1-p}. \tag{51}$$

Note that $\lim_{x\to\infty}\frac{p\sigma_s\phi(x)}{p\Phi(x)+1-p}=0$. Combining this with the fact that $\Delta_v<1$, we have that $\Psi(T,z)$ converges pointwise in z to ∞ as $T\to\infty$. Note further that, from the proof of the previous lemma, $\Psi(T,z)$ is increasing in T. Consequently, $\Psi(T,z)-\Psi(0,z)>0$. Thus, we can apply Fatou's lemma, and arrive at:

$$\lim_{T\to\infty}\mathbb{E}\left[\Psi(T,\tilde{z})-\Psi(0,\tilde{z})\right]\geq\mathbb{E}\left[\lim_{T\to\infty}\Psi(T,\tilde{z})-\Psi(0,\tilde{z})\right]=\infty.$$

This verifies the second inequality in (50).

Next, to prove the first inequality in (50), applying the fact that $h(x)\to -x$ as $x\to-\infty$, we have:

$$\lim_{x\to-\infty}\left(x+\frac{p\phi(x)}{p\Phi(x)+1-p}\right)=\begin{cases}0 & \text{when } p=1\\ -\infty & \text{when } p\in(0,1).\end{cases}$$

Thus, when $p\in(0,1)$ ($p=1$), we have that $\Psi(T,z)$ converges pointwise in z to $-\infty$ (to 0) as $T\to-\infty$. Applying Fatou's lemma as above, this immediately implies that, when $p\in(0,1)$, $\lim_{T\to-\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right]=-\infty$, which verifies the inequality holds in this case. Next, when $p=1$, as in the proof of the previous lemma, we may apply the dominated convergence theorem to interchange limit and expectations to immediately arrive at $\lim_{T\to-\infty}\mathbb{E}\left[\Psi(T,\tilde{z})\right]=\mathbb{E}\left[\lim_{T\to-\infty}\Psi(T,\tilde{z})\right]=0$. Moreover, given the assumption that one of the disclosure frictions is always present, we have that, if $p=1$, then $c>0$. Thus, we once again have verified the inequality holds.

$\square$

$\square$

Proof of Proposition 3. Part i. Since $\mathbb{E}[\tilde{z}]=0$, we can write the expected price given non-disclosure, $\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]$, as:

$$\begin{aligned}\mathbb{E}\left[P_{ND}|\tilde{v}<T\right] &= \mathbb{E}\left[P_U(\tilde{v},\tilde{z})|\tilde{v}<T\right]-\sigma_s\mathbb{E}\left[h\left(\frac{T-P_U(\tilde{v},\tilde{z})}{\sigma_s}\right)|\tilde{v}<T\right]\\ &= \int_i\mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di-\sigma_s\mathbb{E}\left[h\left(\frac{T-\int_i\tilde{\mu}_i di-\frac{z}{\tau}\sigma_s^2}{\sigma_s}\right)|\tilde{v}<T\right]. \end{aligned}\tag{52}$$

Now, we may write the firm's expected value conditional on non-disclosure as:

$$\begin{aligned}\mathbb{E}\left[\tilde{v}|\tilde{v}<T\right] &= \mathbb{E}\left\{\mathbb{E}\left[\tilde{v}|\tilde{v}<T,\tilde{s}_j,\tilde{s}_p\right]|\tilde{v}<T\right\} \\ &= \mathbb{E}\left[\tilde{\mu}_j|\tilde{v}<T\right]-\sigma_s\mathbb{E}\left[h\left(\sigma_s^{\ 1}\left(T-\tilde{\mu}_j\right)\right)|\tilde{v}<T\right], \end{aligned} \tag{53}$$

for an arbitrary investor j. Given that investors' signals are homogeneously distributed, $\int_i \mathbb{E}\left[\tilde{\mu}_i|\tilde{v}<T\right]di=\mathbb{E}\left[\tilde{\mu}_j|\tilde{v}<T\right]$. Thus, combining equations (52) and (53) yields:

$$\begin{aligned}&\mathbb{E}\left[P_{ND}|\tilde{v}<T\right]-\mathbb{E}\left[\tilde{v}|\tilde{v}<T\right] \\ \propto\ &\mathbb{E}\left[h\left(\sigma_s^{\ 1}\left(T-\tilde{\mu}_j\right)\right)|\tilde{v}<T\right]-\mathbb{E}\left[h\left(\sigma_s^{\ 1}\left(T-\int_i\tilde{\mu}_i di-\frac{\tilde{z}}{\tau}\sigma_s^2\right)\right)|\tilde{v}<T\right].\end{aligned}$$

