Robust Corporate Signaling with Heterogeneous Beliefs

Thomas J. Rivera*

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Abstract

We study the ability of a manager to use costly corporate signaling to reveal private information about firm characteristics prior to entering a new round of equity financing. We assume that the manager and the investor that provides financing may have heterogeneous and privately known beliefs regarding the value of firm characteristics. We show that equity prices under any informative signaling scheme must necessarily rely on the investor's perceived distribution of managerial beliefs. Therefore, incentive compatibility of signaling requires the manager to form precise beliefs regarding their prospective investor's higher order beliefs. Given the practical difficulty of forming correct beliefs regarding the higher order beliefs of other agents, we study robust signaling which remains incentive compatible for all higher order beliefs of the investor. Any robust disclosure mechanism is completely uninformative casting doubt on the efficacy of dynamic corporate signaling.

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*Desautels Faculty of Management, McGill University, thomas.rivera@mcgill.ca

Capital markets are typically plagued with asymmetric information whereby "insiders" have better information about a firm's assets than the external investors they tap for financing. For this reason there has been a large interest in how corporate policy (e.g. IPO underpricing, dividend policy, retaining equity, etc.) can help to credibly signal a firm's private information to the market. While this literature has generated many important insights, the aim of this paper is to demonstrate how the standard assumption of homogeneous beliefs among contracting parties can lead to severe robustness concerns regarding the efficacy of corporate signaling.

In this paper we study the ability of a manager to use costly signaling to disclose private information about firm characteristics prior to seeking new equity financing. As motivated by earlier literature, the aim of the manager is to obtain cheaper future financing by engaging in costly actions that signal when they have characteristics that prospective investors value more. In contrast to this earlier literature, we account for the fact that the firm's manager and their prospective investors may have different beliefs regarding which firm characteristics are most valuable.

To explain the difficulties that heterogeneous beliefs create for corporate signaling, we first show that equity prices under any informative signaling scheme must necessarily rely on the investor's perceived distribution of managerial beliefs (i.e. the prior probability they attach to managers having certain beliefs). Yet, when this is the case, incentive compatibility of informative signaling requires the manager to form precise beliefs regarding the investor's perceived distribution of managerial beliefs (i.e. beliefs over beliefs). The reason for this is that while the manager knows their own beliefs, they must anticipate the equity price offered by the investor when deciding their signaling strategy. Given that the equity price must depend on the investor's perceived distribution of managerial beliefs, then incentive compatibility requires the manager to have a precise belief regarding these higher order beliefs of the investor. If the manager has difficulty forming precise estimates of the higher order beliefs of their prospective investors (e.g. due to ambiguity), then it is not even clear how to define incentive compatibility in the context of dynamic signaling.[1]

[1]While one can define incentive compatibility with respect to any fixed higher order beliefs, those beliefs must be common knowledge among the manager and investor in order for them to support credible disclosure.

In light of this problem, we study the ability of the manager to engage in *robust signaling* which remains incentive compatible for any higher order beliefs of the investor (i.e. for any prior distribution over managerial beliefs that the investor might have). This type of signaling would ensure that any inconsistency in beliefs does not affect the incentive compatibility of signaling and therefore could allow for informative disclosure with heterogeneous beliefs. Unfortunately, we show how any robust signaling scheme must necessarily be uninformative, casting doubt on the efficacy and robustness of dynamic corporate policy to credibly signal firm characteristics.

We study a model whereby a manager must issue equity to an outside investor when the manager has private information about firm characteristics. For example, the manager may need funding for an expansion of their business to a new market and have private information regarding customer tastes in that market. Importantly, we assume that the manager and investor have common beliefs regarding the distribution of firm characteristics but may have different beliefs regarding how firm characteristics map to expected firm value. Continuing with the previous example, this would imply that the manager may attach a higher value to a market where customers have certain tastes than the investor would and vice-versa. In particular, we assume a fixed number of firm characteristics and a fixed number of belief types. Each belief type has single peaked preferences, valuing a firm with a specific characteristic most and valuing firms with characteristics closest to their most preferred characteristic higher than those that are further from their most preferred characteristic.

