The effect of insurance on pricing strategies and fraud in markets for repair goods

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Abstract

We study the effect of insurance and claims auditing on pricing and overcharging behavior of suppliers in oligopolistic repair markets. Consumers' reduced price sensitivity with insurance coverage and informational advantages enable repair service suppliers to include high markups and lower the inhibition threshold for issuing fraudulent invoices. Insurers typically make use of claims auditing to prevent overcharging by repair firms. Our results indicate that low insurance coverage leads to a uniform price for different repair services and thus prevents overcharging of repair service suppliers. High insurance coverage results in treatment-specific repair prices that come along with occasional overcharging and frequent audits. Claims auditing leads to a welfare loss for consumers. Given an arbitrary coinsurance rate, consumers' expected utility is higher with a uniform repair price and no auditing than with repair-dependent prices and auditing. To prevent overcharging and costly auditing also for high insurance coverage, insurers could use capitation instead of bill-dependent reimbursement.

Keywords: Repair markets · Insurance · Overcharging · Costly state verification

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1 Introduction

Private insurance contracts are mostly incomplete. A complete insurance contract would assign an indemnity payment to each possible "state of the world". For instance, it would define a specific indemnity for any possible disease or injury. The incompleteness of insurance contracts arises from the immensity of possible states of nature and for this reason insurance contracts typically define indemnities based on repair or treatment costs. The reimbursement of repair costs reduces insured consumers' price sensitivity, which causes suppliers in imperfect repair markets to increase markups on prices. This phenomenon is known as external moral hazard. As a result, insurers' expenses and insurance premiums increase (Nell et al., 2009). The incompleteness of insurance contracts and the resulting indemnity schedule not only allow for legal adjustments of repair firms (i.e. markup increases in imperfect repair markets), but also enhance fraudulent overcharging by repair firms. Repair firms may use information asymmetries to inflate repair bills by charging for services that were not provided. According to Chen et al. (2018), 2222 physicians (0.3% of US physicians in 2017) were temporarily or permanently banned from Medicare and state public insurance programs owing to fraud, waste, and substance abuse in the United States between 2007 and 2017. By applying a field experiment in the area of automobile insurance, Tracy and Fox (1989) show that auto body repair estimates and customers' insurance coverage are positively related. In a field experiment, Kerschbamer et al. (2016) find that Austrian computer repair shops charge, on average, a price 80% higher when customers are insured for resulting repair costs. The economic consequences of both forms of moral hazard (legal and illegal) include higher premiums and out-of-pocket expenses for consumers. However, insurance companies put measures into place to combat external moral hazard and third party insurance fraud including claims auditing. Consumers' coverage level influences the pricing and billing behavior of repair firms and therefore the likelihood of an audit being carried out. An increase in insurance coverage intensifies external moral hazard, but increases auditing incentives on the insurer side, which results in a trade-off between risk allocation, pricing incentives for repair firms, and auditing incentives for insurance companies.

The contribution of this paper consists of studying the effect of insurance and claims auditing on pricing and overcharging behavior in markets for repair goods. After the occurrence of a loss, (insured) consumers visit a repair firm for the reparation of the loss. In many types of repair markets, e.g. in car repair and healthcare markets, consumers often do not know what kind of service they need and therefore have to rely on the recommendation from the repair firm. The repair firms, on the other hand, adjust their pricing on consumers' price sensitivity, on which insurance coverage has a strong impact, and may use their information advantage by issuing fraudulent invoices. Consumers with insurance coverage forward the bill to the insurer that may audit suspicious claims to detect overcharging bills. We examine the interaction of consumers, insurance, and repair markets. The likelihood of an audit being carried out strongly depends on consumers' chosen coverage level and therefore also influences pricing and billing in repair markets. A basic understanding of the underlying effects is important to understand why insurers and regulators frequently intervene in repair markets. In contrast to existing insurance economic theory, the proposed model makes predictions about (fraudulent) repair market behavior with insured demand side and explains why insurers increasingly resort to capitation instead of bill-dependent reimbursement.

Our approach combines costly state verification, the characteristics of credence goods markets, and insurance-induced price effects in repair markets. Existing literature on insurance fraud mainly focuses on fraudulent behavior by the policyholder. For instance, Boyer (2000), Picard (2000), and Schiller (2006) study fraudulent policyholder behavior using costly state verification models. This game-theoretic approach allows the policyholder to decide between "opportunistic fraud" and "no fraud" and the insurer has the strategies "costly audit" and "no audit". Fraud refers to claiming an insured damage that never happened and insurers are able to detect and fine fraud by conducting a costly audit. The equilibrium is in mixed strategies. Insurance contracts entail over-insurance of losses in order to give insurers incentives to audit claims. Picard and Wang (2015) and Bourgeon et al. (2008) incorporate a third party in the analysis of insurance fraud. In both papers fraud takes the form of collusion between policyholders and repair service providers. The former paper analyzes the effect of the insurance distribution channel on fraudulent behavior. Picard and Wang (2015) find that the collusion between policyholders and dealer-owned agents increases fraud in the Taiwanese car insurance market. Bourgeon et al. (2008) theoretically show that the implementation of a network of providers by an insurer may improve customers' surplus when insurers and providers have repeated relationship. The existing costly state verification literature, however, does not examine fraudulent third party behavior that affects policyholders and insurers.

The credence goods literature deals with markets in which the seller knows more about the good/service that the customer needs than the customer himself.[1] Typical examples are healthcare and car repair markets. The information asymmetry between buyer and seller causes strong fraud incentives on the seller side. Pitchik and Schotter (1987), Wolinsky (1995), Emons (2001), Sülzle and Wambach (2005), and Dulleck and Kerschbamer (2006) provide theoretical contributions to this branch of literature. One basic model feature underlies them all: Consumers either face a minor or a major problem (e.g. a car damage or disease) and require a repair to fix the problem. Although a consumer is aware that there

[1] The term "credence good" was introduced by Darby and Karni (1973). Credence goods are goods and services about which an expert knows more about the quality or scope a consumer needs than the consumer himself. The information asymmetry holds the danger that expert sellers overcharge, overtreat, or undertreat their customers.

is a problem, he is unable to identify its severity. Expert sellers either provide a minor treatment, that only repairs the minor problem, or a major treatment, that repairs the minor and the major problem. The costs for provision of the major treatment exceed the costs for the minor one. Typically, experts first give a repair recommendation before the actual treatment is performed. Three forms of fraud may occur: Undertreatment (the minor treatment is provided although the problem is major), overtreatment (the major treatment is provided although the problem is minor), and overcharging (the expert charges the price for the major treatment although the minor one was conducted). If the expert is liable to solve a customer's problem, undertreatment is ruled out. If the customer is able to verify which kind of treatment was conducted, overcharging is ruled out. Otherwise, overcharging dominates overtreatment.

As we are concerned with overcharging, we restrict ourselves to results from the credence goods literature relating to overcharging. Dulleck and Kerschbamer (2006) assume endogenous prices, constant transaction costs per visited expert, and a monopolistic or perfectly competitive repair market framework. Under the assumptions of consumer homogeneity, liability, and commitment to the first visited expert, they find that the expert(s) set(s) an identical price for both types of treatment, efficiently serve(s) all customers, and thus overcharging does not occur in equilibrium. Emons (2001) proves a similar result when a monopoly expert is capacity constrained and consumers are able to observe the monopolist's capacity. Inefficiency arises when consumers are able to reject an expert's offer and search for a second opinion as it is shown by Dulleck and Kerschbamer (2006) and Pitchik and Schotter (1987). Sülzle and Wambach (2005) study the impact of insurance arrangements on the degree of fraud in markets for credence goods. The authors assume fixed repair prices with the price for the major treatment exceeding the one for the minor treatment. A consumer can accept a given recommendation or get a second opinion from another expert. Due to the non-observability of the provided treatment, overtreatment is ruled out. The model has three possible equilibria. Experts either always diagnose dishonestly and consumers always accept the first diagnosis (equilibrium in pure strategies) or the market level of fraud is either relatively low or relatively high with consumers sometimes searching for a second opinion (equilbrium in mixed strategies). Whereas the former equilibrium is efficient, the latter two are not. In accordance with Dulleck and Kerschbamer (2006), Sülzle and Wambach (2005) show that inefficiency only arises with the search for a second opinion. In summary, insurance arrangements per se do not change suppliers' overcharging strategy in credence goods markets when prices are fixed and consumers do not search for a second option. In this case, experts charge a uniform price for any kind of treatment such that overcharging cannot occur. This result applies regardless of the availability of insurance. Considering the credence goods literature, it is not evident how insurance and claims auditing affect the equilibrium if repair firms determine prices endogenously, which we address in this paper.

The third branch of literature relating to insurance and repair markets in the one concerning legal external moral hazard. In this case, consumers' insurance-induced decreased price sensitivity leads to price increases in repair markets (i.e. repair firms increase the markup). For instance, Nell (2001) reports significant price differences between the general rental car market with uninsured consumers and the accident replacement car market, where insured consumers rent cars. Nell et al. (2009) reveal by means of a theoretical approach that an increase in insurance coverage leads to higher prices and additional market entry in repair markets. We extend this model by adding the components overcharging and auditing. To the best of our knowledge, the literature has not considered costly state verification in credence goods markets so far. We fill this gap in the literature, as pricing and billing in repair markets as well as claims auditing by insurers have a major impact on consumers' out-of-pocket expenses and therefore on their choice of a coverage level.

We set up a model in which consumers can purchase a coinsurance contract against possible losses. The indemnity is based on consumers' actual expenses. The loss can either be small or large, but consumers are unable to identify its severity. After the occurrence of a loss, consumers visit a repair firm for the reparation of the loss. The repair market is an oligopoly with suppliers competing in prices. Repair firms are identical and offer a small (repairs the small loss) and a large repair (repairs both types of loss). The repair firms adequately repair the loss and forward the bill, that indicates the provided treatment, to the respective consumer and insurer. An insurer can conduct audits to identify overcharging by repair firms. In the case of a detected fraud attempt, the repair firm must pay a fine. For large coinsurance rates, consumers exhibit a high price sensitivity, as they bear the major part of the repair bill. Consequently, the price level in the repair market is low, as the suppliers adjust their pricing to consumers' high price sensitivity. If repair firms set a tariff with two different prices, overcharging can occur by charging the higher price for the small repair, but an insurer would conduct no audits, as expected costs with audit exceed costs without audit. However, consumers anticipate that repair firms will always charge the maximum of the two prices and choose their strategy accordingly. Thus, in equilibrium, repair firms set a uniform price for both types of repair. By decreasing the coinsurance rate, consumers' price sensitivity decreases, which allows repair firms to increase the price level in repair market. If repair firms set a two-price tariff for sufficiently large coinsurance rates, an insurer would audit each large-repair claim (given an overcharging probability of one). The large share in the repair costs and the high price level in repair market results in claims costs without auditing exceeding claims costs with auditing. Consequently, the equilibrium is in mixed strategies with a two-price tariff, a positive auditing probability, and a positive overcharging probability for sufficiently small coinsurance rates.

Claims auditing leads to a welfare loss for consumers. Given an arbitrary coinsurance rate, consumers' expected utility is higher with a uniform repair price and no auditing than with repair-dependent prices and auditing. This observation together with the presence of external moral hazard explain our findings with respect to the utility-maximizing insurance contract: Full insurance is never optimal for risk-averse consumers and partial insurance is only utility-maximizing if consumers are sufficiently risk-averse. The price effect in the repair market outweighs risk aversion, unless risk aversion is sufficiently large. Moreover, only if the audit costs are sufficiently small, consumers' risk aversion is sufficiently high, and the loss probability is sufficiently large, the utility-maximizing contract entails a small coinsurance rate for which the equilibrium with two different prices and auditing arises. Otherwise, a large coinsurance rate, for which the equilibrium with a uniform repair price arises, is utility-maximizing. Our results highlight the inefficiency of market insurance, as consumers could be better off without claims auditing. To prevent this inefficiency, insurers could establish their own network of repair service providers or provide own repair services. In the case of an own network, an insurer negotiates a fixed repair price (capitation) for any type of repair with the repair firms. The negotiation artificially induces the equilibrium with uniform repair price and without overcharging and auditing for any coinsurance rate. Therefore, a high level of insurance coverage is in demand if consumers are sufficiently risk-averse, which resolves the inefficiency with claims auditing.

The paper is organized as follows. The next section introduces the model framework. Subsequently, the equilibrium is derived in section three. We provide a comparative static analysis of the equilibrium set and a welfare analysis in section four. Section five addresses the optimal insurance contract. The final section concludes.

2 Model framework

Overcharging by repair firms constitutes insurance fraud if an insurer is participated in the repair bill. To study insurance fraud, we can apply a costly state verification or a costly state falsification approach (Picard, 2000). While insurers can verify damages at a cost under the costly state verification approach, policyholders (or in our case third parties) may invest in falsification activities to build up or exaggerate a claim, making the fraud attempt undetectable for the insurer. We use a costly state verification approach, as it seems reasonable that insurers can detect fraud attempts. For instance, insurance companies check the correctness of physicians' bills on a random basis. In this article, we focus on the overcharging dimension of fraud, i.e. repair firms may charge for more services than actually provided. To gain insights into the interaction between insurance and repair markets, we apply a sequential game between insurers, consumers, and suppliers of repair services. First, we derive the market equilibrium (repair prices, overcharging probability, audit probability). In a second step, we apply a comparative static analysis to study the influence of exogenous factors such as the coinsurance rate on the equilibrium. Finally, the optimal insurance contract for consumers is established.

The sequence of the game is as follows: First, insurers offer zero-profit expected utility maximizing contracts. Each consumer chooses an amount of coinsurance (it is possible to choose a coinsurance rate of one which corresponds to no insurance). A consumer who suffered a loss goes to a supplier to receive a treatment that repairs the loss. The external moral hazard problem arises due to the non-existence of a contractual relationship between insurer and supplier. The supplier observes the loss size, repairs the loss, and makes out a (correct or overcharging) invoice. If the consumer did not purchase insurance, he has to pay the bill on his own. Otherwise, the bill is submitted to the insurer that may reveal fraud by conducting a costly audit. For example, prepayment audits allow Medicare auditors to verify the legitimacy of a claim before the provider is paid. In the end, the payoffs are distributed. Subsequently, we introduce the model components.

Repair market

The repair market consists of n risk-neutral suppliers, denoted by $j = 1, \ldots, n$, that offer a good, i.e. a repair service. Each repair firm provides either one or two units of the repair service to solve a customer's problem and suppliers compete in prices. Every supplier incurs constant marginal costs of $c \geq 0$ to provide one unit of the repair service.

The repair market model is based upon an approach introduced by Salop (1979) and refined by Nell et al. (2009): Consumers are uniformly and continuously distributed along a circle with perimeter 1. The n suppliers are equidistantly distributed around the circle. A consumer faces transportation costs of t per unit of the distance x to the supplier.

Consumers/Insurees

Risk-averse consumers having a twice-differentiable utility function u with $u' > 0, u'' < 0$, are endowed with an initial wealth of w_0, and face the risk of a loss with probability $\pi \in (0,1)$. Given that a loss event took place, the loss can either be small with probability $\pi_s \in (0,1)$ or big with probability π_b, i.e. $\pi_s + \pi_b = 1$. A consumer can observe whether a loss occurred, but he is unable to identify the loss size. Given the complexity of medical diseases or car repair procedures, consumers' ignorance regarding the loss size turns out to be realistic. The game ends if the consumer did not suffer a loss. It is always beneficial for a consumer to have the damage repaired. Thus, each consumer who suffered a loss goes to a repair shop to have the damage repaired. The repair shop observes the loss size (small or big) free of cost: Exactly one repair unit is necessary to repair a small damage and exactly two repair units are necessary to repair a big damage. An expert firm is liable to repair a customer's damage. In a medical context, for instance, the Hippocratic oath obliges physicians to heal their patients. It is not possible to sell more than two repair units for the repair of one damage and experts maximize the profit per customer implying that cross-subsidization between small-loss and big-loss customers is ruled out. The supplier informs the customer (and the customer's insurer) about the quantity of repair units used and therefore about the reparation costs after the repair has been conducted. In the German private health insurance, for instance, policyholders receive an invoice after the utilization of a medical treatment. The insurance company reimburses the policyholder after the receipt of the bill.