Next, note that the inverse-mills ratio $h(\cdot)$ is convex. Thus, to show that the above expression is negative, it is sufficient to show that, conditional on $\tilde{v}<T$, $\tilde{\mu}_j \succ_{SSD} \int_i\tilde{\mu}_i di+\frac{z}{\tau}\sigma_s^2$, where $\succ_{SSD}$ denotes second-order stochastic dominance. It is straightforward to verify that the coefficients on $\tilde{v}$ in $\tilde{\mu}_j$ and $\int_i\tilde{\mu}_i di+\frac{z}{\tau}\sigma_s^2$ are identical. Therefore, the components of variation driven by $\tilde{v}$ in both $\tilde{\mu}_j$ and $\int_i\tilde{\mu}_i di+\frac{z}{\tau}\sigma_s^2$ are identical. Together with the normality of the error terms $\{\tilde{\varepsilon}_i\}$ and $\tilde{z}$ and their independence of $\tilde{v}$, this implies that second-order stochastic dominance reduces to the relative variance conditional on $\tilde{v}$, i.e.,

$$\tilde{\mu}_j \succ_{SSD} \int_i\tilde{\mu}_i di+\frac{\tilde{z}}{\tau}\sigma_s^2 \Longleftrightarrow \operatorname{var}\left[\tilde{\mu}_j|\tilde{v}\right]<\operatorname{var}\left[\int_i\tilde{\mu}_i di+\frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right].$$

Calculating these variances, we have:

$$\begin{aligned}\operatorname{var}\left[\tilde{\mu}_j|\tilde{v}\right] &= \operatorname{var}\left[\frac{\frac{1}{\sigma_\varepsilon^2}\tilde{\varepsilon}_j+\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau}\tilde{z}}{\frac{1}{\sigma_\varepsilon^2}+\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}+\frac{1}{\sigma_v^2}}\right]=\frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)^2}; \\ \operatorname{var}\left[\int_i\tilde{\mu}_i di+\frac{\tilde{z}}{\tau}\sigma_s^2|\tilde{v}\right] &= \operatorname{var}\left[\frac{\left(\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}\frac{\sigma_\varepsilon^2}{\tau}+\frac{1}{\tau}\right)\tilde{z}}{\frac{1}{\sigma_\varepsilon^2}+\frac{\tau^2}{\sigma_\varepsilon^4\sigma_z^2}+\frac{1}{\sigma_v^2}}\right]=\frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^2\sigma_\varepsilon^4\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)^2}{\left(\tau^2\sigma_v^2+\sigma_z^2\sigma_\varepsilon^2\left(\sigma_v^2+\sigma_\varepsilon^2\right)\right)^2}.\end{aligned}$$

Taking the difference yields $-\frac{1}{\tau^2}\frac{\sigma_v^4\sigma_z^4\sigma_\varepsilon^6\left(\tau^2+\sigma_z^2\sigma_\varepsilon^2\right)}{\left(\tau^2\sigma_v^2+\sigma_v^2\sigma_z^2\sigma_\varepsilon^2+\sigma_z^2\sigma_\varepsilon^4\right)^2}<0$.

Part ii. We show that the firm is over-valued when $\sigma_\varepsilon\to\infty$; by continuity, this ensures that the firm is over-valued for σ_ε sufficiently large. Note that $\sigma_\varepsilon\to\infty$, $P_U(v,z)\to\frac{\sigma_v^2}{\tau}(z-\kappa)$ and $\sigma_s\to\sigma_v$. Thus, in this limit, the non-disclosure price given a threshold T does not depend directly on the firm's value; denote this price by $\hat{P}_{ND}(z;T)$. In this limit, we have that the equilibrium condition reduces to:

$$
\begin{aligned}
0 &= \mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right] \\
\Longleftrightarrow 0 &= \mathbb{E}\left\{\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa) + \frac{p\phi\left(\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa)\right)}{p\Phi\left(\frac{T}{\sigma_v} - \frac{\sigma_v}{\tau}(\tilde{z}-\kappa)\right) + 1 - p}\right\}.
\end{aligned}
$$