We then ask whether it is possible to construct a system of costly signals that create incentives for the manager to reveal their private information, taking into account that revealing this information will directly affect their cost of financing. Rather than studying a specific type of signal, we focus on a more general problem of designing a system of ex-ante transfers paid to finance such signals. These transfers are meant to capture the cost of signaling via a number of signaling tools proposed by the literature — summarized in Daniel and Titman (1995) as *money burning* — such as overpaying for underwriting services (e.g. Titman and Trueman (1986)), underpricing equity (e.g. Allen and Faulhaber (1989)), or the opportunity cost of retaining a portion of the equity issuance (Leland and Pyle (1977)) or of issuing dividends (e.g. Battacharya (1979)). If the mechanism is facilitated by a regulatory body, then these transfers can also represent government imposed penalties, subsidies, and taxes (Rivera (2021)) along with stricter financial reporting rules and supervision.[2] Thus, it is not the existence of signals with particular costs that drives our results, but rather a fundamental issue with corporate signaling when agents have heterogeneous beliefs.

The underlying *single-crossing* property that would allow such transfers to incentivize the manager to credibly reveal their private information comes from the fact that some managers may be willing to pay more to reveal to the market that their firm has a particular characteristic — thus benefiting from the associated price change — than other managers would be willing to pay to mimic them. The reason for this is that when issuing equity at a higher price, the manager sells a smaller portion of the firm's cashflows in exchange for financing and therefore retains a larger ownership share of those proceeds. Accordingly, the manager values retaining a larger fraction of proceeds more when the firm has higher value according to her beliefs. When this property holds, then one can design a system of transfers such that each type is willing to pay the transfer that reveals their type, but unwilling to pay any other transfer that leads their equity to be priced as if they were some other type.

In the case of heterogeneous beliefs, the key distinction is that differences in beliefs can lead the manager and investor to have different *rankings* of the value of the firm as a function of firm characteristics. In particular, if there were only two firm characteristics, e.g. high risk and low risk, then it is possible to provide incentives for each type to report truthfully only if whenever the firm's manager values the high risk equity more than the low risk equity, then the investor *also* values the high risk equity more than the low risk equity (and vice-versa). This is due to the fact that when the manager and investor disagree on the ranking of firm characteristics then managers with firm characteristics that they value most may not be willing to pay the most to reveal this

[2] As long as the cost to the firm of these activities are known and do not change ex-post then they can be modeled as ex-ante transfers.

information as the investor will not be willing to offer them the necessary price increase for this to be the case: the single-crossing property breaks down. Thus, whenever this type of disagreement occurs for any two distinct firm characteristics, then it is impossible for any incentive compatible disclosure mechanism to fully disclose the risk of those two types, even when risk preferences are public information.

In addition to heterogeneous beliefs, we also study the natural case where beliefs are private information. To simplify the problem, we assume that the investor's beliefs are publicly known in order to focus solely on unknown managerial beliefs.[3] We also consider the design of partial disclosure whereby the mechanism generates pools of firm types that signal to the investor that the firm is in a particular pool, but not which type they are directly.

We then proceed to study the ability to design a costly signaling scheme that is informative: for some signal the investor learns more than their prior belief regarding firm characteristics. Our first main result states that under any costly signaling scheme, the posterior beliefs of the investor cannot be a function of only the prior beliefs of project characteristics. Therefore, the equity pricing of the investor must depend on their perceived prior distribution of managerial beliefs. The reason for this is that designing a signaling scheme that generates such posterior beliefs puts a requirement on the pools of types that violate the aforementioned single-crossing property. Therefore, it is impossible to design signaling scheme where equity prices only depend on the distribution of firm characteristics.