Insurance Market

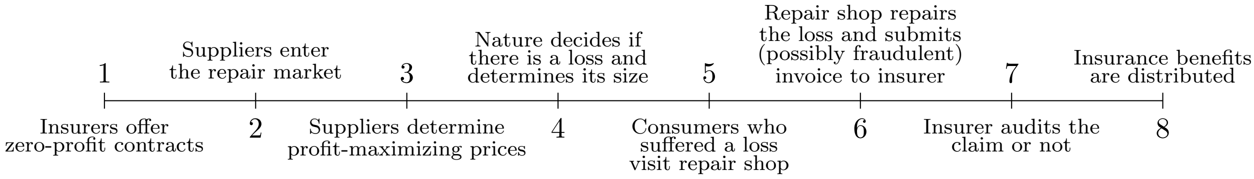
The insurance market is based upon the model introduced by Nell et al. (2009): m risk-neutral insurers, denoted by $i = 1 \ldots, m$, simultaneously offer contracts $C_i = (\alpha_i, I_i^s, I_i^b)$, which consist of an indemnity I_i^s in the small-loss case and of an indemnity I_i^b in the big-loss case. Consumers participate with a share of $\delta_i \in [0,1]$ in the repair costs (coinsurance rate) and α_i denotes an actuarially fair zero-profit premium, i.e. premiums are equal to expected benefits plus expenses due to fraud. Since the consumer does not know the actual loss size, the indemnity that is paid in the case of a loss is based on the information provided by the repair shop.

Sequence of the game

The sequence of the considered game between insurers, consumers and suppliers is as follows:

1. All m insurance companies simultaneously offer a zero-profit insurance contract C_i and each consumer purchases one contract (the contract with zero coverage and zero premium counts among all other insurance contracts and is offered in any case).
2. The potential suppliers decide on entry in the repair market facing fixed entry costs of $f > 0$. The equilibrium profit of entering firms is zero because market entry is free.

Figure 1: Sequence of the game between insurers, repair shops, and consumers



3. Suppliers, that entered the repair market, determine profit-maximizing tariffs. From the credence goods literature we know that a repair firm may either set a uniform price $\bar{p_j}$ for both treatments (provision of one or two repair units) or charge different prices for the two types of repair. The repair-dependent tariff corresponds to setting p_j for the provision of one repair unit and $2p_j$ for the provision of two repair units in our case. After the determination of prices, suppliers set their capacity according to the expected number of repair units needed. If a repair firm has a market share of $y \in [0, 1]$, the capacity is set to $yc(1 + \pi_b)$. Due to shortage of resources, capacity expansion is not feasible.[2]

4. Nature decides if there is a loss and determines its size.

5. After a loss event, a consumer visits a supplier to receive a repair or treatment. As overcharging can only occur if the expert differentiates between prices, the game ends at stage 5 if the expert charges identical prices for both treatments. In this case, the expert observes the loss size free of cost, provides the cheapest sufficient treatment, and the invoice is paid.

6. Otherwise, the supplier observes the loss size free of cost. As the expert is liable to repair the damage adequately, each consumer receives a treatment that fully restores the loss. A consumer with a big loss always receives and charged for two repair units. As the consumer is not able to verify if one or two repair units are provided, the repair firm has three possibilities to serve a small-loss customer: The expert can either provide and charge for one repair unit (no fraud), provide one repair unit and charge for two repair units (overcharging), or provide and charge for two repair units (overtreatment). As repair firms are capacity-constrained and prefer to serve all customers, overtreatment is not an option for repair firms. Hence, a small-loss customer is either charged correctly or overcharged. We denote the fraud probability by η_j. The supplier submits the (fraudulent) bill to the consumer and respective insurer (if necessary) after the repair. In the case the consumer did not purchase insurance, he pays the bill immediately.

[2] The assumption of capacity restriction allows us to focus on the overcharging dimension of fraud. Otherwise, overtreatment would be an attractive option for repair firms, especially if overtreatment cannot or hardly be revealed by insurance audits. If the assumption of capacity restriction is relaxed, overtreatment will occur in equilibrium if insurers are unable to detect and fine overtreatment.

7. Otherwise, the insurer audits the claim or not. A claim requiring one repair unit cannot be fraudulent by the liability assumption. Therefore, the corresponding indemnity is paid out. A claim requiring two repair units can either originate from a small-loss or big-loss customer. The insurer can verify the legitimacy of a two-repair-units claim by conducting a costly audit. An audit costs the insurer $k > 0$. If a fraudulent claim is audited the fraud is revealed with certainty. We denote by v_i the probability that a two-repair-units claim is audited. In the case of a detected fraudulent claim, the small-loss indemnity is paid and the supplier must pay a fine $r > 0$.
8. Finally, at stage 8 the insurance indemnity is paid out.[3]

Auditing procedures observed in health and auto insurance serve as a basis for our model: Policyholders first receive the treatment/repair and afterwards submit the bill to the insurer. The insurer has the opportunity to review the bill and verify its correctness. All above listed quantities as well as the sequence of the game are common knowledge. Asymmetric information only exists in three ways: The consumer does not know the loss size if a loss occurred, the insurer does not know the loss size unless it audits a claim, and the repair shop does not know a priori if a big-loss claim will be audited or not.

As we face a game with asymmetric information and observable actions, we look for a Nash equilibrium, i.e. a stable situation where none of the players has an incentive to deviate from his chosen strategy assuming the other players do not deviate. We only consider symmetric equilibria, i.e. all suppliers set the same tariff and all insurers offer contracts with the same coinsurance rate δ. Consequently, consumers are passive players, as the strategy "go to the nearest repair shop to have the damage repaired" minimizes their expected costs, implying that no decision has to be made at stage 5.[4] Unless otherwise specified, δ is treated as an exogenous parameter, which is determined by insurers before the start of the game. Thus, an insurer's only strategy is "offer zero-profit contract" at stage 1. This contract with the exogenously given coinsurance rate $\delta \in [0, 1]$ is purchased by each consumer. Before we focus on finding the solution to the sequential game, we briefly look at a benchmark situation: A situation where repair service suppliers do not inflate the repair costs by accounting for not rendered services serves as a reference point for our further analysis:

[3] The structure of the claiming game, i.e. stages 4 to 7, is based upon the claiming game introduced by Boyer (2000). The game played in stages 1 to 3 is based upon the market entry game introduced by Nell et al. (2009).

[4] We implicitly assume that consumers are unable to repair the damage on their own and consumers do not conduct a costly audit on their own (e.g. through an investigator) to figure out the actual loss size.

Situation without fraud (Benchmark situation)

The sequence of the game remains unchanged with the following exceptions: Insurers are able to observe the loss size free of cost which implies information symmetry between insurer and repair firm. Consequently, repair shops report the true loss size at stage 6, i.e. if the consumer suffered a small loss, a small loss is indeed reported to the insurer. Consumers know that repair shops always report the true loss size to the insurer. As we face a situation without fraud, insurers audit no claims and pay the insurance indemnity according to the repair shop's indication about the loss size. We determine the profit-maximizing tariff and the number of suppliers in a no-fraud setting in order to compare these results with the quantities in a fraud setting. In this context, we address the question how the opportunity to commit fraud affects the price level in the repair market. Furthermore, the comparison with the benchmark situation allows us to derive welfare effects of fraud in repair markets.

Proposition 1. *In a no-fraud setting, repair firms are indifferent between a uniform price and differentiated prices for the two types of treatment. In any case,* $n_{nf} = \sqrt{\frac{t\pi}{\delta f}}$ *suppliers enter the repair market. The profit-maximizing uniform price for both treatments is given by* $\bar{p}_{nf} = (1 + \pi_b)c + \sqrt{\frac{tf}{\delta\pi}}$*. The profit-maximizing price for one repair unit in the case of differentiated prices is given by* $p_{nf} = c + \frac{1}{1+\pi_b}\sqrt{\frac{tf}{\delta\pi}}$*.* $\bar{p}_{nf}$ *equals the expected revenue per customer when repair firms set the tariff* $(p_{nf}, 2p_{nf})$*.*

Proof. See Appendix. □

As consumers know that repair firms always reveal their private information and therefore there is information symmetry between buyer and seller, prices do not contain information costs. Thus, consumers' expected costs in the case of a uniform average price equal the expected costs in the case of a tariff with differentiated prices. As we assume that consumers choose a repair firm solely on the basis of expected costs, the profit of a repair firm is independent of the chosen tariff. Moreover, consumers' insurance coverage leads to an external moral hazard effect: Even in the absence of ex-post moral hazard, consumers' insurance coverage significantly influences the price in repair markets. With decreasing δ, consumers' price sensitivity decreases so that the price level in the repair market increases (Nell et al., 2009).

3 Overcharging equilibrium analysis

Returning to the overcharging model, we first note that insurers are not interested in conducting audits at all in some circumstances. If audits costs exceed insurance payments, insurers forego costly auditing.

Proposition 2. *In equilibrium, repair firms either set a tariff with different prices for the two types of treatment (E1) or a uniform price is charged for both types of treatment (E2). In the first case, small-loss customers are overcharged with probability* $\eta \in (0, 1)$ *and* *big-repair claims are audited with probability $v \in (0,1)$. In the latter case, no audits are conducted. There exists some $a \in (0,1)$ (that depends on π, π_s, k, t, r, f and c) such that for $\delta \in (0,a)$ the equilibrium is of type (E1). For $\delta \in [a,1]$ the equilibrium is of type (E2).*

Proof. See Appendix. □

The proposition reveals that repair firms' pricing-overcharging strategy and insurers' auditing strategy depend on consumers' level of insurance coverage. In accordance with Tennyson and Salsas-Forn (2002), we find that claims are only audited if potential benefits of audits outweigh their costs. If the insurance coverage is too small, insurance companies do not conduct audits, as audit costs exceed insurance payments.

Repair firms' tariff setting depends on insurers' auditing strategy and consumers' preferences. If insurers conduct no audits, consumers expect to pay the highest treatment price in any case which results in a uniform treatment price. This observation applies to high coinsurance rates, as audit costs exceed insurance benefits in this case. If insurers are generally willing to audit claims, repair firms need to decide between a tariff with differentiated prices and a tariff with a uniform price. In the benchmark situation, repair firms are indifferent between the two options due to information symmetry between buyer and seller. This attitude changes when overcharging comes into play and consumers are not fully insured. Suppose that repair firms set a uniform treatment price. Consequently, insurers conduct no audits (as overcharging is not possible) and consumers would be sure that they have to pay the retention of the specified price in the case of a loss. Consumers' willingness to pay to learn the actual loss size would be zero. In contrast, suppose that repair firms set a tariff with differentiated prices and insurers conduct audits on a random basis. In this case, consumers are exposed to risk, as they do not know the repair price in advance and if the repair firm overcharges. Consequently, consumers' willingness to pay to avoid overcharging is positive. Therefore, repair firms can add a "credibility" surcharge on the price for one repair unit to signal honesty without repelling customers. This surcharge represents the price that consumers are willing to pay to overcome the information asymmetry. In summary, repair firms prefer to set a tariff with differentiated prices, as such a tariff generates higher expected profits than a tariff with a uniform price. The difference in profits can be attributed to the credibility surcharge in the case of tariff with differentiated prices. This case applies if insurers are generally willing to audit claims which is true for contracts with smaller coinsurance rates.

Our results extend and complement the existing theoretical literature concerning overcharging in credence goods markets in several ways. In accordance with Dulleck and Kerschbamer (2006) and Sülzle and Wambach (2005), we find that an efficient result arises, when repair firms charge a uniform price for any kind of treatment and provide the cheapest sufficient treatment. This result applies regardless of the competition degree in the repair market and the availability of insurance. Second, inefficiency and welfare losses arise through claims auditing and the associated costs. Contrary to intuition, claims auditing does not improve consumers' welfare, but burdens them with additional costs. As well as searching for a second opinion (Dulleck and Kerschbamer, 2006; Sülzle and Wambach, 2005), claims auditing leads to less efficiency in the market. Third, claims auditing inhibits cross-subsidization from minor repairs to major repairs. Dulleck and Kerschbamer (2006) prove that no-fraud equilibria are characterized by cross-subsidization from minor treatments to major treatments. A uniform price or similar prices for both treatments prevent fraud. We observe the same situation when insurers conduct no audits. Cross-subsidization makes small-loss customers the more profitable ones for repair firms. Claims auditing favors treatment-specific prices and high markups for each kind of treatment and therefore each type of treatment is equally profitable or rather major repairs bring in more profit. Moreover, we find that the results of Nell et al. (2009) remain to hold under the introduction of overcharging and claims auditing. Regardless of the overcharging possibility, repair firms adjust prices to consumers' insurance scheme. Thereby, the marginal price effect is increasing in coverage.

In fact, we can observe many of our model's predictions in reality which underlines the practical relevance of the model. For instance, some healthcare providers in the United States offer treatments that include accommodation, nursing care, diagnostic tests, drugs, and many further medical services for one fixed price to people who have no health insurance or to those whose insurance does not cover their particular illness (e.g. Northern Devon Healthcare, Doktoroo). Furthermore, the empirical findings of Erdem et al. (2002) demonstrate that brand credibility moderates consumers' price sensitivity under uncertainty. The larger the uncertainty about the product to purchase, the larger the reduction in price sensitivity. In our approach, credibility refers to repair firms foregoing fraud. Consumers' uncertainty about the treatment they need reduces their price sensitivity yielding to surcharges on the price for the repair good. Repair firms signal credibility (i.e. no overcharging) by increasing prices. The positive correlation between audit probability and claim amount is empirically captured by Tennyson and Salsas-Forn (2002). This result coincides with our theoretical implication that insurers only audit claims up to a certain critical coinsurance rate and forego claims auditing for larger coinsurance rates.

In the following subsection, we derive and study the overcharging equilibrium (E1). For this purpose we focus on $\delta \in (0, a)$ for the remainder of section 3. The game is solved using backward induction. The result is the optimal strategy combination (a sequential equilibrium) for every given coinsurance rate $\delta \in (0, a)$. After the calculation of the overcharging equilibrium (E1), we study both equilibrium sets (E1) and (E2) in more detail. Finally, we consider δ as an endogenous parameter to determine the utility-maximizing insurance contract.

3.1 Solution of the sequential game

Since we only consider symmetric equilibria, it is sufficient to solve the game for one representative insurer and one representative supplier. At stage 4, δ, n, and p are already determined and known to the insurer, the consumer, and the repair shop. The game played in stages 4 to 7 yields a unique Perfect Bayesian Nash Equilibrium (PBNE) in mixed strategies.

Proposition 3. *For any coinsurance rate $\delta \in (0, a)$ the following holds: The game played in stages 4 to 7 yields a unique Perfect Bayesian Nash Equilibrium. The PBNE is given by:*
1. The insured consumer always goes to a repair shop if a loss occurred.
2. The insurer never audits a small-loss claim.
3. The repair shop plays a mixed strategy between filing a big-loss claim (fraudulent) and filing a small-loss claim if the insuree suffered a small loss (η = fraud probability).
4. The insurer plays a mixed strategy between auditing (with probability v) and not auditing when a big-loss claim is filed.
In equilibrium, η and v are given by:

$$\eta(p) = \frac{\pi_b}{\pi_s} \frac{k}{(1-\delta)p - k} \quad \text{and} \quad v(p) = \frac{p}{p+r}. \tag{1}$$

Here, p denotes the equilibrium price for one repair unit that is determined at stage 3 of the game.

Proof. See Appendix. □

We find that audit probability and the price for the repair good are positively correlated. This observation is intuitive: The higher the price for the repair good, the larger the financial damage to insurers and consumers caused by a potential fraud attempt. Furthermore, we observe that price and fraud probability are inversely proportional. Repair shops either offer the repair good for a comparably high price and rarely defraud or they offer the repair service for a low price and often commit fraud. Consequently, legal external moral hazard and illegal overcharging are particularly complementary effects. It is intuitively clear that repair shops cannot charge high prices and defraud frequently at the same time, as this strategy would entail high auditing incentives and consequently lead to profit losses. When providers charge a low price for the repair service, insurers do not conduct many audits, as paying the indemnity might be less expensive than auditing. Therefore, repair shops use this opportunity to commit fraud with a high frequency. Conversely, a high price for the repair service induces many audits by insurers. Thus, repair shops are well-advised to reduce the fraud frequency in order to prevent profit losses caused by fines.

Now we go back one stage in the sequence of the play. At stage 3, the suppliers that entered the repair market determine the profit-maximizing price. Potential suppliers decide on market entry in the repair market at stage 2. New suppliers enter the repair market until each supplier's expected profit is zero. Proposition 4 presents the equilibrium price and number of suppliers.

Proposition 4. *For any given coinsurance rate $\delta \in (0, a)$, the number of operating suppliers in a symmetric equilibrium is implicitly given by:*

$$n^* = \sqrt{\frac{t\pi(1+\pi_b)}{\delta f\left[1+\pi_b - \pi_s \eta\Big(p(n^*)\Big)\Big(1 - v\Big(p(n^*)\Big)\Big)\Big(\frac{\pi_s}{\pi_b}\eta\Big(p(n^*)\Big) + v\Big(p(n^*)\Big)\Big)\right]}}. \tag{2}$$

Here, $p(n)$ is implicitly given by:

$$p = c + \frac{t}{\delta n\left[1+\pi_b - \pi_s\eta(p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p) + v(p)\Big)\right]}. \tag{3}$$

The profit-maximizing price in a symmetric equilibrium is implicitly given by:

$$p^* = c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\left[1+\pi_b - \pi_s\eta(p^*)\Big(1-v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*) + v(p^*)\Big)\right]}}. \tag{4}$$

p^* satisfies the following conditions: $p^* > c$ and $p^* > \frac{k}{\pi_s(1-\delta)}$.