Now, since $\frac{\phi^{0}(x)}{\phi(x)} = -x$, note that:

$$
\begin{aligned}
\left[\frac{\partial}{\partial T}\hat{P}_{ND}(0;T)\right]_{T=\hat{T}} = 0 \quad &\Longleftrightarrow \quad \frac{p\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{p\Phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right) + 1 - p} = \frac{\phi^{0}\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)} \\
&\Longleftrightarrow \quad \frac{p\phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right)}{p\Phi\left(\frac{\hat{T}}{\sigma_v} - \frac{\kappa\sigma_v}{\tau}\right) + 1 - p} = -\frac{\hat{T}}{\sigma_v} + \frac{\kappa\sigma_v}{\tau} \\
&\Longleftrightarrow \quad \hat{T} - \hat{P}_{ND}(0;\hat{T}) = 0.
\end{aligned}
$$

Consistent with the minimum principle in Acharya, DeMarzo, and Kremer (2011), this implies that the equilibrium threshold when there is no noise trade $\hat{T}$ satisfies $\hat{T} = \arg\min_x \hat{P}_{ND}(0;x)$ (the second-order condition is straightforward to verify). Now, note that this implies:

$$
\begin{aligned}
\mathbb{E}\left[\hat{T} - \hat{P}_{ND}(\tilde{z};\hat{T})\right] &= \hat{T} - \mathbb{E}\left[\hat{P}_{ND}\left(0;\hat{T} - \frac{\sigma_v^2}{\tau}\tilde{z}\right)\right] \\
&< \hat{T} - \mathbb{E}\left[\hat{P}_{ND}\left(0;\hat{T}\right)\right] = 0.
\end{aligned}
$$

Now, as shown in Lemma A.4, $\mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right]$ strictly increases in T. Thus, the equilibrium threshold with noise trade T (i.e., the solution to $\mathbb{E}\left[T - \hat{P}_{ND}(\tilde{z};T)\right] = 0$), satisfies $T > \hat{T}$. Now, note that when $\kappa = 0$, $\hat{P}_{ND}(0;T) = \mathbb{E}[\tilde{v}|ND]$ (where ND here refers to the event of non-disclosure in the equilibrium in which the manager discloses when $v > T$), and thus:

$$
\begin{aligned}
\mathbb{E}\left[\hat{P}_{ND}(\tilde{z};T)\right] - \mathbb{E}[\tilde{v}|ND] &= \mathbb{E}\left[\hat{P}_{ND}(\tilde{z};T)\right] - \hat{P}_{ND}(0;T) \\
&= T - \hat{P}_{ND}(0;T),
\end{aligned}
$$

because T by definition satisfies the equilibrium condition $\mathbb{E}\left[\hat{P}_{ND}(\tilde{z};T)\right] = T$. Now, from the proof of Lemma A.4, $x - \hat{P}_{ND}(0;x)$ is increasing in x. Thus, since $T > \hat{T}$ and

$\hat{T} - \hat{P}_{ND}(0;\hat{T}) = 0$, we have that $T\ \ - \hat{P}_{ND}\left(0; T\ \ \right) > 0$. $\square$

Proof of Proposition 4. The public signal is observable to all agents in the model prior to the disclosure and trading stages. Thus, the proofs of Propositions 1 and 2 directly extend to this case upon replacing the prior mean and variance parameters 0 and σ_v^2 with the mean and variance parameters conditional on the public signal, $\mathbb{E}[\tilde{v}|\tilde{y}]$ and $\text{var}[\tilde{v}|\tilde{y}]$. $\square$

Proof of Lemma 2. Note we can rewrite the equilibrium condition as:

$$\begin{aligned} 0 &= T - c - \mathbb{E}\left[P_{ND}(T,\tilde{z},y)\right] \\ &= \mathbb{E}\left[T - c - \left(P_U\left(T,\tilde{z},y\right) - \sigma_s \frac{p\phi\left(\frac{T\ \ P_U(T,z,y)}{\sigma_s}\right)}{p\Phi\left(\frac{T\ \ P_U(T,z,y)}{\sigma_s}\right) + 1 - p}\right)\right] \\ &\propto \mathbb{E}\left[\frac{T - P_U\left(T,\tilde{z},y\right)}{\sigma_s} + \frac{p\phi\left(\frac{T\ \ P_U(T,z,y)}{\sigma_s}\right)}{p\Phi\left(\frac{T\ \ P_U(T,z,y)}{\sigma_s}\right) + 1 - p} - \frac{c}{\sigma_s}\right]. \end{aligned}$$

