Psychological Distance and Deviations from Rational Expectations*

Harjoat S. Bhamra Raman Uppal Johan Walden

December 12, 2021

Abstract

Empirical evidence shows that households' beliefs deviate from rational expectations. Combining concepts from psychology and robust control, we develop a model where the deviations of beliefs about stock returns from rational expectations are an *endogenous* outcome of household-firm psychological distance, which encompasses temporal, spatial, and social distance. To make the model testable, we establish the relation between unobservable beliefs and observable portfolio choices. We use portfolio holdings for 405,628 Finnish households and 125 firms to show that household-firm spatial distance has a significant distortionary effect on beliefs and welfare, which leads to substantial inequality across households.

Keywords: behavioral finance, construal level theory, inequality, belief heterogeneity, household finance.
JEL: D84, E03, G11, G41, G5.

*Harjoat Bhamra is affiliated with Imperial College Business School and CEPR; Email: h.bhamra@imperial.ac.uk. Raman Uppal is affiliated with Edhec Business School and CEPR; Email: raman.uppal@edhec.edu. Johan Walden is affiliated with Haas School of Business, University of California, Berkeley; Email: walden@haas.berkeley.edu. We thank David Sraer for valuable suggestions, and Nicholas Bel and Zhimin Chen for research assistance. We are grateful for comments from Hossein Asgharian, Claes Bäckman, Olga Balakina, Peter Bossaerts, Markus Brunnermeier, Adrian Buss, Veronika Czellar, Victor DeMiguel, Peter Drake, Lorenzo Garlappi, Nicola Gennaioli, Lars Hansen, Winifred Huang, Abraham Lioui, Kaustia Markku, Massimo Massa, Ernst Maug, David Newton, Kim Peijnenburg, Joël Peress, Piet Sercu, Sophie Shive, David Thesmar, Fabio Trojani, Argyris Tsiaras, Laura Veldkamp, Michael Weber, Ania Zalewska, Luigi Zingales, and seminar participants at BI Norwegian Business School, Bocconi University, EDHEC Business School, Florida State University, INSEAD, Lund University, Mannheim University, Tilburg University, University of Bath, University of Cambridge, University of Miami, University of Geneva, 6th Annual Conference on Network Science in Economics Conference, 8th HEC-McGill Winter Finance Workshop, American Finance Association Meeting, Australasian Finance and Banking Conference, European Economic Association Meeting, FMA Annual Meeting, Northern Finance Association Meeting, and RCFS/RAPS Winter Conference.

1 Introduction and Motivation

Since the work of Muth (1961), and especially Lucas (1976), rational expectations has been the workhorse assumption in both macroeconomics and finance. However, survey expectations of stock returns are inconsistent with rational return expectations (Greenwood and Shleifer, 2014, Gennaioli and Shleifer, 2018, Adam, Matveev, and Nagel, 2021), "data on households' expectations about future macroeconomic outcomes reveal significant systematic biases" (Bhandari, Borovička, and Ho, 2019), and in experiments "rational expectations are strongly rejected" (Afrouzi, Kwon, Landier, Ma, and Thesmar, 2021). The overwhelming evidence against rational expectations necessitates taking a stance on how to deviate from rational expectations, while still imposing rigor on the manner in which beliefs are formed, with the importance of developing an empirically grounded model of belief formation highlighted in Brunnermeier et al. (2021). In this paper, we combine ideas from psychology and robust control to develop a theoretical framework for deviating from rational expectations in a disciplined fashion.[1] Then, using Finnish data on the location of firms and the portfolio holdings of households, we establish strong empirical support for our theoretical framework, which shows how psychological distance affects household beliefs about stock returns, and through that, household welfare.

In social psychology, *construal level theory* (Trope and Liberman, 2010, page 440) describes how the psychological distance between an individual and an object affects her decision making.

> "People directly experience only the here and now. It is impossible to experience the past and the future, other places, other people, and alternatives to reality. And yet, memories, plans, predictions, hopes, and counterfactual alternatives populate our minds, influence our emotions, and guide our choice and action. . . . How do we plan for the distant future, understand other people's point of view, and take into account hypothetical alternatives to reality? Construal level theory (CLT) proposes that we do so by forming abstract mental construals of distal objects. . . . Psychological distance is a subjective experience that something is close or far away from the self, here, and now. Its reference point is the self, here and now, and the different ways in which an object might be removed from that point—in time, space, social distance, and hypotheticality—constitute different distance dimensions."

[1] Excellent surveys of psychological biases of investors are provided by Shleifer (2000), Barberis and Thaler (2003), Hirshleifer (2001, 2015), Shefrin (2007, 2010), Statman (2010, 2011), and Barberis (2018). For other models based on psychological evidence, see Barberis, Shleifer, and Vishny (1998), Barberis (2011), and Barberis, Jin, and Wang (2021).

Construal plays a crucial role in situations where the decision maker is obliged to venture beyond the information immediately provided by the direct observation of an event. In the context of financial decision making, we assume that when a household forms its beliefs, it does so with an inbuilt sense of caution that leads it to deviate from rational expectations. Just as in the theory of robust control (Hansen and Sargent, 2001, 2008), the household faces a penalty for deviating from the rational-expectations benchmark. However, in contrast to the robust-control literature, where the penalty depends on the Kullback and Leibler (1951) divergence, we introduce a novel penalty that depends on the psychological distances of a household from each firm in the economy. Consequently, differences in psychological distances of households from firms lead to belief heterogeneity across households.[2]

We use our framework to derive the endogenous deviation of each household's beliefs from rational expectations. We show that the beliefs of household h, denoted by $\mathbb{P}^h$, are bounded between two probability measures: the rational beliefs under which the risk premia on stocks are computed using the objective physical probability measure $\mathbb{P}$, and the *risk-neutral* beliefs, computed under the risk-neutral measure $\mathbb{Q}$, where the perceived risk premia are zero. We then establish the relation between the unobservable beliefs of a household and its observable portfolio choice so that the model can be taken to the data. Finally, we prove that the welfare loss to households in deviating from rational expectations is directly proportional to the relative entropy between their distorted beliefs and the rational beliefs.

In our empirical tests, we focus on spatial distance as the measure of psychological distance between investors and firms. Indeed, spatial distance, together with temporal, social, and hypothetical distance, are the most commonly studied ingredients of psychological distance. We argue that spatial distance was important during the time period we study (1990s). As discussed in Walden (2019), Finland's population density is one of the lowest in the European Union, at seventeen people per square kilometer, a majority of the population resided in rural areas, and internet penetration was low (about one-third of the population in 2000). It is therefore plausible that spatial distance would have been an important psychological distance dimension at that point in time.

We use the portfolio holdings of 405,628 Finnish households living in 2,923 postal codes to infer the effect of spatial distance on household beliefs about 125 stock returns. We find that spatial distance between households and firms has a statistically and economically significant effect on the beliefs of households about firms' stock returns. In particular, the sensitivity coefficient representing spatial distance is highly statistically significant in influencing beliefs in all

[2] Meeuwis, Parker, Schoar, and Simester (2018) and Giglio, Maggiori, Stroebel, and Utkus (2021) provide empirical evidence of substantial heterogeneity in beliefs across households that is linked to portfolio decisions.

regressions—univariate regressions, regressions including risk-aversion fixed effects, regressions including risk-aversion and stock-characteristic fixed effects, and panel regressions with robust standard errors double clustered at the firm and postal code level. Our results are also *economically* significant: a one standard deviation decrease in distance to a firm's headquarters predicts an effect on beliefs that more than doubles the wealth invested in that firm.

We also calculate the loss in mean-variance welfare resulting from the deviation of beliefs from rational expectations and show how this varies depending on the location of households. We show that the welfare loss can be expressed in the same units as an annualized Sharpe ratio. If households had rational expectations, this measure of welfare loss would be zero, while the maximum value it can reach is equal to the Sharpe ratio for the Finnish stock market, which, during the period of our study, is about 0.59 per year. We find that the smallest value of the welfare loss when measured as a Sharpe ratio is 0.25 per year, which is for households in the Helsinki region. It is striking that even for households close to Helsinki, where a majority of the firms are headquartered, the value is so much greater than 0. As one moves away from Helsinki, the welfare loss increases and is about 0.57 per year for households farther away from Helsinki, which is almost equal to the Sharpe ratio of the Finnish market portfolio. Therefore, for these households, beliefs under the subjective probability measure $\mathbb{P}^h$ almost coincide with the risk-neutral measure $\mathbb{Q}$.

A possible concern with these empirical results is that they may be driven by households in Helsinki, the main conurbation in Finland, and there may be differences in behavior between rural and urban households. To check if this is indeed the case, we run regressions *excluding* stocks and postal codes in the Helsinki area and find that the results are still both statistically and economically highly significant. Another potential concern is that it may not be spatial distance per se that drives beliefs, but rather employment; that is, households may tend to invest in the firms they work for, which are also likely to be close to where they live.[3] To investigate if this is the case, we *exclude* observations for which the household and firm headquarters are close to each other. We consider three cutoffs: 8 miles (13 km), 24 miles (39 km), and 40 miles (64 km). We find that the results remain qualitatively similar, and thus, an "employment" effect does not seem to be driving the results.

Our paper is related to the literature on robust decision making in finance and economics, which is described in Hansen and Sargent (2008). The key idea we take from this literature is that decision makers are uncertain about the benchmark model they use to make decisions, and consequently, they consider a range of models around the benchmark; however, there is a

[3]See Cohen (2009) for empirical evidence on how loyalty to a company can influence portfolio choice.

penalty for deviating from the benchmark model. Trojani and Vanini (2004) study the implications of model uncertainty for portfolio choice in continuous-time economies with heterogeneous investors. Gagliardini, Porchia, and Trojani (2008) provide an application of robust decision making to study the implications for the term structure of interest rates. Bhandari et al. (2019) develop a model where agents' subjective beliefs are endogenous consequences of model misspecification. Chen, Hansen, and Hansen (2020) show how one can obtain information about investor beliefs based on data and asset-pricing models using moment conditions on the stochastic discount factor. Our approach extends the robust control literature by using a penalty that depends on the psychological distance of a household relative to firms.

Our work also contributes to the empirical literature on deviations from rational expectations by providing a unifying framework for belief distortions that is micro-founded and based on psychological distance.[4] The following empirical papers in financial economics can be interpreted as providing evidence on how individuals' decisions are influenced by the different dimensions of psychological distance: spatial distance (French and Poterba, 1991, Coval and Moskovitz, 1999a,b, Huberman, 2001, Grinblatt and Keloharju, 2001, Kuchler and Zafar, 2018), cultural and linguistic distance (Grinblatt and Keloharju, 2001, Guiso, Sapienza, and Zingales, 2006, Laudenbach, Malmendier, and Niessen-Ruenzi, 2018, Badarinza, Ramadorai, and Shimizu, 2019, D'Acunto, Prokopczuk, and Weber, 2019), temporal distance (Kaustia and Knüpfer, 2008, 2012, Greenwood and Nagel, 2009, Chiang, Hirshleifer, Qian, and Sherman, 2011, Malmendier and Nagel, 2011, 2015, Knüpfer, Rantapuska, and Sarvimäki, 2017), and social distance (Shive, 2010, Das, Kuhnen, and Nagel, 2017, Bailey, Cao, Kuchler, and Stroebel, 2018, Hirshleifer, 2020, Pedersen, 2021).

Our focus on estimating beliefs about expected stock returns for 125 individual firms distinguishes our work from that of Giglio et al. (2021), who use survey data and focus on beliefs about aggregate equity-market returns as opposed to individual stock returns. Our use of data on the portfolio holdings of Finnish households to study their beliefs also distinguishes our work from experimental studies of beliefs, which have been criticized on the grounds that in experiments the stakes are low, the subjects typically have limited experience with the experimental setting, and there is little incentive to pay attention to details of the experiment.

A separate strand of literature has focused on an information channel, rather than biased expectations, for explaining individual behavior, see, e.g., Van Nieuwerburgh and Veldkamp (2009). In such a framework, risk averse agents associate higher risk with firms they have less

[4]For reviews of the literature on modeling beliefs that deviate from rational expectations, see Camerer (1995, 1998), and Manski (2004, 2018). For the literature that discusses how information rigidities affect expectations, see, for instance, Coibion and Gorodnichenko (2012, 2015).

information about (e.g., in a Bayesian framework), and therefore invest less in such firms. But, as we show formally in equation (23) below, there are three distinct differences between the portfolios predicted by an information-based model and our model. First, an information-based model would predict an increase in the portfolio weight if the investor's information leads to either an increase in the expected returns or a decrease in stock-return volatility; in contrast, in our model, psychological distance leads only to a decrease in the portfolio weight toward zero for assets with a positive expected risk premium. Two, in an information-based model, if the information about a particular stock's mean return is negative, it can lead to a short position, but in our model, investor's never take a short position in an asset whose true expected risk premium is positive. Three, in an information-based model, there will be non-zero demand for all risky assets, while out model leads to sparse portfolio holdings with zero holdings in stocks with firm headquarters located beyond a threshold distance. Empirically, our data supports all three predictions of the model based on psychological deviations in beliefs.[5]

The rest of this paper is organized as follows. In Section 2, we describe the main features of our model and describe the choice problem of an individual household whose beliefs are affected by psychological distance. In Section 3, we present our main theoretical results about the effect of psychological distance on deviation of beliefs from rational expectations and establish the link between the beliefs, portfolio decision, and welfare of a household. In Section 4, we evaluate the predictions of the model empirically. In Section 5, we measure the welfare costs of deviating from rational expectations and show how they are related to spatial distance. We conclude in Section 6. Proofs for all results, theoretical results for the model extended to Epstein and Zin (1989) preferences, and additional empirical tests are reported in the appendices.

2 The Model

The principal objective of this section is to develop a parsimonious framework that links psychological distance to belief deviations from rational expectations. We consider a dynamic continuous-time model of a finite number of households and firms. The first part of this section describes our model for stock returns and the investment opportunities of households. The second part describes the effects of psychological distance on household decisions. The third part explains how households form their subjective beliefs as distortions of rational beliefs. We will use this framework in the next section to derive the relation between the deviation of house-

[5]Also theoretically, within our continuous-time framework, there is no role for a volatility channel, because individuals must agree on all second moments of asset returns (otherwise there would be arbitrage opportunities).

hold beliefs from rational expectations, portfolio weights under these distorted beliefs, and the resulting welfare loss.

2.1 Stock Returns and the Investment Opportunities of Households

We assume there are N firms indexed by $n \in \{1, \ldots, N\}$, whose stock returns are given by

$$dR_{n,t} = \alpha_n \, dt + \sum_{k=1}^{N} \sigma_{n,k} dZ_{k,t},$$

where α_n is the expected rate of return on firm n, $Z_{k,t}$ is a standard Brownian motion under the reference probability measure $\mathbb{P}$ that represents rational beliefs, and $\sigma_{n,k}$ is the loading of stock return n on the k'th Brownian motion. Observe that the N Brownian motions are pairwise orthogonal, that is, $E_t^{\mathbb{P}}[dZ_{k,t} dZ_{k',t}] = 0$ for $k' \neq k$. The volatility of the nth stock return is given by $\sigma_n = \sqrt{\boldsymbol{\sigma}_n^\top \boldsymbol{\sigma}_n}$, where $\boldsymbol{\sigma}_n = (\sigma_{n,1}, \ldots, \sigma_{n,N})^\top$ is the $N \times 1$ column vector of volatility loadings for stock n. The correlation between returns on stocks n and m is given by $\rho_{nm} = \frac{\boldsymbol{\sigma}_n^\top \boldsymbol{\sigma}_m}{\sigma_n \sigma_m}$. The parameters α_n, σ_n, and ρ_{nm} are constant over time. The $N \times N$ variance-covariance matrix of stock returns is given by $V = [V_{nm}]$, where

$$V_{nm} = \begin{cases} \sigma_n^2, & n = m \\ \rho_{nm} \sigma_n \sigma_m, & n \neq m. \end{cases}$$

There are H households, indexed by $h \in \{1, \ldots, H\}$. Households can invest their wealth in a risk-free asset that has an interest rate i, which is constant. Additionally, households can invest their wealth in the N risky stocks. Denoting the proportion of a household's wealth invested in stock n by ω_{hn}, the portfolio return for household h is given by

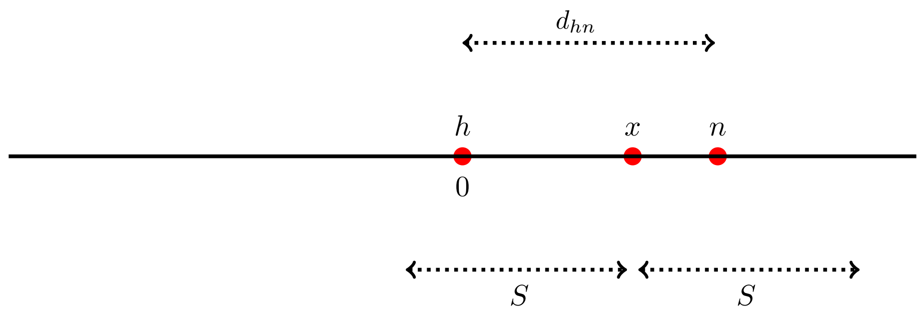
$$dR_{h,t} = \left(1 - \sum_{n=1}^{N} \omega_{hn,t}\right) i \, dt + \sum_{n=1}^{N} \omega_{hn,t} \, dR_{n,t}. \tag{1}$$

2.2 Psychological Distance and Trust

It is well-documented in the psychology literature that people's actions and decisions are influenced by the psychological distance between them and the object they are making a decision about. Specifically, we build on Construal Level Theory (Trope and Liberman, 2010). When making choices and taking actions about objects that are not immediately here and now, people form abstract mental construals, and the more distal the object, the more abstract the construal. Trope and Lieberman identify four main sources of psychological distance: spatial, temporal, social, and hypothetical (the likeliness of an event). Other potential sources of psychological

Figure 1: Trust region of a household

The figure shows that each household has a trust region of length $2S$ centered on x, where S is a random variable that is identically and independently distributed across households with maximum size $\overline{d}$. The psychological distance of household h from firm n is d_{hn}.



distance have also been discussed in the literature, for example, experiential and informational distance. Formally, we introduce a psychological distance function $d(x_{\mathcal{O}}, x_A)$, which describes the psychological distance between an observer located at $x_{\mathcal{O}}$ and an object located at x_A, which is nondecreasing in the aforementioned sources of psychological distance.

Psychological distance has been shown to affect people's decisions in several ways. For example, Hartzmark, Hirshman, and Imas (2021) find that owning a good affects beliefs about its quality. Darke, Brady, Benedicktus, and Wilson (2016) find that trust is affected by psychological distance; specifically, consumers' trust in retailers decrease as the psychological distances to these retailers increase. That trust in an important component for people's decisions is also well-known in the economics literature; for example, Guiso, Sapienza, and Zingales (2008) show that the level of trust in the stock market influences an investor's decision to invest in the market. Li, Turmunkh, and Wakker (2019) provide experimental evidence that a subject's ambiguity aversion influences the amount of trust the subject places in their counterparties, with more ambiguity averse subjects expressing less trust.

These results suggest a mechanism through which psychological distance impacts agents' financial decisions, for instance in deciding how much to invest in a company's stock. Consider an agent who is choosing how much to invest in the stocks of companies A and B. Company A is in near psychological proximity to the agent, that is, $d(x_{\mathcal{O}}, x_A)$ is small, whereas company B is psychologically distant, that is, $d(x_{\mathcal{O}}, x_B)$ is large. As a consequence, the agent puts less trust in company B than A.

To quantify the link between psychological distance, trust and, ultimately, beliefs, we start by defining the trust region of a household, using an approach similar to that in Shepard (1987).

Definition 1. *A* trust region *is an interval of length* $2S$ *centered at some unknown point* x *(see Figure 1), where* S *is a random variable with maximum size* $\overline{d}$*, with a known mean* $\mu = E[S]$ *but with unknown distribution.*

The household must decide how much trust to place in measurements about a firm, which is located at psychological distance d from itself. We define the level of trust to be placed in such a firm as the probability that it lies in the same trust region as the household.

Definition 2. *The level of trust household* h *places in measurements about firm* n *is the probability that the firm lies in the household's trust region,*

$$\phi_{hn} = \Pr(d_{hn} \leq S). \tag{2}$$

It is clear from the above definition that when a household h's psychological distance from the trust region's center is zero, then $\phi_{hn} = 1$ and that this is the maximum possible level of trust.

To compute the probability in (2), we assume that households begin with the least-informative prior according to the Principle of Maximum Entropy (Jaynes, 1957, 1986). The prior is then updated using Bayes' Law, as in Shepard (1987). Using the resulting posterior, we compute the probability in (2) to obtain the expression for trust, which is given in the proposition below. The argument underlying this proposition is completely general, as long as psychological distance can be defined by a metric function on a finite-dimensional vector space.

Proposition 1. *The trust that a household* h *places in a firm that is at psychological distance* d_{hn} *is well approximated by the function*

$$\phi_{hn} = e^{-\kappa d_{hn}} I_{d_{hn} \leq \bar{d}}, \tag{3}$$

where, $\kappa > 0$ *is a constant governing how quickly trust decreases in psychological distance and* $\overline{d}$ *is the maximum size of a household's trust region.*

In our theoretical model and empirical work, we will consider the beliefs of households $h \in \{1, \ldots, H\}$ and firms $n \in \{1, \ldots, N\}$, which leads to $H \times N$ trust parameters, $\phi_{hn} = e^{-\kappa d_{hn}} I_{d_{hn} \leq \bar{d}}$, where d_{hn} is the psychological distance from household h to firm n. We can conveniently represent these parameters using the matrix $\Phi = [\phi_{hn}]_{hn} \in \mathbb{R}_{+}^{H \times N}$. We note that (3) provides an empirically tractable specification for a set of households and firms.

As discussed in the introduction, in our empirical work we will focus on spatial distance as the key dimension of psychological distance. The psychological distance from a household located at coordinates (x_h, y_h) to a firm located at (x^n, y^n) is in this context defined in terms of the Euclidean distance

$$d_{hn} = \sqrt{(x_h - x^n)^2 + (y_h - y^n)^2}.$$

But, our framework is sufficiently flexible to allow for more general distance functions. For instance, social distance measures can easily be incorporated into a more general psychological distance, e.g., by the difference between the predominant language of the household and the CEO of the firm, then one could generalize the distance function to include this. For instance, in a bilingual region, one could represent the predominant language of the household h and firm n's CEO by $\ell_h \in \{0, 1\}$, respectively, and the set $\{0, 1\}$ represents the two spoken languages in the region. Then the generalized distance function could take the form

$$d_{hn} = \sqrt{(x_h - x^n)^2 + (y_h - y^n)^2} + \text{constant}|\ell_h - \ell^n|,$$

where the first term on the right-hand side represents spatial distance and the second term social distance.