Next, we note that our first finding implies that incentive compatibility of signaling necessarily requires the manager to have a precise belief regarding the investor's perceived distribution of managerial beliefs. We find this to be highly unlikely in many corporate signaling contexts and therefore proceed to study the ability to disclose information in a way that is robust to these higher order beliefs as motivated by the economic theory literature on higher order beliefs and robustness (e.g. Bergemann and Morris (2005), Carroll (2015), and more specifically Carroll (2017)).

[3]Note that having to solicit the investor's beliefs would only add constraints to our problem and therefore our results would still hold if we relax this assumption.

In particular, we assume a common prior over firm characteristics and ask whether there exists a signaling scheme that remains incentive compatible for any joint distribution of investor beliefs over firm characteristics and managerial beliefs that is consistent with the common prior over firm characteristics. Unfortunately, while this criteria would be necessary to facilitate robust informative disclosure in many practical settings, we show that any such robust disclosure must always be uninformative.

In order to derive these results (illustrated via Example 3.1 below), we first show that for any fixed prior beliefs of the investor, heterogeneous beliefs puts certain restrictions on which types can and cannot be pooled together given the way that the investor values each of the pools generated by the signaling mechanism (determined by the prior). For example, we show that whenever one pool of types is valued more by the investor than another, then each type in that pool must value their firm's characteristic (given their beliefs) more than each type in the pool that the investor values less. This result effectively represents the analog single-crossing property for heterogeneous beliefs. Then, after deriving these necessary conditions for incentive compatibility for any fixed prior belief of the investor, our main result is to show that no such signaling scheme can obey all of our necessary conditions for all possible (full support) priors of the investor. Thus, for any informative signaling scheme we can find a full support prior distribution over managerial beliefs which induces a violation of the incentive compatibility conditions (meaning some agent will find it optimal to deviate from the signaling scheme).

These results highlight an issue with the heavily utilized paradigm in agency problems whereby it is assumed that beliefs are homogenous and publicly known. This also casts doubt onto the efficacy of corporate policy as a signaling device when managers and prospective investors have different beliefs. While our results do not state that informative signaling is impossible, they highlight how any such signaling scheme will crucially rely on managers and prospective investors having common knowledge higher order beliefs, something that we do not expect to be true in practice. Finally, we mention in the next section that these findings add support to the empirical literature which has struggled to confirm the hypothesis that different types of corporate policy are linked to signaling efforts.

1 Literature Review

This paper relates to many of the early papers studying the ability of firms to use costly signaling to reveal their private information prior to raising capital.[4] Leland and Pyle (1977) show how firms can signal their types to investors by committing to retain a portion of the new equity issued. Allen and Faulhaber (1989), Grinblatt and Hwang (1989), Welch (1989) study the ability of equity underpricing in initial public offerings and seasoned equity offerings to signal firm characteristics. Battacharya (1979), Miller and Rock (1985), Ambarish, et. al. (1987), and Williams (1988) study the ability of dividend and investment policy to signal firm type. Signaling is achievable in these contexts given that the underlying problems satisfy the single-crossing property and it is assumed that agents have homogeneous beliefs (see e.g. Guesnerie and Laffont (1984)). Daniel and Titman (1995) survey this signaling literature when managers are better informed about firm characteristics and show that many of the models cited above are equivalent to a more general model where a firm signals information by *burning money* prior to contracting or commits to burn money after contracting. This paper considers general transfers and we study the effect that heterogeneous beliefs will have on the incentives of the contracting parties.[5] Further, it is important to note that we make no restriction whether transfers be positive or negative. Therefore, our results show that no matter how subsidized the disclosure mechanism is, it must still be uninformative if it is to be robust to changes in higher order beliefs of the investor.

Costly signaling of unverifiable information has been explored in many economic applications

[4] Myers and Majluf (1984) among others study the preference of financing with equity v.s. debt when the adverse selection problem cannot be resolved through signaling.