The second condition implies that the fraud probability is strictly between 0 and 1. A priori it is not evident that both conditions are fulfilled. As the audit probability is chosen in such a way that the repair shop is indifferent between defrauding and not defrauding, the profit of the repair firm is independent of any fraud attempts, i.e. the profit with fraud equals the profit without any fraud and audit. Therefore, a price below or equal the marginal costs c would imply a loss. The second condition is only fulfilled in equilibrium (E1) where insurers conduct audits with positive probability. Since we only consider coinsurance rates for which equilibrium (E1) arises, we know that the second condition is fulfilled as well. But it is important to keep in mind that the price given by (4) is *not* the profit-maximizing price for *every* coinsurance rate $\delta \in [0, 1]$. In equilibrium (E2), the uniform profit-maximizing price is given by $\bar{p}_{nf}$.

Proof. See Appendix. □

In both equilibria (E1) and (E2) we observe an external moral hazard effect: Prices are increasing in insurance coverage. This can be again attributed to consumers' decreasing price sensitivity and repair firms' increasing market power with increasing coverage. However, one important feature distinguishes the equilibrium price in (E1) from the price in (E2), i.e. the price in the benchmark situation: The term $\pi_s\eta(p^*)\Big(1-v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*)+v(p^*)\Big)$ in the denominator of the equilibrium price in (E1). This term represents consumers' willingness

to pay to avoid fraud or, in other words, repair firms' credibility surcharge. The willingness to avoid fraud increases with the fraud probability $\eta(p^*)$ and decreases with the audit probability $v(p^*)$. Consequently, small coinsurance rates entail great external moral hazard effects, high auditing incentives, and rare overcharging whereas an increase in δ imply less auditing incentives, more frequent overcharging, and therefore a higher willingness to avoid fraud on consumers' side.
In summary, the equilibrium set (E1) consists of $n^*, p^*, \eta^* := \eta(p^*)$ and $v^* := v(p^*)$. The equilibrium set (E2) consists of $n_{nf}, \bar{p}_{nf}$. Before we determine the optimal insurance contract and therefore consider δ as an endogenous variable, a comparative static analysis of the equilibrium set enables us to uncover the driving factors behind the players' strategies.

4 The interaction between insurance and repair markets

Our results shed a new light on the interaction between insured consumers and expert sellers in repair markets. Whereas Sülzle and Wambach (2005) study overcharging in credence goods markets by fixing prices and allowing for second opinions, we focus on the impact of claims auditing and the interdependence of pricing and overcharging enabling us to disentangle legal and illegal external moral hazard. Sülzle and Wambach (2005) find that overcharging is inevitable in equilibrium. As consumers try to avoid additional transportation costs and fear that searching for a second opinion does not improve their situation, experts exploit the information asymmetry by (occasional) overcharging. The equilibrium is either in pure strategies (full-fraud and never-searching equilibrium) or mixed (occasional overcharging and searching for second opinions). The former outcome corresponds to equilibrium (E2) in our setting where repair suppliers always demand the highest possible price and the overcharging issue is bypassed. The equilibrium in mixed strategies corresponds to (E1) where repair firms overcharge with a strictly positive probability smaller than 1. However, two important characteristics distinguish our results from those of Sülzle and Wambach (2005) and therefore our insights extend the existing theoretical literature about the impact of insurance on credence goods markets in different ways. First, the change in the overcharging frequency with an increase in the coinsurance rate is a consequence of physicians' adjustment reaction to patients' implicit change in behavior in the context of Sülzle and Wambach (2005). In our model a change in the overcharging frequency with increasing coinsurance rate is caused by consumers' change in preferences, suppliers' adjustment reaction, *and* insurers' change in the audit frequency. Second, consumers' price sensitivity and repair firms' pricing strategy have strong impact on the equilibrium outcome in our model. Whereas fixed prices cause patients to only care about potential overcharging in the approach from Sülzle and Wambach (2005), our model reveals the trade off between price effect and fraud aversion effect in consumers' decision. Taking price and overcharging/audit probability into account, consumers may accept higher prices in order to avoid overcharging.

We now turn to a closer analysis of both equilibrium sets (E1) and (E2) and their interdependence. First, we apply a comparative static analysis of the equilibrium sets to uncover the impact of certain parameters. This enables us to explore how consumers, repair firms, and insurers respond to changes in the insurance structure or to changes in cost structures. Subsequently, we compare the situations with and without fraud and derive welfare effects of overcharging and auditing. The following proposition summarizes the results obtained from the comparative static analysis.

Proposition 5. *The comparative static analysis of the equilibrium sets (E1) and (E2) yields the following results:*

- *In (E1), p^* is either monotonously decreasing in δ or p^* decreases until a minimum is reached for some $\delta_{min} \in (0, 1)$ and increases for $\delta \in (\delta_{min}, a)$.*
- *In (E2), the price is monotonously decreasing in δ.*
- *In (E1), p^* is monotonously decreasing in π_b and π.*
- *In (E2), the price is monotonously increasing in π_b and decreasing in π.*
- *In (E1), p^* and η^* are increasing in k.*

Proof. See Appendix. □

4.1 Consumers' insurance scheme

Consumers' decision to go to a particular repair firm depends on four factors: the transportation costs of visiting the supplier, the price for the repair good, the probability of getting overcharged by the repair firm, and the probability that a fraud attempt is detected by the insurer. The proposition shows that the coinsurance rate determines which of these factors are predominant in consumers' decision. Full coverage/a very small coinsurance rate diminishes consumers' price sensitivity and their willingness to avoid fraud (fraud aversion), as insurers pay the major fraction of the repair bill. Furthermore, insurers' willingness to conduct audits is very high which causes consumers to even care less about overcharging bills. Hence, consumers can only be attracted by small transportation costs, which is reached by a large density of suppliers. Due to consumers' low price sensitivity and their preference for low transportation costs and insurers' high audit tendency, it is beneficial for repair firms to charge high repair prices (external moral hazard effect) and only defraud little. High profits attract many suppliers resulting in a high supplier density. An increasing coinsurance rate raises consumers' price sensitivity as well as their willingness to avoid overcharging. Simultaneously, insurers become less interested in auditing claims which also increases consumers' concerns about potential overcharging. As price and fraud probability are inverse to each other, consumers trade off the price effect and the fraud aversion effect when choosing a repair firm. If the insurer's share in the repair bill is still sufficiently large, the price effect outweighs the fraud aversion effect. As a consequence, consumers prefer smaller repair prices with decreasing insurance coverage. Repair suppliers' optimal strategy consists of reducing prices and increasing the overcharging probability to keep insurers indifferent. Due to the decreasing price, insurers need to decrease the audit probability to keep repair firms indifferent.

For δ approaching 1, we have to distinguish between two different cases. It can either be the case that expected audit costs exceed expected insurance payments when the price effect still outweighs the fraud aversion effect or audits are still worthwhile when the fraud aversion effect outweighs the price effect. The first case occurs when audit costs and big-loss probability are relatively high. The low probability of detecting an actual fraud attempt and the high audit costs force the insurer to already give up auditing for a relatively small δ. We observe the transition from (E1) to (E2) when the price in (E1) is falling in the coinsurance rate. In the latter case, relatively low audit costs and a comparably high small-loss probability allow the insurer, on the one hand, to also conduct audits for larger coinsurance rates. On the other hand, consumers become more and more concerned about potential overcharging with increasing share in the repair costs. In comparison to the price effect, the fraud aversion effect increases faster with decreasing insurance coverage due to the potential duplication of repair costs. As consumers trade off the price effect and the fraud aversion effect, there exists some critical coinsurance rate for which the fraud aversion effect outweighs the price effect. In other words, consumers prefer to pay more for the repair good itself with increasing coinsurance rate to avoid overcharging. A price increase with increasing coinsurance rate goes along with an increasing audit probability. Consequently, it is beneficial for repair firms to increase the price with increasing coinsurance rate even if consumers' price sensitivity further increases. We observe the transition from (E1) to (E2) when the price in (E1) is rising in the coinsurance rate.

With regard to (E1), we find surprising results. One might assume that overcharging in repair markets declines when consumers have less insurance coverage due to their increasing price awareness. In general, this does not hold in equilibrium. Starting from full insurance, it is rather the case that repair firms decrease displayed repair prices due to consumers' increasing price sensitivity which makes insurance audits rarer. The increased chance of a successful fraud attempt leads to repair firms overcharging more often. Furthermore, the comparative static analysis allows us disentangle legal and illegal moral hazard on the side of repair firms. Whereas full insurance coverage and low coinsurance rates primarily induce external moral hazard and few overcharging bills, moderate and lower insurance coverage lead to frequent overcharging and moderate price effects.

4.2 Demand, audit costs, and welfare effects

The comparative static analysis w.r.t. the loss probability π indicates that an increase in demand leads to a price decline. In both equilibria, an increase in demand intensifies competition between suppliers (i.e. more suppliers enter the repair market) which entails a price decline. However, the price decline has an impact on the overcharging probability in (E1). Due to the price decline, insurers have to decrease the audit probability to keep repair firms indifferent between their strategies. The potential gain from a fraud attempt decreases which gives repair firms less incentives to commit fraud. Simultaneously, insurers have less incentives to conduct audits such that repair firms need to increase the fraud probability to keep insurers indifferent between auditing and not auditing. The current COVID-19 pandemic provides evidence for this phenomenon: Due to the fast spread of the pandemic in spring 2020, the demand for medical providers increased. (Verisk, 2020, p.1) announces that "the pandemic is giving rise to a new wave of questionable behavior showing up in insurance claims. In April 2020, Verisk data showed a 14 percent increase in claims linked to providers with suspicious billing practices".

An increase in the big-loss probability π_b has different effects. While experts increase the price in equilibrium (E2) due to the higher expected costs, repair firms decrease the price in equilibrium (E1). As the price in equilibrium (E2) represents an average price between minor and major repairs, repair firms need to increase the price with increasing demand for major repairs due to the increased average costs. The price decline in (E1) can be explained by the possibility of overcharging. It is more beneficial for repair firms to reduce the price and defraud more often than to increase the price and defraud less. As more legitimate claims requiring two repair units are filed, the probability of detecting a fraudulent claim decreases. From a repair firm's point of view, a price increase would not exploit this advantage. A price decline gives insurers even less incentives to audit claims. To keep the insurer indifferent between auditing and not auditing, experts need to increase the fraud probability.

The costs of an audit have an indirect impact on the situation in the repair market, as they are not related to demand or supply. If insurers succeed in reducing the audit costs (e.g. through the use of machine learning in fraud detection), the willingness to audit increases and consumers anticipate this attitude. To keep the insurer indifferent between auditing and not auditing, repair firms need to decrease the fraud probability. Consequently, consumers' willingness to pay to avoid fraud decreases such that repair firms also need to reduce the price for one repair unit. The price decline makes overcharging less attractive for repair firms leading insurers to decrease the audit probability.

By comparing the overcharging equilibrium with the benchmark situation, it turns out that welfare does not change if (E2) is established, i.e. the coinsurance rate is sufficiently large. For smaller coinsurance rates, however, equilibrium (E1) arises where overcharging and audits take place. From a social welfare point of view, (E1) is inefficient due to audit costs and fines for repair firms. Taking consumers' perspective, (E1) reduces utility in comparison to the benchmark situation. The loss in utility is caused by the credibility surcharge on the price for the repair good and an increase in the insurance premium due to audit and fraud costs. The lower the audit costs, the lower the loss in utility. If an insurer manages to keep audit costs low, the overcharging frequency in the repair market is low due to frequent audits and insurance coverage stays attractive for consumers due to the low fraud-caused surcharge on the premium.

The next section addresses the question of optimal insurance coverage. Due to insurance-induced price effects and premiums accounting for fraud costs, it remains to check if consumers should purchase insurance at all. And if so, what amount of insurance coverage is optimal?

5 Optimal insurance contract

The computation of the equilibrium set in dependence of the coinsurance rate δ finally allows us to determine the optimal insurance contract. The optimal insurance contract maximizes the expected utility of an average consumer under the constraint of an actuarially fair premium (Nell et al., 2009). Since insurers are unable or it is to costly to define each state of the nature, the insurance indemnity depends on policyholders' actual expenses. Furthermore, the indemnity is only paid out when the claim is found to be legitimate, i.e. either an audit has been conducted or the insurance company assumes legitimacy without audit. We further assume that transportation costs are uninsurable. The insurance-induced external moral hazard effect, the welfare loss due to fraud costs, and the benefits from an improved risk allocation influence the optimal insurance contract, i.e. the optimal coinsurance rate δ^*. In the following we denote by $\alpha_1(\delta)$ the premium function in the case $\delta < a$ and by $\alpha_2(\delta)$ the premium function in the case $\delta \geq a$. Accordingly, we denote by $p_1(\delta)$ the price function for one repair unit in the case $\delta < a$ and by $p_2(\delta)$ the price function in the case $\delta \geq a$. $tc(\delta)$ denote the average transportation costs in the case $\delta < a$. Consequently, the function to maximize is given by:

$$
\begin{aligned}
\max_{\delta} EU(\delta) = \mathbb{1}_{\{\delta<a\}} \Bigg(&(1-\pi)u\Big(\underbrace{w_0 - \alpha_1(\delta)}_{=:w_1(\delta)}\Big) + \pi\pi_b u\Big(\underbrace{w_0 - \alpha_1(\delta) - 2\delta p_1(\delta) - tc(\delta)}_{=:w_2(\delta)}\Big) \\
&+ \pi\pi_s\Big(1-\eta(\delta)+\eta(\delta)v(\delta)\Big)u\Big(\underbrace{w_0 - \alpha_1(\delta) - \delta p_1(\delta) - tc(\delta)}_{=:w_3(\delta)}\Big) \\
&+ \pi\pi_s\eta(\delta)\Big(1-v(\delta)\Big)u\Big(\underbrace{w_0 - \alpha_1(\delta) - 2\delta p_1(\delta) - tc(\delta)}_{=:w_2(\delta)}\Big)\Bigg) \\
+\mathbb{1}_{\{\delta\geq a\}} \Bigg(&(1-\pi)u\Big(w_0 - \alpha_2(\delta)\Big) + \pi u\Big(w_0 - \alpha_2(\delta) - \delta p_2(\delta) - 0.25\sqrt{\frac{\delta t f}{\pi}}\Big)\Bigg).
\end{aligned} \tag{5}
$$

The following two propositions characterize the optimal insurance contract.

Proposition 6. *The optimal insurance contract cannot entail full coverage ($\delta^* > 0$).*

Proof. See Appendix. □

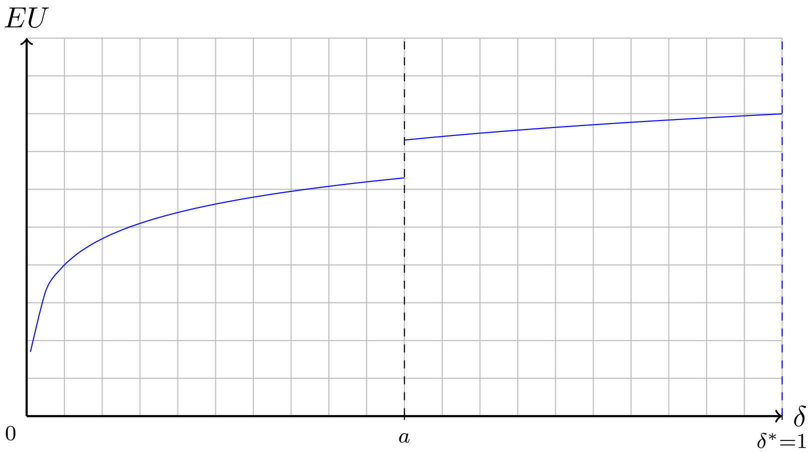
Intuitively, the optimal insurance contract trades off the benefit of an improved risk allocation, the costs of the insurance-induced price effect, and the costs of overcharging by suppliers. Whereas the marginal benefit from improved risk allocation and the benefit from increased fraud detection due to an additional increase in insurance coverage vanish with increasing coverage, the marginal price effect is strictly increasing in coverage and tends to infinity as δ approaches zero. Consequently, the optimal contract cannot provide full coverage. Our theoretical analysis reveals that full insurance coverage entails strong external moral hazard effects in repair markets which manifest in significantly higher prices compared to partial insurance. Overcharging, on the other hand, does not play an important role. As insurers audit a high number of claims, repair firms largely forego overcharging. Next, we turn our attention to the question of whether the optimal contract entails any coverage at all ($\delta^* < 1$).