2.3 Beliefs of Households

The less trust a household h puts in a company n (i.e., the lower is ϕ_{hn}), the more cautious it will be, and therefore the more willing it will be to revise its beliefs about the prospects of the firm in a conservative direction. Within a portfolio choice setting, there are of course multiple forces at play that impact the costs and benefits of such belief revisions, for example the covariance structure of firms' returns, that in turn determine the benefits of portfolio diversification. Our next objective is to solve for these effects jointly, and derive an agent's belief revisions as the optimal response to these tradeoffs, that is, to formally model how Φ affects households' beliefs.

Each household h has its own beliefs—for the time being taken as given—about expected returns on the N stocks, represented by a personal distorted probability measure $\mathbb{P}^h$ that differs from the physical (objective) probability measure $\mathbb{P}$, which represents the rational-expectations benchmark. Below, we define the beliefs of household h relative to the rational expectations benchmark. We do so by using an exponential martingale $M_{h,t}$, which distorts the rational-expectations probabilities in order to furnish the household's subjective probabilities.

Definition 3. *If we consider an event A which can occur at time $T > t$, household h's personal subjective expectation that event A could occur, conditional on date-t information, is given by*

$$E_t^{\mathbb{P}^h}[I_A] = E_t^{\mathbb{P}}\left[\frac{M_{h,T}}{M_{h,t}} I_A\right], \tag{4}$$

where I_A is the indicator function associated with event A, and $M_{h,t}$ is an exponential martingale (the Radon-Nikodym derivative of $\mathbb{P}^h$ with respect to $\mathbb{P}$) defined by

$$\frac{dM_{h,t}}{M_{h,t}} = \boldsymbol{\nu}_{h,t}^\top \Sigma^{-1} d\boldsymbol{Z}_t, \tag{5}$$

where $\boldsymbol{Z}_t = (Z_{1,t}, \ldots, Z_{N,t})^\top$, Σ is the N by N volatility matrix such that $\Sigma_{nk} = \sigma_{n,k}$,[6] *and $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \ldots, \nu_{hN,t})^\top$ is the vector of divergences of household expectations from rational expectations. For the special case where the stock-return correlations are zero, (5) reduces to*

$$\frac{dM_{h,t}}{M_{h,t}} = \sum_{n=1}^{N} \frac{\nu_{hn,t}}{\sigma_n} dZ_{n,t}.$$

Observe that in Equation (4) multiplication by the exponential martingale changes the expectation of event A under the measure $\mathbb{P}$ to the expectation under household h's measure, $\mathbb{P}^h$. We show below that applying this definition to the stock return for firm n (instead of event A) changes the expected rate of return from α_n under the measure $\mathbb{P}$ to $\alpha_n - \nu_{hn,t}$ under household h's measure, $\mathbb{P}^h$, which is an application of Girsanov's Theorem.

Lemma 1. *The expected return for stock n under household h's measure, $\mathbb{P}^h$ is the expected return under rational expectations, $\alpha_n dt$ with a distortion $\nu_{hn} dt$:*

$$\begin{aligned} E_t^{\mathbb{P}^h}[dR_{n,t}] &= E_t^{\mathbb{P}}\left[dR_{n,t}\frac{M_{h,t+dt}}{M_{h,t}}\right] \\ &= E_t^{\mathbb{P}}[dR_{n,t}] + E_t^{\mathbb{P}}\left[dR_{n,t}\frac{dM_{h,t}}{M_{h,t}}\right] \\ &= (\alpha_n + \nu_{hn,t})dt. \end{aligned}$$

Therefore, we can summarize household h's beliefs relative to the rational expectations benchmark for the N stock returns via the vector $\boldsymbol{\nu}_{h,t} = (\nu_{h1,t}, \ldots, \nu_{hN,t})^\top$. In the next section, we study the divergence in beliefs $\boldsymbol{\nu}_{h,t}$ that depend on psychological distance and the impact of this on the portfolio weights, $\boldsymbol{\omega}_{h,t}$.

3 Endogenous Beliefs, Portfolio Weights, and Welfare

This section contains our main theoretical results. It is divided into three parts. First, we study how psychological distance influences the belief formation of a household; in particular, how beliefs deviate endogenously from rational expectations. Second, we examine the implications of these belief distortions for portfolio choice. We conclude by proving that the household's

[6] Note that $\Sigma\Sigma^\top = V$, where V is the variance-covariance matrix of returns.

welfare loss resulting from distorted beliefs can be expressed in terms of the relative entropy of distorted beliefs from rational beliefs.

3.1 Psychological Distance and the Deviation from Rational Expectations

In our framework, a household's beliefs are influenced by their caution and distrust toward objects that are psychologically distant and a penalty that is increasing in its deviations from rational expectations (under which the belief deviations are $\boldsymbol{\nu}_{h,t} = 0$). Households are assumed to have mean-variance preferences defined over portfolio returns. We choose mean-variance preferences to make our novel theoretical insights more transparent; it is straightforward to extend all our results to the case of Epstein and Zin (1989) preferences, as shown in Appendix C.

Specifically, in a situation with full trust, $\phi_{hn} = 1$ for all n, household h would solve the standard mean-variance problem

$$\max_{\boldsymbol{\omega}_h} E_t^{\mathbb{P}}[dR_{h,t}] - \frac{1}{2}\gamma_h \mathrm{Var}_t[dR_{h,t}], \tag{6}$$

where $dR_{h,t}$, given in (1), is the return on the household's portfolio. The first term in (6) is the expected portfolio return under the household's subjective beliefs, which, for the case with full trust, coincide with rational beliefs, $\mathbb{P}$. The second term is the penalty for portfolio risk, which depends on the household's risk aversion parameter, γ_h, and the variance of the portfolio return.[7]

When there is less than full trust by household h for some firm n, $\phi_{h,n} < 1$, the household trades off the benefits of choosing conservative beliefs against the losses associated with deviating from rational expectations. Specifically, household h faces the following max-min problem:

$$\max_{\boldsymbol{\omega}_h} \min_{\boldsymbol{\nu}_h} E_t^{\mathbb{P}^h}[dR_{h,t}] - \frac{1}{2}\gamma_h \mathrm{Var}_t[dR_{h,t}] + \frac{1}{\gamma_h} L_h dt, \tag{7}$$

The novel third term in (7) represents a penalty for deviating from the rational expectations benchmark. To specify the form of the penalty we note that we would like it to have the following properties. When the psychological distance of household h from a particular firm n is zero, and hence, there is full trust ($\phi_{hn} = 1$) we would like the penalty for deviating from rational expectations to be large. On the other hand, when the psychological distance of household h from a particular firm n is large and exceeds the threshold (i.e., $d_{hn} > \bar{d}$), then trust is zero ($\phi_{hn} = 0$) and so in this case we would like the cost of deviating from rational expectations to be small. In general, we would like the cost of deviating from rational expectations to increase as

[7] Note that in continuous time the change in probability measure impacts expected returns but not the variance of returns.

trust increases. Based on these considerations, we specify that the penalty takes the following form; in Appendix D we provide an alternative justification for this penalty function.

Definition 4. *The* loss function *for household h, $L_{h,t}$, is*

$$L_{h,t} = \frac{1}{2}\sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2}, \tag{8}$$

which can be expressed more compactly as

$$L_{h,t} = \frac{1}{2}\boldsymbol{\nu}_{h,t}^{\top} D_{\sigma}^{-1} F_h D_{\sigma}^{-1} \boldsymbol{\nu}_{h,t},$$

where $D_\sigma = diag(\sigma_1, \ldots, \sigma_N)$ is the $N \times N$ diagonal matrix of stock return volatilities and F_h is the $N \times N$ diagonal matrix $F_h = diag\left(\frac{\phi_{h1}}{1-\phi_{h1}}, \ldots, \frac{\phi_{hN}}{1-\phi_{hN}}\right), \quad \phi_{hn} \in [0,1], \quad n \in \{1, \ldots, N\}$.

Within the robust control context, the belief adjustment a household makes from $\mathbb{P}$ to $\mathbb{P}^h$ can be viewed as driven by the desire to be cautious when forming beliefs, and the loss function L represents the cost of deviating from the rational-expectation benchmark. Specifically, we can interpret the term $\nu_{hn,t}^2/(2\sigma_n^2)$ in (8) as a measure of the deviation (or divergence) of the household h's personal beliefs solely about firm n from the rational beliefs $\mathbb{P}$. The term $\frac{\phi_{hn}}{1-\phi_{hn}}$ in the loss function represents how much weight the household puts on this deviation when assessing how costly it is. If ϕ_{hn} is close to one, trust in measurements of expected returns is high, a great deal of weight is put on the deviation from rational expectations and the household's resulting deviation in beliefs is small. If ϕ_{hn} is close to zero, trust is low, the household is eager to be conservative, and the costs of deviation from rational expectations are downplayed, leading to potentially large deviations.

Our specification differs from Hansen and Sargent (2008) in not using the entropy of beliefs relative to rational expectations (i.e., the Kullback-Leibler divergence) as our penalty. Instead, the penalty in our framework is directly related to the trust a household places in measurements about firms—the less trust it has, the more it is willing to deviate from rational expectations.

We obtain the household's belief deviations from rational expectations by solving its max-min optimization problem. First, we solve for the minimization problem to obtain the belief deviations for a given portfolio. Then, using the expression for the chosen beliefs for a given portfolio, we solve the entire max-min problem to obtain the belief deviations and portfolio weights in terms of exogenous variables.

Using the fact that $E_t^{\mathbb{P}^h}[dR_t] = i + (\boldsymbol{\alpha} + \boldsymbol{\nu}_{h,t} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t}$, the max-min problem in (7) can be rewritten as

$$\max_{\boldsymbol{\omega}_{h,t}} \min_{\boldsymbol{\nu}_{h,t}} \quad i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{\gamma_h}{2}\boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} + \boldsymbol{\omega}_{h,t}^\top \boldsymbol{\nu}_{h,t} + \frac{1}{2\gamma_h}\boldsymbol{\nu}_{h,t}^\top D_\sigma^{-1} F_h D_\sigma^{-1} \boldsymbol{\nu}_{h,t}, \tag{9}$$

which is equivalent to

$$\begin{aligned} \max_{\boldsymbol{\omega}_{h,t}} \min_{\boldsymbol{\nu}_{h,t}} \quad & i + \sum_{n=1}^{N} (\alpha_n - i)\omega_{hn,t} - \frac{\gamma_h}{2} \sum_{n=1}^{N}\sum_{m=1}^{N} \omega_{hn,t} V_{nm} \omega_{hm,t} \\ & + \sum_{n=1}^{N} \nu_{hn,t}\omega_{hn,t} + \frac{1}{2\gamma_h} \sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2}. \end{aligned} \tag{10}$$

From (10) we see that at time t, a household selects its personal beliefs $\mathbb{P}^h$ by choosing the vector of personal divergences $\boldsymbol{\nu}_{h,t}$ that minimizes the sum of the deviation in the expected portfolio return relative to rational expectations and the cost of deviating from rational expectations,

$$\boldsymbol{\nu}_{h,t} = \arg\min_{\boldsymbol{\nu}_{h,t}} \left(\sum_{n=1}^{N} \nu_{hn,t}\omega_{hn,t} + \frac{1}{2\gamma_h} \sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2} \right). \tag{11}$$

When a household chooses its beliefs by choosing a vector of divergences $\boldsymbol{\nu}_{h,t}$, it does so in order to select a *reasonable worst-case* belief. The worst-case aspect of the belief is captured by the minimization with respect to $\boldsymbol{\nu}_{h,t}$ and the influence of the term $\sum_{n=1}^{N} \nu_{hn,t}\omega_{hn,t}$. For a long position in a given risky asset, that is, $\omega_{hn,t} > 0$, making $\nu_{hn,t}$ more negative reduces the household's expectations for portfolio returns. How unreasonable the beliefs are is captured by the term $\sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2}$, which is larger when beliefs deviate further from rational expectations. By optimizing this tradeoff, the household selects its beliefs, as shown in the lemma below.

Lemma 2. *A household's vector of deviations from rational expectations is given by*

$$\boldsymbol{\nu}_{h,t} = -\gamma_h (D_\sigma^{-1} F_h D_\sigma^{-1})^{-1} \boldsymbol{\omega}_{h,t}, \tag{12}$$

with each element being

$$\nu_{hn,t} = -\gamma_h \sigma_n^2 \frac{1-\phi_{hn}}{\phi_{hn}} \omega_{hn,t}, \tag{13}$$

and the household's max-min problem reduces to the following mean-variance portfolio maximization problem

$$\max_{\boldsymbol{\omega}_{h,t}} \quad i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma_h \boldsymbol{\omega}_{h,t}^\top (V + D_\sigma F_h^{-1} D_\sigma) \boldsymbol{\omega}_{h,t}. \tag{14}$$

We can now see how psychological distance leads to deviations from rational expectations. For example, household h will change the expected rate of return for firm n from α_n to $\alpha_n+\nu_{hn,t}$, thereby reducing the magnitude of the firm's expected risk premium ($\nu_{hn,t} \leq 0$ if $\omega_{hn,t} > 0$ and $\nu_{hn,t} \geq 0$ if $\omega_{hn,t} < 0$). Thus, when the psychological distance between a household and firm, d_{hn}, is larger, trust ϕ_{hn} is smaller and the household diverges further from rational expectations. Thus, differences in psychological distance d_{hn} across households lead them to use different estimates of expected returns in their decision making.

Solving the mean-variance portfolio optimization problem in (14) yields the optimal portfolio vector under distorted beliefs

$$\boldsymbol{\omega}_h = \frac{1}{\gamma_h}(V + D_\sigma F_h^{-1} D_\sigma)^{-1}(\boldsymbol{\alpha} - i\mathbf{1}), \tag{15}$$

which when substituted into (13) gives the chosen vector of deviations from rational expectations in terms of exogenous variables, as shown in the following proposition.

Proposition 2. *The chosen vector of deviations from rational expectations is*

$$\boldsymbol{\nu}_h = -(I + V D_\sigma^{-1} F_h D_\sigma^{-1})^{-1}(\boldsymbol{\alpha} - i\mathbf{1}). \tag{16}$$

For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, the chosen deviations in household h's beliefs about the expected returns for firm n is

$$\nu_{hn} = -(\alpha_n - i)(1 - \phi_{hn}), \tag{17}$$

which can be expressed in terms of the household's psychological distance from the firm:

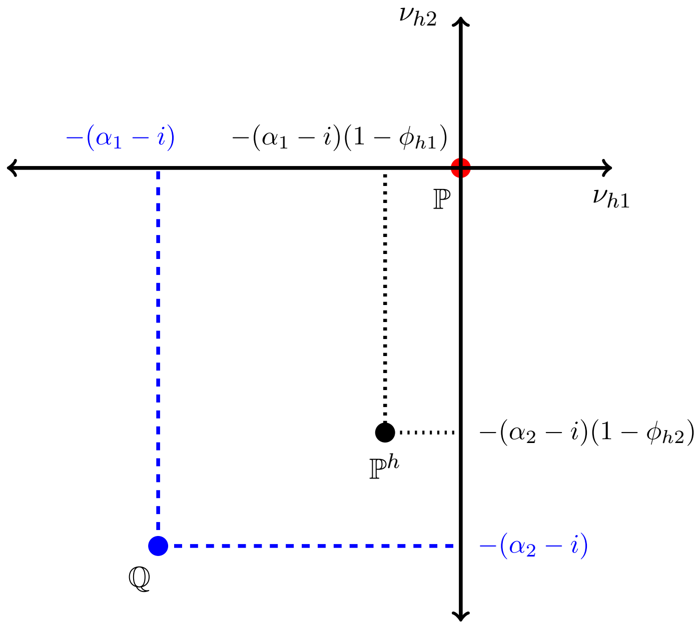
$$\nu_{hn} = \begin{cases} -(\alpha_n - i)(1 - e^{-\kappa d_{hn}}), & d_{hn} \leq \overline{d} \\ -(\alpha_n - i), & d_{hn} > \overline{d}. \end{cases} \tag{18}$$

From (17), we can see that the size of a household's deviation in a firm's expected return is smaller when trust, ϕ_{hn}, is higher; if $\phi_{hn} = 1$, then the deviation vanishes altogether and the household holds the portfolio weight that would be optimal under rational expectations. Figure 2 illustrates the beliefs given by the expression in (17).

Equation (18) shows how the divergence in beliefs from rational expectations increases in magnitude with psychological distance. In particular, beliefs diverge rapidly from rational expectations because of the exponential-decay function. Household h's beliefs about the expected return for firm n are given by $\alpha_n + \nu_{hn}$. In the case where $d_{hn} > \overline{d}$ for a particular firm n, the household's perceived expected rate of return about stock n equals the risk-free rate; that is, $\alpha_n + \nu_{hn} = \alpha_n - (\alpha_n - i) = i$, and therefore, the household does not hold this stock in its portfolio. In the extreme case where the psychological distance for a household h exceeds the

Figure 2: Visualizing probability measures

In this figure, we show how to visualize different probability measures, and hence, beliefs. In particular, we show the deviations from rational expectations possible in our framework. The figure is drawn for the case of two risky assets whose returns are uncorrelated. Under rational expectations, denoted by $\mathbb{P}$, the divergence in beliefs is $(\nu_{h1}, \nu_{h2}) = (0, 0)$, thereby placing the rational expectations beliefs at the origin. The maximum possible deviation from rational expectations is when $(\nu_{h1}, \nu_{h2}) = (-(\alpha_1 - i), -(\alpha_2 - i))$, implying that the subjective risk premium is 0 for both assets, in which case the subjective probability measure coincides with the risk-neutral probability measure, $\mathbb{Q}$. In general, the subjective probability measure, $\mathbb{P}^h$, lies in the rectangle defined by the points $(0, 0)$, $(-(\alpha_1 - i), 0)$, $(-(\alpha_1 - i), -(\alpha_2 - i))$ and $(0, -(\alpha_2 - i))$. The precise location of $\mathbb{P}^h$ will depend on its psychological distances from the two firms, d_{h1} and d_{h2}.



threshold distance for every firm, that is, $d_{hn} > \overline{d}$ for all n, the household does not participate in the stock market at all. In this case, its subjective probability measure $\mathbb{P}^h$ coincides with the risk-neutral probability measure $\mathbb{Q}$, implying that households perceive all expected risk premia are zero. If $d_{hn} \leq \overline{d}$, the perceived expected risk premium for a given stock n halves every $\frac{\ln 2}{\kappa}$ units of psychological distance. Therefore, the impact of psychological distance on expected risk premia is extremely strong, geometric rather than arithmetic. The same effect holds for Sharpe ratios.

3.2 Portfolio Weights when Beliefs Diverge from Rational Expectations

In the section above, we have explained how psychological distance influences the beliefs of a household. However, Equation (17) is given in terms of a household's trust in each firm, ϕ_{hn}, which is unobservable; in particular, we do not know the sensitivity parameter κ. We now show in the proposition below how ϕ_{hn} can be related to portfolio weights that, in contrast to beliefs, are observable. If variation in portfolio weights is driven by variation in psychological distances, then we will be able to use data on portfolio weights to accurately estimate κ.

Proposition 3. *The vector of chosen portfolio weights is*

$$\begin{aligned} \boldsymbol{\omega}_h &= \frac{1}{\gamma_h} V^{-1}\big(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h\big), \\ &= \frac{1}{\gamma_h}(V + \Sigma^2 F_h^{-1})^{-1}\big(\boldsymbol{\alpha} - i\mathbf{1}\big), \end{aligned} \tag{19}$$

where $\boldsymbol{\nu}_h$ is the vector of deviations in beliefs from rational expectations, given in (16).

For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, the optimal proportion of wealth invested by household h in firm n is

$$\omega_{hn} = \frac{1}{\gamma_h} \frac{\alpha_n - i + \nu_{hn}}{\sigma_n^2}, \tag{20}$$

$$= \frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2} \phi_{hn}, \tag{21}$$

which, in terms of psychological distance, is given by

$$\omega_{hn} = \begin{cases} \frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa d_{hn}}, & d_{hn} \leq \overline{d} \\ 0, & d_{hn} > \overline{d}. \end{cases} \tag{22}$$

In our model, a household makes portfolio decisions using its personal beliefs. Consequently, one can see from the expression in (20) for the portfolio weight in stock n when stock returns are uncorrelated, the expected risk premium under rational expectations, $\alpha_n - i$, has been replaced by the perceived risk premium, $\alpha_n - i + \nu_{hn}$.

Equation (21), which is obtained by substituting (17) into (20), shows that the standard mean-variance portfolio weight for firm n when stock returns are uncorrelated, $\frac{1}{\gamma_h} \frac{\alpha_n - i}{\sigma_n^2}$, is scaled by the psychological proximity measure for household h with respect to firm n, ϕ_{hn}. As a household's proximity measure with respect to a particular firm decreases, the proportion of its wealth that it chooses to invest in that firm also decreases. That is, from (18) we know that if a household is psychologically more distant from a firm (a reduction in psychological proximity), its beliefs about the firm's return diverge further from the objective physical expectation, and hence, it tilts its portfolio away from the one under rational expectations, as shown by Equation (22).

Proposition 3 suggests that households may hold severely underdiversified portfolios, in line with what has been observed empirically. Specifically, a household will put so low trust in firms that are psychologically more distal than $\overline{d}$, that they completely avoid investing in them. If many of the firms in the market are further away than $\overline{d}$, the household's portfolio will consequently be severely underdiversified.

We also see from Equation (22) that the key parameter for understanding the quantitative impact of psychological distance on beliefs is κ. In our empirical analysis, we will estimate κ by using the variation in ω_{hn}, the portfolio weights of households located at different psychological distances from firms. Using our estimate for κ, we will then examine how households' probability measures vary spatially across Finland and drive the welfare costs of deviating from rational expectations.