[5] Unlike our model, many of these papers rely on more dynamic signaling aspects (e.g. a dynamic dividend payout strategy) and therefore the robustness of these results to homogeneous beliefs could potentially be compounded by the robustness of the modeled dynamics. We focus on a simple static model in order to derive our no disclosure result, noting that adding dynamics will only affect the problem if information is somehow resolved over time. John (1987) shows that the way in which information asymmetries are resolved over time will drastically affect the optimal corporate signaling policy, which further motivates our static approach.

starting primarily with Spence (1973). The closest signaling model to ours in the economics literature is that of Milgrom and Roberts (1986) who study the ability of costly but uninformative advertising to signal information about product quality. In our corporate finance application, the manager's payoff is directly affected by the investor's valuation of the firm given their perceived characteristics of the firm, as is similar to the model of Milgrom and Roberts (1986). In contrast, a key feature of our problem — unique to the corporate financing setting — is that the manager's payoff is also affected by the manager's valuation of the firm's equity (given that they retain a fraction of the firm's equity) which need not be the same as the investor's valuation of equity when their beliefs differ.

Given the reliance of incentive compatibility on higher order beliefs we proceed to study robustness of signaling incentives to those beliefs in the spirit of Bergemann and Morris (2005), Carroll (2015), and more specifically Carroll (2017) (see Carroll (2019) for a survey of this literature). In particular, Carroll (2017) looks at a multidimensional screening problem where agent's types have multiple components but the principle only knows the marginal distribution of each component, not their joint distribution. In this paper, we similarly assume that the manager knows the investor's beliefs regarding the marginal distribution of firm characteristics but has uncertainty regarding the investor's beliefs regarding the joint distribution of firm characteristics and managerial beliefs.

Finally, this paper is motivated by the mixed empirical literature on firm signaling through corporate policy. Jegadeesh, et. al. (1992) empirically test whether IPO underpricing acts as a signal of firm quality and show that IPO underpricing is better explained by alternative mechanisms (e.g. that the market is better informed about firm value than the firm's managers) rather than signaling motives. Michaely and Shaw (1994) and Spiess and Pettway (1997) similarly reject the signaling theories of IPO underpricing by showing that there is little evidence that the costs of IPO underpricing are recouped by the firms over time as the theory would suggest if IPO underpricing actually signals firm characteristics. Krinsky and Rotenberg (1989) reject the hypothesis of Leland and Pyle (1977) that the retention of entrepreneurial ownership signals firm value. Finally, Lang and Litzenberger (1989) reject the dividend signaling hypothesis, developed by Battacharya (1979) and others, over the Jenson (1986) over-investment hypothesis which postulates that firm's use dividends to commit to not over-invest when free cash flows are high.

2 Model

We assume a manager presides over a firm which has one of n possible firm characteristics $\theta_1, ..., \theta_n$ and looks to sell equity to an investor in order to raise funding $I > 0$ to finance a new project.[6] Both the manager and investor have the ex-ante common prior that projects are distributed according to the full support distribution $q \in \Delta(\Theta)$. We assume that prior to signaling, the manager privately learns its firm's characteristic $\theta \in \Theta := \{\theta_1, ..., \theta_n\}$ but that this characteristic remains unknown to the investor.

We assume that the manager and investor may have different beliefs regarding the value of the firm given its characteristic. For simplicity, we assume that all agents have a belief type γ which can be one of n possible types $\gamma_1, ..., \gamma_n$ whereby an agent with belief type $\gamma_k \in \Gamma := \{\gamma_1, ..., \gamma_n\}$ has single-peaked preferences in the sense that they value the firm with characteristic θ_k strictly greater than all other firm types. For modeling simplicity we further assume that agents with beliefs γ_k value firms with characteristics closer to θ_k more than firms with characteristics further from θ_k. In order to formalize this, we endow each agent with a value function $v(\theta, \gamma)$ which is the value they attach to a firm with characteristic θ when they have beliefs γ. We make the following assumption on the functional form of v.

Assumption 1. *The valuation function v satisfies the following conditions:*

1.) $v(\theta_k, \gamma_k) > v(\theta_j, \gamma_k)$ for all $j \neq k$.