Proposition 7. *If consumers are sufficiently risk-averse, the optimal insurance contract entails partial coverage $\delta^* \in (0, 1)$. Only if consumers are risk-averse to a high degree, the audit costs k are sufficiently low, and the loss probability π is sufficiently large, the optimal contract provides high coverage ($\delta^* < a$). If the audit costs k are sufficiently large and consumers are sufficiently risk-averse, the optimal contract entails small coverage ($\delta^* \geq a$). Otherwise the optimal contract does not provide any coverage ($\delta^* = 1$).*

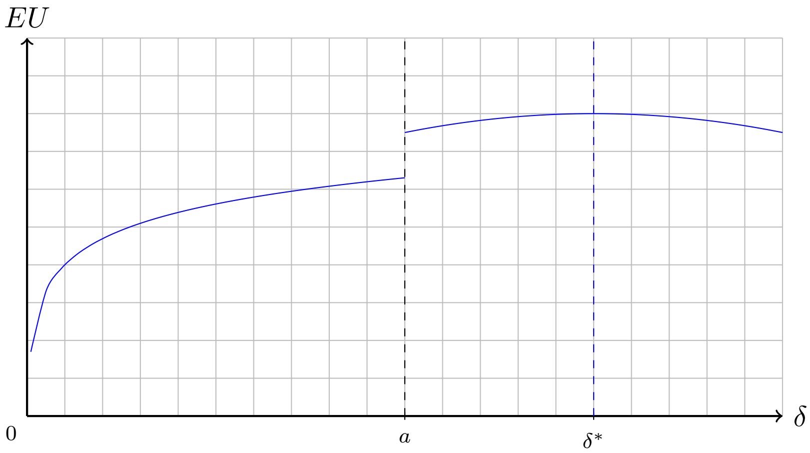
Proof. See Appendix. □

By comparing the situation without insurance coverage ($\delta = 1$) and small coverage ($1 > \delta > a$), we infer that consumers incur higher costs in the case of small insurance coverage. This increase in costs results from the repair good price increase effect that outweighs the transportation costs decrease effect. Only if the benefit from improved risk allocation exceeds the total costs increase, purchasing small insurance coverage is advantageous for consumers. By comparing the situation without insurance coverage ($\delta = 1$) and high coverage ($0 < \delta < a$), we again observe that total costs are higher in the case of high insurance coverage. This cost difference is caused by the repair good price increase effect and the additional fraud costs. Only if the benefit from improved risk allocation exceeds the increase in repair and fraud costs, consumers purchase high insurance coverage. The benefit from improved risk allocation may only exceed the costs increase if the fraud costs are below a certain threshold, as otherwise the utility derived from no coverage always exceeds the utility derived from high coverage. By comparing the situation with small insurance coverage ($\delta > a$) and high coverage ($\delta < a$), it turns out that total costs are lower in the case of small insurance coverage due to the additional fraud costs in the case of high coverage. Consequently, only in the case of low audit costs and a large loss probability the benefit from improved risk allocation can exceed the total cost difference. Figure 2 shows the optimal insurance contract for different levels of risk aversion and different parameter combinations.

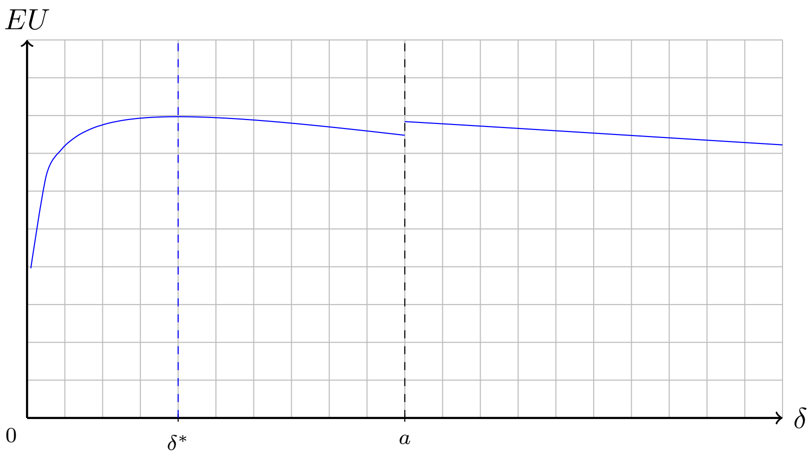
The results obtained in this paper provide interesting insights into the interaction between consumers, insurance, and repair markets and highlight the impact of consumers' insurance scheme on the resulting equilibrium. The following main findings characterize the effect of insurance on pricing strategies and fraud in markets for repair goods. First, consumers with minor damages are likely to pay a surcharge which can be either due to fraudulent overcharging or due to a lack of price differentiation between minor and major repairs. For example, the repair of a broken car part is often as expensive as the replacement of the defective part. In contrast, consumers with major damages mostly receive the treatment for the appropriate price. The use of many available resources for the repair leaves little room for fraudulent overcharging. Second, consumers would be better off without auditing. Foregoing audits would raise consumers' price sensitivity and their willingness to avoid fraud. Due to the oligopolistic framework, repair service suppliers are forced to react to consumers' preferences and adapt their prices and billing behavior accordingly. In accordance with the credence goods literature, we find that overcharging is not an issue as long as the interaction between consumer and repair firm is not disturbed from outside (e.g. through an audit or the search for a second opinion). Third, external moral hazard and overcharging are *complementary* effects. On the one hand, external moral hazard is *always* present in repair markets and decreases with consumers' increasing price sensitivity. On the other hand, overcharging only arises when insurers conduct audits and increases with consumers' price sensitivity.



(a) Low risk aversion



(b) Moderate risk aversion



(c) High risk aversion, low audit costs, high loss probability

Figure 2: The optimal insurance contract δ^* for different levels of risk aversion and different parameter combinations

6 Conclusion

In this article, we study the effect of insurance and claims auditing on pricing and overcharging behavior in oligopolistic markets for repair goods. On the one hand, incomplete insurance contracts (e.g. in car and health insurance) and market imperfections lead repair firms to set prices containing high markups. On the other hand, asymmetric information regarding loss amounts gives repair firms strong overcharging incentives. To limit third party insurance fraud, insurance companies mostly make use of claims auditing. Consumers' choice of a coverage level influences their price sensitivity and therefore the pricing in repair markets. The price setting determines overcharging possibilities for suppliers and thus also auditing incentives for insurers. We combine the characteristics of credence goods markets, costly state verification, and legal insurance-induced price effects in repair markets to uncover the interaction of insurance and repair markets. The assumptions of an imperfect repair market and information asymmetry between repair firm and insurer allow us to examine the joint effects of illegal and legal moral hazard in repair markets on consumers.

We find that the price setting of repair service suppliers as well as overcharging and auditing probability depend on consumers' coverage level. For a sufficiently small coverage level, the repair firms set a uniform price for different types of repair, which inhibits overcharging and auditing. For a sufficiently high coverage level, repair firms set different prices for different types of repair, which enables overcharging. The equilibrium in the claiming game is in mixed strategies with positive overcharging and positive auditing probability. In any case, the markup over cost increases in consumers' level of insurance coverage, which shows an external moral hazard effect. Claims auditing leads to a welfare loss for consumers. Given an arbitrary coinsurance rate, consumers' expected utility is higher with a uniform repair price and no auditing than with repair-dependent prices and auditing. This observation together with the presence of external moral hazard explain our findings with respect to the utility-maximizing insurance contract: Full insurance is never optimal for risk-averse consumers and partial insurance is only utility-maximizing if consumers are sufficiently risk-averse. The price effect in the repair market outweighs risk aversion, unless risk aversion is sufficiently large. Moreover, only if the audit costs are sufficiently small, consumers' risk aversion is sufficiently high, and the loss probability is sufficiently large, the utility-maximizing contract entails a small coinsurance rate for which the equilibrium with different prices and auditing arises. Otherwise, a large coinsurance rate, for which the equilibrium with a uniform repair price arises, is utility-maximizing.

Our results highlight the inefficiency of market insurance, as consumers could be better off without claims auditing. To prevent this inefficiency, insurers could establish their own network of repair service providers or provide own repair services. In the case of an own network, an insurer negotiates a fixed repair price (capitation) for any type of repair with the repair firms. As capitation prevents overcharging, auditing is not needed. The negotiation artificially induces the equilibrium with uniform repair price and without overcharging and auditing for any level of insurance coverage. Therefore, a high level of insurance coverage is in demand if consumers are sufficiently risk-averse, which resolves the inefficiency with claims auditing. Especially, the American healthcare market (Managed Care) profits from the contractual relationship between insurers and suppliers (Nell et al., 2009).

This paper provides a starting point for further research in the area of fraudulent repair market behavior with insured demand side. However, we make some simplifying assumptions. Firstly, we only consider two types of loss, but real-world insurance contracts capture a great range of different losses. The argument of limited range also applies to the fraudulent exaggeration amount. A possible path forward would be to examine a setting with a continuously distributed loss size where repair firms could arbitrarily choose the overcharging amount. Furthermore, providers of repair services also take ethical concerns into account before committing fraud. This aspect was also disregarded in our model. For these reasons, we suggest to extend the analysis by including ethical preferences. Finally, and maybe most importantly, we put the focus on the overcharging dimension of fraud. However, overtreatment represents another problematic issue in the context of repair market interaction with insurance that needs to be tackled by insurance providers. Therefore, the investigation of the interdependence of overtreatment and insurance would be an interesting continuation of the theoretical third party insurance fraud literature. We leave these aspects for future research.

Appendix

Proof of Proposition 1. Let us start with the case where all suppliers set a tariff with two different prices for the treatments. Each supplier has two surrounding competitors. In order to derive a single supplier's demand function, let us consider supplier j. A consumer located between supplier j and one of its neighbors (offering a repair unit at the price p) at the distance $x \in [0,1]$ from supplier j is indifferent between the two competitors if

$$\pi_s \delta p_j + \pi_b 2\delta p_j + tx = \pi_s \delta p + \pi_b 2\delta p + t\left(\frac{1}{n} - x\right) \Leftrightarrow x = \frac{1}{2t}\left(\frac{t}{n} + \delta(p - p_j) + \pi_b \delta(p - p_j)\right). \quad \text{(A.1)}$$

We obtain that the resulting demand function of supplier j is given by:

$$D_j(p_j, p) = 2x\pi = \frac{\pi}{n} + \frac{\pi\delta(1+\pi_b)(p-p_j)}{t}. \quad \text{(A.2)}$$

The objective function is given by the supplier's profit function:

$$\begin{aligned} \Pi_j(p_j, p) &= (p_j - c)\left(\frac{\pi}{n} + \frac{\pi\delta(1+\pi_b)(p-p_j)}{t} + \frac{\pi\pi_b}{n} + \frac{\pi\pi_b\delta(1+\pi_b)(p-p_j)}{t}\right) - f \\ &= (p_j - c)(1+\pi_b)\left(\frac{\pi}{n} + \frac{\pi\delta(1+\pi_b)(p-p_j)}{t}\right) - f. \end{aligned} \quad \text{(A.3)}$$

The first order condition for a profit maximum in a symmetric equilibrium with $p_j = p$ is:

$$p = c + \frac{t}{n\delta(1+\pi_b)}. \quad \text{(A.4)}$$

Suppliers enter the repair market until each supplier's profit is zero:

$$\Pi_j(p) = \frac{t\pi}{n^2\delta} - f = 0. \quad \text{(A.5)}$$

Hence, the number of firms in the no-fraud equilibrium is given by:

$$n_{nf} = \sqrt{\frac{t\pi}{\delta f}}. \quad \text{(A.6)}$$

The equilibrium price in the repair market is given by:

$$p_{nf} = c + \frac{1}{1+\pi_b}\sqrt{\frac{tf}{\delta\pi}}. \quad \text{(A.7)}$$

Let us now study the case where all suppliers set a uniform price for both treatments. Applying the same technique as in the first case, we obtain that $n_{nf} = \sqrt{\frac{t\pi}{\delta f}}$ suppliers enter the repair market and set $\bar{p}_{nf} = (1+\pi_b)c + \sqrt{\frac{tf}{\delta\pi}}$ as the profit-maximizing price. Consequently, profits are independent of the chosen tariff for any $n > 0$.

Proof of Proposition 2. The game is solved using backward induction. Since we only consider symmetric equilibria, each insurance company will offer contracts with the same coinsurance rate and each supplier will set the same price for one repair unit. Thus, it is sufficient to solve the game for one representative insurer and one representative supplier. At stage 4, δ, n, and p are already determined and known to the insurer, the insuree, and the repair shop. At stage 7, the insurer has to decide whether to audit a big-loss claim without knowing the repair shop's chosen strategy at stage

6. We know that the insurer will never audit a small-loss claim. At stage 6, the repair shop has to decide whether to defraud (in the case that the customer suffered a small loss) without knowing the insurer's strategy at stage 7. Therefore, both players choose their strategies simultaneously. As the insurer's auditing strategy depends on the indemnity, we have to distinguish between three different cases:

- $p < \frac{k}{\pi_s(1-\delta)}$: Regardless of the fraud strategy chosen by the repair shop, the insurer will never audit a big-loss claim, since the expected costs associated with an audit are strictly larger than the expected costs without audit. Let η be the probability that the repair shop wrongfully claims a big loss instead of a small loss, then we have:[5]

$$\underbrace{\frac{\pi_s\eta}{\pi_s\eta+\pi_b}\Big((1-\delta)p+k\Big)+\frac{\pi_b}{\pi_s\eta+\pi_b}\Big(2(1-\delta)p+k\Big)}_{\text{Expected costs with audit}} > \underbrace{2(1-\delta)p}_{\substack{\text{Costs without}\\ \text{audit}}} \tag{A.8}$$

  The repair shop anticipates the insurer's strategy and will therefore charge a uniform price for both treatments.

- $p > \frac{k}{\pi_s(1-\delta)}$: In the next proof we will show that the audit probability (v) and the fraud probability (η) are given by:

$$\begin{aligned} \eta(p) &= \frac{\pi_b}{\pi_s}\frac{k}{(1-\delta)p-k}, \\ v(p) &= \frac{p}{p+r}. \end{aligned} \tag{A.9}$$

  Both the repair shop and the insurer play a mixed strategy. The audit and the fraud probability are chosen such that the insurer is indifferent between auditing and not auditing and the repair shop is indifferent between committing fraud and not committing fraud. This implies that a repair shop's expected profit is independent of whether it defrauds or not.

- $p = \frac{k}{\pi_s(1-\delta)}$: If the fraud probability is strictly smaller than 1, the insurer will audit no claim, since the expected costs associated with an audit are strictly larger than the expected costs without audit. Then it is optimal for the supplier to commit fraud whenever it is possible. In turn, the insurer is indifferent between auditing and not auditing a big-loss claim. The insurer anticipates that the supplier commits fraud whenever it is possible and will therefore audit every big-loss claim. But then it would be optimal for the supplier never to defraud the insurer etc.
  Therefore, the price $p = \frac{k}{\pi_s(1-\delta)}$ can never be part of an equilibrium.