We conclude this section by highlighting the difference between the expression for portfolio weights in our model and what one would get from a model where investors had access to private information when estimating the mean $(\alpha_n|\text{info})$ or the volatility $(\sigma_n|\text{info})$ of stock returns:

$$\omega_{hn} = \frac{1}{\gamma_h}\frac{(\alpha_n|\text{info}) - i}{(\sigma_n|\text{info})^2}. \tag{23}$$

The portfolio weights in our model, given in (22), differ from those from an information-based models, given in (23), in at least three respects. One, information can lead to an *increase* in the portfolio weight if the the investor's information leads her to either increase the mean return or decrease stock-return volatility; in contrast, in our model, psychological distance leads only to a decrease in the portfolio weight toward zero. Two, even for assets with a positive expected risk premium, if information about the mean stock return is sufficiently negative, it can lead to a *short* (i.e., negative) portfolio weight; in our model, an asset that has a positive expected risk premium will never be shorted by an investor whose beliefs deviate from rational expectations, but instead will decrease to zero. Finally, in the information-based model, there will be non-zero demand for all risky assets, while our model leads to sparse portfolios with zero holdings in stocks for which the distance between the investor and firm exceeds the threshold; i.e., $d_{hn} > \overline{d}$. In the data, there is much more support for the model based on psychological deviations in beliefs; in particular, in the data we find that portfolios are sparse (the median investor holds only two stocks) and that there are no short positions.

3.3 Household Welfare when Beliefs Deviate from Rational Expectations

In order to study the impact of psychological distance on beliefs, and through that on welfare, we compute the *welfare loss* to households in deviating from rational expectations and show that it is related to the relative entropy from the distorted belief $\mathbb{P}^h$ to the rational expectations belief $\mathbb{P}$.

The mean-variance utility of a household with rational expectations ($\boldsymbol{\nu}_{hn} = \mathbf{0}$) for an arbitrary portfolio vector $\boldsymbol{\omega}_h$ is given by

$$U^{MV}\big(\boldsymbol{\omega}_h\big) = i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_h - \frac{1}{2}\gamma_h \boldsymbol{\omega}_h^\top V \boldsymbol{\omega}_h, \tag{24}$$

which is the expected portfolio rate of return minus a penalty for portfolio variance. Substituting the optimal portfolio under rational expectations, which is

$$\boldsymbol{\omega}_h(\mathbb{P}) = \frac{1}{\gamma_h} V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}),$$

into the mean-variance utility in (24) and simplifying gives

$$U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P})\big) = i + \frac{1}{2}\frac{1}{\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}),$$

which for uncorrelated stock returns reduces to

$$= i + \frac{1}{2}\frac{1}{\gamma_h}\sum_{n=1}^{N}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2.$$

The mean-variance welfare for a household with distorted beliefs, $U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big)$, given by

$$U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big) = i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_h(\mathbb{P}^h) - \frac{1}{2}\gamma_h \boldsymbol{\omega}_h(\mathbb{P}^h)^\top V \boldsymbol{\omega}_h(\mathbb{P}^h),$$

where $\boldsymbol{\omega}_h(\mathbb{P}^h)$ is the optimal portfolio under distorted beliefs given in (19). That is, $U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big)$ is obtained by taking the portfolio under distorted beliefs, denoted by $\boldsymbol{\omega}_h(\mathbb{P}^h)$, and substituting it into the expression for mean-variance utility for a household whose beliefs are given by the reference probability measure under rational expectations, $\mathbb{P}$, in (24). We can then compute the *welfare loss* in deviating from rational expectations

$$\Delta U^{MV}(\mathbb{P}, \mathbb{P}^h) = U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P})\big) - U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big).$$

An elegant way to express this welfare loss will be to use the concept of relative entropy, also known as Kullback-Leibler divergence.[8] We have the following definition for conditional relative entropy per unit time.

[8]In a discrete state space of dimensionality $S \in \mathbb{Z}^+$, where the physical probability of being in state $s \in \{1, \ldots, S\}$ is denoted by p_s and the corresponding subjective distorted probability by p_s^h, the relative entropy from $\mathbb{P}^h$ to $\mathbb{P}$ is given by

$$D^{KL}[\mathbb{P}|\mathbb{P}^h] = -\sum_{s=1}^{S} p(s) \ln \frac{p^h(s)}{p(s)} = -E^{\mathbb{P}}\left[\ln \frac{\tilde{p}^h}{\tilde{p}}\right],$$

where $\tilde{p}$ is the discrete random variable which takes the value p_s in state s and $\tilde{p}^h$ is the discrete random variable which takes the value p_s^h in state s. In continuous time with a continuous state space, we replace the discrete random variable $\frac{\tilde{p}^h}{\tilde{p}}$ with the Radon-Nikodym derivative

$$\frac{d\mathbb{P}^h(s_{t+dt}|s_t)}{d\mathbb{P}(s_{t+dt}|s_t)} = \frac{M_{h,t+dt}}{M_{h,t}},$$

where $s_{t+dt}|s_t$ represents the state of the economy at time $t + dt$ conditional on the state at time t.

Definition 5. *The conditional relative entropy per unit time from the personal belief* $\mathbb{P}^h$ *to the objective belief* $\mathbb{P}$ *is*

$$D^{KL}[\mathbb{P}|\mathbb{P}^h] = -\frac{1}{dt}E_t^{\mathbb{P}}\left[\ln\frac{M_{h,t+dt}}{M_{h,t}}\right],$$

which using Ito's Lemma can be rewritten as

$$\begin{aligned} D^{KL}[\mathbb{P}|\mathbb{P}^h] &= \frac{1}{2}\frac{1}{dt}E_t^{\mathbb{P}}\left[\left(\frac{dM_{h,t}}{M_{h,t}}\right)^2\right] \\ &= \frac{1}{2}\boldsymbol{\nu}_{h,t}^{\top}V^{-1}\boldsymbol{\nu}_{h,t}. \end{aligned} \tag{25}$$

The concept of entropy has been used at least as early as Marschak (1959); Cabrales, Gossner, and Serrano (2013, Sec. IV) present an excellent review of the related literature, and Garleanu and Pedersen (2019, Prop. 9) show how entropy can be used to measure the inefficiency of a portfolio in a two-date asymmetric-information model. We show that the welfare loss from distorted beliefs is proportional to relative entropy, link it to psychological distance, and then provide, in the next section, an empirical estimate of it.

Proposition 4. *The mean-variance* welfare loss *from having distorted beliefs instead of rational expectations is given by*

$$\Delta U^{MV}(\mathbb{P},\mathbb{P}^h) = \frac{1}{\gamma_h}D^{KL}[\mathbb{P}|\mathbb{P}^h]. \tag{26}$$

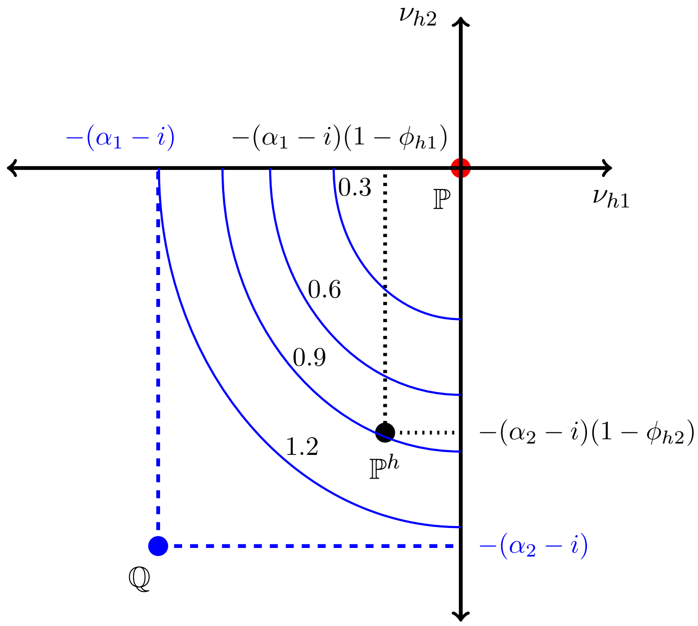
Proposition 4 shows that the mean-variance welfare loss is given by the relative entropy measure (i.e., Kullback-Leibler divergence defined in Equation (25)) multiplied by the household's risk tolerance, $1/\gamma_h$. The reason that the welfare loss depends on risk tolerance is because households that have low risk tolerance (i.e., high risk aversion) invest little in risky assets, and hence, their welfare loss from distorted beliefs about the return distribution of risky assets is smaller.

To get some intuition for relative entropy and hence the welfare loss, observe that for the special case where stock returns are uncorrelated, the expression in (25) simplifies to

$$\begin{aligned} D^{KL}[\mathbb{P}|\mathbb{P}^h] &= \frac{1}{2}\sum_{n=1}^{N}\left(\frac{\nu_{hn,t}}{\sigma_n}\right)^2 \\ &= \frac{1}{2}\sum_{n=1}^{N}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2(1-\phi_{hn})^2 \qquad (27) \\ &= \frac{1}{2}\sum_{n:\, d_{hn}<\bar{d}}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2(1-e^{-\kappa d_{hn}})^2 + \frac{1}{2}\sum_{n:\, d_{hn}\geq\bar{d}}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2, \qquad (28) \end{aligned}$$

Figure 3: Iso-entropy contours

In this figure, we superimpose iso-entropy contours on the previous plot in which we illustrated the various probability measures for the special case of two risky assets with uncorrelated returns. Each iso-entropy contour represents the set of probability measures with the same level of relative entropy from the rational expectations benchmark, $\mathbb{P}$. In this plot, we assume that the annualized volatility of each asset is equal to 0.30. The numbers for each iso-entropy contours represent the Kullback-Leibler divergence.



where $\nu_{hn,t}$ is the change in the expected rate of return for stock n, and $\nu_{hn,t}/\sigma_n$ is the change in the Sharpe ratio, when moving from the rational-expectation probability measure $\mathbb{P}$ to household h's probability measure $\mathbb{P}^h$. For the case where psychological distances are all zero (implying that there is full trust for all firms), we see from (27) and (28) that relative entropy $D^{KL}[\mathbb{P}|\mathbb{P}^h]$ reduces to zero, because households' subjective beliefs coincide with the beliefs under rational expectations. However, when psychological distances are not all zero, then households' beliefs diverge from rational expectations. In the extreme case where $d_{hn} \geq \bar{d}$ for all N firms, a household's subjective probability measure coincides with the risk-neutral measure $\mathbb{Q}$ and the Kullback-Leibler divergence reaches its upper limit, given by $(1/2)\sum_{n=1}^{N}\left((\alpha_n - i)/\sigma_n\right)^2$. Figure 3 shows iso-entropy contours, each of which represents the set of probability measures with the same level of relative entropy from the rational-expectation benchmark, $\mathbb{P}$.

Trust, ϕ_{hn}, which appears in (27) is not directly observable. However, Proposition 1 shows how we can link ϕ_{hn} to psychological distances, d_{hn}, via the parameter κ, which then allows us to express relative entropy in terms of psychological distances, as shown in (28). Empirically, we shall show in Section 4 that the magnitude of the welfare loss increases rapidly as psychological distances increase and, for most of the households that are located away from Helsinki, is close to the maximum because their subjective beliefs are closer to the risk-neutral measure instead of rational expectations.

4 Empirical Estimation of Belief Distortions

Psychological distances between households and firms, from construal level theory (Trope and Liberman, 2010), can be temporal, spatial, social, or hypothetical distances. In the empirical work that we report in this section, we use spatial distance as a measure of psychological distance. Our empirical analysis is described in three parts. We describe the data that we use for our empirical analysis in Section 4.1. The results of our empirical analysis of the relation between beliefs and distance are reported in Section 4.2. We conclude by undertaking a series of robustness tests in Section 4.3. In the next section, we will use the results of our empirical analysis to estimate the spatial distribution of deviations from rational expectations and its consequences for welfare and household inequality.

4.1 Data

We obtained portfolio holdings for all accounts on the Helsinki Stock Exchange, as of January 2, 2003, from Euroclear, which acquired the Finnish Central Securities Depositary in 2008.[9] The data contain portfolio holdings and postal-code information, as well as further characteristics (age, gender, and sector-code classification) of all account holders in the market. There are altogether 3,036 valid postal codes in the data set and the data contains over 60 million trades during the time period 1995-2004, and about 1.2 million accounts, most of which represent the household sector.

We obtain spatial coordinates for each postal code area from the Finnish postal services company, Posti Group Corporation. These postal codes make up a fine-grained representation of Finland, as shown in Figure 4. We represent each postal code spatially by its center of gravity (CoG).

We obtain information about the postal codes of company headquarters from Thomson One Reuters and exclude companies headquartered outside of Finland. We also exclude companies with shares that were not traded within the previous month. Finally, we exclude the telecommunications company Elisa Oyj, which until 1999 had been a privately held mutual association with broad ownership among its association members, and therefore had a quite different own-

[9] This dataset is similar to the one used in Grinblatt and Keloharju (2000, 2001) and Walden (2019).

Figure 4: Postal codes, firms, and population in Finland

This figure shows the 3036 postal code regions (at the five-digit level), which are obtained from the Finnish postal services company, Posti Group Corporation. The figure also shows the location of firm headquarters (orange circles) and the major cities of Finland (yellow diamonds). Finally, the choropleth map shows the population of Finland across different postal codes. We see from the map that the population is dispersed across postal codes, while a majority of firm headquarters (80%) are in the Helsinki region.

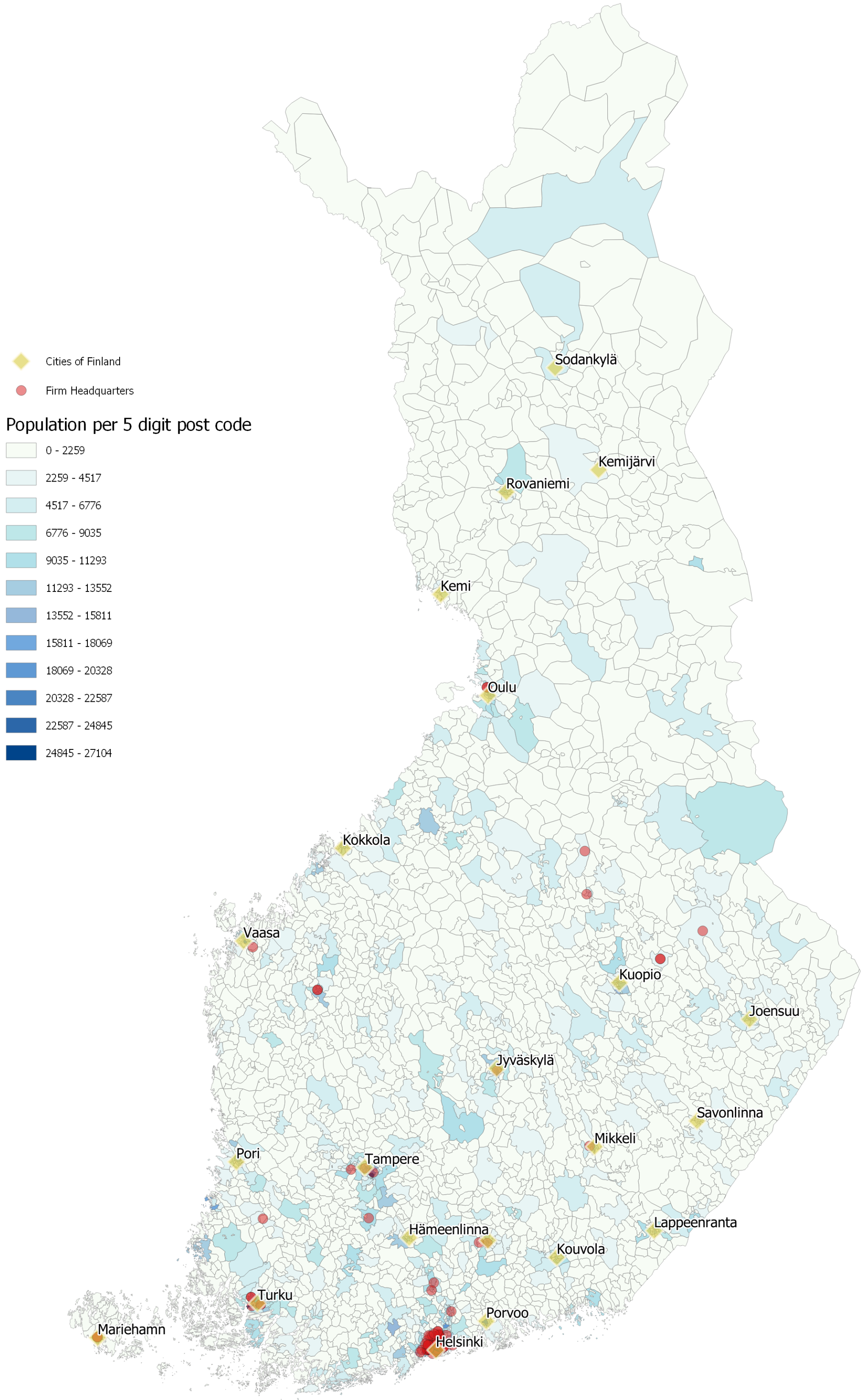


Table 1: Summary statistics

This table gives the summary statistics for the data used in our empirical analysis.

Item	Value
Number of stocks	125
Number of household accounts	405,628
Number of postal codes	2,923
Average number of accounts in postal code	139
Total number of observations	364,750
Number of nonzero observations	132,811
Maximum portfolio holdings	FIM 178.7 million
Minimum portfolio holdings	FIM 0
Average stock holdings by postal code	FIM 30,617
Median number of stocks held by an account	2
Mean number of stocks held by an account	2.75

ership history and structure than the rest of the firms.[10] This leaves us with 125 stocks, which are listed in Table B1 of Appendix B.[11]

We include accounts that are classified as households (these are accounts associated with sector codes between 500 and 599), that are associated with a valid postal code, and that owned shares in at least one of the 125 stocks on January 2, 2003. This leaves us with 405,868 households associated with altogether $P = 2,923$ postal codes.

The postal code associated with household h is denoted p_h. We assume that each agent resides at the center of gravity of his/her respective postal code and also that each firm is headquartered at the center of gravity of its postal code. Thus, all agents within a postal code are assumed to be at the same distance from each of the firms, with d_{pn} denoting the distance between the center of gravity of postal code area p, (x_p, y_p), and the center of gravity of the postal code area in which firm n, (x^n, y^n), is located,

$$d_{pn} = D((x_p, y_p), (x^n, y^n)) = \sqrt{(x_p - x^n)^2 + (y_p - y^n)^2}. \tag{29}$$

[10] Elisa Oyj was formed on July 1, 2000, with the merger of the Helsinki Telephone Corporation (in Finnish, "Helsingin Puhelin") and its holding company, HPY Holding Corporation. Helsinki Telephone Corporation had been a privately held telephone cooperative with broad ownership among its 550,000 "association members." Subscribers to its telephone services automatically became association members. When the company was listed on the Helsinki Stock Exchange in 1997, these owners became shareholders, which then carried over to Elisa Oyj after the merger (Source: Annual Reports, 1997-2000). Most of these shareholders only held this one stock, and thus seem to be shareholders for different reasons than the rest of the investor population. Except for Predictions 2 and 3, on underdiversification and similarity of holdings of individual households, the results are very similar when Elisa Oyj is included.

[11] Some of these stocks represent A and B shares in the same company. An A share in Finland typically come with greater voting rights compared with a B share. There were significant differences in share prices and returns between A and B shares of the same companies and we therefore include both A and B shares in our sample for companies with both types of shares.

We normalize the distance function, so that all the spatial coordinates lie in the unit square, $[0, 1] \times [0, 1]$, making the interpretation of the sensitivity coefficient κ straightforward (1 unit is approximately equal to 794 miles (1278 km)).

We use spatial distance as a proxy for psychological distance. As discussed in Walden (2019), this is a reasonable assumption for the time period and market that the data covers because over half of Finland's population resided in rural areas—as illustrated in Figure 4—making it one of the most rural countries in the European Union. One might view spatial distance as an unimportant hurdle in the present time due to almost universal access for households to the Internet. However, only about one third of the Finnish population used the Internet in 2000. We therefore view it as plausible that spatial distance was an important component of total psychological distance in the early 2000's.

The spatial coordinates of postal codes and firms (orange circles) are shown in Figure 4. As can be seen in the figure, most firms (about 80%) are headquartered in the far south around the capital, Helsinki (with associated five-digit postal codes between 00100 and 09900), whereas about 20% of the firms have headquarters elsewhere. Summary statistics of the data we use are provided in Table 1.

Despite the detailed dataset that we have, it has some limitations, which have implications for how we undertake our empirical analysis. One, the portfolios of the retail investors we study are, at the postal code level (which is the level of resolution we use for our tests), very persistent, to a degree that they are almost perfectly correlated in the time series. Therefore, we focus on a single point in time: January 2003. So, our panel is defined across postal codes and stocks, which is used to draw inferences about the parameters in (22), but not across time. Two, our dataset does not contain information about households' total wealth, bond holdings, or savings in retirement accounts and mutual funds, information that would allow us to draw further inferences about risk aversion and allocation between risky and risk-free assets. Three, although postal codes, in principle, should have been updated for households that moved during the period, this almost never happened in our dataset. Postal codes for fewer than 100 households changed during the entire period 1996-2004, corresponding to less than 0.04% of the households in the sample.[12] So, we were not able to study whether portfolios changed for households that moved, even thought that would be an interesting analysis to undertake.[13] Finally, our data contains no information that would allow the identification of

[12]Moreover, several of these changes seem to be erroneous, for example, a switch for one trade but then a switch back for the next and onward.

[13]However, the effect of a change in residence on investors' portfolio is studied in Bodnaruk (2009). Consistent with our hypothesis that beliefs are influenced by psychological distance, he finds that the farther investors move

individual households, and thereby, the possibility of using other data sources to gain additional information at the individual household level.

4.2 Estimating Belief Distortions

We explore the empirical implications of our model in three stages. First, in Predictions 1 to 3, we test the general economic mechanism whereby household-firm distances impact beliefs, and hence, portfolio choices. Then, in Prediction 4, we estimate the threshold $\bar{d}$. Finally, in Predictions 5 and 6, we exploit our theoretical model to empirically estimate the parameter κ, which measures the sensitivity of household beliefs to spatial distance, and hence, the extent to which household beliefs about firm-level expected returns are distorted relative to the rational-expectations benchmark. By doing so we show how household beliefs vary across each of the 2,923 postal codes in Finland. Finally, we undertake a series of robustness tests.

4.2.1 Test of Prediction 1

If beliefs are determined by rational expectations, then the household-firm distance should be irrelevant for the ownership of stocks. In particular, the center of gravity of ownership for each individual stock should be identical and equal to the center of gravity for the market. On the other hand, if the beliefs of households are impacted by their distance from firms, then the center of gravity (CoG) of ownership of stocks should be influenced by firm location. This leads to the first prediction of our model.