2.) $v(\theta_j, \gamma_k) > v(\theta_{j'}, \gamma_k)$ for all j, j', k such that $|j - k| < |j' - k|$.

3.) $v(\theta_k, \gamma_k) = v(\theta_j, \gamma_j)$ for all $j, k \in [0, 1]$

4.) $v(\theta_k, \gamma_j) = v(\theta_{k'}, \gamma_{j'})$ whenever $|j - k| = |j' - k'|$.

[6] Note that the firm characteristic can be influenced by the project type, the firm's existing assets, or both.

Assumptions 1.1 and 1.2 formalize the fact that agents have single peaked preferences while Assumptions 1.3 and 1.4 add a regularity condition stating that while each belief type has a favorite firm type, no belief type gets a higher benefit from their most preferred firm than any other belief type (1.3) and the decrease in firm value when its characteristic is further from your optimum is the same for all belief types provided that the distance from the optimal firm characteristic is the same (1.4). In order to ease the number of cases we must study we will assume without loss that $n \geq 4$.

We assume all agents are risk neutral and normalize the risk free rate to zero. Further, throughout we will assume that the outside investor has the belief type γ_1 and that this is common knowledge among the investor and manager.[7] In contrast, we assume that the manager's belief type is private information, only known by the manager. We will frequently use the notation that an agent's type is given by $\omega \in \Omega := \Theta \times \Gamma$ where $\omega = (\theta, \gamma)$ refers to the firm type and belief type of the agent. In that case, we endow the investor with beliefs over Ω that are consistent with the common prior, q, over firm characteristics as summarized in the following definition.

Definition 1. *Let $q \in \Delta(\Theta)$ be the common prior over firm characteristics. The joint probability distribution $p \in \Delta(\Omega)$ is q-admissible if*

$$\sum_{\gamma \in \Gamma} p(\theta, \gamma) = q(\theta)$$

for all $\theta \in \Theta$. Further, p is full support if $p(\omega) > 0$ for all $\omega \in \Omega$. We denote by $\Delta^+(\Omega)$ the set of full support admissible priors.

We will be interested in prior beliefs of the investor, which represents a joint probability distribution, $p \in \Delta^+(\Omega)$, that is consistent with the prior q (i.e. the marginal distribution of p with respect to θ is equal to q). We use the notation that $q_k := q(\theta_k)$ is the ex-ante probability of characteristic k and further assume without loss that q is full support so that $q_i > 0$ for all $i = 1, ..., n$.

[7] As mentioned in Footnote 3, this is without loss as introducing uncertainty regarding the investor's beliefs would only add constraints to the design problem.

We will now introduce our definition of a disclosure mechanism.

Definition 2. *A disclosure mechanism is a menu $\mathcal{M} := \{Y, T\}$ where $Y(\omega) \subset \Omega$ represents the pool of types that send the same signal as type ω and $T(\omega) \in \mathbb{R}$ the cost of the signal sent by types $\omega \in Y(\omega)$. Under any admissible disclosure mechanism $\{Y(\omega)\}_{\omega\in\Omega}$ represents a partition of Ω.*

Under the disclosure mechanism $\mathcal{M}$ the investor with prior $p \in \Delta^+(\Omega)$ requires a share of the firm $\alpha_p(Y(\omega))$ in exchange for financing I determined as:

$$\alpha_p(Y(\omega)) := \frac{I}{\mathbb{E}_p[v(\theta, \gamma_1)|\ \exists\gamma : (\theta, \gamma) \in Y(\omega)] - T(\omega)}$$