At stage 3, the suppliers that entered the repair market determine the profit-maximizing price. In equilibrium, each supplier charges the same profit-maximizing price. When is it optimal to choose a price below $\frac{k}{\pi_s(1-\delta)}$, which implies that that price is uniform and the audit probability is 0? When is it optimal to choose a price above $\frac{k}{\pi_s(1-\delta)}$, which implies that the fraud probability is below 1 and the audit probability is strictly greater than 0?
In order to answer those questions, we have to compare the supplier's profit in the two cases. Therefore, we calculate the profit-maximizing price in each case. At stage 3, n is a fixed and known number. Anticipating an audit probability of 0, the profit-maximizing price is uniform and given

[5] For further details see proof of Proposition 3.

by:

$$p_{pm}(n,\delta) = \frac{t}{\delta n} + c(1+\pi_b). \tag{A.10}$$

This can be shown using the technique in the proof of Proposition 4.
Anticipating a fraud probability ¡ 1 and an audit probability ¿ 0, the profit-maximizing price per repair unit is given by:

$$\hat{p}_{pm}(n,\delta) = c + \frac{t}{\delta n\Big[1+\pi_b - \pi_s\eta(\hat{p}_{pm})\Big(1-v(\hat{p}_{pm})\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\hat{p}_{pm})+v(\hat{p}_{pm})\Big)\Big]}. \tag{A.11}$$

We will show how to obtain this formula in the Proof of Proposition 4. $v(p)$ and $\eta(p)$ are given by formula (A.9).
We observe that $\hat{p}_{pm}(n,\delta) > c + \frac{t}{\delta n(1+\pi_b)}$ for every given n and every $\delta \in (0,1)$. It follows that $(1+\pi_b)\hat{p}_{pm}(n,\delta) > p_{pm}(n,\delta)$ for every given δ and every n.
We denote by $\mathbb{E}[\Pi(p_{pm}(n,\delta))]$ the expected profit of one supplier if the market price is given by $p_{pm}(n,\delta)$. Correspondingly, we denote by $\mathbb{E}[\Pi(\hat{p}_{pm}(n,\delta))]$ the expected profit of one supplier if the market price is given by $\hat{p}_{pm}(n,\delta)$.
At stage 3, suppliers that entered the repair market prefer to set the price $\hat{p}_{pm}(n,\delta)$ (that induces audits by insurers and a fraud probability ¡ 1) because the resulting profit is higher:

$$\begin{aligned}\mathbb{E}[\Pi(p_{pm}(n,\delta))] &= \left(\frac{t}{\delta n} - c\pi_s\right)\frac{\pi}{n} + c\frac{\pi\pi_s}{n} - f = \frac{t\pi}{\delta n^2} - f \\ &= \left(\frac{t}{\delta n(1+\pi_b)} + c - c\right)\frac{\pi(1+\pi_b)}{n} - f < \mathbb{E}[\Pi(\hat{p}_{pm}(n,\delta))].\end{aligned} \tag{A.12}$$

This implies the following: We are at stage 3 and n is known to the repair shops in the market, i.e. n is a fixed number. In general, suppliers in the market prefer to charge the price $\hat{p}_{pm}(n,\delta)$ for one repair unit and defraud with a probability ¡ 1. But n and the resulting competition between the suppliers can be too high such that a price larger than $\frac{k}{\pi_s(1-\delta)}$ is not feasible. If $\hat{p}_{pm}(n,\delta) > \frac{k}{\pi_s(1-\delta)}$, suppliers will set $\hat{p}_{pm}(n,\delta)$ as the profit-maximizing price per repair unit at stage 3 and defraud with a probability ¡ 1. If $p_{pm}(n,\delta) \leq \frac{k}{\pi_s(1-\delta)}$, suppliers will set $p_{pm}(n,\delta)$ as the profit-maximizing uniform price at stage 3 and no audits will be conducted.
At stage 2, suppliers enter the repair market until the expected profit is 0. We define the following:

- Let $n_1 > 0$ be such that $\mathbb{E}[\Pi(p_{pm}(n_1,\delta))] = 0$.
- Let $n_2 > 0$ be such that $\hat{p}_{pm}(n_2,\delta) = \frac{k}{\pi_s(1-\delta)}$.
- Let $n_3 > 0$ be such that $\mathbb{E}[\Pi(\hat{p}_{pm}(n_3,\delta))] = 0$.

Then we know that $n_1 < n_3$, as $(1+\pi_b)\hat{p}_{pm}(n,\delta) > p_{pm}(n,\delta)$ for every n and every δ.
It can be easily shown that $p_{pm}(n_1,\delta)$ and $\hat{p}_{pm}(n_3,\delta)$ are given by the following formulas:

$$\begin{aligned}p_{pm}(n_1,\delta) &= \sqrt{\frac{tf}{\delta\pi}} + c(1+\pi_b), \\ \hat{p}_{pm}(n_3,\delta) &= c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b - \pi_s\eta(\hat{p}_{pm})\Big(1-v(\hat{p}_{pm})\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\hat{p}_{pm})+v(\hat{p}_{pm})\Big)\Big]}}.\end{aligned} \tag{A.13}$$

We observe that $p_{pm}(n_1, \delta) < (1+\pi_b)\hat{p}_{pm}(n_3, \delta)$. It is obvious that no more than n_3 suppliers will enter the repair market, as otherwise each supplier's expected profit would be smaller than 0. At stage 3, suppliers set the profit-maximizing price after n suppliers have entered the repair market. If $n < n_2$, the profit-maximizing price is given by $\hat{p}_{pm}(n, \delta)$: In this case, it is possible to set a price strictly higher than $\frac{k}{\pi_s(1-\delta)}$, which is preferred by repair shops. If $n \geq n_2$, the profit-maximizing price is given by $p_{pm}(n, \delta)$: The n suppliers would like to set a price higher than $\frac{k}{\pi_s(1-\delta)}$. But if the market price for one repair unit was strictly higher than $\frac{k}{\pi_s(1-\delta)}$, each supplier would have an incentive to deviate from this price (lower the price), as a lower price would generate more profits. As suppliers enter the repair market until the expected profit is 0, we have to distinguish between the following three cases:

- Case $n_1 < n_3 < n_2$: As no more than n_3 suppliers will enter the repair market, the profit-maximizing price per repair unit determined at stage 3 is given by $\hat{p}_{pm}(n, \delta)$. Thus, exactly n_3 suppliers will enter the repair market at stage 2. Finally, the equilibrium price per repair unit is given by $p^* = \hat{p}_{pm}(n_3, \delta)$, as $\hat{p}_{pm}(n_3, \delta) > \frac{k}{\pi_s(1-\delta)}$.
- Case $n_1 \leq n_2 \leq n_3$: For every $n < n_2$ the profit-maximizing price is given by $\hat{p}_{pm}(n, \delta)$. But we observe that an equilibrium where the price is higher than $\frac{k}{\pi_s(1-\delta)}$ is not possible: To obtain an equilibrium set with a price strictly higher than $\frac{k}{\pi_s(1-\delta)}$, less than n_2 suppliers are needed to enter the repair market. But as each supplier's profit is still higher than 0 if less than n_2 suppliers enter the repair market, new entrants are attracted and will enter the market. Therefore, a price higher than $\frac{k}{\pi_s(1-\delta)}$ cannot be the case in an equilibrium. By repeating the game, for instance, suppliers anticipate at stage 3 that it cannot be optimal to set a price higher than $\frac{k}{\pi_s(1-\delta)}$, even if $n < n_2$. Therefore, the profit-maximizing price determined at stage 3 is always given by $p_{pm}(n, \delta)$ for every $n > 0$. Thus, exactly n_1 suppliers enter the repair market such that the equilibrium price is given by $p_{pm}(n_1, \delta)$.
- Case $n_2 < n_1 < n_3$: The equilibrium price is given by $p^* = p_{pm}(n_1, \delta)$, as an equilibrium where the price is higher than $\frac{k}{\pi_s(1-\delta)}$ is not possible: To obtain an equilibrium set with a price strictly higher than $\frac{k}{\pi_s(1-\delta)}$, less than n_2 suppliers are needed to enter the repair market. As potential entrants anticipate that it is possible that more than n_2 suppliers enter the repair market and make zero profits, n_1 suppliers enter the market.

It is left to show that the case $n_1 < n_3 < n_2$ does not apply for every $\delta \in (0,1)$, i.e. $\hat{p}_{pm}(n_3, \delta) \leq \frac{k}{\pi_s(1-\delta)}$ for $\delta > 0$ large enough. Assume that $\hat{p}_{pm}(n_3, \delta) > \frac{k}{\pi_s(1-\delta)}$ for every $\delta \in (0,1)$. This would imply that $\eta(\hat{p}_{pm}) < 1$ for every $\delta \in (0,1)$ and therefore $\hat{p}_{pm}(n_3, \delta) < c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)(1+\pi_b-\pi_s)}}$ for δ large enough, as $(1 - v(\hat{p}_{pm})) \to 0$ for $\delta \to 1$. This implies that $c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)(1+\pi_b-\pi_s)}} > \frac{k}{\pi_s(1-\delta)}$ for δ close to 1. This is a contradiction.
For sufficiently small δ, audit and fraud probability are strictly between 0 and 1 and for sufficiently large δ, the audit probability is 0 suppliers charge a uniform price:
Let a be the smallest number in $(0,1)$ such that $\hat{p}_{pm}(n_3, a) = \frac{k}{\pi_s(1-a)}$. Then we have:

- For every $\delta < a$ the number of suppliers in the repair market is given by n_3 and the equilibrium price for one repair unit in the repair market is given by $\hat{p}_{pm}(n_3, \delta)$.
- For every $\delta \geq a$ the number of suppliers in the repair market is given by $n_1 = \sqrt{\frac{t\pi}{\delta f}}$ and the equilibrium price for one repair unit in the repair market is given by $p_{pm}(n_1, \delta)$.

Proof of Proposition 3. 1. It is true according to the assumption.
2. If the insuree suffered a big loss, a repair shop always sells two repair units, since two repair units

are needed to repair a big damage. Accordingly, a big loss is reported to the insurer. The strategy "sell two repair units" dominates the strategies "sell one repair unit" and "sell no repair unit".
An insurer never audits a small-loss claim, since such a claim cannot be fraudulent. The strategy "no audit" dominates the strategy "audit".
3. & 4. We consider one representative insurer and one representative supplier. First, let us assume that the insurance company ex ante commits bindingly on an auditing strategy. The repair shop will defraud if the expected profit per small-loss customer is higher with fraud than without fraud. Let v be the probability that the insurer audits a big-loss claim.

$$\underbrace{v(p-c-r)+(1-v)(2p-c)}_{\text{Expected profit with fraud}} > \underbrace{p-c}_{\substack{\text{Profit without}\\ \text{fraud}}} \Leftrightarrow v < \frac{p}{p+r}. \tag{A.14}$$

If the insurer ex ante commits on $v < \frac{p}{p+r}$, the repair shop will defraud with probability 1. Hence, it would be optimal for the insurer to audit every big-loss claim. Then it would be optimal for the repair shop to never commit fraud. Etc. This cannot be an equilibrium.
If the insurer ex ante commits on $v > \frac{p}{p+r}$, the repair shop will defraud with probability 0. Hence, it would be optimal for the insurer to audit no claim. Then it would be optimal for the repair shop to always commit fraud. Etc. This cannot be an equilibrium.
If the insurer ex ante commits on $v = \frac{p}{p+r}$, the repair shop is indifferent between committing fraud and not committing fraud. If the repair shop chooses a strategy such that the insurer's expected costs with an audit are strictly smaller than the expected costs without audit, the insurer will choose $v = 1$. We already know that this cannot be an equilibrium. If the repair shop chooses a strategy such that the expected costs of an audit are strictly higher than the expected costs of no audit, the insurer will choose $v = 0$. We already know that this cannot be the case in an equilibrium.
If the repair shop chooses the strategy such that expected costs with audit equal the expected costs without audit, the insurer will be indifferent between auditing and not auditing a big-loss claim. Therefore, the insurer has no incentive to deviate from the strategy $v = \frac{p}{p+r}$. The same equilibrium arises when the insurer and the repair shop choose their strategies simultaneously. Subsequently, we examine the game with simultaneous actions.
If the players choose their strategies simultaneously, they do not know the other player's strategy. It is obvious that no equilibrium in pure strategies exists. Therefore, there has to be an equilibrium in mixed strategies: The insurer chooses its strategy such that the repair shop is indifferent between defrauding and not defrauding. The repair shop chooses its strategy such that the insurer is indifferent between auditing and not auditing a big-loss claim. We already know that the repair shop is indifferent if the audit probability is given by $v = \frac{p}{p+r}$. Let η be the probability that the repair shop files a big loss claim given that the insuree has a small loss, i.e. the fraud probability. By Bayes' rule γ, the insurer's posterior belief that a big loss occurred given that the repair shop has filed a big-loss claim, is given by:

$$\gamma = \frac{\pi\pi_b}{\pi\pi_b + \pi\pi_s\eta} = \frac{\pi_b}{\pi_b + \pi_s\eta}. \tag{A.15}$$

The only strategy for which the insurer is indifferent between auditing and not auditing must be such that η solves:

$$\begin{aligned}&\gamma(-2p(1-\delta)-k)+(1-\gamma)(-p(1-\delta)-k)=-2p(1-\delta)\\&\Leftrightarrow \gamma=\frac{p(1-\delta)-k}{p(1-\delta)} \Leftrightarrow \frac{p(1-\delta)}{p(1-\delta)-k}=\frac{\pi_b+\pi_s\eta}{\pi_b} \Leftrightarrow \eta=\frac{\pi_b}{\pi_s}\frac{k}{p(1-\delta)-k}.\end{aligned} \tag{A.16}$$

Finally, we check that this strategy combination is a Perfect Bayesian Nash Equilibrium: Given the players' beliefs, the strategies are sequentially rational, as the repair shop's expected profit with fraud equals the expected profit without fraud and the insurer's expected costs associated with an audit equals the expected costs without audit. Therefore, no player has an incentive to deviate from the chosen strategy, i.e. the strategy combination is a sequential equilibrium. For Bayesian extensive games with observable actions, every sequential equilibrium is a Perfect Bayesian equilibrium (Osborne and Rubinstein, 1994). The equilibrium is unique because there exists no equilibrium in pure strategies.

Proof of Proposition 4. Each firm has only two surrounding competitors. In order to derive a single supplier's demand function, let us consider supplier j.
Supplier j offers one repair unit at the price p_j. A policyholder that suffered a loss and is located between supplier j and one of its neighbors at the distance $x \in [0,1]$ from supplier j has to decide whether to go to supplier j or its competitor in order to have the damage repaired. The loss magnitude is unknown to the insuree and he does not know whether supplier j or its competitor will defraud. Therefore, the insuree builds expectations about all possible outcomes. If the insuree decides to go to supplier j for the repair of the damage, the expected costs are given by:

$$\pi_s\underbrace{\left[\Big((1-\eta_j)+\eta_j v_j\Big)\delta p_j+\eta_j(1-v_j)2\delta p_j+tx\right]}_{\text{Expected costs in the case of a small loss}}+\pi_b\underbrace{\Big[2\delta p_j+tx\Big]}_{\substack{\text{Costs in the}\\ \text{case of a big loss}}}, \tag{A.17}$$

where

$$\begin{aligned}&\eta_j:=\eta(p_j)=\frac{\pi_b}{\pi_s}\frac{k}{(1-\delta)p_j-k} \text{ is the probability that supplier j defrauds and}\\&v_j:=v(p_j)=\frac{p_j}{p_j+r} \text{ is the probability that the fraud is detected.}\end{aligned} \tag{A.18}$$

The policyholder is indifferent between going to supplier j or its competitor (the neighbor is offering a repair unit at price p) if:

$$\begin{aligned}&\pi_s\left[\Big((1-\eta_j)+\eta_j v_j\Big)\delta p_j+\eta_j(1-v_j)2\delta p_j+tx\right]+\pi_b\Big[2\delta p_j+tx\Big]=\\&\pi_s\left[\Big((1-\eta)+\eta v\Big)\delta p+\eta(1-v)2\delta p+t\Big(\frac{1}{n}-x\Big)\right]+\pi_b\left[2\delta p+t\Big(\frac{1}{n}-x\Big)\right]\\&\Leftrightarrow \delta p_j+\Big(\pi_s\eta_j(1-v_j)+\pi_b\Big)\delta p_j+tx=\delta p+\Big(\pi_s\eta(1-v)+\pi_b\Big)\delta p+t\Big(\frac{1}{n}-x\Big)\\&\Leftrightarrow x=\frac{1}{2t}\left[\frac{t}{n}+\delta(1+\pi_b)(p-p_j)+\delta\pi_s\Big(\eta(1-v)p-\eta_j(1-v_j)p_j\Big)\right],\end{aligned} \tag{A.19}$$

where

$$\begin{aligned} \eta &:= \eta(p) = \frac{\pi_b}{\pi_s}\frac{k}{(1-\delta)p-k} \text{ is the probability that the competitor defrauds and} \\ v &:= v(p) = \frac{p}{p+r} \text{ is the probability that the fraud is detected.} \end{aligned} \tag{A.20}$$

The resulting demand function of supplier j is given by:

$$D_j(p_j,p) = 2x\pi = \frac{\pi}{n} + \frac{\pi\delta(1+\pi_b)(p-p_j)}{t} + \frac{\pi\delta\pi_s\Big(\eta(1-v)p - \eta_j(1-v_j)p_j\Big)}{t}. \tag{A.21}$$

Supplier j can now observe which of the customers suffered a big loss and which of the customers suffered a small loss, namely $\pi_s D_j(p_j,p)$ customers suffered a small loss and $\pi_b D(p_j,p)$ customers suffered a big loss.
Two repair units are always sold to customers with a big loss. Small-loss customers are sold one repair unit with probability $1-\eta_j$. With probability η_j the repair shop tries to charge the price for two repair units, but the fraud is detected with probability v_j. The expected profit of supplier j is independent of whether it defrauds or not (otherwise the supplier would not be indifferent between committing fraud and not committing fraud). Hence, it is sufficient to analyze the profit function in the case that the supplier never defrauds: Two repair units are sold to big-loss customers and one repair unit is sold to small-loss customers. The expected profit function of supplier j is given by:

$$\begin{aligned} \mathbb{E}[\Pi_j(p_j,p)] &= (p_j-c)D_j(p_j,p) + (p_j-c)D_j(p_j,p)\pi_b - f \\ &= (p_j-c)\pi(1+\pi_b)\left(\frac{1}{n} + \frac{\delta(1+\pi_b)(p-p_j) + \pi_s\delta\Big(\eta(1-v)p - \eta_j(1-v_j)p_j\Big)}{t}\right) - f. \end{aligned} \tag{A.22}$$