Prediction 1. *The center of gravity (CoG) of ownership of stocks is influenced by the distance between households and firms.*

To evaluate this prediction, we calculate the center of gravity of ownership in each stock. Specifically, denote the set of households that have invested in stock n by $\mathcal{H}_n$. Let x_{p_h} and y_{p_h} denote the x- and y-coordinates, respectively, for the location of the postal code of household h. The center of gravity of ownership in stock n is then the spatial coordinate (x_n^O, y_n^O), where

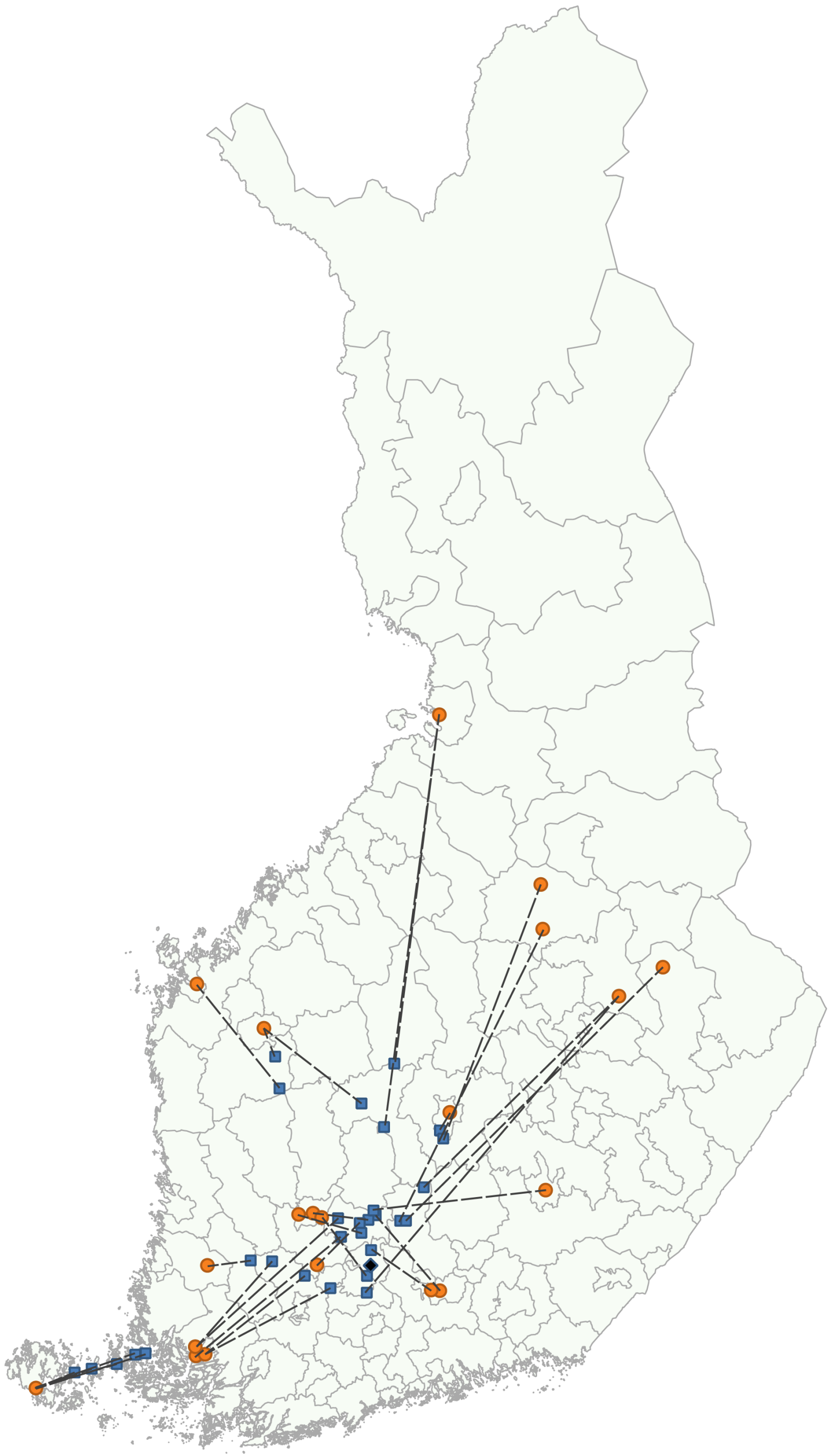
$$x_n^O = \frac{1}{|\mathcal{H}_n|} \sum_{h \in \mathcal{H}_n} x_{p_h} \quad \text{and} \quad y_n^O = \frac{1}{|\mathcal{H}_n|} \sum_{h \in \mathcal{H}_n} y_{p_h}.$$

The center of gravity of ownership as well as the headquarters of the firms outside of the Helsinki area are shown in Figure 5. Under rational expectations ($\boldsymbol{\nu}_{hn} = \mathbf{0}$), the center of gravity of ownership for each stock (given by the blue squares) should be the same across

away from a firm, the more of its shares they sell compared to investors who do not move. He also finds that, after moving, investors increase ownership in the firms that are closer to their new location.

Figure 5: Center of gravity for stocks headquartered outside Helsinki

This figure shows the center of gravity of ownership (blue squares) and firm headquarters (orange circles) for stocks headquartered outside of the Helsinki area, connected by (black) lines. The market's total center of gravity of ownership is given by the solid (black) diamond. Under rational expectations the blue squares should coincide with the black diamond. Instead, the blue squares are closer to the orange circles.



stocks, coinciding with the market's total center of gravity of ownership—shown as a solid black diamond in the figure. This is not what we observe. Instead, the centers of gravity of ownership of firms tend to be closer to the firm headquarters (given by the orange circles). This finding indicates not just that spatial distance matters for beliefs, but also that it matters in a particular way—the distance of a household from the headquarters of a firm impacts the household's beliefs in such a way that households are more likely to invest in firms that have a smaller psychological distance.

4.2.2 Test of Prediction 2

If beliefs are impacted by household-firm distances, then households that are farther away from clusters of firms should hold portfolios that are less well diversified. This leads to the second prediction of our model.

Prediction 2. *Households that are farther away from clusters of firms should hold portfolios that are less well diversified.*

To test if households tend to be more underdiversified the farther away they are located from firm headquarters, we define the center of gravity of all the firms in our data set to be the mean of their individual locations

$$(x^C, y^C) = \frac{1}{\sum_{n=1}^{N} W^n} \left(\sum_{n=1}^{N} W^n x^n, \sum_{n=1}^{N} W^n y^n \right),$$

where (x^n, y^n) are the coordinates for firm n. We use both equally-weighted CoG ($W^n = 1$ for all firms) and value-weighted CoG, (a firm's weight is defined as the total value of household portfolio holdings in that firm).

We further define q_p, the average number of stocks in households' portfolios within postal code area p, and $d_p = D((x_p, y_p), (x^C, y^C))$, the distance between households in postal code area p and firm CoG, and estimate how they are related. The relation, shown in Table 2, is significantly negative, both economically and statistically. A household located at the maximum distance from the firms' CoG is predicted to hold about one fewer stock in its portfolio than a household located right at the CoG—a major effect because the median number of stocks held in a household portfolio is only two. The results are very similar regardless of whether the value-weighted or equally weighted definition of CoG is used.

We also control for portfolio size. Specifically, if wealthier households tend to be better diversified and are also located closer to firms on average, similar results would arise, but

Table 2: Test of Prediction 2: Underdiversification versus distance from firms

This table shows how the average number of stocks held by households in a postal code is related to the distance of the postal code to firm center-of-gravity (CoG), d_p. Columns 1 and 2 use an equally weighted definition of CoG, whereas Columns 3 and 4 use a value-weighted definition. Univariate regressions are used in Columns 1 and 3, and bivariate regressions, also including log-average portfolio size, $\ln(W_p)$, are used in Columns 2 and 4.

	Equally weighted		Value weighted	
	(1)	(2)	(3)	(4)
Distance, d_p	−1.011***	−0.306***	−1.024***	−0.313***
Standard error	0.084	0.072	0.082	0.070
Portfolio size, $\ln(W_p)$		0.446***		0.445***
Standard error		0.012		0.012

the results would be influenced by this omitted variable. We therefore include the logarithm of average portfolio-size within a postal code area, $\ln(W_p)$, in the regression. The estimated coefficient decreases by about two thirds when $\ln(W_p)$ is included, both in the value-weighted and equally weighted CoG specification, but still remains highly significant.

4.2.3 Test of Prediction 3

Our model suggests that households in proximity to the same firms should have similar beliefs, and hence, should hold similar investment portfolios. That is, households located in the same postal code should have similar beliefs, and thus, should hold portfolios with more overlap (i.e., more common stocks) than if the portfolios were randomly chosen.

For example, for two households that each randomly and independently invest in one of N stocks, with probability $\frac{1}{N}$ of choosing each stock, the probability that the two portfolios overlap would also be $\frac{1}{N}$. In other words, the expected value of the random value $\tilde{N}^c$, which denotes the number of common stocks in their portfolios, would be $\frac{1}{N}$. More generally, if household h_1 randomly chooses N_{h_1} stocks and household h_2 randomly and independently chooses N_{h_2} stocks, then the expected overlap is

$$E\left[\tilde{N}^c_{h_1,h_2}\right] = \frac{N_{h_1} N_{h_2}}{N}.$$

This leads to the following prediction.

Prediction 3. *Two households in proximity should have similar beliefs, and hence, should hold more similar investment portfolios than two randomly chosen households.*

Prediction 3 can then be tested by studying the actual versus expected overlap of the portfolios of all households located within the same postal code area, and summing up these overlaps over all postal codes, that is, by calculating the actual total overlap

$$N^c_{TOT} = \sum_p \sum_{\substack{h_1,h_2 \in \mathcal{H}_p \\ h_1 \neq h_2}} N^c_{h_1,h_2},$$

where $\mathcal{H}_p = \{h : p_h = p\}$ is the set of households in postal code area p, and comparing it with the expected total overlap.

The actual total overlap in the data is $N^c_{TOT} = 51,604,247$, whereas the expected total overlap under the independent, $\frac{1}{N}$ probability-per-stock, assumption is $3,478,029$. Thus, the likelihood that two randomly chosen households within the same randomly chosen postal code area invest in the same stock is almost 15 times higher than expected. This strongly suggests that households in the same location hold similar beliefs about expected stock returns, while households in different locations do not.

A limitation of the above test is that it assumes that each stock is chosen with the same probability, $\frac{1}{N}$, whereas in practice some stocks (for example, the telecommunications company Nokia in our dataset) are much more broadly held than others. The higher overlap will therefore partly be a consequence of the $\frac{1}{N}$ assumption. We therefore use a more sophisticated bootstrapping method, based on the actual distribution of portfolio holdings among the households to test the prediction.

Briefly, we assume that the empirical distribution of household portfolio holdings represent the underlying data generating process, and compare the expected overlap if households and their portfolios were randomly assigned to postal codes with what is observed in the data. An interesting property of this test is that the maximum portfolio size, measured by number of stocks in the portfolio, can be capped, so that the test is applied to a subpopulation of households whose portfolios contain no more than 1, 2, 5, etc., stocks. Because location, via personal beliefs, drives both underdiversification and the choice of similar portfolios for nearby households, we expect higher-than-expected overlaps to be more pronounced among households that hold few stocks.[14] Further details of the test are provided in Appendix B.

The results of the bootstrapping method are shown in Table 3. The ratio of actual-to-expected overlap (the right-most column in the table) is the highest, 2.387, when only households holding one stock are included in the test, and then gradually decreases as the maximum

[14] There is also a purely mechanical effect in that the actual overlap cannot be much higher than the expected overlap when households hold large portfolios. For example, if a household holds *all* stocks in its portfolio, *every* stock in the other household's portfolio is common for the two portfolios, so actual and expected overlap must coincide in that case.

Table 3: Test of Prediction 3: Overlap of portfolios within a postal code area

This table tests whether portfolio overlaps within a postal code area is higher than expected if households were randomly assigned to the area. Column 1 shows the maximum portfolio size included in test, Column 2 the number of households included in the sample, Column 3 the expected number of overlaps, Column 4 the actual number of overlaps, and Column 5 the ratio between actual and expected overlaps.

Max. portf. size	Number of households	Actual overlap	Expected overlap	Ratio
1	188,535	6,324,570	2,649,371	2.387
2	268,268	12,989,232	6,096,314	2.130
5	357,772	28,450,235	16,782,239	1.695
10	393,606	42,315,889	27,935,744	1.515
20	404,277	49,978,463	34,481,911	1.449
125	405,628	51,604,247	35,952,390	1.436

portfolio size increases. The ratio is 1.436 when the full sample of households is included. These ratios are thus lower than the ratio based on the $\frac{1}{N}$ assumption, but remain highly significant, in support of Prediction 3.

4.2.4 Test of Prediction 4

According to our model, a household's belief distortions about firms that are included in its investment portfolio are smaller than the belief distortions about firms that are not included in its portfolio; in particular, if a household-firm distance exceeds the threshold $\overline{d}$, then the household will not include that firm in its portfolio. This leads to the following prediction.

Prediction 4. *A household's belief distortions about firms that are included in its portfolio are lower than the belief distortions about firms not included in its portfolio; thus, the distance between postal codes and headquarters for stocks with positive holdings of a stock should be smaller than that for zero holdings of a stock.*

To test this prediction, for each stock, we perform a two-sample t-test, comparing the average distances of postal codes with zero and with positive holdings from the firms' headquarters. The results are shown in Table 4. The average distance for postal codes with zero holdings is about

Table 4: Test of Prediction 4: Zero versus positive portfolio holdings

This table shows results from two-sample t-tests of difference in means between the distance from postal codes with zero holdings to firm headquarters and from postal codes with positive holdings.

	Zero holding	Positive holding	Difference
Average distance to HQ	0.332	0.224	0.108
Average t-stat	−3.047	14.080	13.258
Number of firms	6	119	

Table 5: Estimated size of trust region

This table shows the estimated size of the threshold distance, $\overline{d}$.

Item	Value
Average threshold, $\overline{d}$	0.1799
Standard deviation	0.2165
Fraction of zero holdings	0.6359
Fraction correctly classified	0.7534

0.332, whereas the average distance for postal codes with positive holdings is about 0.224, corresponding to a difference of 0.108—about 85 miles (137 km). The average t-statistic for the difference of means being positive is 13.3.

At the individual firm level, the average distance from a postal code with zero holdings is higher than the distance for positive holdings for 119 of the 125 stocks. For 116 of these firms, the difference is statistically significant at the 0.01% level. For the remaining 6 firms, for which the average distance is higher for postal codes with positive holdings compared to the distance for zero holdings, only one is significant at the 0.01% level. Thus, the data strongly support the prediction that the farther away a household is from a firm's headquarters, the more distorted are its beliefs, and hence, the higher the likelihood of zero portfolio holdings.

We also estimate the size of the threshold distance, $\overline{d}$. Specifically, for each of the 125 stocks, we choose the $\overline{d}$ that maximizes the number of correctly classified postal codes with respect to whether the stock holdings in the firm are positive or zero. The results are shown in Table 5. The average estimated $\overline{d}$ is 0.1799, corresponding to a threshold distance of about 143 miles (230 km). Beyond this threshold distance from a firm's headquarters, household beliefs are predicted to be so strongly distorted, that the household completely avoids investing in that stock.

The fraction of holdings that are zero among all postal code/firm observations is 64%. When the estimated threshold distances at the firm level are used to predict whether a postal code/firm portfolio holdings are zero, the fraction of correct classifications is about 75%. The model is good at identifying whether households in a particular postal code invest in a stock or not.

4.2.5 Test of Prediction 5

The previous four predictions were about the general implications of our theoretical framework for belief formation. We now test the specific relation between beliefs and household-firm distance implied by Equation (18) of our model. In order to simplify the estimation, we assume that stock-return correlations are zero. This assumption is reasonable given that the average correlation across stocks over our sample period is less than 0.10 and the median number of stocks held by a household is smaller than 3, suggesting that correlations do not play a major role in a household's portfolio choice.

The central difficulty in estimating the sensitivity of beliefs to distance is that beliefs are not observable. To circumvent this problem, we rely on our theoretical model that identifies the relation between beliefs and portfolio choices. In particular, we use Equation (22) of Proposition 3 to estimate κ. Our estimate for κ then allows us to compute the belief distortion using Equation (18) of Proposition 2, and the effect of these beliefs on welfare using Proposition 4.

The relation between the belief distortion of household h at the postal code level (indicated by replacing subscript h by p) about expected returns for firm n, ν_{pn}, and the distance between the postal code and the firm, d_{pn}, is, from Equation (18)

$$\nu_{pn} = \begin{cases} -(\alpha_n - i)(1 - e^{-\kappa\, d_{pn}}), & d_{pn} \leq \overline{d} \\ -(\alpha_n - i), & d_{pn} > \overline{d}. \end{cases}$$

The above equation makes clear that our theory allows one to estimate the belief distortions once we have estimated κ. Specifically, we will estimate the decay factor κ and use it with the estimated threshold value $\overline{d}$ to measure the belief distortions of households in each postal code about all stocks in the market.

We do so by exploiting Equation (20), which when expressed at the postal code level is

$$\omega_{pn} = \frac{\text{amt}_{pn}}{W_p} = \begin{cases} \frac{1}{\gamma_p} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{pn}}, & d_{pn} \leq \overline{d} \\ 0, & d_{pn} > \overline{d}, \end{cases} \tag{30}$$

where amt_{pn} is the Finnish mark (FIM) amount invested in stock n by households in postal code p, W_p is the wealth of the households in that postal code, and γ_p is the harmonic mean of the relative risk aversion coefficients for agents living in postal code p, $\frac{1}{\gamma_p} = \sum_{\{h:p_h=p\}} \frac{1}{\gamma_h}$. The equation above implies that

$$\text{amt}_{pn} = \begin{cases} \frac{W_p}{\gamma_p} \frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa\, d_{pn}}, & d_{pn} \leq \overline{d} \\ 0, & d_{pn} > \overline{d}. \end{cases}$$

Defining $a_{pn} = \ln(\text{amt}_{pn})$, $s_n = \ln\left(\frac{\alpha_n - i}{\sigma_n^2}\right)$, and $g_p = \ln(\gamma_p / W_p)$ then implies that, when the psychological distance is less than $\overline{d}$, we can rewrite (30) as

$$a_{pn} = -g_p + s_n - \kappa\, d_{pn}. \tag{31}$$

Equation (31) thus consists of $P \times N$ postal code/company holdings. The κ coefficient in (31), which is constant across postal codes and firms, acts as a cross-equation restriction across both dimensions of the data, which allows us to estimate κ from portfolio-holdings data. Similarly, g_p imposes a cross-equation restriction across firms and s_n a cross-equation restriction across postal codes. A significantly positive value of κ suggests that our proposed representation of household beliefs is informative, which leads to the following prediction.

Prediction 5. *The estimated decay factor κ in Equation (31) is strictly positive, $\kappa > 0$.*

To test Prediction 5, we estimate the κ coefficient in Equation (31) using portfolio-holdings data. This equation can be viewed as a panel with fixed effects for postal codes (representing the log-risk aversion coefficients, g_p) and companies (representing the log-return characteristics, s_n). We therefore use panel regressions.

The model is not completely identified because the mapping $g_p \mapsto g_p + c$, $s_n \mapsto s_n + c$ yields the same portfolio weights for an arbitrary constant c. Intuitively, higher risk aversion coefficients are offset by more favorable investment opportunities. We obtain unique identification by normalizing the results so that the average s_n coefficient is 0 ($s_n = 0$ is, for example, obtained with $\alpha_n - i = 0.09$, $\sigma_n = 0.3$ so that the ratio $\frac{\alpha_n - i}{\sigma_n^2} = 1$).

Finally, we consider two approaches for handling portfolio amounts of zero, for which the logarithm is not defined. There are a large number of such observations in the data, even after aggregation at the postal code level. Under the first approach, we include these observations but replace the zero with a small positive threshold, namely one Finnish mark (corresponding to about USD 0.17). Under the second approach, we exclude such observations and run unbalanced panel regressions. The disadvantage of the former approach is that it introduces an arbitrary lower threshold, whereas the disadvantage of the latter is that it does not use information about zero holdings.

The results are shown in Table 6. We see that the decay factor, κ, is highly significant in all four regression specifications (univariate, including risk-aversion fixed effects, including risk-aversion and stock-characteristic fixed effects, and panel regression with robust standard errors double clustered at the firm and postal-code level). For the coefficient estimate that includes risk-aversion, stock distribution, and all observations (first row in Panel A, Column (3)), we estimate $\kappa = 3.18$.

The results are also economically significant. The standard deviation of the distance between headquarters and household is 0.312. For the estimation that includes risk-aversion, stock distribution, and all observations (first row in Panel A, Column (3), $\kappa = 3.18$), a one standard deviation decrease in distance to a firm's headquarters predicts an increase in portfolio holdings

Table 6: Test of Prediction 5: Estimate of decay factor κ

This table estimates the decay factor, κ. Panel A includes observations with $\text{amt}_{pn} = 0$ replaced with an investment of one Finnish mark, corresponding to about USD 0.17. Panel B excludes observations with $\text{amt}_{pn} = 0$. Univariate in Column (1), including risk aversion in Column (2), including risk aversion and stock-return distributions in Column (3), panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in Column (4). Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)
Panel A				
Sensitivity coefficient, κ	5.873***	3.861***	3.188***	3.185***
Standard error	0.037	0.058	0.056	0.434
log risk aversion, g				
-average		-4.330	-4.120	
-max		-0.382	-0.325	
-min		-12.887	-12.834	
log distribution, s				
-average			0	
-max			7.258	
-min			-2.658	
R^2	0.065	0.385	0.591	0.591
Adj. R^2	0.065	0.380	0.588	0.588
$N = 368,298$				
Panel B				
Sensitivity coefficient, κ	2.251***	2.123***	2.669***	2.669***
Standard error	0.035	0.061	0.046	0.403
log risk aversion, g				
-average		-8.572	-7.979	
-max		-4.563	-2.638	
-min		-12.845	-12.883	
log return distribution, s				
-average			0	
-max			4.166	
-min			-3.985	
R^2	0.030	0.210	0.626	0.626
Adj. R^2	0.030	0.191	0.618	0.618
$N = 134,902$				

by a factor $e^{3.1880\times 0.312} = 2.645$. The R^2 for the univariate regression in Panel A is 0.0654, corresponding to a correlation between proximity and log-portfolio holdings of about 0.26.