Now, we can manipulate equations (20), (22), and (23) to arrive at:

$$\begin{aligned} \frac{T - P_U\left(T,z,y\right)}{\sigma_s} &= \frac{1}{\sigma_s}\left[T - \frac{\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\mathbb{E}\left[\tilde{v}|\tilde{y}=y\right] + \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\left(T + \frac{\sigma_\varepsilon^2}{\tau}z\right)}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}}\right] - \frac{\sigma_s\kappa}{\tau} \\ &= \frac{1}{\sigma_s}\frac{\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\left(T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) - \left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}\right)\frac{\sigma_\varepsilon^2}{\tau}z}{\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_p^2}} - \frac{\sigma_s\kappa}{\tau} \\ &\equiv A_1\left(T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) + A_2 z + A_3\kappa. \end{aligned}$$

Thus, the equilibrium condition may be written:

$$\mathbb{E}\left[A_1\left(T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) + A_3\kappa + \frac{p\phi\left(A_1\left(T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) + A_2\tilde{z} + A_3\kappa\right)}{p\Phi\left(A_1\left(T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right]\right) + A_2\tilde{z} + A_3\kappa\right) + 1 - p} - \frac{c}{\sigma_s}\right] = 0.$$

Now, this implies that:

$$T - \mathbb{E}\left[\tilde{v}|\tilde{y}=y\right] = t\ \ , \tag{54}$$

where $t\ \ $ solves:

$$\mathbb{E}\left[A_1 t\ \ + A_3\kappa + \frac{p\phi\left(A_1 t\ \ + A_2\tilde{z} + A_3\kappa\right)}{p\Phi\left(A_1 t\ \ + A_2\tilde{z} + A_3\kappa\right) + 1 - p} - \frac{c}{\sigma_s}\right] = 0.$$

We now have that the probability of disclosure given $\tilde{y} = y$ satisfies:

$$\Pr(\tilde{v} > T(y) \,|\tilde{y} = y) = \Pr(\tilde{v} > t \; + \mathbb{E}[\tilde{v}|\tilde{y} = y] \,|\tilde{y} = y)$$
$$= \Pr(\tilde{v} - \mathbb{E}[\tilde{v}|\tilde{y} = y] > t \; |\tilde{y} = y) = \Phi\left(\frac{t}{\sqrt{\mathrm{var}(\tilde{v}|\tilde{y})}}\right),$$

Since t does not depend upon y, this is independent of y. Finally, the result that $T(y) = T(0) + \mathbb{E}[\tilde{v}|\tilde{y} = y]$ follows from equation (54) and the fact that $\mathbb{E}[\tilde{v}|\tilde{y} = 0] = 0$.

□

Proof of Proposition 5. Part i. Note first that, applying Lemma 2, the probability of disclosure equals:

$$\begin{aligned}
\Pr(\tilde{v} > T(\tilde{y})) &= \int \Pr(\tilde{v} > T(\tilde{y}) \,|\tilde{y} = x) \, dF_y(x) \\
&= \Pr(\tilde{v} > T(0) \,|\tilde{y} = 0) \\
&= 1 - \Phi\left(\frac{T(0)}{\sqrt{\mathrm{var}(\tilde{v}|\tilde{y})}}\right) \\
&= 1 - \Phi\left(\sqrt{\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}} T(0)\right). \qquad (55)
\end{aligned}$$

Let $\Omega(T, \sigma_\eta)$ denote net expected benefit from disclosure when $y = 0$ as a function of T and σ_η:

$$\Omega(T, \sigma_\eta) \equiv T - c - \mathbb{E}[P_{ND}(T, \tilde{z}, 0; T)],$$

and let Ω_1 and Ω_2 denote the derivatives of Ω with respect to its first and second arguments. Then, note that:

$$\begin{aligned}
&\frac{\partial}{\partial \sigma_\eta}\left[1 - \Phi\left(T(0)\sqrt{\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}}\right)\right] \\
&= -\phi\left(T(0)\sqrt{\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}}\right)\left[-\frac{\Omega_2(T(0), \sigma_\eta)}{\Omega_1(T(0), \sigma_\eta)}\sqrt{\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}} - \frac{1}{\sigma_\eta^3}\left(\frac{1}{\sigma_\eta^2} + \frac{1}{\sigma_v^2}\right)^{\frac{1}{2}} T(0)\right] \\
&\propto \sigma_\eta^3\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\eta^2}\right)\Omega_2(T(0), \sigma_\eta) + T(0)\,\Omega_1(T(0), \sigma_\eta).
\end{aligned}$$