Given that we fix the investor's belief type as γ_1, then $\mathbb{E}_p[v(\theta, \gamma_1)|\ \exists\gamma : (\theta, \gamma) \in Y(\omega)]$ represents the investor's valuation of a firm type after learning they are in $Y(\omega)$. We will regularly use the short hand that $\mathbb{E}_p[v(\theta, \gamma_1)|\ \exists\gamma : (\theta, \gamma) \in Y(\omega)] := \mathbb{E}_p[v(\theta, \gamma_1)|Y(\omega)]$. Next, note that the construction of the share α_p implies that the investor prices the firm's equity competitively (given their information) which is equivalent to the investor breaking even on their investment: the value of the share of the firm obtained by the investor is worth exactly I given their information. Similarly, signaling is costly to the firm which is why the transfer $T(\omega)$ is deducted from the value of the firm when calculating the competitive equity share α_p in exchange for financing I. Finally, for simplicity we assume without loss that the signal reported by type ω to the investor is the *pool* of types that they belong to $Y(\omega)$.

Definition 3. *For any $p \in \Delta^+(\Omega)$ the disclosure mechanism $\mathcal{M}$ satisfies incentive compatibility if*

$$(v(\omega) - T(\omega)) \cdot (1 - \alpha_p(Y(\omega))) \geq (v(\omega) - T(\omega')) \cdot (1 - \alpha_p(Y(\omega'))) \tag{1}$$

for all $\omega, \omega' \in \Omega$. We say the mechanism $\mathcal{M}$ satisfies robust incentive compatibility if (1) is satisfied for all $p \in \Delta^+(\Omega)$. We call a mechanism robust if it satisfies robust incentive compatibility.

The left hand side of Equation (1) is the pay-off to the manager of type ω when truthfully reporting $Y(\omega)$ and paying $T(\omega)$.[8] In contrast, the right hand side of (1) is the pay-off to the manager of type ω who reports $Y(\omega')$ and pays $T(\omega')$. Note that the definition of robust incentive compatibility is equivalent to stating that the mechanism should be incentive compatible for type $\omega \in \Omega$, and therefore satisfy (1), under the worst possible prior in line with the literature on ambiguity aversion (

$$\tilde{p} \in \underset{p \in \Delta^+(\Omega)}{\operatorname{argmax}} \ \alpha_p(\omega) - \alpha_p(\omega')$$

for all $\omega' \in \Omega$. To see why this is the case, note that a larger value of $\alpha_p(\omega) - \alpha_p(\omega')$ implies that the manager must give away a larger share of the firm when reporting ω as opposed to ω' and therefore has a stronger incentive to mis-report their type as ω'.

In what follows we will use the convention that if $\alpha_p(\omega) = \alpha_p(\omega')$, then $Y(\omega) = Y(\omega')$. This is without loss given that whenever $\alpha_p(\omega) = \alpha_p(\omega')$, then incentive compatibility requires $T(\omega) = T(\omega')$ (see Lemma 1.3 below) and therefore the investor offers the same financing terms after learning the manager's type is in $Y(\omega)$ when compared to learning the manager's type is in $Y(\omega')$.[9] Thus, the incentives of all types in $Y(\omega)$ are identical to those of the types in $Y(\omega')$ and it is without loss to assume then that $Y(\omega) = Y(\omega')$.

3 Results

We will now present our first main result.

Proposition 1. *Let $\mathcal{M}$ be a mechanism that credibly reveals information regarding firm characteristics. Then, equity pricing under $\mathcal{M}$ must necessarily depend on the investor's beliefs regarding the distribution of managerial beliefs. In particular, the investor's posterior beliefs under any informative mechanism cannot only be a function of the prior beliefs over firm characteristics: there*

[8]Note that this construction implicitly assumes the manager's objective is to signal in a way that minimizes the share of the firm they must sell for financing I.

[9]Namely, when $\alpha_p(\omega) = \alpha_p(\omega')$ then reporting ω or ω' implies selling the same equity share in exchange for financing. In, that case if $T(\omega) \neq T(\omega')$ then it must be the case that all types in $Y(\omega)$ and $Y(\omega')$ will optimally report the type that requires paying the lowest fee.

exists $\omega \in \Omega$ such that the posterior $p(\cdot|Y(\omega)) \in \Delta(\Theta)$ is not just a function of the prior, q, over project characteristics.