4.2.6 Test of Prediction 6

We now test if the data on portfolio holdings and location are informative also about out-of-sample stock-return distributions. If the predictions of our model with distorted beliefs are superior to those under rational expectations, it would provide significant support for our belief-based explanation of investor behavior because we are using only data on portfolio holdings and location to predict out-of-sample stock-return distributions.

Prediction 6. *The estimated* $\{s_n\}_n$ *coefficients in* (31) *are informative about stock-return distributions*

$$\ln\left(\frac{\alpha_n - i}{\sigma_n^2}\right), \qquad n = 1, \dots N.$$

To test whether the estimated s_n coefficients (the firm fixed effects) from the belief-based model are informative about actual stock-return distributions out of sample, we calculate daily mean excess returns, $\hat{\alpha}_n - i$, and volatility, $\hat{\sigma}_n$, over a five-year period subsequent to the period used in the panel regressions, from 2003–2008.

The results are shown in Table 7. We first compare the relation between estimated s_n and realized volatility, $\ln(\hat{\sigma}_n)$, because realized volatility is the part of $\ln\left(\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}\right)$ that is easiest to estimate. This relation should be negative. As shown in Column 1 in Panel A of the table, the relation is strongly negatively significant, with an R^2 of over 50%.

We compare this with the prediction of the rational-expectations model. In the rational-expectations model, all households choose the same risky portfolio, $\phi_{hn} \equiv 1$ for all h and n, and by summing (31) over households, it follows that s_n is directly related to a firm's log-size. Column 2 in Panel A of Table 7 shows that firm log-size also is informative about volatility in our data, but with lower explanatory power than the belief-based model. Moreover, when both the belief-based and rational-expectations estimates are included in a bivariate regression, as done in Column 3 of Panel A, the belief-based estimate dominates, remaining significant, whereas the log-firm size coefficient switches sign and becomes insignificant.

In Panel B of the table, we do the same estimation for $\ln\left(\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}\right)$ as we did for $\ln(\hat{\sigma})$, including the 108 stocks that had positive excess realized returns during the period (for which the logarithm is defined). The explanatory power is lower both for the belief-based coefficient and the coefficient based on the rational-expectations model, due to the well-known challenges of estimating stocks' expected returns from realized returns. However, the belief-based estimate remains significant at the 0.1% level in the univariate regression, and at the 0.01% level in the bivariate regression, while the rational-expectations coefficient switches sign in the bivariate regression. As a robustness check, we also run the same tests using three years of realized returns. The results (not reported) are similar, the main difference being that the belief-based coefficient in the bivariate regression in Panel B is significant only at the 10% level.

Table 7: Test of Prediction 6: Predictive power of estimated s_n coefficients

This table tests whether the s_n coefficients estimated from portfolio holdings predict realized volatility (Panel A) and return over variance (Panel B) in subsequent 5-year period. Column 1 uses the belief-based model, Column 2 the rational-expectations model, and Colum (3) both models. Realized volatility and returns measured using daily data over 3-year period, 2003-2005. Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)
Panel A: $\ln(\hat{\sigma}_n)$			
Rational expectations, s_n^{RE}		−0.149***	0.036
Belief-based, s_n^{BB}	−0.185***		−0.219***
R^2	0.506	0.356	0.510
N	125		
Panel B: $\ln\left(\frac{\hat{\alpha}_n - i}{\hat{\sigma}_n^2}\right)$			
Rational expectations, s_n^{RE}		0.075	−0.296*
Belief-based, s_n^{BB}	0.149**		0.426***
R^2	0.068	0.019	0.121
N	108		

We emphasize that only data on portfolio holdings and location is used in the estimation of the s_n coefficients. That these are informative about out-of-sample stock-return distributions therefore lends significant support for our belief-based explanation of investor behavior.

These results complement the ones in Grinblatt and Keloharju (2001). Relative to Grinblatt and Keloharju, we develop a theory linking psychological distance to beliefs, and hence, portfolios. In particular, we show that deviations from rational expectations are based on psychological distance, and we provide an explicit test for a specific functional form that links beliefs to distance that is supported empirically. Grinblatt and Keloharju do not link portfolio holdings to beliefs. Moreover, the main regression tests in Grinblatt and Keloharju arise naturally in our framework, but our theoretical model allows us to provide economic interpretations for their fixed effects. We show that the firm fixed effects in their tests represent distributional properties of stock returns (excess return per unit of variance risk), and postal-code fixed effects represent household preferences (risk aversion). Our belief-based model is supported by the positive relation between the estimated excess return per unit of variance risk and out-of-sample firm performance.

4.3 Robustness tests

A possible concern with the test of our key result reported in Table 6, which tests the relation between beliefs and household-firm distance (Prediction 5), is that it may be driven by different behaviors of households in urban areas (specifically, around Helsinki) relative to rural areas.

For example, households in the Helsinki area may have a preference for stocks headquartered in an urban area for some reason than the effect of distance on beliefs, which will then lead to results similar to those reported above. To check if this is indeed the case, we run the regressions *excluding* stocks and postal codes in the Helsinki area (postal codes with fewer than 5 significant digits). The results are reported in Table B2 in Appendix B. We see that the results are qualitatively similar to before, and specifically, still both statistically and empirically highly significant. We also exclude households in the Helsinki area but not stocks, with similar results (not reported).

A potential alternative explanation for the results is that it may not be psychological distance per se that drives beliefs, but rather employment. That is, if households tend to invest in the firms they work for—which they likely also live close to—similar results may arise. To rule out such an explanation, we exclude observations for which the postal codes of account holder and firm headquarters are close. Specifically, we exclude all observations for which the normalized distance is less than some d_0. Table B3 in Appendix B, shows that the results remain qualitatively similar when $d_0 = 0.01$ (corresponding to a minimal distance of about 8 miles between the postal code of the account holder and firm headquarters for an observation to be included), and $d_0 = 0.03$ (corresponding to a minimal distance of about 24 miles). Thus, an employment effect does not seem to be driving the results.

Our approach provides a tightly specified model that links households' beliefs about firms and their portfolio choices to their psychological distances from the firms—in our empirical specification captured by spatial distance. Other explanations for such portfolio choices that have been put forward in the literature are transaction costs and hedging demand.

We argue that transaction costs are unlikely to play a major role within our sample, which is focused on one country with all stocks traded on the same exchange. With respect to hedging demand, an alternative explanation for why agents prefer to invest in nearby stocks is that they provide a hedge against local shocks. For example, if a local firm performs well, prices of services and goods (e.g., housing) may increase because of increased demand from the employees at the firm who are now wealthier. Investing in the local firm provides a hedge against such price shocks. Inasmuch as such hedging demand is related to the age and gender of the population, we can assess its affect, because account-level information is available in the data. For example, younger investors—who are less likely to own their home—are likely to be more exposed to real estate price shocks than older investors.

We create variables for the average birth year (YOB) of the investors in a specific stock and postal code, and for their gender (GEN, which is 1 for male and 2 for female). The correlations

Table 8: Estimate of sensitivity coefficient with additional controls

This table estimates the sensitivity coefficient, κ, when including investor year of birth (YOB) and gender (GEN). Panel A sets $d_0 = 0$, whereas Panel B sets $d_0 = 0.05$, corresponding to a threshold distance of about 40 miles (64 km). No fixed effects in Column (1), including risk aversion in Column (2), including risk aversion and stock distributions in Column (3), panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in Column (4). Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)
Panel A				
Sensitivity coefficient, κ	2.298***	2.056***	2.658***	2.658***
Standard error	0.034	0.060	0.046	0.398
Year of birth coefficient, YOB	−0.031***	−0.040***	−0.013***	−0.013***
Standard error	0.001	0.001	0.001	0.001
Gender coefficient, GEN	0.597***	0.445***	−0.157***	−0.157***
Standard error	0.019	0.018	0.013	0.033
R^2	0.065	0.249	0.630	0.630
Adj. R^2	0.065	0.233	0.622	0.622
$N = 134,825$				
Panel B				
Sensitivity coefficient, κ	0.809***	0.704***	1.377***	1.377***
Standard error	0.040	0.073	0.056	0.263
Year of birth coefficient, YOB	−0.031***	−0.037***	−0.012***	−0.012***
Standard error	0.001	0.001	0.001	0.001
Gender coefficient, GEN	0.461***	0.345***	−0.169***	−0.169***
Standard error	0.019	0.019	0.014	0.033
R^2	0.039	0.200	0.606	0.606
Adj. R^2	0.039	0.180	0.596	0.596
$N = 115,256$				

between the distance to firm headquarters and these variables are both low, $\rho_{d,\text{YOB}} = -0.049$, and $\rho_{d,\text{GEN}} = -0.056$, suggesting that any hedging demand that varies with age and/or gender is not captured by the distance to firm headquarters. To further explore a potential relation, we rerun the tests from Table 6, Panel B, but including birth year and gender, and also from Table B3 with $d_0 = 0.05$, to rule out hedging demand for local shocks within a 40-mile (64-km) radius.

As seen in Table 8, the estimates for the decay coefficient κ barely change and are still highly significant. We conclude that it is unlikely that our results are driven by hedging demand against local shocks, at least within a 40-mile (64-km) radius of the investor, and against shocks that are related to age and/or gender.

Another concern may be that headquarters provides a very rough measure of a firm's location. For example, some firms have operations spread out over the whole country and will be familiar to households far away from its headquarters. The challenges of developing an objectively superior alternative measure of firm location are significant though, which is why we use the well-established headquarters measure of location. As a robustness check, we ensure that the results do not change when excluding the two "least local" companies during the period: the global telecommunication company Nokia, which made up over half of the stock market value in the early 2000s, and the retail store chain Stockmann, which had stores all over Finland. The results (not reported) are very similar when excluding those two companies.

5 Estimating the Effect of Belief Distortions on Welfare

In this section, we evaluate the consequences for household welfare of deviations from rational expectations driven by psychological distance. In our estimation of the welfare costs, we use spatial distance as a proxy for psychological distance. We then use the spatial distribution of welfare losses to describe how the location of a household drives inequality in portfolio returns.

From Proposition 4, the mean-variance *welfare loss* from having distorted beliefs $\mathbb{P}^p$ (at the postal-code level) instead of rational expectations, is given by

$$\Delta U^{MV}(\mathbb{P}, \mathbb{P}^p) = \frac{1}{\gamma_p} D^{KL}[\mathbb{P}|\mathbb{P}^p] = \frac{1}{2}\frac{1}{\gamma_p} \boldsymbol{\nu}_{p,t}^{\top} V^{-1} \boldsymbol{\nu}_{p,t}.$$

In order to interpret the magnitude of the welfare loss resulting from the deviation of beliefs from rational expectations, it will be convenient to define *transformed relative entropy*, which is a useful measure for financial economists because it has the same units as a Sharpe ratio. This measure of welfare loss is attractive also because it is independent of leverage and the risk aversion of the household.

Definition 6. *The transformed relative entropy, at the level of postal codes, is given by*

$$TD^{KL}[\mathbb{P}|\mathbb{P}^p] = \sqrt{2\frac{D^{KL}[\mathbb{P}|\mathbb{P}^p]}{N}}. \tag{32}$$

For the special case in which stock returns are uncorrelated, the transformed relative entropy is the root mean square of the changes in individual stock Sharpe ratios:

$$\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p] = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\frac{\nu_{pn,t}^2}{\sigma_n^2}}. \tag{33}$$

$$= \sqrt{\frac{1}{N}\left[\sum_{n:\, d_{hn}<\bar{d}}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2 (1-e^{-\kappa d_{hn}})^2 + \sum_{n:\, d_{hn}\geq\bar{d}}\left(\frac{\alpha_n - i}{\sigma_n}\right)^2\right]},$$

To compute $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ at the level of each postal code, we first compute in miles the spatial distance between firm headquarters and the CoG for each postal code using Equation (29). We standardize this distance by dividing it by 794 miles so that it is in the same units as our empirical work. We then compute the trust of each postal code to the 125 firms in our dataset, using the value of $\bar{d} = 0.1799$ estimated in Table 5 and the value of $\kappa = 3.185$ estimated in Table 6. Then, we take the moments of daily stock returns and annualize them. With these annualized moments, we compute the deviation in beliefs at the postal code level, ν_{pn}, using the expression in Equation (18). Finally, using the estimates of ν_{pn} and σ_n^2, we compute $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ as defined above in (33).

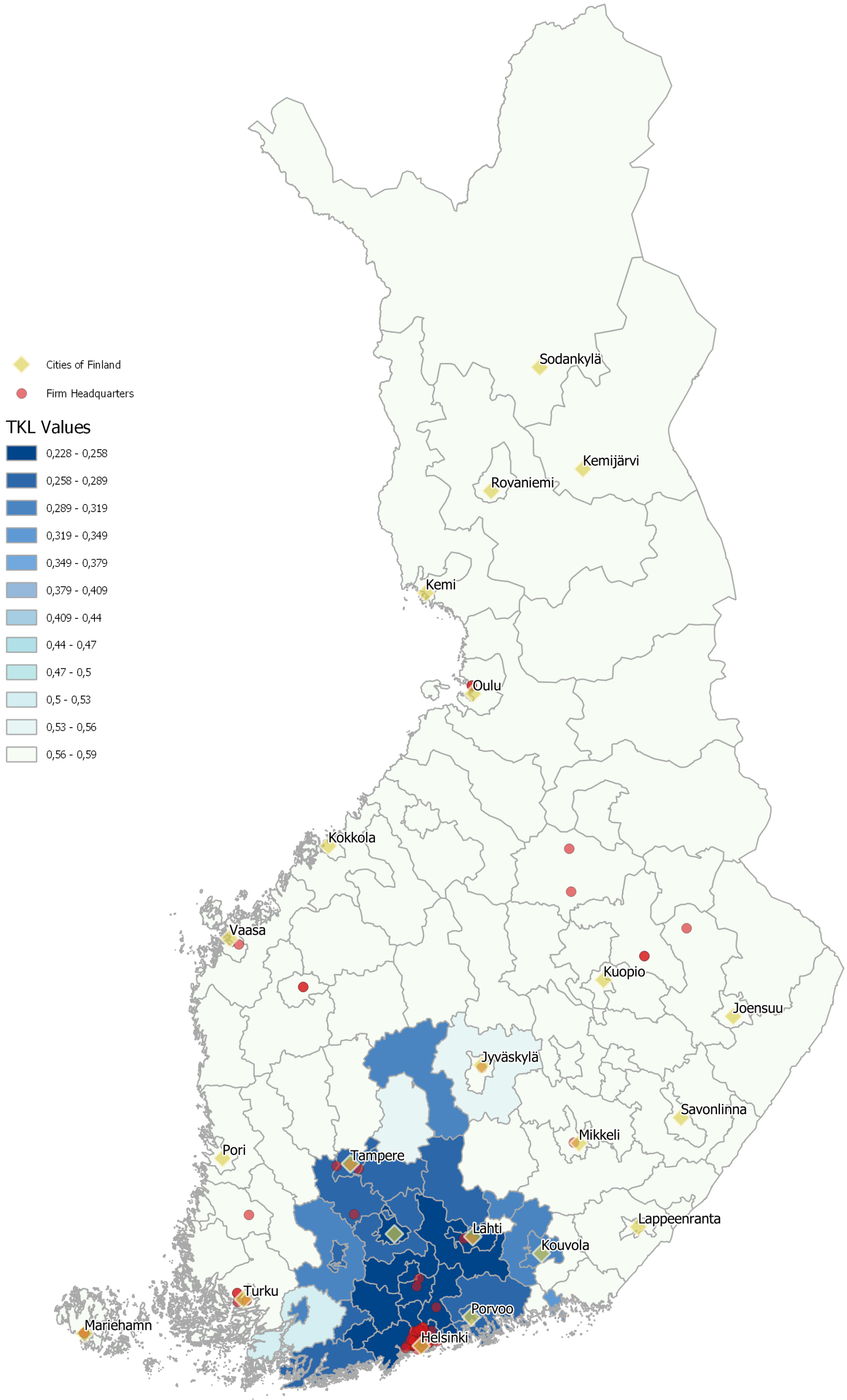
Computing $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$, we find that the smallest value it takes is about 0.25, which is for the Helsinki region (postal codes below 00990). It is striking that even for postal codes close to Helsinki, the value of $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ is much greater than 0, which is the value under rational expectations. As one moves away from Helsinki, $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ increases: for postal codes greater than 20000 we find that $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ is about 0.50, double that of the value for the Helsinki region. For postal codes greater than 40000, $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ is about 0.57, which is close to its maximum value of 0.59.[15] This indicates that for these postal codes $\mathbb{P}^p$ is very close to the *risk-neutral* measure, $\mathbb{Q}$. That is, the effect of psychological distance is so great that households in these postal codes perceive the risk-premium from investing in risky assets to be zero, and hence, do not invest much in any of the risky assets. This is a direct consequence of trust decaying exponentially with respect to psychological distance, as shown in (1).

A choropleth map displaying these values of $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ for different postal codes is given in Figure 6. Recall from Figure 4 that during the period of our study a significant proportion of the Finnish population (more than 70%) resided outside of Helsinki, with over half of Finland's population in rural areas. From Figures 6, it is apparent that there is significant variation in welfare levels across the population of Finland. In particular, the segment of the population

[15] From the definition in (32), the $\mathrm{TD}^{KL}[\mathbb{P}|\mathbb{P}^p]$ quantities of $\{0.25, 0.50, 0.57, 0.59\}$ reported in the text can be expressed also in terms of the Kullback-Leibler divergence; the corresponding $\mathrm{D}^{KL}[\mathbb{P}|\mathbb{P}^p]$ quantities are $\{3.90, 15.63, 20.31, 21.76\}$.

Figure 6: Transformed relative entropy across postal codes

This choropleth map displays for different postal codes (that have been aggregated to the 2-digit level) the values of transformed relative entropy, $TD^{KL}[\mathbb{P}|\mathbb{P}^p]$, which is defined in Equation eqrefeqn:DKL-trans-new2.



residing close to Helsinki has beliefs that are much less distorted compared to the population residing in regions that are distant from Helsinki. As a result of this belief distortion based on spatial distance from firms, Figure 6 paints a stark picture of the substantial inequality in the welfare levels of the population residing close to Helsinki relative to the population in the rest of the country.

6 Conclusion

Motivated by empirical evidence that rejects the rational-expectations hypothesis, we combine concepts from psychology and robust control, to develop a framework in which a household's beliefs about stock returns are an *endogenous* outcome of the household's psychological distance from firms. In the theoretical framework, psychological distance encompasses temporal, spatial, and social distance. To make the model testable, we establish the relation between unobservable household beliefs and observable portfolio choices.

We evaluate the model empirically using data on portfolio holdings of Finnish households to infer household beliefs and welfare. The empirical evidence indicates that spatial distance between the locations of households and firms, which we use as a proxy for psychological distance, has a statistically and economically significant effect on the beliefs of households about stock returns. We calculate the reduction in household welfare resulting from the deviation of beliefs from rational expectations and show that households located close to firms have substantially higher welfare than those that are located far away from firms. Thus, psychological distance has a strong bearing on beliefs, and through that, on household welfare.

A Proofs

In this appendix, we provide all derivations for the results in the main text.

Proof of Proposition 1

Psychological space is represented by a finite dimensional vector space, V_P, as in Shepard (1987). Suppose there is a norm $\|\cdot\|$: $V_P \to \mathbb{R}^+$. We can then define the metric d: $V_P \times V_P \to \mathbb{R}^+$ via $d(x,y) = \|x-y\|$, where $x, y \in V_P$.

We assume that households and firms can be placed at points within the psychological space V_P. Denote the location of a household h in psychological space via the vector $v_h \in V_P$ and the location of firm n in V_P via $v_n \in V_P$. The psychological distance of h from n is $d(v_h, v_n)$, which we denote by d_{hn}.

As in Shepard (1987), a household faces the task of inferring whether the properties of a firm closer to it in terms of psychological distance can be generalized to a firm farther away, at some point $v_n \in V_P$. A household believes objects within a trust region can be trusted, but is unsure about the size and location of the trust region, S. A trust region $R(x, S)$, centered on $x \in V_P$ and with size S, is defined by $R(x,S) = \{v \in V_P : d(x,v) \leq S\}$, that is, the closed ball in V_P, centered on x with radius S, where the exact size of S is unknown, but its mean is known and it is known that $S \leq \overline{d}$, for some fixed $\overline{d} > 0$. By symmetry, any x is assumed to be equally likely for the center of the trust region $R(x, S)$. For simplicity, we now assume $V_P = \mathbb{R}$, but note that our results can easily be extended to the multidimensional case, as shown in Shepard (1987).

We now apply Definition 2. The trust a household has in observations about a firm n at psychological distance d_{hn} from itself is equal to the likelihood that the object lies within its trust region multiplied by the probability that the size of the trust region lies in the interval $[s, s+ds)$ summed over all possible values of the trust region:

$$\phi(d) = \int_0^\infty \max\left(\frac{s-d}{s}, 0\right) p_1(s) ds,$$

where $p_1(s)$ is the probability density for the trust region.

All that is known about the trust region size S is its mean value and its maximum size and that $d(v_h, x) \leq S$. The Principle of Maximum Entropy (Jaynes, 1957, 1986) states that if nothing is known about a distribution except that it belongs to a certain class, then the distribution with the largest entropy should be chosen as the least informative. Consequently, the following static optimization problem determines the prior distribution of S, the size of the trust region.

$$\max_{p(s)} - \int_0^{\overline{d}} p(s) \ln p(s) ds$$

subject to

$$\int_0^{\overline{d}} p(s) ds = 1, \tag{A1}$$

and

$$\int_0^{\overline{d}} s p(s) ds = \mu, \tag{A2}$$

where $\mu = E[S]$ is the known mean of S.