In order to derive the optimal p_j for supplier j, we calculate the first order condition:

$$\begin{aligned} \frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j} &= \pi(1+\pi_b)\left(\frac{1}{n} + \frac{\delta(1+\pi_b)(p-p_j) + \pi_s\delta\Big(\eta(1-v)p - \eta_j(1-v_j)p_j\Big)}{t}\right) \\ &\quad + (p_j-c)\pi(1+\pi_b)\left(-\frac{\delta(1+\pi_b)}{t} - \frac{\pi_s\delta\Big(\frac{d\eta_j}{dp_j}(1-v_j)p_j - \eta_j\frac{dv_j}{dp_j}p_j + \eta_j(1-v_j)\Big)}{t}\right). \end{aligned} \tag{A.23}$$

Since we only consider symmetric equilibria, i.e. $p_j = p$, we look for a solution of the following equation:

$$\begin{aligned} &\left.\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\right|_{p_j=p} = 0 \\ \Leftrightarrow\ &\frac{1}{n} + (p-c)\left(-\frac{\delta(1+\pi_b)}{t} - \frac{\pi_s\delta\Big(\frac{d\eta}{dp}(1-v)p - \eta\frac{dv}{dp}p + \eta(1-v)\Big)}{t}\right) = 0. \end{aligned} \tag{A.24}$$

By observing that

$$\begin{aligned}
&\frac{d\eta}{dp}(1-v)p-\eta\frac{dv}{dp}p+\eta(1-v)\\
&=-\eta\frac{(1-\delta)p(1-v)}{p(1-\delta)-k}-\eta(1-v)\frac{p}{p+r}+\eta(1-v)\\
&=-\eta(1-v)\left(1+\frac{\pi_s}{\pi_b}\eta\right)-\eta v(1-v)+\eta(1-v)\\
&=-\eta(1-v)\left(\frac{\pi_s}{\pi_b}\eta+v\right),
\end{aligned} \tag{A.25}$$

equation (A.24) simplifies to:

$$\frac{1}{n}+(p-c)\left(-\frac{\delta(1+\pi_b)-\pi_s\delta\eta(1-v)\left(\frac{\pi_s}{\pi_b}\eta+v\right)}{t}\right)=0. \tag{A.26}$$

Since v and η are functions of p, the profit-maximizing p solves:

$$\begin{aligned}
&\frac{1}{n}+(p-c)\left(-\frac{\delta(1+\pi_b)-\pi_s\delta\eta(p)\Big(1-v(p)\Big)\left(\frac{\pi_s}{\pi_b}\eta(p)+v(p)\right)}{t}\right)=0\\
&\Leftrightarrow p=c+\frac{t}{\delta n\left[1+\pi_b-\pi_s\eta(p)\Big(1-v(p)\Big)\left(\frac{\pi_s}{\pi_b}\eta(p)+v(p)\right)\right]}.
\end{aligned} \tag{A.27}$$

In the following we denote by $p(n)$ the profit-maximizing price in dependence of n. It can easily be shown that equation (A.27) has a unique solution that is a global maximum: $\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\Big|_{p_j=p}$ is a rational function with two vertical asymptotes. The vertical asymptotes are given by $p_1=-r$ and $p_2=\frac{k}{1-\delta}$. The numerator is a polynomial of degree 5 (if $c=\frac{k}{1-\delta}$, the polynomial is of degree 4) and in order to solve $\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\Big|_{p_j=p}=0$, we have to find the roots of this polynomial. In a symmetric equilibrium, where one repair unit is offered at the price $p(n)$, each supplier's profit function is given by:

$$\mathbb{E}[\Pi(p(n))]=\Big(p(n)-c\Big)\frac{\pi+\pi\pi_b}{n}-f \tag{A.28}$$

It is obvious that a price $<\frac{k}{1-\delta}$ cannot be profit-maximizing, as such a low price would be only optimal if the fraud probability equals 1. Applying the Intermediate Value Theorem, one can show that either one or two roots of the polynomial are larger than $\frac{k}{1-\delta}$. If two roots are in the interval $(\frac{k}{1-\delta},\infty)$, it follows that the smaller one is a local minimum by applying the second derivative. Therefore, the profit-maximizing price $p(n)$ is given by the largest real root of the polynomial in the numerator of $\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\Big|_{p_j=p}$. Applying the Intermediate Value Theorem in combination with $\left(\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\Big|_{p_j=p}\right)\Big|_{p=c}=\frac{1}{n}>0$ and $\lim_{p\to\infty}\frac{d\mathbb{E}[\Pi_j(p_j,p)]}{dp_j}\Big|_{p_j=p}<0$, we see that the largest root of this polynomial is strictly larger than c.

New suppliers enter the market until each supplier's expected profit is zero. We denote by $\mathbb{E}[\Pi(p(n))]$

a supplier's expected profit when n suppliers enter the market. The optimal n^* solves:

$$\begin{aligned}
&\mathbb{E}[\Pi(p(n^*))] = 0 \Leftrightarrow (p(n^*) - c)\Big(\frac{\pi + \pi\pi_b}{n^*}\Big) - f = 0 \\
&\Leftrightarrow \frac{t(\pi + \pi\pi_b)}{\delta(n^*)^2\Big[1 + \pi_b - \pi_s\eta\Big(p(n^*)\Big)\Big(1 - v\Big(p(n^*)\Big)\Big)\Big(\frac{\pi_s}{\pi_b}\eta\Big(p(n^*)\Big) + v\Big(p(n^*)\Big)\Big)\Big]} = f \\
&\Leftrightarrow n^* = \sqrt{\frac{t\pi(1 + \pi_b)}{\delta f\Big[1 + \pi_b - \pi_s\eta\Big(p(n^*)\Big)\Big(1 - v\Big(p(n^*)\Big)\Big)\Big(\frac{\pi_s}{\pi_b}\eta\Big(p(n^*)\Big) + v\Big(p(n^*)\Big)\Big)\Big]}}.
\end{aligned} \tag{A.29}$$

Finally, we will show that equation (A.29) has a solution: Obviously, $\mathbb{E}[\Pi(p(n))]$ is a continuous function. Thus, we can use the Intermediate Value Theorem. As η and v are probabilities, it follows that

$$\mathbb{E}\Big[\Pi\Big(p\Big(\sqrt{\frac{t\pi}{\delta f}}\Big)\Big)\Big] = \Big(p\Big(\sqrt{\frac{t\pi}{\delta f}}\Big) - c\Big)\Big(\frac{\pi + \pi\pi_b}{\frac{t\pi}{\delta f}}\Big) - f > \frac{t}{\delta(1 + \pi_b)}\Big(\frac{\pi + \pi\pi_b}{\frac{t\pi}{\delta f}}\Big) - f = 0. \tag{A.30}$$

This implies that all suppliers still make positive expected profits when $\sqrt{\frac{t\pi}{\delta f}}$ suppliers enter the repair market.

$p(n)$ is bounded from above by $max\Big\{\frac{2k}{1-\delta}, c + \frac{t}{\delta n\pi_b}\Big\}$. This implies that

$$\mathbb{E}[\Pi(p(n))] \le \Big(max\Big\{\frac{2k}{1-\delta}, c + \frac{t}{\delta n\pi_b}\Big\} - c\Big)\Big(\frac{\pi + \pi\pi_b}{n}\Big) - f \overset{n\to\infty}{\longrightarrow} -\infty. \tag{A.31}$$

Therefore, there exists $n_1 > 0$ such that $\mathbb{E}[\Pi(p(n_1))] < 0$. Finally, we can find n^* such that $\mathbb{E}[\Pi(p(n^*))] = 0$.

The equilibrium n^* is given by the smallest positive number solving equation (A.29). If more than one positive number solves equation (A.29), all numbers greater than n^* cannot be considered as realistic: This would imply that suppliers enter the repair market even if the expected profit is negative. Hence, the optimal n^* is unique.

Inserting n^* in the price equation (A.27) yields that the equilibrium price p^* is given by:

$$p^* = c + \sqrt{\frac{tf}{\delta\pi(1 + \pi_b)\Big[1 + \pi_b - \pi_s\eta(p^*)\Big(1 - v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*) + v(p^*)\Big)\Big]}}. \tag{A.32}$$

Proof of Proposition 5. In this proof, we completely focus on equilibrium set (E1), as the results for (E2) are trivial. Before we start with the actual proof, we briefly have a look at the notation:

- p^* denotes the equilibrium price for one repair unit in (E1). It is a function of the parameters $\delta, \pi, \pi_b, f, c, t, r$ and k. In order to conduct a comparative static analysis, we will write $p^*(\delta)$, for instance, to state that we consider p^* solely as a function of δ in this specific context. We will do the same for the big-loss probability π_b.
- n^* denotes the equilibrium number of suppliers in (E1). It is also a function of the parameters.
- $v(p) = \frac{p}{p+r}$ for an arbitrary price $p > 0$. $\eta(p) = \frac{\pi_b}{\pi_s}\frac{k}{(1-\delta)p-k}$ for an arbitrary price $p > 0$.
- η^* and v^* denote the equilibrium fraud and audit probability in (E1). They solely depend on the given parameters, i.e. $\eta^* = \eta(p^*)$ and $v^* = v(p^*)$.

1. We start with a comparative static analysis with respect to the coinsurance rate δ. The equilibrium price p^* is a function of the coinsurance rate δ, i.e. $p^* = p^*(\delta)$. In the following we write $p^*(\delta)$ instead of p^* in order to make the dependence on δ clear. We know that the equilibrium price is implicitly given by:

$$p^* = c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b - \pi_s\eta(p^*)\Big(1-v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*)+v(p^*)\Big)\Big]}}. \tag{A.33}$$

We will examine how $p^*(\delta)$ changes when δ increases, i.e. we are interested in the sign of $\frac{dp^*}{d\delta}$.
We start with the following definition:

$$\Gamma(\delta,p) := p - c - \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b - \pi_s\eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big)\Big]}}. \tag{A.34}$$

As η is a function of δ and p, we write $\eta(\delta,p)$.
For every $\hat{\delta} \in (0,a)$, there exists a unique $\hat{p} > \frac{k}{\pi_s(1-\delta)}$ such that $\Gamma(\hat{\delta},\hat{p}) = 0$.We further know that $\hat{p} = p^*(\hat{\delta})$. Applying the Implicit Function Theorem yields

$$\frac{dp^*}{d\delta} = -\frac{\frac{\partial\Gamma}{\partial\delta}\Big|_{p=p^*(\delta)}}{\frac{\partial\Gamma}{\partial p}\Big|_{p=p^*(\delta)}}. \tag{A.35}$$

Let us start with determining the sign of the denominator:

$$\begin{aligned}
\frac{\partial\Gamma}{\partial p} &= 1 - \frac{\Big(\frac{\partial\eta}{\partial p}\Big(1-v(p)\Big) - \eta(\delta,p)\frac{\partial v}{\partial p}\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big) + \eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\frac{\partial\eta}{\partial p}+\frac{\partial v}{\partial p}\Big)}{2(\sqrt{tf}\pi_s)^{-1}\sqrt{\delta\pi(1+\pi_b)}\Big(1+\pi_b-\pi_s\eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big)\Big)^{1.5}} \\
&\geq 1 - \frac{\sqrt{tf}\pi_s\Big(-\eta(\delta,p)\frac{1}{p}\frac{(1-\delta)p}{(1-\delta)p-k}\Big(1-v(p)\Big)v(p) + \eta(\delta,p)\Big(1-v(p)\Big)^2\frac{1}{p}v(p)\Big)}{2\sqrt{\delta\pi(1+\pi_b)}\Big(1+\pi_b-\pi_s\eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big)\Big)^{1.5}} \\
&\geq 1 \quad \text{for every } p > \frac{k}{\pi_s(1-\delta)} \text{ and every } \delta\in(0,1)
\end{aligned} \tag{A.36}$$

The denominator of the right hand side of equation (A.35) is always positive. Let us continue with the numerator:

$$\frac{\partial\Gamma}{\partial\delta} = -\frac{-\Big[1+\pi_b-\pi_s\eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big) - \delta\pi_s\Big(1-v(p)\Big)\frac{\partial\eta}{\partial\delta}\Big(2\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big)\Big]}{2(\sqrt{tf})^{-1}\sqrt{\pi(1+\pi_b)}\delta^{1.5}\Big(1+\pi_b-\pi_s\eta(\delta,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\delta,p)+v(p)\Big)\Big)^{1.5}}. \tag{A.37}$$

The sign of the numerator is ambiguous. We obtain that the sign of $\frac{dp^*}{d\delta}$ is given by:

$$sgn\Big(\frac{dp^*}{d\delta}\Big) = -sgn\Bigg[1+\pi_b-\pi_s\eta^*\big(1-v^*\big)\Big(\frac{\pi_s}{\pi_b}\eta^*+v^*\Big)+-\delta\pi_s\big(1-v^*\big)\Big(\frac{\partial\eta}{\partial\delta}\Big)^*\Big(2\frac{\pi_s}{\pi_b}\eta^*+v^*\Big)\Bigg],$$
$$\text{where } \eta^*=\eta(\delta,p)\Big|_{p=p^*(\delta)}, v^*=v(p)\Big|_{p=p^*(\delta)} \text{ and}$$
$$\Big(\frac{\partial\eta}{\partial\delta}\Big)^* = \frac{\partial\eta}{\partial\delta}\Big|_{p=p^*(\delta)}. \tag{A.38}$$

If we can find an upper bound for $\big(\frac{\partial\eta}{\partial\delta}\big)^*$, we will obtain that $\lim_{\delta\to 0}\delta(-\pi_s)\big(1-v^*\big)\Big(2\frac{\pi_s}{\pi_b}\eta^*\big(\frac{\partial\eta}{\partial\delta}\big)^*+v^*\big(\frac{\partial\eta}{\partial\delta}\big)^*\Big)=0$. This implies that $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}<0$ for $\hat\delta$ small enough.

$$\Big(\frac{\partial\eta}{\partial\delta}\Big)^* = \eta^*\frac{p^*(\delta)}{(1-\delta)p^*(\delta)-k} \leq \frac{\eta^*}{1-\delta}max\Big\{2,\frac{2k}{(1-\delta)p^*(\delta)-k}\Big\}\leq\frac{\eta}{1-\delta}max\Big\{2,2\frac{\pi_s}{\pi_b}\Big\}. \tag{A.39}$$

This implies that $\lim_{\delta\to 0}\big(\frac{\partial\eta}{\partial\delta}\big)^*\leq const.<\infty$ and we obtain $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}<0$ for small $\hat\delta>0$.
Further observations:

- $\lim_{\delta\to 0}p^*(\delta)\geq\lim_{\delta\to 0}\Big(c+\sqrt{\frac{tf}{\delta(1+\pi_b)^2}}\Big)=\infty$.
- For $\hat\delta\in(0,a)$ small enough and fixed, we know that $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}<0$.
- We further know that $p^*(\delta)>\frac{k}{1-\delta}$ for every $\delta\in(0,a)$ (see proof of Proposition 4).
- As $\lim_{\delta\to 1}\big(\frac{k}{1-\delta}\big)=\infty$, it follows that p^* has to reach a local minimum for a certain $\delta_{min}\in(0,a]$ (as otherwise the price falls under $\frac{k}{1-\delta}$). This minimum will be either reached for a or for some $\delta_{min}<a$.
- Furthermore, every price ¿ 0 can be reached at most with two different δs. This follows from equation (A.38).
- This implies that $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}>0$ for $\delta_{min}<\hat\delta<a$ for some $\delta_{min}>0$ *if* a is close to 1.
- Consequently, we have to distinguish between two different cases:
- 1. Case: $\Big(\frac{\partial\Gamma}{\partial\delta}\Big|_{p=p^*(\delta)}\Big)\Big|_{\delta=a}\leq 0$: In this case, the equilibrium price is monotonously decreasing in δ.
- 2. Case: $\Big(\frac{\partial\Gamma}{\partial\delta}\Big|_{p=p^*(\delta)}\Big)\Big|_{\delta=a}>0$: In this case, the equilibrium price as a function of δ has a convex shape at reaches its minimum for some $\delta_{min}\in(0,a)$. It follows that $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}<0$ for $\hat\delta<\delta_{min}$ and $\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}>0$ for $\hat\delta>\delta_{min}$.

The next step is the examination of the sign of $\frac{dn^*}{d\delta}$. For this purpose we can use the above results for $p^*(\delta)$.
The optimal $n^*(\delta)$ solves $n^*(\delta)=\frac{\big(p^*(\delta)-c\big)\big(\pi(1+\pi_b)\big)}{f}$. This implies that $\frac{dn^*}{d\delta}=\frac{\pi(1+\pi_b)}{f}\frac{dp^*}{d\delta}$.
We have to pay attention: We have to distinguish again between the cases $a=\delta_{min}$ and $a>\delta_{min}$. In the first case, the equilibrium price and number of suppliers are monotonously decreasing in δ. In the latter case, there exist a minimum price and a minimum number of suppliers that are reached for some δ_{min}. From this point on, the equilibrium price and the number of suppliers are monotonously increasing in δ. Hence, it follows that $sgn\Big(\frac{dp^*}{d\delta}\Big|_{\delta=\hat\delta}\Big)=sgn\Big(\frac{dn^*}{d\delta}\Big|_{\delta=\hat\delta}\Big)$ for all $\hat\delta\in(0,a)$.