Analogous arguments to those in the proof of Proposition 2 enable us to interchange the order of limits/derivatives and expectations. Also, from the proof of Proposition 4, $\frac{\partial}{\partial T}\Omega(T, \sigma_\eta) > 0$. Together with the fact that $\frac{\partial}{\partial c}\Omega(T, \sigma_\eta) < 0$, we have that $\frac{\partial T(0)}{\partial c} > 0$. Furthermore, since, for any finite T, $\lim_{c\to\infty}\Omega(T,\sigma_\eta)<0$, we have that $\lim_{c\to\infty}T(0)=\infty$. Therefore,

$$\begin{aligned}
&\lim_{c\to\infty}\left[\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2\left(T\left(0\right),\sigma_\eta\right)+T\left(0\right)\Omega_1\left(T\left(0\right),\sigma_\eta\right)\right]\\
=&\lim_{T\to\infty}\left[\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2\left(T,\sigma_\eta\right)+T\Omega_1\left(T,\sigma_\eta\right)\right].
\end{aligned}$$

Now, note that:

$$\begin{aligned}
\lim_{T\to\infty}\Omega_2\left(T,\sigma_\eta\right)&=\lim_{T\to\infty}\mathbb{E}\left\{\frac{\partial}{\partial\sigma_\eta}\left[T-P_U\left(T,\tilde{z},0\right)-\sigma_s h\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\right]\right\}\\
&=\mathbb{E}\left[\begin{array}{c}-\sigma_s\frac{\partial\left(\sigma_s^{-1}(1-\Delta_v)\right)}{\partial\sigma_\eta}*\lim_{T\to\infty}\left[T*h'\left(\frac{T-P_U(T,z,0)}{\sigma_s}\right)\right]\\ -\frac{\partial\sigma_s}{\partial\sigma_\eta}\lim_{T\to\infty}h\left(\frac{T-P_U(T,z,0)}{\sigma_s}\right)-\frac{\partial\Delta_v}{\partial\sigma_\eta}\lim_{T\to\infty}T\end{array}\right],
\end{aligned}$$

where Δ_v is the coefficient on T in $P_U(T,z,0)$ (which can immediately be derived by substituting $\mathrm{var}\left[\tilde{v}|\tilde{y}\right]$ for σ_v^2 in equation (42)). Note that the inverse-mills ratio satisfies $h\left(x\right),h'(x)\to 0$ and $xh'\left(x\right)\to 0$ as $x\to\infty$. Thus, the limits of the first two terms in the above expression equal zero, such that we have:

$$\lim_{T\to 0}\Omega_2(T,\sigma_\eta)=-\frac{\partial\Delta_v}{\partial\sigma_\eta}\lim_{T\to\infty}T. \tag{56}$$

Next, note:

$$\begin{aligned}
&\lim_{T\to\infty}\left[T\Omega_1\left(T,\sigma_v\right)\right]\\
=&\ \mathbb{E}\left\{\lim_{T\to\infty}\left(T*\frac{\partial}{\partial T}\left[T-P_U\left(T,\tilde{z},0\right)-\sigma_s h\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\right]\right)\right\}\\
=&\ (1-\Delta_v)*\mathbb{E}\left\{\lim_{T\to\infty}T\left[1-h'\left(\frac{T-P_U\left(T,\tilde{z},0\right)}{\sigma_s}\right)\right]\right\}\\
=&\ (1-\Delta_v)*\lim_{T\to\infty}T,
\end{aligned}\tag{57}$$

where we again applied the fact that $\lim_{x\to\infty}h'(x)=0$. Combining equations (56) and (57), we have:

$$\begin{aligned}
&\lim_{T\to\infty}\left[\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\Omega_2\left(T,\sigma_\eta\right)+T\Omega_1\left(T,\sigma_\eta\right)\right]\\
=&\ sign\left[-\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\frac{\partial\Delta_v}{\partial\sigma_\eta}+1-\Delta_v\right]*\infty.
\end{aligned}$$

Now, explicitly calculating $-\sigma_\eta^3\left(\frac{1}{\sigma_v^2}+\frac{1}{\sigma_\eta^2}\right)\frac{\partial\ \ v}{\partial\sigma_\eta}+1-\Delta_v$, we find it is proportional to:

$$\frac{1}{\mathrm{var}\left[\tilde{v}|\tilde{y}\right]}-\frac{1}{\sigma_\varepsilon^2}-\frac{1}{\sigma_p^2}.$$