To solve the above constrained maximization problem, we form the Lagrangian

$$\mathcal{L} = - \int_0^{\overline{d}} p(s) \ln p(s) ds - \kappa_0 \left(1 - \int_0^{\overline{d}} p(s) ds \right) - \kappa \left(\mu - \int_0^{\overline{d}} s p(s) ds \right).$$

We have the first order condition

$$0 = - \ln p(s) - 1 + \kappa_0 + \kappa s,$$

and so

$$p(s) = e^{-(1-\kappa_0)} e^{-\kappa s}, \ s \in [0, \overline{d}].$$

From (A1), we see that

$$p(s) = A e^{-\kappa s}, \ s \in [0, \overline{d}],$$

where

$$A = \frac{\kappa}{1 - e^{-\kappa \overline{d}}}.$$

We now use the constraint (A2) to determine κ, starting with

$$\mu = \frac{\kappa}{1 - e^{-\kappa \overline{d}}} \int_0^{\overline{d}} s e^{-\kappa s} ds.$$

It follows that

$$\frac{1}{\kappa} = \mu + \frac{\overline{d}}{e^{\kappa \overline{d}} - 1}. \tag{A3}$$

We note that although a closed form solution is not available, the behavior of κ as a function of μ and $\overline{d}$ is straightforward to characterize. Specifically, if we define $z = \kappa \overline{d}$ and $\hat{\mu} = \frac{\mu}{\overline{d}}$, (A3) may be written as

$$\frac{1}{q} = \hat{\mu} + \frac{1}{e^q - 1}.$$

It follows that $q(\hat{\mu})$ is a decreasing function of $\hat{\mu}$ in the interval $\hat{\mu} \in (0, 1/2)$, that $\lim_{z \searrow 0} q(\hat{\mu}) = \infty$, and $\lim_{z \nearrow 1/2} q(\hat{\mu}) = 0$.

Starting from the prior density $p(s)$, the posterior density for S is given by Bayes' Law via

$$p_1(s) \propto sp(s),$$

where s is the likelihood of an object being in its trust region. Therefore

$$p_1(s) = \frac{\kappa^2}{1 - e^{-\overline{d}\kappa}(1 + \kappa\overline{d})} se^{-\kappa d} 1_{s \leq \overline{d}}.$$

Hence,

$$\begin{aligned}
\phi(d) &= \int_0^\infty \max\left(\frac{s-d}{s}, 0\right) p_1(s) ds \\
&= \frac{\kappa^2}{1 - e^{-\overline{d}\kappa}(1 + \kappa\overline{d})} \int_d^{\overline{d}} (s-d) e^{-\kappa d} ds \\
&= e^{-\kappa d} I_{d \leq \overline{d}} f(\kappa(\overline{d} - d)) \\
&\approx e^{-\kappa d} I_{d \leq \overline{d}},
\end{aligned}$$

where $f(y) = \frac{1-e^{-y}(1+y)}{1-e^{-q}(1+q)}$, and $q = \kappa\overline{d}$ as defined above. The approximation of $\phi(d)$ by $e^{-\kappa d} I_{d \leq \overline{d}}$ is very close, as long as $\mu < \frac{1}{4}\overline{d}$. We therefore use the approximation:

$$\phi(d) = e^{-\kappa d} I_{d \leq \overline{d}}$$

to link psychological distance to trust, where $\overline{d}$ is the threshold psychological distance at which trust declines to zero. □

Proof of Lemma 1

We start by observing that the stock return for firm n over the interval $[t, t+dt)$, that is, $dR_{n,t}$, is a date-$(t + dt)$ random variable. Thus,

$$\begin{aligned}
E_t^{\mathbb{P}^h}[dR_{n,t}] &= E_t^{\mathbb{P}}\left[dR_{n,t} \frac{M_{h,t+dt}}{M_{h,t}}\right] \\
&= E_t^{\mathbb{P}}\left[dR_{n,t} \frac{M_{h,t} + dM_{h,t}}{M_{h,t}}\right] \\
&= E_t^{\mathbb{P}}[dR_{n,t}] + E_t^{\mathbb{P}}\left[dR_{n,t} \frac{dM_{h,t}}{M_{h,t}}\right]. \qquad \text{(A4)}
\end{aligned}$$

The distortion in probability mass is captured by the second term in (A4), which is the covariance between the stock return and the percentage change in the martingale. Simplifying this term, one gets:

$$\begin{aligned} E_t^{\mathbb{P}}\left[dR_{n,t}\frac{dM_{h,t}}{M_{h,t}}\right] &= \left(\boldsymbol{\sigma}_n^\top d\boldsymbol{Z}_t\right)\left(d\boldsymbol{Z}_t^\top(\Sigma^{-1})^\top\boldsymbol{\nu}_{h,t}\right) \\ &= \boldsymbol{\sigma}_n^\top(\Sigma^{-1})^\top\boldsymbol{\nu}_{h,t}dt \\ &= (\boldsymbol{e}_n^\top\Sigma^\top)(\Sigma^{-1})^\top\boldsymbol{\nu}_{h,t}dt \\ &= \boldsymbol{e}_n^\top\boldsymbol{\nu}_{h,t}dt \\ &= \nu_{hn,t}dt, \end{aligned} \tag{A5}$$

where $\boldsymbol{e}_n$ is the N by 1 column vector with a one in the n'th entry and zeros everywhere else. Hence, substituting (A5) in (A4), we see that

$$E_t^{\mathbb{P}^h}[dR_{n,t}] = E_t^{\mathbb{P}}[dR_{n,t}] + E_t^{\mathbb{P}}\left[dR_{n,t}\frac{dM_{h,t}}{M_{h,t}}\right] = (\alpha_n + \nu_{hn,t})dt.$$

Thus, the deviation in household h's beliefs about the expected rate of return on firm n (relative to the physical probability measure) is ν_{hn}. □

Proof of Lemma 2

Expression (11) can be written in vector-matrix form as

$$\boldsymbol{\nu}_{h,t} = \arg\min_{\boldsymbol{\nu}_{h,t}}\left(\boldsymbol{\omega}_{h,t}^\top\boldsymbol{\nu}_{h,t} + \frac{1}{2\gamma_h}\boldsymbol{\nu}_{h,t}^\top D_\sigma^{-1}F_hD_\sigma^{-1}\boldsymbol{\nu}_{h,t}\right).$$

Fixing $\boldsymbol{\omega}_{h,t}$, we can see that the above optimization problem is linear-quadratic with a unique solution given by the vector (12), the n'th element of which is given in (13). Substituting (12) into (9) and simplifying gives (14). □

Proof of Proposition 2

Substituting (15) into (12) gives

$$\begin{aligned} \boldsymbol{\nu}_{h,t} &= -(D_\sigma^{-1}F_hD_\sigma^{-1})^{-1}(V + D_\sigma F_h^{-1}D_\sigma)^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) \\ &= -[(V + D_\sigma F_h^{-1}D_\sigma)(D_\sigma^{-1}F_hD_\sigma^{-1})]^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) \\ &= -[(V(D_\sigma^{-1}F_hD_\sigma^{-1}) + D_\sigma F_h^{-1}D_\sigma(D_\sigma^{-1}F_hD_\sigma^{-1}))]^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) \\ &= -[(V(D_\sigma^{-1}F_hD_\sigma^{-1}) + D_\sigma F_h^{-1}D_\sigma(D_\sigma F_h^{-1}D_\sigma)^{-1})]^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) \\ &= -(VD_\sigma^{-1}F_hD_\sigma^{-1} + I)^{-1}(\boldsymbol{\alpha} - i\mathbf{1}), \end{aligned}$$

which gives (16). For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, we have $V = D_\sigma^2$. Using the fact that D_σ and F_h are diagonal matrices and hence commute, we see that

$$VD_\sigma^{-1}F_hD_\sigma^{-1} = D_\sigma^2D_\sigma^{-1}F_hD_\sigma^{-1} = F_h,$$

and so

$$\boldsymbol{\nu}_{h,t} = -(I + F_h)^{-1}(\boldsymbol{\alpha} - i\mathbf{1}),$$

which implies (17). Using Proposition 1 we hence obtain (18). □

Proof of Proposition 3

We aim to show that (19) is true. Defining, for ease of notation, $A \equiv D_\sigma F_h^{-1} D_\sigma$, we see by comparison with (15) that (19) is true if and only if

$$I - V(V + A)^{-1} = (I + VA^{-1})^{-1}. \tag{A6}$$

To prove (A6), we start from the observation that

$$I = (A + V)(V + A)^{-1}.$$

Therefore,

$$I = A(V + A)^{-1} + V(V + A)^{-1},$$

and so

$$I - V(V + A)^{-1} = A(V + A)^{-1} = (A^{-1})^{-1}(V + A)^{-1} = ((V + A)A^{-1})^{-1} = (VA^{-1} + I)^{-1},$$

and thus we obtain (A6). For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, we have $V = D_\sigma^2$, and so we obtain (20) from (19) and (21) from (15). Using Proposition 1 we obtain (22) from (21). □

Proof of Proposition 4

The mean-variance utility of a household with rational expectations for an arbitrary portfolio vector $\boldsymbol{\omega}_h$ is given by

$$U^{MV}\left(\boldsymbol{\omega}_h\right) = i + (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_h - \frac{1}{2}\gamma_h \boldsymbol{\omega}_h^\top V \boldsymbol{\omega}_h. \tag{A7}$$

The above utility function is maximized when

$$\boldsymbol{\omega}_h = \frac{1}{\gamma_h} V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}).$$

Substituting the above expression into (A7) and simplifying gives

$$\begin{aligned} U^{MV}\left(\boldsymbol{\omega}_h(\mathbb{P})\right) &= \max_{\boldsymbol{\omega}_h} U^{MV}\left(\boldsymbol{\omega}_h\right) \\ &= i + \frac{1}{2\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}). \end{aligned} \tag{A8}$$

The optimal portfolio for a household with distorted beliefs is given by (19). Substituting this expression into (A7) gives

$$
\begin{aligned}
U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big) &= i + \frac{1}{\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h) - \frac{1}{2\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h)^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h) \\
&= i + \frac{1}{\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) + \frac{1}{\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}\boldsymbol{\nu}_h \\
&\quad - \frac{1}{2\gamma_h}\Big[(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) + 2(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}\boldsymbol{\nu}_h + \boldsymbol{\nu}_h^\top V^{-1}\boldsymbol{\nu}_h\Big] \\
&= i + \frac{1}{2\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top V^{-1}(\boldsymbol{\alpha} - i\mathbf{1}) - \frac{1}{2\gamma_h}\boldsymbol{\nu}_h^\top V^{-1}\boldsymbol{\nu}_h.
\end{aligned}
$$

From (A8) and the definition of the Kullback-Leibler divergence given in (25), it follows that

$$
U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P}^h)\big) = U^{MV}\big(\boldsymbol{\omega}_h(\mathbb{P})\big) - \frac{1}{\gamma_h}D^{KL}[\mathbb{P}|\mathbb{P}^h].
$$

Hence, we obtain (26). □

B Additional Details for the Empirical Results

In this appendix, we provide additional information about the empirical tests and results.

B.1 Bootstrapping Method

Consider a finite multi-set of vectors, $\mathcal{V} = \{\mathbf{v}_m\}_{m\in\mathcal{M}}$, $|\mathcal{M}| \in \mathbb{N}$, where $\mathbf{v}_m \in \{0,1\}^N$, $N \in \mathbb{N}$. In our test, m will be associated with a household and $(\mathbf{v}_m)_n$ will represent whether it holds stock n (in which case $(\mathbf{v}_m)_n = 1$) or not (in which case $(\mathbf{v}_m)_n = 0$). The object $\mathcal{V}$ is a multi-set, since there may be multiple households with the same stocks in their portfolios. The nonzero elements in $\mathbf{v} \in \mathcal{V}$ are denoted the vector's *constituents*, and $c_{\mathbf{v}} = |\{\ell : (\mathbf{v})_\ell = 1\}|$ is defined as the number of such constituents.

For two elements, $\mathbf{v}^a \in \mathcal{V}$, $\mathbf{v}^b \in \mathcal{V}$, define the vector of overlaps, $\mathcal{O}(\mathbf{v}^a, \mathbf{v}^b) \in \{0,1\}^N$, such that $\mathcal{O}(\mathbf{v}^a, \mathbf{v}^b)_n = (\mathbf{v}^a)_n \times (\mathbf{v}^b)_n$, and the overlap $o(\mathbf{v}^a, \mathbf{v}^b) = \sum_n \mathcal{O}(\mathbf{v}^a, \mathbf{v}^b)_n \in \{0, 1, \dots, N\}$.

Consider a multi-set $\mathcal{A}$ of (at least two) elements in $\mathcal{V}$, ordered from 1 to $r = |\mathcal{A}|$, $\mathbf{a}_1, \dots, \mathbf{a}_r$. This set will correspond to the set of household portfolios in a postal code area in our test. The overlap in $\mathcal{A}$ is

$$o(\mathcal{A}) = \frac{1}{2} \sum_{\substack{k,\ell=1,\dots,r \\ k\neq\ell}} o(\mathbf{a}_k, \mathbf{a}_\ell). \tag{B1}$$

Define $\mathcal{Z}^k = \{\mathbf{v} \in \mathcal{V} : c_{\mathbf{v}} = k\}$, the set of all elements in $\mathcal{V}$, with k constituents, and $z^k = |\mathcal{Z}^k|$, the number of such elements. Also define $K = \max\{k : z^k > 0\}$. For simplicity, we assume that $z^k > 0$ for all $k < K$. Our analysis also holds if this is assumption is not satisfied, but with the extra notational burden of excluding such k's for which $z^k = 0$ in some of the sums below, which is why we make this assumption.

Consider a random data generating process to generate a vector $\tilde{\mathbf{a}}$ of k constituents from the elements in $\mathcal{Z}^k$, where each element in $\mathcal{Z}^k$ is chosen with the same probability. Thus, if the elements in $\mathcal{Z}^k$ are ordered as $\mathbf{v}_1^k, \dots, \mathbf{v}_{z_k}^k$, then

$$\mathbb{P}(\tilde{\mathbf{a}} = \mathbf{v}_\ell^k) = \frac{1}{z_k}, \qquad \ell = 1, \dots, k.$$

Now, consider two randomly chosen such elements, $\tilde{\mathbf{a}} \in \mathcal{Z}^k$ and $\tilde{\mathbf{b}} \in \mathcal{Z}^\ell$, $1 \leq k, \ell \leq K$. The expected overlap of these two elements, given the above data generating process, is then

$$o_{k,\ell}^{\text{Exp}} = E\left[o(\tilde{\mathbf{a}}, \tilde{\mathbf{b}})\right] = \frac{1}{z^k z^\ell} \sum_{s=1}^{z_k} \sum_{t=1}^{z_\ell} o(\mathbf{v}_s^k, \mathbf{v}_t^\ell).$$

Moreover, given the set $\mathcal{A}$ with $r \geq 2$ elements, and the ordered elements, $\mathbf{a}_1, \ldots, \mathbf{a}_r$, the expected overlap, when $\mathcal{A}$ is chosen from the above data generating process, is

$$o^{\text{Exp}}(\mathcal{A}) = \frac{1}{2} \sum_{\substack{k,\ell=1,\ldots,r \\ k \neq \ell}} o^{\text{Exp}}_{c_{\mathbf{a}_k}, c_{\mathbf{a}_\ell}}. \tag{B2}$$

By comparing the actual overlap given in Equation (B1) with the expected overlap given in (B2), we can thus draw inferences about whether the elements in $\mathcal{A}$ were randomly selected according to the above data generating process, or whether they have too much (or too little overlap) compared with what would be expected.

Note that it is straightforward to restrict the above test to portfolios up to a specific size, S, by replacing the set $\mathcal{A}$ with $\mathcal{A}^S = \{\mathbf{v} \in \mathcal{A} : c_{\mathbf{v}} \leq S\}$. This allows us to draw additional inferences about whether potential nonrandomness stems from elements with a low number of constituents or not. Of course, $\mathcal{A}^N = \mathcal{A}$.

For the portfolio data, we use the set of portfolios of households in different postal code areas, $\mathcal{A}_p = \{\mathbf{v}_h : h \in \mathcal{H}_p\}$, $p = 1, \ldots, P$. Our null hypothesis is that portfolios of households within a postal code are randomly selected from the total set of portfolios, $\mathcal{V} = \cup_p \mathcal{A}_p$. We compare the actual and expected total overlap across postal codes areas, controlling for portfolio size,

$$N^c_{TOT} = \sum_p o(\mathcal{A}^S_p) \qquad \text{with} \qquad E\left[\tilde{N}^c_{TOT}\right] = \sum_p o^{\text{Exp}}(\mathcal{A}^S_p),$$

where we vary the portfolio size, S, between 1 and N.

The advantage of this method, which is admittedly more complex than the simple test in the main paper, is that instead of assuming that each household picks each stock with probability $\frac{1}{N}$, we use the empirical distribution function of portfolios to test whether the actual portfolios observed within postal codes are consistent with a "random allocation" of these portfolios. This approach is therefore robust to some stocks being much more widely held than others. Note that our total test contains $P = 2,923$ postal codes, with an average of 139 households each. The overrepresentation of overlap of 43.6%-139% we find is therefore extremely statistically significant.

B.2 Additional Tables

The stocks in our sample are shown in Table B1. The estimate sensitivity coefficient when excluding the Helsinki area is shown in Table B2. The estimated sensitivity coefficient when including only distant observations are shown in Table B3.

Table B1: Stocks in sample

This table lists the 125 stocks issued by companies that are headquartered in Finland and are included in our sample. For companies with A- and B-shares, both shares are included in our sample.

1	Bank of Aland Plc A	2	Pohjola Group Plc
3	Norvestia Plc	4	Kesko Corporation B
5	Stockmann Plc A	6	Stockmann Plc B
7	Tieto Corporation	8	Amer Sports Corporation
9	Fiskars Corporation	10	Fiskars Corporation K
11	Huhtamki Oyj	12	Instrumentarium
13	Kone Corporation B	14	Metsa Board Oyj A
15	Metsa Board Oyj B	16	Nokia Corporation
17	Tamro Oyj	18	Tamfelt Corp.
19	Tamfelt Corporation Ord. shares	20	Bank of Aland Plc B
21	Uponor Oyj	22	Outokumpu Oyj
23	Citycon Oyj	24	Polar Real Estate Corp.
25	Raisio Plc Vaihto-osake	26	Birka Line Abp B
27	Pohjola Bank A	28	Finnair Oyj
29	Sampo Plc A	30	Stromsdal Corporation
31	Apetit Plc	32	Rautaruukki Corporation
33	Finnlines Plc	34	Silja Oyj Abp
35	Wartsila Corporation A	36	Wartsila Corporation
37	Tiimari Plc	38	Kemira Oyj
39	Ponsse Oyj	40	Viking Line Abp
41	Nokian Tyres Plc	42	Biohit Oyj B
43	Konecranes Plc	44	Stora Enso Oyj A
45	Stora Enso Oyj R	46	UPM-Kymmene Corporation
47	HKScan Oyj A	48	PKC Group Oyj
49	Incap Corporation	50	Atria Plc A
51	Payry PLC	52	Sponda Plc
53	Technopolis Plc	54	Valoe Oyj
55	Alma Media Corporation 1	56	Alma Media Corporation 2
57	Ramirent Plc	58	Fortum Corporation
59	Bittium Corporation	60	Yomi Plc
61	Rapala VMC Corporation	62	Sonera Oyj
63	Eimo Oyj	64	Innofactor Plc
65	Marimekko Corporation	66	SanomaWSOY Corporation A
67	Sanoma Corporation	68	Teleste Corporation
69	Oral Hammaslakarit Plc	70	Perlos Corporation
71	Metso Corporation	72	Talentum Oyj
73	Kesko Corporation A	74	Aldata Solution Oyj
75	Digia Plc	76	Solteq Oyj
77	Ixonos Plc	78	Aspo Plc
79	Aspocomp Group Plc	80	Dovre Group Plc
81	Trainers House Plc	82	Comptel Corporation
83	SSH Communications Security Oyj	84	Basware Corporation
85	Wulff Group Plc	86	Saunalahti Group Oyj
87	Etteplan Oyj	88	QPR Software Plc
89	eQ Oyj	90	Tekla Corporation
91	Sievi Capital plc	92	Sentera Plc
93	Okmetic Oyj	94	CapMan Plc B
95	Vacon Plc	96	eQ Oyj
97	Evox Rifa Group Plc	98	Componenta Corporation
99	Glaston Corporation	100	Tecnotree Corporation
101	Lassila & Tikanoja Plc	102	Suominen Oyj
103	Revenio Group Corporation	104	Biotie Therapies Corp.
105	Ilkka-Yhtyma Oyj 2	106	Neo Industrial Oyj
107	Orion Corporation A	108	Orion Corporation B
109	Raisio Plc K	110	Saga Furs Oyj C
111	YIT Corporation	112	Stonesoft Corporation
113	F-Secure Corporation	114	Chips Corporation B
115	Efore Plc	116	Hackman Oyj Abp
117	Honkarakenne Oyj B	118	Lemminkainen Corporation
119	Evia Oyj	120	Martela Oyj A
121	Olvi Plc A	122	Cramo Oyj
123	Tulikivi Oyj A	124	Elecster Oyj A
125	Vaisala Corporation A		

Table B2: Estimate of sensitivity coefficient excluding Helsinki area

This table estimates the sensitivity coefficient, κ, using only firms and households outside of the Helsinki area (with 5-digit postal codes). Panel A includes observations with $\text{amt}_{pn} = 0$, replaced with $\text{amt}_{pn} = 1$ (one Finnish mark, corresponding to approximately USD 0.17). Panel B excludes observations with $\text{amt}_{pn} = 0$. Univariate in Column (1), including risk aversion in Column (2), including risk aversion and stock distributions in Column (3), panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in Column (4). Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)
Panel A				
Sensitivity coefficient, κ	4.228***	3.868***	3.771***	3.771***
Standard error	0.069	0.071	0.073	0.630
log risk aversion, g				
-average		-3.806	-3.771	
-max		-1.097	-1.073	
-min		-12.393	-12.355	
log return distribution, s				
-average			0	
-max			5.418	
-min			-1.797	
R^2	0.051	0.308	0.487	0.487
Adj. R^2	0.051	0.283	0.469	0.469
$N = 69,368$				
Panel B				
Sensitivity coefficient, κ	1.824***	1.892***	2.754***	2.754***
Standard error	0.074	0.082	0.071	0.325
log risk aversion, g				
-average		-8.072	-8.084	
-max		-2.681	-2.282	
-min		-12.392	-12.757	
log return distribution, s				
-average			0	
-max			1.787	
-min			-3.197	
R^2	0.028	0.249	0.501	0.501
Adj. R^2	0.028	0.159	0.440	0.440
$N = 20,787$				

Table B3: Estimate of sensitivity coefficient including only distant observations

This table estimates the sensitivity coefficient, κ, including only postal code/firm observation at distances larger than d_0, and excluding $\text{amt}_{pn} = 0$ observations. Panel A sets d_0 to 0.01, corresponding to a minimum distance between postal codes of account holder and firm headquarters of about 8 miles, whereas in Panel B $d_0 = 0.03$, corresponding to a minimal distance of about 24 miles (39 km). Univariate in Column (1), including risk aversion in Column (2), including risk aversion and stock distributions in Column (3), panel regression with postal code and stock fixed effects and robust standard errors double-clustered at the postal code and firm level in Column (4). Statistical significance levels: * = 0.01, ** = 0.001, *** = 0.0001.