Let $\delta_1 \in (0,1)$ be arbitrary but fixed.
From Proposition 2 we know: If $p^*(\delta_1) > \frac{k}{\pi_s(1-\delta_1)}$, the equilibrium price is given by $p^*(\delta_1)$ and if $p^*(\delta_1) \leq \frac{k}{\pi_s(1-\delta_1)}$, the equilibrium price is strictly smaller than $\frac{k}{\pi_s(1-\delta_1)}$. Therefore, $a \in (0,1)$ is implicitly given by $p^*(a) = \frac{k}{\pi_s(1-a)}$. We assume that insurers will only offer a coinsurance rate strictly smaller than a.[6] It is very important to distinguish between two different cases:

- $a = \delta_{min}$: In this case $\frac{dp^*}{d\delta}\Big|_{\delta=\hat{\delta}} < 0$ for every $\hat{\delta} \in (0,a)$.
- $a > \delta_{min}$: In this case $\frac{dp^*}{d\delta}\Big|_{\delta=\hat{\delta}} < 0$ for $\hat{\delta} \in (0,\delta_{min})$ and $\frac{dp^*}{d\delta}\Big|_{\delta=\hat{\delta}} > 0$ for $\hat{\delta} \in (\delta_{min}, a)$

It does strongly depend on the parameters, e.g. the big-loss probability, which of the two cases applies. The higher the small-loss probability, the later the "critical" price $\frac{k}{\pi_s(1-\delta)}$ is reached, the larger is a. Thus, for high small-loss probabilities, e.g. $\pi_s = 0.9$, the case $a > \delta_{min}$ applies whereas for low small-loss probabilities, e.g. $\pi_s = 0.1$, the case $a = \delta_{min}$ applies.
Since $\frac{\partial v}{\partial p} > 0$, it follows directly that $sgn\Big(\frac{dv^*}{d\delta}\Big|_{\delta=\hat{\delta}}\Big) = sgn\Big(\frac{dp^*}{d\delta}\Big|_{\delta=\hat{\delta}}\Big)$ for every $\hat{\delta} \in (0,a)$. Furthermore, we obtain that $sgn\Big(\frac{d\eta^*}{d\delta}\Big|_{\delta=\hat{\delta}}\Big) = 1$ for $\hat{\delta} \leq \delta_{min}$.

2. We continue with a comparative static analysis with respect to the big-loss probability π_b: The profit-maximizing price is a function of the big-loss probability π_b and therefore we write $p^*(\pi_b)$ in the following.
We use again the Implicit Function Theorem. Therefore, we define again:

$$\Gamma(\pi_b, p) := p - c - \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b-\pi_s\eta(\pi_b,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)\Big]}}. \tag{A.40}$$

As in 1., the fraud probability η is a function of π_b and p. Therefore, we write $\eta(\pi_b, p)$.
Applying the Implicit Function Theorem yields

$$\frac{dp^*}{d\pi_b} = -\frac{\frac{\partial\Gamma}{\partial\pi_b}\Big|_{p=p^*(\pi_b)}}{\frac{\partial\Gamma}{\partial p}\Big|_{p=p^*(\pi_b)}}. \tag{A.41}$$

We already know that the denominator is positive. The partial derivative in the numerator is given by:

$$\begin{aligned}\frac{\partial\Gamma}{\partial\pi_b} &= \frac{\sqrt{tf}\Big[2+2\pi_b-\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+2\pi_s\eta(\pi_b,p)\Big)\Big]}{2\sqrt{\delta\pi}\Big(1+\pi_b-\pi_s\eta(\pi_b,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)\Big)^{1.5}} \\ &= \frac{2-\frac{\pi_s}{\pi_b}\eta(\pi_b,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)+2\pi_b\Big(1-\frac{\pi_s}{\pi_b}\eta(\pi_b,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)\Big)}{2\sqrt{tf}^{\,-1}\sqrt{\delta\pi}\Big(1+\pi_b-\pi_s\eta(\pi_b,p)\Big(1-v(p)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(\pi_b,p)+v(p)\Big)\Big)^{1.5}}\end{aligned} \tag{A.42}$$

[6] Otherwise suppliers will commit fraud with probability 1. Insurers aim to prevent this situation.

Observations:

- $\frac{dp^*}{d\pi_b}\Big|_{\pi_b=\underline{\pi_b}} < 0$ for $\underline{\pi_b} \geq \frac{1}{2}$.
- Suppose there exits some $\hat{\pi_b}$ such that $\frac{dp^*}{d\pi_b}\Big|_{\pi_b=\hat{\pi_b}} > 0$. Then we can find $\tilde{\pi_b}, \bar{\pi_b}$, $\tilde{\pi_b} < \bar{\pi_b}$, such that $p^*(\tilde{\pi_b}) = p^*(\bar{\pi_b})$ and $\frac{dp^*}{d\pi_b}\Big|_{\pi_b=\tilde{\pi_b}} > 0$ and $\frac{dp^*}{d\pi_b}\Big|_{\pi_b=\bar{\pi_b}} < 0$.
- This implies that $\left(\frac{\partial\Gamma}{\partial\pi_b}\Big|_{p=p^*(\pi_b)}\right)\Bigg|_{\pi_b=\tilde{\pi_b}} < 0$ and $\left(\frac{\partial\Gamma}{\partial\pi_b}\Big|_{p=p^*(\pi_b)}\right)\Bigg|_{\pi_b=\bar{\pi_b}} > 0$.
- For the sake of simplicity we write $\hat{p}$ instead of $p^*(\tilde{\pi_b})$ and define $\mu(\hat{p}) = \frac{k}{(1-\delta)\hat{p}-k}$.
- First case: $\Big(1 - \mu(\hat{p})\Big(1 - v(\hat{p})\Big)\Big(\mu(\hat{p}) + v(\hat{p})\Big)\Big) \geq 0$:

  This would imply that $\left(\frac{\partial\Gamma}{\partial\pi_b}\Big|_{p=p^*(\pi_b)}\right)\Bigg|_{\pi_b=\tilde{\pi_b}} \geq 0$. This is a contradiction.
- Second case: $\Big(1 - \mu(\hat{p})\Big(1 - v(\hat{p})\Big)\Big(\mu(\hat{p}) + v(\hat{p})\Big)\Big) < 0$:

  This would imply that $\left(\frac{\partial\Gamma}{\partial\pi_b}\Big|_{p=p^*(\pi_b)}\right)\Bigg|_{\pi_b=\bar{\pi_b}} < 0$. This is a contradiction.
- Hence, we obtain that $\frac{dp^*}{d\pi_b}\Big|_{\pi_b=\hat{\pi_b}} < 0$ for all $\hat{\pi_b} \in (0, 1)$.
- This implies that $\frac{dp^*}{d\pi_s}\Big|_{\pi_s=\hat{\pi_s}} > 0$ for all $\hat{\pi_s} \in (0, 1)$.

Finally, we will examine how $n^*(\pi_b)$ changes when π_b increases, i.e. we determine the sign of $\frac{dn^*}{d\pi_b}$. For this purpose we use again the results, which we have already obtained for p^*.
We know that the optimal n^* solves

$$n^* = \sqrt{\frac{t\pi(1+\pi_b)}{\delta f\left[1 + \pi_b - \pi_s\eta\Big(p(n^*)\Big)\Big(1 - v\Big(p(n^*)\Big)\Big)\Big(\frac{\pi_s}{\pi_b}\eta\Big(p(n^*)\Big) + v\Big(p(n^*)\Big)\Big)\right]}}. \tag{A.43}$$

Since p^* decreases in π_b, we use equation (A.36) to show that $\pi_s\eta(p^*)\Big(1 - v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*) + v(p^*)\Big)$ increases in π_b. It follows that n^* increases in π_b as a marginal increase in π_b leads to an increase in the numerator of equation (A.43) and to a marginal increase in the denominator that is smaller than the increase in the numerator, i.e. the numerator grows by the factor 1 whereas the denominator grows by a factor strictly smaller than 1. We obtain that $\frac{dn^*}{d\pi_b}\Big|_{\pi_b=\hat{\pi_b}} > 0$ for every $\hat{\pi_b} \in (0, 1)$.
Applying the Implicit Function Theorem again, it follows directly that $\frac{dp^*}{d\pi}\Big|_{\pi=\hat{\pi}} < 0$ and $\frac{dn^*}{d\pi}\Big|_{\pi=\hat{\pi}} > 0$ for every $\hat{\pi} \in (0, 1)$.
Furthermore, we obtain that

$$sgn\left(\frac{dv^*}{d\pi_b}\right) = sgn\left(\frac{dv^*}{d\pi}\right) = -1 \quad \text{and} \quad sgn\left(\frac{d\eta^*}{d\pi_b}\right) = sgn\left(\frac{d\eta^*}{d\pi}\right) = 1. \tag{A.44}$$

3. Using the same technique as in (1) and (2), it easily follows that

$$\begin{aligned} &sgn\left(\frac{dp^*}{d\pi}\right) = -1 \text{ and} \\ &sgn\left(\frac{dp^*}{dk}\right) = sgn\left(\frac{d\eta^*}{dk}\right) = 1. \end{aligned} \tag{A.45}$$

Proof of Proposition 6. For the sake of simplicity we write p^* for $p^*(\delta)$, η^* for $\eta^*(\delta)$ and v^* for $v^*(\delta)$. Furthermore, we will use the following notation:

$$\alpha(\delta) = \underbrace{\pi\pi_b 2(1-\delta)p^*(\delta)}_{=:A(\delta)} + \underbrace{\pi\pi_s(1-\delta)p^*(\delta)\Big(1+\eta^*(\delta)\Big(1-v^*(\delta)\Big)\Big)}_{=:B(\delta)} + \underbrace{kv^*(\delta)[\pi\pi_b + \pi\pi_s\eta^*(\delta)]}_{=:C(\delta)}. \tag{A.46}$$

First of all, we show that $w_1(\delta)$ is decreasing for sufficiently small δ: We already know that $\frac{dp^*}{d\delta}\Big|_{\delta=\hat{\delta}} < 0$ for $\delta_{min} > \hat{\delta} > 0$. Therefore, we obtain that $\frac{d\eta^*}{d\delta}\Big|_{\delta=\hat{\delta}} > 0$ for $\delta_{min} > \hat{\delta} > 0$. We analyze the parts of α separately:

$$\begin{aligned}
\frac{dB}{d\delta} &= \pi\pi_s\Big[-p^*\Big(1+\eta^*(1-v^*)\Big) + (1-\delta)\frac{dp^*}{d\delta}\Big(1+\eta^*(1-v^*)\Big) + (1-\delta)p^*\Big(\frac{d\eta^*}{d\delta}(1-v^*) - \frac{dv^*}{d\delta}\eta^*\Big)\Big] \\
&\leq \pi\pi_s\Big[-p^*\Big(1+\eta^*(1-v^*)\Big) + (1-\delta)\frac{dp^*}{d\delta}\Big(1+\eta^*(1-v^*)\Big) \\
&+ (1-\delta)p^*\Big((1-v^*)\frac{\eta^*}{1-\delta}max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\} - (1-v^*)\frac{\eta^*}{p^*}\frac{dp^*}{d\delta}max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\} - \eta^*v^*(1-v^*)\frac{\frac{dp^*}{d\delta}}{p^*}\Big)\Big] \\
&\leq \pi\pi_s\Big[-p^*\Big(1+\eta^*(1-v^*) - \eta^*(1-v^*)max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\}\Big) \\
&+ (1-\delta)\frac{dp^*}{d\delta}\Big(1+\eta^* - \eta^*v^* - v^*(1-v^*)\eta^* - \eta^*(1-v^*)max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\}\Big)\Big].
\end{aligned} \tag{A.47}$$

As $\lim_{\delta\to 0} p^*(\delta) \geq \lim_{\delta\to 0}\Big(c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)}}\Big) = \infty$, we obtain that $\lim_{\delta\to 0}\eta^*(\delta) = 0$ and therefore $\Big(1+\eta^*(1-v^*)-\eta^*(1-v^*)max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\}\Big)$ and $\Big(1+\eta^*(1-v^*)-v^*(1-v^*)\eta^*-\eta^*(1-v^*)max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\}\Big)$ will be positive for δ small enough. Therefore, $\frac{dB}{d\delta}\Big|_{\delta=\hat{\delta}} < 0$ for $\delta_1 > \hat{\delta} > 0$ for some $\delta_{min} \geq \delta_1 > 0$. Subsequently, we analyze the other parts of α in more detail:

$$\begin{aligned}
\frac{dA}{d\delta} + \frac{dC}{d\delta} &= 2\pi\pi_b\Big[-p^* + (1-\delta)\frac{dp^*}{d\delta}\Big] + k(1-v^*)\frac{\frac{dp^*}{d\delta}}{p^*+r}\Big[\pi\pi_b + \pi\pi_s\eta^*\Big] \\
&+ kv^*\Big[\pi\pi_s\eta^*\Big(\frac{p^* - (1-\delta)\frac{dp^*}{d\delta}}{(1-\delta)p^* - k}\Big)\Big] \\
&\leq 2\pi\pi_b\Big[-p^* + (1-\delta)\frac{dp^*}{d\delta}\Big] + k(1-v^*)\frac{\frac{dp^*}{d\delta}}{p^*+r}\Big[\pi\pi_b + \pi\pi_s\eta^*\Big] \\
&+ kv^*\pi\pi_s\eta^*\Big[\frac{1}{1-\delta}max\Big\{2, 2\frac{\pi_s}{\pi_b}\Big\} - \frac{(1-\delta)}{(1-\delta)p^* - k}\frac{dp^*}{d\delta}\Big].
\end{aligned} \tag{A.48}$$

As $\lim_{\delta\to 0}\eta^*(\delta) = 0$, we obtain that $A(\delta)+C(\delta)$ is decreasing for δ small enough. Putting everything together yields $\frac{d\alpha}{d\delta}\Big|_{\delta=\hat{\delta}} < 0$ and therefore $\frac{dw_1}{d\delta}\Big|_{\delta=\hat{\delta}} > 0$ for sufficiently small $\hat{\delta} > 0$. We also obtain that $\frac{dw_1}{d\delta} \geq \pi\pi_b p^*(\delta)$ for δ small enough. This implies that α is decreasing in δ.
We know that p^* is implicitly given by:

$$p^* = c + \sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b - \pi_s\eta^*(p^*)\Big(1-v^*(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta^*(p^*) + v^*(p^*)\Big)\Big]}}. \tag{A.49}$$

One can show easily that $\lim_{\delta\to 0}\delta p^*(\delta)=0$:

$$0 \le \delta p^*(\delta) \le \delta max\left\{\frac{2k}{1-\delta}, c+\sqrt{\frac{tf}{\delta\pi\pi_b}}\right\} \overset{\delta\to 0}{\longrightarrow} 0. \tag{A.50}$$

Since $w_3 > w_2$ for every $\delta > 0$ we obtain that

$$-\pi\pi_s u\big(w_3\big)\frac{d\eta^*}{d\delta}+\pi\pi_s\frac{d\eta^*}{d\delta}\Big[(1-v^*)u\big(w_2\big)+v^*u\big(w_3\big)\Big]-\pi\pi_s\eta^*u\big(w_2\big)\frac{dv^*}{d\delta}+\pi\pi_s\eta^*u\big(w_3\big)\frac{dv^*}{d\delta} < 0. \tag{A.51}$$

Since $\lim_{\delta\to 0}\delta p^*(\delta)=0$, we obtain that $\lim_{\delta\to 0}|u(w_2)-u(w_3)|=0$. Thus, expression (A.51) tends to 0 for $\delta\to 0$. As $\frac{dw_1}{d\delta} \ge \pi\pi_b p^*(\delta)$ for δ small enough and $\lim_{\delta\to 0} p^*(\delta)=\infty$, there exists some $\delta^l > 0$ such that

$$\begin{aligned}\Bigg(&(1-\pi)u'\big(w_1\big)\frac{dw_1}{d\delta}-\pi\pi_s u\big(w_3\big)\frac{d\eta^*}{d\delta}+\pi\pi_s\frac{d\eta^*}{d\delta}\Big[(1-v^*)u\big(w_2\big)+v^*u\big(w_3\big)\Big]\\ &-\pi\pi_s\eta^*u\big(w_2\big)\frac{d\eta^*}{d\delta}+\pi\pi_s\eta^*u\big(w_3\big)\frac{dv^*}{d\delta}\Bigg)\Bigg|_{\delta=\hat{\delta}} > 0.\end{aligned} \tag{A.52}$$

for every $\hat{\delta} < \delta^l$. Finally, we examine w_2 and w_3 more closely. One can easily show that $\lim_{\delta\to 0} tc(\delta) = \lim_{\delta\to 0}\frac{1}{2n^*(\delta)}=0$, as $\lim_{\delta\to 0} n^*(\delta)=\infty$.
As $\lim_{\delta\to 0}\delta p^*(\delta)=0$, we obtain that $\lim_{\delta\to 0}|w_2-w_1| = \lim_{\delta\to 0}|w_3-w_1|=0$. Therefore, $w_2(\delta)$ and $w_3(\delta)$ are decreasing for sufficiently small δ. This is due to the fact that $w_1(\delta)$ is decreasing for sufficiently small δ.
Putting it all together yields $\frac{dEU}{d\delta}\Big|_{\delta=\hat{\delta}} > 0$ for sufficiently small $\hat{\delta} > 0$, i.e. full insurance is not optimal.