Part ii. We show that, when investors' information is not too precise, crowding in arises on a set of values of σ_η of positive measure. Recall from the proof of part ii in Proposition 3 that, as $\sigma_\varepsilon\to\infty$, the equilibrium threshold T exceeds the threshold in which there is no noise trade, $\hat{T}$. This result extends to the case in which there is a public signal $\tilde{y}$ because its derivation holds for any prior mean and variance parameters. That is, letting the equilibrium thresholds in the presence and absence of noise trade be $T\ \ (\tilde{y})$ and $\hat{T}(\tilde{y})$, respectively, we have $T\ \ (\tilde{y})>\hat{T}(\tilde{y})$, and thus $\Pr(\tilde{v}>T\ \ (\tilde{y})|\tilde{y})-\Pr\left(\tilde{v}>\hat{T}(\tilde{y})|\tilde{y}\right)<0$. To complete the proof, we show that:

$$\lim_{\sigma_\eta\ !\ 0}\left[\Pr(\tilde{v}>T\ \ (\tilde{y})|\tilde{y})-\Pr\left(\tilde{v}>\hat{T}(\tilde{y})|\tilde{y}\right)\right]=0.$$

This immediately implies that $\frac{\partial\Pr(v>T\ \ (y)jy)}{\partial\sigma_\eta}<0$ for σ_η in a set of positive measure.[35] From equation (55), we have:

$$\Pr(\tilde{v}>T(\tilde{y})|\tilde{y})=1-\Phi\left(\frac{T(0)}{\sqrt{\mathrm{var}\left[\tilde{v}|\tilde{y}\right]}}\right),$$

and thus:

$$\lim_{\sigma_\eta\ !\ 0}\left[\Pr(\tilde{v}>T\ \ (\tilde{y})|\tilde{y})-\Pr\left(\tilde{v}>\hat{T}(\tilde{y})|\tilde{y}\right)\right]=\lim_{\sigma_\eta\ !\ 0}\left[\Phi\left(\frac{\hat{T}(0)}{\sqrt{\mathrm{var}\left[\tilde{v}|\tilde{y}\right]}}\right)-\Phi\left(\frac{T\ \ (0)}{\sqrt{\mathrm{var}\left[\tilde{v}|\tilde{y}\right]}}\right)\right].$$

So, letting $t_n(\sigma_\eta)\equiv\frac{T\ \ (0)}{\sqrt{\mathrm{var}[vjy]}}$ and $\hat{t}_n\equiv\frac{\hat{T}(0)}{\sqrt{\mathrm{var}[vjy]}}$, we need to show that $\lim_{\sigma_\eta\ !\ 0}\left[t_n(\sigma_\eta)-\hat{t}_n\right]=0$. Note that, as $\sigma_\varepsilon\to\infty$, $P_U(\tilde{v},\tilde{z},0)\to\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{z\ \ \kappa}{\tau}$ and $\sigma_s^2\to\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}$. Now, let:

$$\gamma(t,\sigma_\eta)\equiv t+\mathbb{E}\left[\frac{p\phi\left(t-\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{z\ \ \kappa}{\tau}\right)}{p\Phi\left(t-\sqrt{\frac{\sigma_v^2\sigma_\eta^2}{\sigma_v^2+\sigma_\eta^2}}\frac{z\ \ \kappa}{\tau}\right)+1-p}\right]$$

denote the limit of the equilibrium condition when $\tilde{y}=0$ as $\sigma_\varepsilon\to\infty$, as a function of σ_η and the "normalized" threshold $t=\frac{T}{\sqrt{\mathrm{var}[vjy]}}$. Then, by definition, we have $\gamma(t_n(\sigma_\eta),\sigma_\eta)=0$ and

[35] To see why, let $q(T,\sigma_\eta)$ denote $\Pr(\tilde{v}>T^*(\tilde{y})|\tilde{y})$ as a function of T and σ_η. Suppose by contradiction that $\frac{\partial q(T^*,\sigma_\eta)}{\partial\sigma_\eta}>0$ a.e. Then, fixing an $x>0$, we have $q(T^*,x)-q\left(\hat{T},x\right)=\delta_x<0$. Thus, $\forall x'\in(0,x)$, $q(T^*,x')-q\left(\hat{T},x'\right)<\delta_x$. This contradicts the fact that $\lim_{\sigma_\eta\to0}q(T^*,\sigma_\eta)=\lim_{\sigma_\eta\to0}q\left(\hat{T},\sigma_\eta\right)$.

$\gamma\left(\hat{t}_n, 0\right) = 0$. So, we need only show that the solution $x(\sigma_\eta)$ to $\gamma\left(x(\sigma_\eta), \sigma_\eta\right) = 0$ is continuous at 0. As Lemma A.4 shows that the equilibrium condition increases in the threshold, we have $\frac{\partial \gamma(t,\sigma_\eta)}{\partial t} > 0$. Thus, the implicit function theorem yields that $x\left(\sigma_\eta\right)$ is continuous.