	(1)	(2)	(3)	(4)
Panel A				
Sensitivity coefficient, κ	1.772***	1.844***	2.397***	2.397***
Standard error	0.036	0.063	0.048	0.392
log risk aversion, g				
-average		−8.480	−7.899	
-max		−4.543	−2.282	
-min		−12.415	−12.609	
log return distribution, s				
-average			0	
-max			4.142	
-min			−3.952	
R^2	0.018	0.186	0.614	0.614
Adj R^2	0.018	0.168	0.605	0.605
$N = 129,802$				
Panel B				
Sensitivity coefficient, κ	0.931***	0.991***	1.662***	1.662***
Standard error	0.039	0.070	0.053	0.326
log risk aversion, g				
-average		−8.187	−7.680	
-max		−4.000	−2.561	
-min		−11.798	−12.117	
log return distribution, s				
-average			0	
-max			4.067	
-min			−3.977	
R^2	0.005	0.160	0.599	0.599
Adj. R^2	0.005	0.139	0.589	0.589
$N = 119,874$				

C Extension of the Model to Epstein-Zin Preferences

In this appendix, we derive our results for the case where investors have Epstein and Zin (1989) preferences. Proposition 1 in the main text is independent of preferences and so is not repeated in this appendix. Propositions 2, 3, and 4 in the main text, which are for the case of mean-variance preferences, are extended below to the case of Epstein and Zin (1989) preferences in Propositions C1, C2, and C3, respectively.

C.1 Certainty-equivalent

We start by defining a household's personal certainty-equivalent, which is simply the standard definition of a certainty equivalent, but taken under the household's personal beliefs, together with a penalty.

Definition C1. *The date-t personal certainty-equivalent of date-t+dt household utility is given by*

$$\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] + U_{h,t}L_{h,t}dt, \tag{C1}$$

where $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ *is defined by*

$$u_{\gamma_h}\left(\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\right) = E_t^{\mathbb{P}^h}[u_{\gamma_h}\left(U_{h,t+dt}\right)], \tag{C2}$$

$$u_{\gamma_h}(x) = \frac{x^{1-\gamma_h}}{1-\gamma_h},$$

and $L_{h,t}$ *is the date-t value of the penalty for deviation from the rational expectations benchmark.*

The first part of the definition of the personal certainty-equivalent in (C1) is just the standard definition of a certainty-equivalent based on power utility with relative risk aversion γ_h, but using the personal belief $\mathbb{P}^h$. The second part of the definition hinges on the choice of the penalty $L_{h,t}$. We can already see that given a portfolio which is long the risky assets, a household will minimize its personal certainty-equivalent by selecting beliefs that are pessimistic, but not so pessimistic that the penalty term becomes overly large, as summarized below:

$$\underbrace{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]}_{\text{becomes smaller}} + \underbrace{U_{h,t}L_{h,t}}_{\text{becomes larger}}\, dt$$

The tradeoff between a smaller power-utility based certainty equivalent $\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ and larger penalty lies at the heart of this model of belief formation. As such, the choice of the penalty function $L_{h,t}$ is crucial, and is given below.

Definition C2. *The expression for* $L_{h,t}$ *is given by*

$$L_{h,t} = \frac{1}{2\gamma_h}\widehat{L}_{h,t},$$

where $\widehat{L}_{h,t}$ *is the* distance-weighted loss *for household* h*:*

$$\widehat{L}_{h,t} = \boldsymbol{\nu}_{h,t}^{\top}\Sigma^{-1}F_h\Sigma^{-1}\boldsymbol{\nu}_{h,t} = \sum_{n=1}^{N}\frac{\phi_{hn}}{1-\phi_{hn}}\frac{\nu_{hn,t}^2}{\sigma_n^2},$$

and F_h *is the* $N \times N$ *diagonal matrix*

$$F_h = diag\left(\frac{\phi_{h1}}{1-\phi_{h1}},\ldots,\frac{\phi_{hN}}{1-\phi_{hN}}\right), \quad \phi_{hn} \in [0,1], \quad n \in \{1,\ldots,N\}.$$

The following lemma shows how a household selects its personal beliefs by choosing a vector of divergences, $\boldsymbol{\nu}_{h,t}$, assuming a constant investment opportunity set.

Lemma C1. *The date-*t *personal certainty equivalent of date-*$t+dt$ *household utility based on the personal belief* $\mathbb{P}^h$ *is given by*

$$\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t} \times \left(\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\boldsymbol{\nu}_{h,t}^{\top}\boldsymbol{\omega}_{h,t} + L_{h,t}\right)dt, \tag{C3}$$

where

$$\mu_{h,t}[U_{h,t+dt}] = E_t[U_{h,t+dt}] - \frac{1}{2}\gamma_h\, U_{h,t}\, E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right], \tag{C4}$$

$U_{W_{h,t}} = \frac{\partial U_{h,t}}{\partial W_{h,t}}$ *is the partial derivative of household* h*'s utility with respect to its wealth, and* E_t *denotes the conditional expectation at* t *under the reference measure. At date* t*, a household selects its personal belief* $\mathbb{P}^h$ *by choosing the vector of personal divergences* $\boldsymbol{\nu}_{h,t}$ *which minimizes its date-*t *personal certainty equivalent,* $\mu_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}]$*,*

$$\boldsymbol{\nu}_{h,t} = \arg\min_{\boldsymbol{\nu}_{h,t}}\left(\frac{W_{h,t}U_{W_{h,t}}}{U_{h,t}}\sum_{n=1}^{N}\nu_{hn,t}\omega_{hn,t} + \frac{1}{2\gamma_h}\sum_{n=1}^{N}\frac{\phi_{hn}}{1-\phi_{hn}}\frac{\nu_{hn,t}^2}{\sigma_n^2}\right). \tag{C5}$$

Proof of Lemma C1

The definition of the certainty equivalent in (C2) implies that

$$\widehat{\mu}_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = E_t^{\mathbb{P}^h}\left[U_{h,t+dt}^{1-\gamma_h}\right]^{\frac{1}{1-\gamma_h}}.$$

Therefore

$$\widehat{\mu}_{h,t}^{\boldsymbol{\nu}}[U_{h,t+dt}] = E_t^{\mathbb{P}^h}\left[U_{h,t+dt}^{1-\gamma_h}\right]^{\frac{1}{1-\gamma_h}} = E_t^{\mathbb{P}^h}\left[U_{h,t}^{1-\gamma_h} + d(U_{h,t}^{1-\gamma_h})\right]^{\frac{1}{1-\gamma_h}}.$$

Applying Ito's Lemma, we obtain

$$\begin{aligned} d(U_{h,t}^{1-\gamma_h}) &= (1-\gamma_h)U_{h,t}^{-\gamma_h}dU_{h,t} - \frac{1}{2}(1-\gamma_h)\gamma_h U_{h,t}^{-\gamma_h-1}(dU_{h,t})^2 \\ &= (1-\gamma_h)U_{h,t}^{1-\gamma_h}\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]. \end{aligned}$$

Therefore

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = E_t^{\mathbb{P}^h}\left[U_{h,t+dt}^{1-\gamma_h}\right]^{\frac{1}{1-\gamma_h}} = U_{h,t}\left(E_t\left[1+(1-\gamma_h)\left[\frac{dU_{h,t}}{U_{h,t}} - \frac{1}{2}\gamma_h\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma_h}}$$

$$= U_{h,t}\left(1+(1-\gamma_h)\left[E_t^{\mathbb{P}^h}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h E_t^{\mathbb{P}^h}\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma_h}}$$

$$= U_{h,t}\left(1+(1-\gamma_h)\left[E_t^{\mathbb{P}^h}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right]\right)^{\frac{1}{1-\gamma_h}}.$$

Hence,

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = U_{h,t}\left(1 + E_t^{\mathbb{P}^h}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right) + o(dt).$$

Therefore, in the continuous time limit, we obtain

$$\frac{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[dU_{h,t+dt}]}{dt} = \frac{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] - U_{h,t}}{dt} = U_{h,t}\left(\frac{1}{dt}E_t^{\mathbb{P}^h}\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h\frac{1}{dt}E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right).$$

From Girsanov's Theorem

$$E_t^{\mathbb{P}^h}\left[\frac{dU_{h,t}}{U_{h,t}}\right] = E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] + E_t\left[\frac{dU_{h,t}}{U_{h,t}}\frac{dM_{h,t}}{M_{h,t}}\right].$$

Therefore

$$\begin{aligned}\frac{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[dU_{h,t}]}{dt} &= \frac{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] - U_{h,t}}{dt} \\ &= U_{h,t}\left(\frac{1}{dt}E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h\frac{1}{dt}E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right] + \frac{1}{dt}E_t\left[\frac{dU_{h,t}}{U_{h,t}}\frac{dM_{h,t}}{M_{h,t}}\right]\right).\end{aligned}$$

It follows from the above expression that the certainty equivalent operator, $\mu_{h,t}[\cdot]$, is given by

$$\begin{aligned}\frac{\mu_{h,t}[dU_{h,t}]}{dt} &= \frac{\mu_{h,t}[U_{h,t+dt}] - U_{h,t}}{dt} \\ &= U_{h,t}\left(\frac{1}{dt}E_t\left[\frac{dU_{h,t}}{U_{h,t}}\right] - \frac{1}{2}\gamma_h\frac{1}{dt}E_t\left[\left(\frac{dU_{h,t}}{U_{h,t}}\right)^2\right]\right).\end{aligned}$$

Therefore

$$\frac{\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[dU_{h,t}]}{dt} = \frac{\mu_{h,t}[dU_{h,t}]}{dt} + \frac{1}{dt}U_{h,t}E_t\left[\frac{dU_{h,t}}{U_{h,t}}\frac{dM_{h,t}}{M_{h,t}}\right],$$

that is,

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[dU_{h,t}] = \mu_{h,t}[dU_{h,t}] + U_{h,t}E_t\left[\frac{dU_{h,t}}{U_{h,t}}\frac{dM_{h,t}}{M_{h,t}}\right],$$

which implies

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t} E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right].$$

If the investment opportunity set is constant, then $U_{h,t}$ is a function solely of $W_{h,t}$ because there are no other state variables. Hence,

$$E_t \left[\frac{dU_{h,t}}{U_{h,t}} \frac{dM_{h,t}}{M_{h,t}} \right] = \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} dt,$$

and so

$$\widehat{\mu}^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] = \mu_{h,t}[U_{h,t+dt}] + U_{h,t} \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} dt.$$

Equation (C3) follows from the above expression and (C1). □

When a household chooses its beliefs by choosing a vector of divergences $\boldsymbol{\nu}_{h,t}$, it does so in order to minimize the impact of its losses on its personalized certainty equivalent. The presence of the term $\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \sum_{n=1}^{N} \nu_{hn,t} \omega_{hn,t}$ in (C5) gives a household the desire to make its divergences in portfolio-return expectations, that is, $\sum_{n=1}^{N} \nu_{hn,t} \omega_{hn,t}$, as negative possible. This desire is clearly a departure from rationality, but it is tempered by the size of its distance-weighted losses, represented by $L_{h,t} = \frac{1}{2\gamma_h} \sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2}$.

In general, distance-weighted costs of deviating from rational expectations are not infinitely large, and so a household faces a trade off between more negative personal divergences in portfolio return expectations and larger distance-weighted losses. Because this optimization problem is linear-quadratic it has the following closed-form solution.

Lemma C2. *A household's personal divergence vector is given by*

$$\boldsymbol{\nu}_{h,t} = -\gamma_h \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} (\Sigma^{-1} F_h \Sigma^{-1})^{-1} \boldsymbol{\omega}_{h,t}, \tag{C6}$$

with each element being

$$\nu_{hn,t} = -\gamma_h \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \sigma_n^2 \frac{1-\phi_{hn}}{\phi_{hn}} \omega_{hn,t}.$$

Proof of Lemma C2

If the investment opportunity set is constant, then the personal divergence vector is obtained from the following optimization problem:

$$\inf_{\boldsymbol{\nu}_{h,t}} \left(\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^{\top} \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma_h} \sum_{n=1}^{N} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}^2}{\sigma_n^2} \right).$$

The FOC for the above problem is

$$0 = \frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \omega_{hn,t} + \frac{1}{\gamma_h} \frac{\phi_{hn}}{1-\phi_{hn}} \frac{\nu_{hn,t}}{\sigma_n^2}.$$

Equation (C6) follows from the above equation. □

C.2 Intertemporal Aggregator with Endogenous Beliefs

Suppose that each household maximizes its date-t utility level, $U_{h,t}$, defined as in Epstein and Zin (1989) by an intertemporal aggregation of date-t consumption flow, $C_{h,t}$, and the date-t personal certainty-equivalent of date $t+dt$ utility:[16]

$$U_{h,t} = \mathcal{A}_h(C_{h,t}, \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]),$$

where $\mathcal{A}_h(\cdot,\cdot)$ is the time aggregator, defined by

$$\mathcal{A}_h(x,y) = \left[(1-e^{-\delta_h dt})x^{1-\frac{1}{\psi_h}} + e^{-\delta_h dt}y^{1-\frac{1}{\psi_h}}\right]^{\frac{1}{1-\frac{1}{\psi_h}}}, \tag{C7}$$

in which $\delta_h > 0$ is the rate of time preference, $\psi_h > 0$ is the elasticity of intertemporal substitution, and $\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ is the date-t personal certainty equivalent of $U_{h,t+dt}$ given in Definition C1.

If a household's beliefs coincided with the objective physical measure, it would choose its consumption rate, $C_{h,t}$, and portfolio policy, $\boldsymbol{\omega}_{h,t}$, to solve the standard choice problem,

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \mu_{h,t}[U_{h,t+dt}]\Big), \tag{C8}$$

where $\mu_{h,t}[U_{h,t+dt}]$ is the standard certainty equivalent, given in (C4).

In general, with endogenous household beliefs that do not coincide with rational expectations, the time aggregator $\mathcal{A}(\cdot)$ in (C7) is unchanged—all we need to do is to replace the maximization of the standard certainty-equivalent in (C8), $\sup_{\boldsymbol{\omega}_{h,t}} \mu_{h,t}[U_{h,t+dt}]$, with the combined maximization and minimization of the personal certainty equivalent, $\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]$ to obtain

$$\sup_{C_{h,t}} \mathcal{A}\Big(C_{h,t}, \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}]\Big). \tag{C9}$$

A household, because of the impact of its location, chooses $\boldsymbol{\nu}_{h,t}$ to *minimize* its personal certainty equivalent. By comparing (C8) and (C9), we can see that once a household has chosen the vector of personal divergences, $\boldsymbol{\nu}_{h,t}$, to adjust the expected returns of each firm to account for its beliefs, it makes consumption and portfolio choices in the standard way.

To solve a household's consumption-portfolio choice problem under subjective beliefs we use Ito's Lemma to derive the continuous-time limit of (C9), which leads to a Hamilton-Jacobi-Bellman equation that is given in the lemma below.

[16]The only difference with Epstein and Zin (1989) is that we work in continuous time, whereas they work in discrete time. The continuous-time version of recursive preferences is known as stochastic differential utility (SDU), and is derived formally in Duffie and Epstein (1992).

Lemma C3. *The utility function of a household with endogenous beliefs is given by the following Hamilton-Jacobi-Bellman equation:*

$$0 = \sup_{C_{h,t}} \left(\delta\, u_\psi \left(\frac{C_{ht}}{U_{ht}} \right) + \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} \frac{1}{U_{h,t}} \mu^{\boldsymbol{\nu}}_{h,t} \left[\frac{dU_{h,t}}{dt} \right] \right), \quad \textit{where} \tag{C10}$$

$$u_\psi(x) = \frac{x^{1-\frac{1}{\psi}} - 1}{1 - \frac{1}{\psi}}, \psi > 0, \quad \textit{and}$$

$$\mu^{\boldsymbol{\nu}}_{h,t}\left[dU_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt} - U_{h,t}\right] = \mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right] - U_{h,t},$$

with $\mu^{\boldsymbol{\nu}}_{h,t}\left[U_{h,t+dt}\right]$ *given in* (C3).

Proof of Lemma C3

Writing out (C9) explicitly gives

$$U_{h,t}^{1-\frac{1}{\psi_h}} = (1 - e^{-\delta_h dt}) C_{h,t}^{1-\frac{1}{\psi_h}} + e^{-\delta_h dt} \left(\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] \right)^{1-\frac{1}{\psi_h}},$$

where for ease of notation sup and inf have been suppressed. Now

$$\begin{aligned}
\left(\mu^{\boldsymbol{\nu}}_{h,t}[U_{h,t+dt}] \right)^{1-\frac{1}{\psi_h}} &= \left(U_{h,t} + \mu^{\boldsymbol{\nu}}_{h,t}[dU_{h,t}] \right)^{1-\frac{1}{\psi_h}} \\
&= U_{h,t}^{1-\frac{1}{\psi_h}} \left(1 + \mu^{\boldsymbol{\nu}}_{h,t} \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right)^{1-\frac{1}{\psi_h}} \\
&= U_{h,t}^{1-\frac{1}{\psi_h}} \left(1 + \left(1 - \frac{1}{\psi_h} \right) \mu^{\boldsymbol{\nu}}_{h,t} \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right) + o(dt).
\end{aligned}$$

Hence,

$$U_{h,t}^{1-\frac{1}{\psi_h}} = \delta C_{h,t}^{1-\frac{1}{\psi_h}} dt + U_{h,t}^{1-\frac{1}{\psi_h}} \left(1 + \left(1 - \frac{1}{\psi_h} \right) \mu^{\boldsymbol{\nu}}_{h,t} \left[\frac{dU_{h,t}}{U_{h,t}} \right] \right) - \delta_h U_{h,t}^{1-\frac{1}{\psi_h}} dt + o(dt),$$

from which we obtain (C10). □

From (C10), we see that the Hamilton-Jacobi-Bellman equation can be decomposed into a portfolio-optimization problem and an intertemporal consumption-choice problem. Given that the investment opportunity set is constant over time, the maximized household utility is a constant multiple of the household's wealth. In this case, the lemma below shows that the optimization problem in the Hamilton-Jacobi-Bellman equation can be decomposed into two parts, a portfolio-optimization problem and an intertemporal consumption-choice problem.

Lemma C4. *The household's optimization problem consists of two parts, a single-period mean-variance optimization*

$$\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}),$$

and an intertemporal consumption-choice problem

$$0 = \sup_{C_{h,t}} \left(\delta_h \, u_\psi \left(\frac{C_{h,t}}{U_{h,t}} \right) - \frac{C_{h,t}}{W_{h,t}} + \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) \right), \quad \textit{where} \tag{C11}$$

$$MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \left(\boldsymbol{\alpha} - i\mathbf{1}\right)^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + \frac{1}{2\gamma_h} \boldsymbol{\nu}_{h,t}^\top \Sigma F_h^{-1} \Sigma \boldsymbol{\nu}_{h,t}, \tag{C12}$$

$\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_N)^\top$*, and* $\mathbf{1}$ *denotes the* $N \times 1$ *unit vector.*

Proof of Lemma C4

From (C3) in Lemma C1, we have

$$\mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] = E_t \left[\frac{dU_{h,t}}{U_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dU_{h,t}}{U_{h,t}} \right)^2 \right] + \left(\frac{W_{h,t} U_{W_{h,t}}}{U_{h,t}} \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + L_{h,t} \right) dt.$$

We use the Ansatz

$$U_{h,t} = u_h W_{h,t}, \tag{C13}$$

where u_h is a constant. Consequently,

$$\frac{dU_{h,t}}{U_{h,t}} = \frac{dW_{h,t}}{W_{h,t}},$$

and so.

$$\mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] = E_t \left[\frac{dW_{h,t}}{W_{h,t}} \right] - \frac{1}{2}\gamma_h E_t \left[\left(\frac{dW_{h,t}}{W_{h,t}} \right)^2 \right] + \left(\boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + L_{h,t} \right) dt.$$

Hence,

$$\frac{1}{dt} \mu_{h,t}^{\boldsymbol{\nu}} \left[\frac{dU_{h,t}}{U_{h,t}} \right] = -\frac{C_{h,t}}{W_{h,t}} + i + (\boldsymbol{\alpha} - i_t \mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma_h \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + L_{h,t}.$$

We now define

$$MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + (\boldsymbol{\alpha} - i_t \mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma_h \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t} + \boldsymbol{\nu}_{h,t}^\top \boldsymbol{\omega}_{h,t} + L_{h,t},$$

which can be rewritten as (C12). Therefore, provided the Ansatz in (C13) is true, our result follows from (C10) in Lemma C3.

We show that the Ansatz in (C13) is true by explicitly solving for u_h. The first part of this proof consists of solving jointly for household beliefs and portfolios, which is done in the proof of Proposition C2. The second part consists of solving for optimal consumption and substituting the optimal controls into (C11) and hence solving for $U_{h,t}$, which is also given in the proof of Proposition C2. □

C.3 Beliefs and Portfolio Weights

Given that the investment opportunity set is constant over time, the maximized household utility is a constant multiple of the household's wealth, which allows us to get the following simple expressions for the choice variables of the household.

Proposition C1. *The chosen vector of personal divergences is*

$$\boldsymbol{\nu}_h = -(I + VF_h\Sigma^{-2})^{-1}(\boldsymbol{\alpha} - i\mathbf{1}). \tag{C14}$$

For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, the chosen divergence in household h's beliefs about the expected return for firm n is

$$\nu_{hn} = -(\alpha_n - i)(1 - \phi_{hn}). \tag{C15}$$

The divergence in household h's beliefs about the expected return for firm n is determined by its psychological distance from the firm:

$$\nu_{hn} = \begin{cases} -(\alpha_n - i)(1 - e^{-\kappa d_{hn}}), & d_{hn} \leq \overline{d} \\ -(\alpha_n - i), & d_{hn} > \overline{d}. \end{cases}$$

Equation (C15) is given in terms of psychological proximities, ϕ_{hn}, which are unobservable. We show in the next proposition how ϕ_{hn} can be related to portfolio weights, which, in contrast to beliefs, are observable. We then provide the proofs for these two propositions.