Proof of Proposition 7. We already know that the type of equilibrium depends on the coinsurance rate δ. For $\delta < a$, equilibrium (E1) arises where repair firms set different prices for the two treatment types and insurers conduct audits. For $\delta \ge a$, equilibrium (E2) arises where repair firms set a uniform price for both treatments and insurers do not conduct audits at all. We recall the price formulas in the two equilibria:

$$\begin{aligned}&(E1)\ \ p^* = c+\sqrt{\frac{tf}{\delta\pi(1+\pi_b)\Big[1+\pi_b-\pi_s\eta(p^*)\Big(1-v(p^*)\Big)\Big(\frac{\pi_s}{\pi_b}\eta(p^*)+v(p^*)\Big)\Big]}},\\ &(E2)\ \ p^* = c(1+\pi_b)+\sqrt{\frac{tf}{\delta\pi}}.\end{aligned} \tag{A.53}$$

We denote by $p_1(\delta)$ the price formula in (E1) and by $p_2(\delta)$ the price formula in (E2) for arbitrary $\delta\in(0,1]$. Correspondingly, we denote by $\alpha_1(\delta)$ the premium formula in (E1) and by $\alpha_2(\delta)$ the premium formula in (E2) for arbitrary $\delta\in(0,1]$. Hence, α_1 and α_2 are given by:

$$\begin{aligned}&\alpha_1(\delta) = \pi\pi_b 2(1-\delta)p_1(\delta)+\pi\pi_s(1-\delta)p_1(\delta)\Big(1+\eta^*(\delta)\Big(1-v^*(\delta)\Big)\Big)+kv^*(\delta)[\pi\pi_b+\pi\pi_s\eta^*(\delta)],\\ &\alpha_2(\delta) = \pi(1-\delta)p_2(\delta).\end{aligned} \tag{A.54}$$

We observe that $\alpha_1(\delta) > \alpha_2(\delta)$ for any $\delta \in (0, 1]$ since

$$\alpha_1(\delta) \geq \pi(1-\delta)p_1(\delta) + \pi\pi_b(1-\delta)p_1(\delta) > \pi(1+\pi_b)(1-\delta)\Big(c + \sqrt{\frac{tf}{\delta\pi}}\frac{1}{1+\pi_b}\Big) = \alpha_2(\delta). \quad \text{(A.55)}$$

We denote by $u_1(\delta)$ the average consumer's utility formula in (E1) and by $u_2(\delta)$ the average consumer's utility formula in (E2) for arbitrary $\delta \in (0, 1]$. We find that u_1 and u_2 are given by:

$$\begin{aligned}
u_1(\delta) &= (1-\pi)u\Big(w_0 - \alpha_1(\delta)\Big) + \pi\pi_b u\Big(w_0 - \alpha_1(\delta) - 2\delta p_1(\delta) - tc(\delta)\Big) \\
&\quad + \pi\pi_s\Big(1 - \eta^*(\delta) + \eta^*(\delta)v^*(\delta)\Big)u\Big(w_0 - \alpha_1(\delta) - \delta p_1(\delta) - tc(\delta)\Big) \\
&\quad + \pi\pi_s\eta^*(\delta)\Big(1 - v^*(\delta)\Big)u\Big(w_0 - \alpha_1(\delta) - 2\delta p_1(\delta) - tc(\delta)\Big), \\
u_2(\delta) &= (1-\pi)u\Bigg(w_0 - \pi(1-\delta)\Big(c(1+\pi_b) + \sqrt{\frac{tf}{\delta\pi}}\Big)\Bigg) \\
&\quad + \pi u\Bigg(w_0 - \pi(1-\delta)\Big(c(1+\pi_b) + \sqrt{\frac{tf}{\delta\pi}}\Big) - \delta\Big(\sqrt{\frac{tf}{\delta\pi}} + c(1+\pi_b)\Big) - \frac{1}{4}\sqrt{\frac{\delta tf}{\pi}}\Bigg).
\end{aligned} \quad \text{(A.56)}$$

$tc(\delta)$ denotes the average transportation costs in (E1). We find that $u_2(\delta) \geq u_1(\delta)$ for every $\delta \in (0, 1]$.

$$\begin{aligned}
u_1(\delta) - u_2(\delta) &\leq (1-\pi)\Big[u\Big(w_0 - \alpha_1(\delta)\Big) - u\Big(w_0 - \alpha_2(\delta)\Big)\Big] \\
&\quad + \pi\Big[u\Big(w_0 - \alpha_1(\delta) - \delta p_1(\delta) - (\pi_b + \pi_s\big(1 - v^*(\delta)\big)\eta^*(\delta)\delta p_1(\delta) - tc(\delta)\Big) \\
&\quad - u\Big(w_0 - \alpha_2(\delta) - \delta p_2(\delta) - 0.25\sqrt{\frac{tf\delta}{\pi}}\Big)\Big] \\
&\leq (1-\pi)\Big[u\Big(w_0 - \alpha_1(\delta)\Big) - u\Big(w_0 - \alpha_2(\delta)\Big)\Big] \\
&\quad + \pi\Big[u\Big(w_0 - \alpha_1(\delta) - \delta p_2(\delta) - \pi_s\big(1 - v^*(\delta)\big)\eta^*(\delta)\delta p_1(\delta)\Big) \\
&\quad - u\Big(w_0 - \alpha_2(\delta) - \delta p_2(\delta)\Big)\Big] \\
&\leq (1-\pi)\Big[u\Big(w_0 - \alpha_1(\delta)\Big) - u\Big(w_0 - \alpha_2(\delta)\Big)\Big] \\
&\quad + \pi\Big[u\Big(w_0 - \alpha_1(\delta) - \delta p_2(\delta)\Big) - u\Big(w_0 - \alpha_2(\delta) - \delta p_2(\delta)\Big)\Big] \leq 0.
\end{aligned} \quad \text{(A.57)}$$

The optimal insurance contract maximizes the expected utility of an average consumer. In the following we denote by δ^* the optimal coinsurance rate that maximizes the expected utility of an average consumer. Consequently, a necessary condition for $\delta^* < 1$ is that the marginal utility expected utility evaluated at $\delta = 1$ is negative. According to Nell et al. (2009)[Proof of Proposition 2], $\delta^* < 1$ only if consumers are sufficiently risk-averse, i.e. the coefficient of absolute risk aversion is sufficiently large.

We have proven that consumers should only purchase insurance coverage if they are sufficiently risk-averse. A question that naturally arises is if the optimal δ^* leads to equilibrium (E1) or (E2). Therefore, we have to differentiate between the following cases:

- If u_2 reaches its maximum in the interval $[a, 1]$, the coinsurance rate for which the maximum is reached is the optimal one and equilibrium (E2) follows.

- If u_2 reaches its maximum in the interval $(0, a)$, the optimal coinsurance rate is not necessarily $\geq a$. It could be the case that $u_1(\bar{\delta}) \geq u_2(\delta)$ for some $\bar{\delta} < a$ and every $\delta \geq a$. In this case, the optimal coinsurance rate is smaller than a and equilibrium (E1) arises.

In the following we prove that consumers' degree of risk aversion determines the equilibrium type. Moreover, equilibrium (E1) only arises under two additional conditions. First of all, we show that the coinsurance rate for which u_2 reaches its maximum is decreasing in absolute risk aversion. For this aim, we fix some arbitrary $\hat{\delta} \in (0, 1]$. Evaluating the marginal expected utility at $\delta = \hat{\delta}$ yields

$$\begin{aligned}\left.\frac{\partial u_2}{\partial \delta}\right|_{\delta=\hat{\delta}} &= (1-\pi)\pi u'(w_n)|_{\delta=\hat{\delta}}\left[\left(c(1+\pi_b)+\sqrt{\frac{tf}{\hat{\delta}\pi}}\right)+\frac{1}{2}(1-\hat{\delta})\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right] \\ &\quad + \pi u'(w_l)|_{\delta=\hat{\delta}}\left[\pi\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right)+\frac{1}{2}\pi(1-\hat{\delta})\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right. \\ &\quad \left. -\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right)+\frac{1}{2}\hat{\delta}\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}-\frac{1}{8}\sqrt{\frac{tf}{\pi\hat{\delta}}}\right]\end{aligned} \tag{A.58}$$

where w_n denotes the consumer's wealth in the no-loss state and w_l denotes the consumer's wealth in the loss state. A necessary condition for $\delta^* < \hat{\delta}$ is that the marginal expected utility evaluated at $\hat{\delta}$ is negative, i.e. $\frac{\partial u_2}{\partial \delta}\big|_{\delta=\hat{\delta}} < 0$. Rearranging terms yields the condition

$$\left.\frac{u'(w_n)}{u'(w_l)}\right|_{\delta=\hat{\delta}} < \frac{(1-\pi)\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right)+\frac{1}{8}\sqrt{\frac{tf}{\pi\hat{\delta}}}-\frac{1}{2}\left(\pi\left(1-\hat{\delta}\right)+\hat{\delta}\right)\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}}{(1-\pi)\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}\right)+\frac{1}{2}(1-\pi)\left(1-\hat{\delta}\right)\sqrt{\frac{tf}{\pi\left(\hat{\delta}\right)^3}}} \tag{A.59}$$

The RHS of equation (A.59) is strictly smaller than one. The LHS is strictly between zero and one and decreases in the consumer's absolute risk aversion. Therefore, equation (A.59) is only to be met if consumers are sufficiently risk-averse.
If consumers are sufficiently risk-averse, the optimal coinsurance rate δ^* is implicitly given by

$$\frac{u'(w_n)}{u'(w_l)} = \frac{(1-\pi)\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\delta^*\right)^3}}\right)+\frac{1}{8}\sqrt{\frac{tf}{\pi\delta^*}}-\frac{1}{2}\left(\pi\left(1-\delta^*\right)+\delta^*\right)\sqrt{\frac{tf}{\pi\left(\delta^*\right)^3}}}{(1-\pi)\left(c(1+\pi_b)+\sqrt{\frac{tf}{\pi\left(\delta^*\right)^3}}\right)+\frac{1}{2}(1-\pi)\left(1-\delta^*\right)\sqrt{\frac{tf}{\pi\left(\delta^*\right)^3}}} \tag{A.60}$$

Let v and w be two concave utility functions with w being a concave transformation of v (i.e. agent w is more risk-averse than agent v). If δ_v is the coinsurance rate for which v reaches is maximum, it follows that $\frac{w'(w_n)}{w'(w_l)}\Big|_{\delta=\delta_v} < \frac{v'(w_n)}{v'(w_l)}\Big|_{\delta=\delta_v}$. Using equation (A.59) yields $\delta_w < \delta_v$, where δ_w is the coinsurance rate for which w reaches its maximum.

Secondly, we show that equilibrium (E1) may never be reached under some circumstances. For $\delta \to 0$ we have that:

- $\alpha_1(\delta) - \alpha_2(\delta) \to \pi\pi_b k$
- $\delta p_1(\delta) \to 0$, $\delta p_2(\delta) \to 0$
- $tc(\delta) \to 0$, $\frac{1}{4}\sqrt{\frac{\delta tf}{\pi}} \to 0$
- $\alpha_2(\delta) - \alpha_2(a) \to \infty$

We already know that equilibrium (E1) can only be reached when u_2 reaches its maximum in the interval $(0, a)$. Let us assume that u_2 reaches its maximum in the interval $(0, a)$. Equilibrium (E1) will be reached if there exists some $\delta_1 \in (0, a)$ such that $u_1(\delta_1) > u_2(a)$. This expression is equivalent to

$$\begin{aligned}(1-\pi)\Big(u(w_0-\alpha_2(a))-u(w_0-\alpha_1(\delta_1))\Big) <\pi\Big(&(\pi_b+\pi_s\eta(1-v))u(w_0-\alpha_1(\delta_1)-2\delta_1 p_1(\delta_1)-tc(\delta_1))\\ &+\pi_s(1-\eta+\eta v)u(w_0-\alpha_1(\delta_1)-\delta_1 p_1(\delta_1)-tc(\delta_1))\\ &-u(w_0-\alpha_2(a)-ap_2(a)-0.25\sqrt{\frac{atf}{\pi}})\Big)\end{aligned} \tag{A.61}$$

The LHS of equation (A.61) is always positive, since $\alpha_2(a) \leq \alpha_2(\delta_1) < \alpha_1(\delta_1)$. If $\alpha_1(\delta_1) > \alpha_2(a) + ap_2(a) + 0.25\sqrt{\frac{atf}{\pi}}$, the RHS of equation (A.61) is negative which would imply that condition (A.61) does not hold. We find that

$$\begin{aligned}&\alpha_1(\delta_1) > \alpha_2(a)+ap_2(a)+0.25\sqrt{\frac{atf}{\pi}}\\ \Leftarrow\ &\alpha_1(\delta_1)-\alpha_2(\delta_1) > ap_2(a)+0.25\sqrt{\frac{atf}{\pi}}\\ \Leftarrow\ &kv(\delta_1)\pi\pi_b > ap_2(a)+0.25\sqrt{\frac{atf}{\pi}}\\ \Leftarrow\ &k\underbrace{v(\delta_{min})}_{=:d}\pi\pi_b > ap_2(a)+0.25\sqrt{\frac{atf}{\pi}}\\ \Leftrightarrow\ &k > \frac{1}{d\pi\pi_b}\left(ap_2(a)+0.25\sqrt{\frac{atf}{\pi}}\right)\\ \Leftarrow\ &k > \frac{1}{d\pi\pi_b}\left(p_2(1)+0.25\sqrt{\frac{tf}{\pi}}\right)\end{aligned} \tag{A.62}$$

where $\delta_{min} \leq a$ is given by the coinsurance rate for which p_1 reaches its minimum in the interval $(0, a]$. Consequently, equilibrium (E1) will never be reached if the audit costs k are sufficiently large. Equation (A.61) also does not hold if the RHS is positive but π is sufficiently small. The expression in the brackets on the RHS of equation (A.61) is uniformly bounded.

Finally, we show that equilibrium (E1) will be reached if k is sufficiently small and π is sufficiently large. We denote by $\delta_2 < a$ the coinsurance rate for which u_2 reaches its maximum. We know that $|\alpha_2(a) - ap_2(a) - \alpha_2(\delta) + \delta p_2(\delta)| \to \infty$ for $\delta \to 0$. Furthermore, $|\alpha_1(\delta) - 2\delta p_1(\delta) - \alpha_2(\delta) + \delta p_2(\delta)| \to k\pi\pi_b$ for $\delta \to 0$. If k is small enough and δ_2 is small enough, we obtain that $\alpha_2(\delta_2) + \delta_2 p_2(\delta_2) < \alpha_1(\delta_2) + 2\delta_2 p_1(\delta_2) < \alpha_2(a) + ap_2(a)$. Consequently, we have $u(w_0 - \alpha_1(\delta_2) - 2\delta_2 p_1(\delta_2) - tc(\delta_2)) > u(w_0 - \alpha_2(a) - ap_2(a) - 0.25\sqrt{\frac{atf}{\pi}})$. If π is sufficiently large, we obtain

$$\begin{aligned}(1-\pi)\Big(u(w_0-\alpha_2(a))-u(w_0-\alpha_1(\delta_2))\Big) <\pi\Big(&(\pi_b+\pi_s\eta(1-v))u(w_0-\alpha_1(\delta_2)-2\delta_2 p_1(\delta_2)-tc(\delta_2))\\ &+\pi_s(1-\eta+\eta v)u(w_0-\alpha_1(\delta_2)-\delta_2 p_1(\delta_2)-tc(\delta_2))\\ &-u(w_0-\alpha_2(a)-ap_2(a)-0.25\sqrt{\frac{atf}{\pi}})\Big)\end{aligned} \tag{A.63}$$

Putting everything together, we obtain that equilibrium (E1) only arises when k is sufficiently small, π is sufficiently large, and consumers are sufficiently risk-averse. The last condition is needed, as δ_2 has to be sufficiently small.

In summary, consumers only purchase insurance if they are sufficiently risk-averse. If this condition is fulfilled, equilibrium (E2) is prevailing. Equilibrium (E1) only arises under a sufficiently high degree of risk aversion, sufficiently small audit costs, and a sufficiently large loss probability.

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