□

B Extension: Post-Disclosure Public Signal

When studying the impact of public information on voluntary disclosure in Section 5, we assume that the public signal arrives prior to disclosure, and is thus observable by the manager. In some contexts, managers may not be able to predict the outcome of public information.[36] Moreover, disclosure regulation likely influences both the amount of existing public information and the amount of public information that is expected to arrive in the future.

To address this issue, in this appendix, we extend our analysis to incorporate a public signal that arrives after the disclosure decision. Suppose now that the firm releases a signal both before (ex-ante) and after (ex-post) the disclosure:

$$\tilde{y}_a = \tilde{v} + \tilde{\eta}_a; \quad \tilde{y}_p = \tilde{v} + \tilde{\eta}_p,$$

respectively, where $\tilde{\eta}_a \sim N(0, \sigma^2_{\eta,a})$ and $\tilde{\eta}_p \sim N(0, \sigma^2_{\eta,p})$ are independent of all other random variables in the model. The key distinction between these signals is that $\tilde{y}_a$ is observable to the manager when disclosing, while $\tilde{y}_p$ is not. In this sense, $\tilde{y}_p$ acts similarly to investors' private information in our model. Thus, the derivation of equilibrium is a straightforward extension of our main analysis, and we summarize the results below.

Proposition B.1. *Suppose that either $p = 1$ and/or $\frac{1}{\sigma^2_{\eta,p}} + \frac{1}{\sigma^2_\varepsilon} + \frac{1}{\sigma^2_p}$ is sufficiently small, and fix a realization of $\tilde{y}_a = y_a$. Then, there exists a unique equilibrium in which the manager discloses if and only if $\tilde{v} \geq T(y_a)$. In this equilibrium, the firm's non-disclosure price takes the same form as in Proposition 4, upon re-defining:*

$$\tilde{\mu}_i = \mathbb{E}\left[\tilde{v} | y_a, \tilde{y}_p, \tilde{s}_i, \tilde{s}_p\right]; \quad \sigma^2_s = \textit{var}\left[\tilde{v} | y_a, \tilde{y}_p, \tilde{s}_i, \tilde{s}_p\right].$$

[36] See Acharya et al. (2011) and Frenkel et al. (2020) for analyses of post-disclosure public information in other settings.

Moreover, the equilibrium disclosure threshold satis es:

$$T(y_a) - c = \mathbb{E}\left[P_{ND} | \tilde{v} = T(y_a), \tilde{y} = y_a\right], \tag{58}$$

where the expectation is taken over $\tilde{s}_p$ *and* $\tilde{y}_p$.