Proposition C2. *The vector of optimal portfolio weights is*

$$\boldsymbol{\omega}_{h,t} = \frac{1}{\gamma_h} V^{-1}\big(\boldsymbol{\alpha} - i\mathbf{1} + \boldsymbol{\nu}_h\big). \tag{C16}$$

For the special case in which the correlation between assets $\rho_{nm} = 0$ for $n \neq m$, the optimal proportion of wealth invested in firm n by household h is

$$\begin{aligned} \omega_{hn} &= \frac{1}{\gamma_h}\frac{\alpha_n - i + \nu_{hn}}{\sigma_n^2} \\ &= \frac{1}{\gamma_h}\frac{\alpha_n - i}{\sigma_n^2}\phi_{hn}. \end{aligned}$$

In terms of psychological distance, the above portfolio weight is given by

$$\omega_{hn} = \begin{cases} \frac{1}{\gamma_h}\frac{\alpha_n - i}{\sigma_n^2} e^{-\kappa d_{hn}}, & d_{hn} \leq \overline{d} \\ 0, & d_{hn} > \overline{d}. \end{cases}$$

Proof of Propositions C1 and C2

Substituting (C13) into (C6) and exploiting the fact that Σ and F_h are diagonal matrices and hence commute with each other, we obtain

$$\boldsymbol{\nu}_{h,t} = -\gamma_h \Sigma^2 F_h^{-1}\boldsymbol{\omega}_{h,t}. \tag{C17}$$

Again, exploiting the commutativity of diagonal matrices, we can write the portfolio choice problem in (C12) as

$$\sup_{\boldsymbol{\omega}_{h,t}} (\boldsymbol{\alpha} - i\mathbf{1})^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma_h \boldsymbol{\omega}_{h,t}^\top \left(V + \Sigma^2 F_h^{-1}\right) \boldsymbol{\omega}_{h,t}.$$

The above linear-quadratic problem has a unique interior solution given by (C16). Substituting (C16) into (C17) and simplifying gives (C14). We can also rewrite the expression for $\boldsymbol{\omega}_h$ in (C16) in terms of the personal divergence measure:

$$\boldsymbol{\omega}_h = \frac{1}{\gamma_h} V^{-1}(\boldsymbol{\alpha} + \boldsymbol{\nu}_h - i\mathbf{1}),$$

where

$$\boldsymbol{\nu}_h = -(I + V F_h \Sigma^{-2})(\boldsymbol{\alpha} - i\mathbf{1}).$$

Substituting the optimal controls (C14) and (C16) into (C12), and simplifying gives

$$\sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}) = i + \frac{1}{2\gamma_h}(\boldsymbol{\alpha} - i\mathbf{1})^\top (V + \Sigma^2 F_h^{-1})^{-1}(\boldsymbol{\alpha} - i\mathbf{1}).$$

From the Hamilton-Jacobi-Bellman equation in (C10) (and also the specialized version in (C11)), the first-order condition with respect to consumption is

$$\delta_h \left(\frac{C_{h,t}}{W_{h,t}}\right)^{-\frac{1}{\psi_h}} = \frac{U_{h,t}}{W_{h,t}}. \tag{C18}$$

From the above expression, we can see that $\frac{U_{h,t}}{W_{h,t}}$ is a constant if and only if $\frac{C_{h,t}}{W_{h,t}}$ is a constant. Substituting the above first-order condition into (C11) to eliminate $U_{h,t}$ and then solving for $\frac{C_{h,t}}{W_{h,t}}$ gives

$$c_h = \frac{C_{h,t}}{W_{h,t}} = \delta_h \psi_h + (1 - \psi_h) \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t}).$$

We can thus see that $\frac{C_{h,t}}{W_{h,t}}$ is indeed a constant. Also, from (C18) we can see that

$$u_h = \left[\frac{\delta_h^{\psi_h}}{c_h}\right]^{\frac{1}{\psi_h - 1}}.$$

Therefore, we obtain the following result for a household's maximised utility

$$u_h = \frac{U_{h,t}}{W_{h,t}} = \left[\frac{\delta_h^{\psi_h}}{\delta_h \psi_h + (1 - \psi_h) \sup_{\boldsymbol{\omega}_{h,t}} \inf_{\boldsymbol{\nu}_{h,t}} MV_h(\boldsymbol{\omega}_{h,t}, \boldsymbol{\nu}_{h,t})}\right]^{\frac{1}{\psi_h - 1}}.$$

□

C.4 Household Welfare

We now compute the welfare loss for a household with Epstein and Zin (1989) utility. To measure the welfare loss for a household with personal beliefs given by $\mathbb{P}^h$ instead of $\mathbb{P}$, we compute the reduction in utility which occurs when the optimal controls for a household which has distorted beliefs are substituted into the lifetime utility function of a household whose beliefs are given by the reference probability measure under rational expectations, $\mathbb{P}$.

Proposition C3. *The utility per unit wealth, v_h (which is also a certainty equivalent), for a household whose beliefs are given by the reference probability measure $\mathbb{P}$, but uses the consumption and portfolio policies computed under distorted beliefs $\mathbb{P}^h$, is given by*

$$v_h\Big(c_h(\mathbb{P}^h), \boldsymbol{\omega}_h(\mathbb{P}^h)\Big) = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + (1-\psi_h)(U_h^{MV}(\boldsymbol{\omega}_h(\mathbb{P}^h)) - c_h(\mathbb{P}^h)}\right]^{\frac{1}{1-1/\psi_h}} c_h(\mathbb{P}^h),$$

where $c_h(\mathbb{P}^h) = C_h(\mathbb{P}^h)/W_h$ is the optimal consumption-wealth ratio and $\boldsymbol{\omega}_h(\mathbb{P}^h)$ is the optimal portfolio choice under a household's distorted beliefs.

Proof of Proposition C3

We measure household-level welfare losses by taking the optimal controls for a household which has biased beliefs and substituting them into the utility of a household whose beliefs are given by the reference probability measure $\mathbb{P}$.

For an exogenously specified consumption and portfolio rule, (C11) reduces to

$$0 = \delta_h \, u_{\psi_h}\left(\frac{C_{h,t}}{U_{h,t}}\right) - \frac{C_{h,t}}{W_{h,t}} + U_h^{MV}(\boldsymbol{\omega}_{h,t}), \tag{C19}$$

where $U_h^{MV}(\boldsymbol{\omega}_{h,t})$ is the utility of a mean-variance investor whose beliefs are given by the reference probability measure $\mathbb{P}$,

$$U_h^{MV}(\boldsymbol{\omega}_{h,t}) = i + \big(\boldsymbol{\alpha} - i\mathbf{1}\big)^\top \boldsymbol{\omega}_{h,t} - \frac{1}{2}\gamma \boldsymbol{\omega}_{h,t}^\top V \boldsymbol{\omega}_{h,t}.$$

By making $U_{h,t}$ the subject of (C19), we see that for a for a household whose beliefs are given by the reference probability measure $\mathbb{P}$, utility per unit wealth, v_h is given in terms of consumption-portfolio choices by

$$v_h(c_h, \boldsymbol{\omega}_h) = \left[\frac{\psi_h \delta_h}{\psi_h \delta_h + (1-\psi_h)(U_h^{MV}(\boldsymbol{\omega}_h) - c_h)}\right]^{\frac{1}{1-1/\psi_h}} c_h,$$

where $c_h = C_h/W_h$ is the consumption-wealth ratio. □

D An Alternative Interpretation of the Penalty Function

Consider a situation where an agent A is to take an action x associated with the object B, for which the agent's trust is ϕ. If B lies in A's trust region, the payoff/utility associated with this action is[17]

$$\Pi_A = c_A x - \frac{b_A}{2} x^2.$$

However, if B is not within A's trust region, then the payoff is

$$\Pi_{\neg A} = -\frac{b_{\neg A}}{2} x^2.$$

The situation is thus such that there is no upside associated with taking an action for an object that does not belong to A's trust region. A special case occurs when $b_A = b_{\neg A}$, in which case the costs associated with actions are the same regardless of whether B belongs to A's trust region or not. The agent, if behaving as an expected payoff/utility maximizer, then solves

$$\max_x \; \phi \left(c_A x - \frac{b_A}{2} x^2 \right) + (1-\phi) \left(-\frac{b_{\neg A}}{2} x^2 \right),$$

leading to the action

$$x = c_A \frac{\phi}{b_A \phi + b_{\neg A}(1-\phi)},$$

and expected payoff/utility

$$E[\Pi] = \frac{c_A^2}{2} \frac{\phi}{b_A \phi + b_{\neg A}(1-\phi)}.$$

An observationally equivalent formulation of the problem solved by the agent is that (s)he trades off how much to adjust the upside associated with an action, by revising down the coefficient c_A, against a loss function of deviating from c_A. Specifically, consider the problem

$$\hat{\Pi} = \max_x \min_\nu \; (c_A - \nu) x - \frac{b_A}{2} x^2 + \frac{1}{b_{\neg A}} \frac{\phi}{1-\phi} \nu^2. \tag{D1}$$

It is straightforward to verify that this problem leads to the same action x as the previous formulation, and $E[\Pi] = \phi \hat{\Pi}$. Note that (D1) technically excludes the case $\phi = 1$. For $\phi = 1$, we interpret the problem as the limit $\phi \nearrow 1$, which will always have $\nu = 0$.

We define the loss function

$$L(\nu) = \frac{1}{2} \frac{\phi}{1-\phi} \nu^2,$$

and rewrite (D1) as

$$\hat{\Pi} = \max_x \min_\nu \; (c_A - \nu) x - \frac{b_A}{2} x^2 + \frac{1}{b_{\neg A}} L(\nu).$$

[17] In a portfolio choice context, the quadratic term could represent a negative contribution to expected utility from variance, for an investor with mean-variance preferences.

The part

$$\min_{\nu}\ (c_A - \nu)x - \frac{b_A}{2}x^2 + \frac{1}{b_{\neg A}}L(\nu)$$

now has the natural interpretation, namely that the agent chooses how much to revise her beliefs about the benefits of action, captured by the coefficient $c_A - \nu$, against the associated losses of deviating from the base beliefs c_A, captured by $L(\nu)$. The loss function L has the intuitive behavior that for ϕ close to zero (low trust), it imposes very mild costs on an agent who chooses conservative belief deviations. In contrast, for ϕ close to one, the costs are very severe, providing strong incentives for the agent to make almost no belief revisions. The specification (D1) is similar to what arises in the robust control literature, see Hansen and Sargent (2008), thereby connecting our model to this strand of literature, and also more broadly to the literature on ambiguity and ambiguity aversion. Indeed, the distrust an agent associates with an object, and the associated belief adjustment may be interpreted as consequences of ambiguity and ambiguity aversion.

We have a complete mapping from psychological distance, d to belief revisions, ν. Our argument has had a rational flavor in that the actions and updating of the agent were derived as optimal. However, whether an agent's perceived uncertainty and distrust induced by psychological distance is rational is, of course, an open question. In the main paper, we use the framework to model how psychological distance induces actions, that deviate from rational expectations, with the interpretation that the agent changes his/her beliefs to adjust for the distrust and uncertainty induced by psychological distance.

References

Adam, Klaus, Dmitry Matveev, and Stefan Nagel, 2021, Do survey expectations of stock returns reflect risk adjustments?, *Journal of Monetary Economics* 117, 723–740.

Afrouzi, Hassan, Spencer Yongwook Kwon, Augustin Landier, Yueran Ma, and David Thesmar, 2021, Overreaction in expectations: Evidence and theory, *Available at SSRN* .

Badarinza, Cristian, Tarun Ramadorai, and Chihiro Shimizu, 2019, Gravity, counterparties, and foreign investment, available at SSRN 3141255.

Bailey, Michael, Rachel Cao, Theresa Kuchler, and Johannes Stroebel, 2018, The economic effects of social networks: Evidence from the housing market, forthcoming in *Journal of Political Economy.*

Barberis, Nicholas, 2011, Psychology and the financial crisis of 2007-2008, working Paper, Yale University.

Barberis, Nicholas, 2018, Psychology-based models of asset prices and trading volume, in *Handbook of Behavioral Economics: Applications and Foundations 1*, volume 1, 79–175 (Elsevier).

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Barberis, Nicholas, and Richard H. Thaler, 2003, A survey of behavioral finance, in George M. Constantinides, Milton Harris, and René M. Stulz, (eds.) *Handbook of the Economics of Finance* (Elsevier, Amsterdam).

Barberis, Nicholas C., Lawrence J. Jin, and Baolian Wang, 2021, Prospect theory and stock market anomalies, forthcoming in *Journal of Finance.*

Bhandari, Anmol, Jaroslav Borovička, and Paul Ho, 2019, Survey data and subjective beliefs in business cycle models, University of Minnesota Working Paper.

Bodnaruk, Andriy, 2009, Proximity always matters: Local bias when the set of local companies changes, *Review of Finance* 13, 629–656.

Brunnermeier, Markus, Emmanuel Farhi, Ralph S. J. Koijen, Arvind Krishnamurthy, Sydney C. Ludvigson, Hanno Lustig, Stefan Nagel, and Monika Piazzesi, 2021, Review article: Perspectives on the future of asset pricing, *forthcoming* Review of Financial Studies.

Cabrales, Antonio, Olivier Gossner, and Roberto Serrano, 2013, Entropy and the value of information for investors, *American Economic Review* 103, 360–77.

Camerer, Colin, 1995, Individual decision making, in J. Kagel and A. Roth, (eds.) *Handbook of Experimental Economics* (Princeton University Press, Princeton, NJ).

Camerer, Colin, 1998, Bounded rationality in individual decision making, *Experimental Economics* 1, 163–183.

Chen, Xiaohong, Lars Peter Hansen, and Peter G. Hansen, 2020, Robust identification of investor beliefs, *Proceedings of the National Academy of Sciences* 117, 33130–33140.

Chiang, Yao-Min, David Hirshleifer, Yiming Qian, and Ann E. Sherman, 2011, Do investors learn from experience? Evidence from frequent IPO investors, *Review of Financial Studies* 24, 1560–1589.

Cohen, Lauren, 2009, Loyalty-based portfolio choice, *Review of Financial Studies* 22, 1213–1245.

Coibion, Olivier, and Yuriy Gorodnichenko, 2012, What can survey forecasts tell us about information rigidities?, *Journal of Political Economy* 120, 116–159.

Coibion, Olivier, and Yuriy Gorodnichenko, 2015, Information rigidity and the expectations formation process: A simple framework and new facts, *American Economic Review* 105, 2644–78.

Coval, J., and T. Moskovitz, 1999a, The geography of investment: Informed trading and asset prices, *Journal of Political Economy* 109, 811–841.

Coval, J., and T. Moskovitz, 1999b, Home bias at home: Local equity preference in domestic portfolios, *Journal of Finance* 54, 2045–2073.

D'Acunto, Francesco, Marcel Prokopczuk, and Michael Weber, 2019, Historical antisemitism, ethnic specialization, and financial development, *Review of Economic Studies* 86, 1170–1206.

Darke, Peter R., Michael K. Brady, Ray L. Benedicktus, and Andrew E. Wilson, 2016, Feeling close from afar: The role of psychological distance in offsetting distrust in unfamiliar online retailers, *Journal of Retailing* 92, 287–299.

Das, Sreyoshi, Camelia M. Kuhnen, and Stefan Nagel, 2017, Socioeconomic status and macroeconomic expectations, Working Paper, National Bureau of Economic Research.

Duffie, J. Darrell, and Larry G. Epstein, 1992, Stochastic Differential Utility, *Econometrica* 60, 353–394.

Epstein, Larry G., and Stanley Zin, 1989, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

French, K., and J. Poterba, 1991, Investor diversification and international equity markets, *American Economic Review* 81, 222–226.

Gagliardini, Patrick, Paolo Porchia, and Fabio Trojani, 2008, Ambiguity aversion and the term structure of interest rates, *Review of Financial Studies* 22, 4157–4188.

Garleanu, Nicolae, and Lasse Heje Pedersen, 2019, Active and passive investing, available at SSRN 3253537.

Gennaioli, Nicola, and Andrei Shleifer, 2018, *A Crisis of Beliefs: Investor Psychology and Financial Fragility* (Princeton University Press).

Giglio, Stefano, Matteo Maggiori, Johannes Stroebel, and Stephen Utkus, 2021, Five facts about beliefs and portfolios, forthcoming in *American Economic Review.*

Greenwood, Robin, and Stefan Nagel, 2009, Inexperienced investors and bubbles, *Journal of Financial Economics* 93, 239–258.

Greenwood, Robin, and Andrei Shleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Grinblatt, Mark, and Matti Keloharju, 2000, The investment behavior and performance of various investor types: A study of Finland's unique data set, *Journal of Financial Economics* 55, 43–57.

Grinblatt, Mark, and Matti Keloharju, 2001, How distance, language, and culture influence stockholdings and trades, *Journal of Finance* 56, 1053–1073.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales, 2006, Does culture affect economic outcomes?, *Journal of Economic Perspectives* 20, 23–48.

Guiso, Luigi, Paola Sapienza, and Luigi Zingales, 2008, Trusting the stock market, *Journal of Finance* 63, 2557–2600.

Hansen, Lars Peter, and Thomas J. Sargent, 2001, Robust control and model uncertainty, *American Economic Review: Papers and Proceedings* 2, 60–66.

Hansen, Lars Peter, and Thomas J. Sargent, 2008, *Robustness* (Princeton University Press, Princeton, NJ).

Hartzmark, Samuel M., Samuel Hirshman, and Alex Imas, 2021, Ownership, learning, and beliefs, forthcoming in *Quarterly Journal of Economics*.

Hirshleifer, David, 2001, Investor psychology and asset pricing, *Journal of Finance* 56, 1533–1597.

Hirshleifer, David, 2015, Behavioral finance, *Annual Review of Financial Economics* 7, 133–159.

Hirshleifer, David, 2020, Presidential address: Social transmission bias in economics and finance, *The Journal of Finance* 75, 1779–1831.

Huberman, Gur, 2001, Familiarity breeds investment, *Review of Financial Studies* 14, 659–680.

Jaynes, Edwin T., 1957, Information theory and statistical mechanics, *The Physical Review* 106, 620–630.

Jaynes, Edwin T., 1986, Monkeys, kangaroos, and N, in J. Justice, (ed.) *Maximum Entropy and Bayesian Methods in Applied Statistics: Proceedings of the Fourth Maximum Entropy Workshop University of Calgary*, 26–58 (Cambridge University Press).

Kaustia, Markku, and Samuli Knüpfer, 2008, Do investors overweight personal experience? Evidence from IPO subscriptions, *Journal of Finance* 63, 2679–2702.

Kaustia, Markku, and Samuli Knüpfer, 2012, Peer performance and stock market entry, *Journal of Financial Economics* 104, 321–338.

Knüpfer, Samuli, Elias Rantapuska, and Matti Sarvimäki, 2017, Formative experiences and portfolio choice: Evidence from the Finnish Great Depression, *Journal of Finance* 72, 133–166.

Kuchler, Theresa, and Basit Zafar, 2018, Personal experiences and expectations about aggregate outcomes, Working Paper, New York University, Stern School of Business.

Kullback, Solomon, and Richard A. Leibler, 1951, On information and sufficiency, *Annals of Mathematical Statistics* 22, 79–86.

Laudenbach, Christine, Ulrike Malmendier, and Alexandra Niessen-Ruenzi, 2018, The long-lasting effects of propaganda on financial risk-taking, Working Paper.

Li, Chen, Uyanga Turmunkh, and Peter P Wakker, 2019, Trust as a decision under ambiguity, *Experimental Economics* 22, 51–75.

Lucas, Jr., Robert E., 1976, Econometric policy evaluation: A critique, in Karl Brunner and Allan H. Meltzer, (eds.) *The Phillips Curve and Labor Markets: Carnegie-Rochester Conference Series on Public Policy*, volume 1, 19–46 (North-Holland).

Malmendier, Ulrike, and Stefan Nagel, 2011, Depression babies: Do macroeconomic experiences affect risk taking?, *Quarterly Journal of Economics* 126, 373–416.

Malmendier, Ulrike, and Stefan Nagel, 2015, Learning from inflation experiences, *Quarterly Journal of Economics* 131, 53–87.

Manski, Charles F., 2004, Measuring expectations, *Econometrica* 72, 1329–1376.

Manski, Charles F., 2018, Survey measurement of probabilistic macroeconomic expectations: Progress and promise, *NBER Macroeconomics Annual* 32, 411–471.

Marschak, Jacob, 1959, Remarks on the economics of information, in *Contributions to Scientific Research in Management*, 79–98 (Western Data Processing Center, University of California, Los Angeles).

Meeuwis, Maarten, Jonathan A. Parker, Antoinette Schoar, and Duncan I. Simester, 2018, Belief disagreement and portfolio choice, Working Paper, National Bureau of Economic Research.

Muth, John F., 1961, Rational expectations and the theory of price movements, *Econometrica* 29, 315–335.

Pedersen, Lasse H., 2021, Game on: Social networks and markets, Working Paper, Copenhagen Business School.

Shefrin, Hersh, 2007, *Beyond Greed and Fear: Understanding Behavioral Finance and the Psychology of Investing* (Oxford University Press).

Shefrin, Hersh, 2010, Behavioralizing finance, *Foundations and Trends in Finance* 4, 1–184.

Shepard, Roger N., 1987, Toward a universal law of generalization for psychological science, *Science* 237, 1317–1323.

Shive, Sophie, 2010, An epidemic model of investor behavior, *Journal of Financial and Quantitative Analysis* 45, 169–198.

Shleifer, Andrei, 2000, *Inefficient Markets: An Introduction to Behavioral Finance* (Oxford University Press, Oxford).

Statman, Meir, 2010, What is behavioral finance, in A. Wood, (ed.) *Behavioral Finance and Investment Management* (Research Foundation of CFA Institute).

Statman, Meir, 2011, *What Investors Really Want* (McGraw-Hill).

Trojani, Fabio, and Paolo Vanini, 2004, Robustness and ambiguity aversion in general equilibrium, *Review of Finance* 8, 279–324.

Trope, Yaacov, and Nira Liberman, 2010, Construal-level theory of psychological distance, *Psychological Review* 117, 440–463.

Van Nieuwerburgh, Stijn, and Laura Veldkamp, 2009, Information immobility and the home bias puzzle, *Journal of Finance* 64, 1187–1215.

Walden, Johan, 2019, Trading, profits, and volatility in a dynamic information network model, *Review of Economic Studies* 86, 2248–2283.