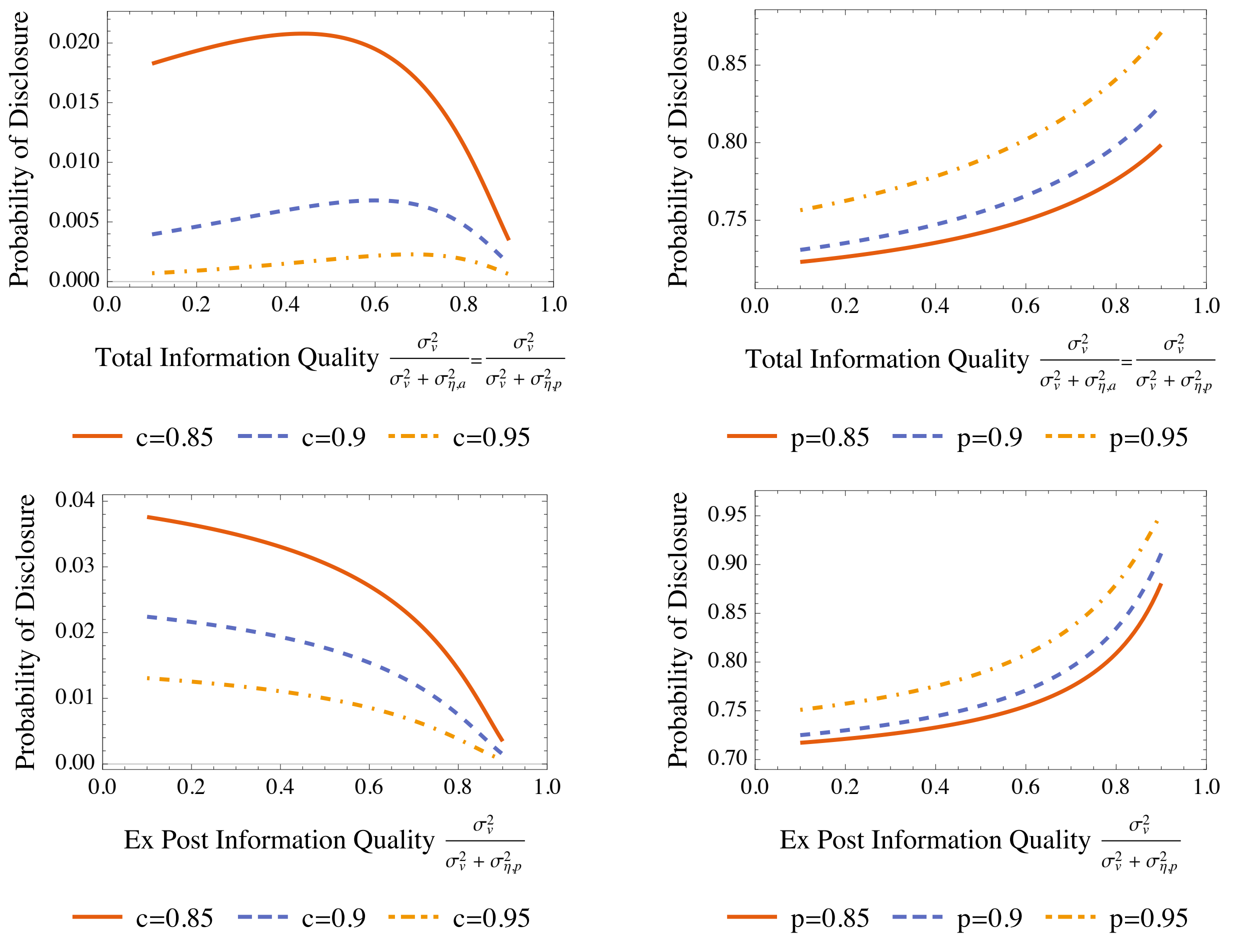
Proof of Proposition B.1. The proof is a straightforward extension of the main analysis in our paper upon adding $\tilde{y}_p$ to investors' conditioning set. The only step that materially differs is the derivation of Lemma A.3, which establishes sufficient conditions on when the manager is more inclined to disclose as the firm's value increases. The reason is that the ex-post public signal raises the sensitivity of the non-disclosure price to the firm's value. It can be verified that the sensitivity of the price to v (conditional on the public signal $\tilde{y}_a$, which is a known constant) is now $\sigma_s^2 \left(\frac{1}{\sigma_{\eta,p}^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)$, as opposed to $\sigma_s^2 \left(\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}\right)$. Thus, the analogous argument to the one in the proof of Lemma A.3 shows that the appropriate sufficient conditions are now that $p = 1$ or $\frac{1}{\sigma_{\eta,p}^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_p^2}$ sufficiently small. □

While an analytical treatment is not tractable, we next numerically study the probability of voluntary disclosure in this setting. We conduct two analyses. First, we consider how varying only ex-post information quality affects voluntary disclosure. We then consider how simultaneously varying the quality of both ex-ante and ex-post information quality affects voluntary disclosure. The goal of the latter analysis is to provide insight into the effects of persistent differences in disclosure quality, such as those driven by disclosure mandates.

Figure B.1 depicts the results. Observe first that, in the probabilistic information benchmark, ex-post public information raises the likelihood of voluntary disclosure. This is consistent with the findings in the main text. On the other hand, in the costly disclosure benchmark, ex-post information quality lowers the likelihood of voluntary disclosure. However, when jointly varying ex-ante and ex-post information quality, public information may again crowd in voluntary disclosure when c is large.

Figure B.1: Probability of Disclosure vs. Information Quality (Ex-Post Signal)

The figure plots the probability of disclosure as a function of the amount of public information σ_η. The upper plots simultaneously vary the quality of both ex-ante and ex-post information, while the lower plots vary the quality of ex-post information only. Other parameters are set to $\kappa = 0$, $\tau = 1$, and $\sigma_v = 2$. In the lower left (right) plot, $\sigma_{\eta,a}$ is set to 1 (2).



(a) Costly Disclosure Benchmark ($p = 1$, $\sigma_\varepsilon = \sigma_z = 1$)

(b) Probabilistic Information Benchmark ($c = 0$, $\sigma_\varepsilon = \sigma_z = 2$)