Proof. See Appendix Section 5.1. □

This result states that the investor's posterior beliefs under any incentive compatible disclosure mechanism cannot only depend on the common prior, q, over project characteristics. Importantly, if this were the case, then the equity pricing of the investor would only depend on the common prior q and therefore would not change as a function of the investor's prior $p \in \Delta^+(\Omega)$ (i.e. equity pricing would only depend on q, regardless of the investor's perceived distribution of managerial beliefs). We show that while there exists signaling schemes that generate posterior beliefs which are only a function of the prior beliefs over firm characteristics (see Example 3.1 below), no such signaling scheme can be incentive compatible. Further, this result holds for any prior $p \in \Delta^+(\Omega)$ and therefore is not specific to our robustness results below. This is an important result as it implies that investor's equity pricing must rely in some way on the investors higher order beliefs whenever signaling is informative. Therefore, changes to those higher order beliefs will produce changes in the investor's equity pricing even though the distribution over project characteristics remains the same.

The reason this result holds is that the single crossing property breaks down if equity prices only depend on the objective prior over firm characteristics q. Namely, if the investor's posterior beliefs over firm characteristics is only a function of the prior q then it would require that for any pool $Y(\theta_k, \gamma)$ it must be the case that $(\theta_k, \gamma') \in Y(\theta_k, \gamma)$ for all γ'. Hence, for each k, the pool $Y(\theta_k, \gamma)$ contains a type that values characteristic θ_k the most and a type that values characteristic k less. Yet, if this holds for all pools $Y(\omega)$ then it will not be possible to for the investor to value one pool more than another as this will create an incentive for some type to deviate. We will now illustrate this point with an example that we will use repeatedly throughout the exposition to demonstrate the intuition of our results.

Example 3.1. *Suppose that $n = 3$, $I = 1$, and that valuations are given by the grid of Figure 1.*

$v(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	3	2	1
θ_2	2	3	2
θ_3	1	2	3

Figure 1: The valuation $v(\theta_k, \gamma_j)$ is the given by the entry in the kth row and jth column (e.g. $v(\theta_1, \gamma_2) = 2$).

Note that a disclosure mechanism consists of a partition of the pairs $(\theta, \gamma) \in \Omega := \{\theta_1, \theta_2, \theta_3\} \times \{\gamma_1, \gamma_2, \gamma_3\}$. Proposition 1 states that it is not possible to design a mechanism that only partitions the rows of the grid in Figure 1 (so that posteriors only depend on the prior q). To see why this is the case, suppose that the mechanism has only two pools Y_1 and Y_2 where $Y_1 = \{(\theta_1, \gamma_1), (\theta_1, \gamma_2), (\theta_1, \gamma_3)\}$ and $Y_2 = \Omega \backslash Y_1$ (i.e. Y partitions the first row from the second two rows). Note that for any $p \in \Delta^+(\Omega)$ it is the case that $Pr(\theta_1|Y_1) = 1$, $Pr(\theta_2|Y_2) = \frac{q_2}{q_2+q_3}$, and $Pr(\theta_3|Y_2) = \frac{q_3}{q_2+q_3}$ and therefore this mechanism is informative and generates posteriors beliefs of the investor that only depend on q. In this case, changes to the prior p do not affect the posterior beliefs of the investor so long as p remains admissible. To see why any disclosure mechanism of this form violates incentive compatibility note that combining the incentive compatibility condition of $\omega \in Y_1$ and the incentive compatibility condition for $\omega' \in Y_2$ we obtain the necessary condition that

$$v(\omega)(\alpha_p(Y_2) - \alpha_p(Y_1)) \geq v(\omega')(\alpha_p(Y_2) - \alpha_p(Y_1)) \tag{2}$$

Further note that this condition will be violated for any values of $\alpha_p(Y_1) \neq \alpha_p(Y_2)$. For example, if $\alpha_p(Y_1) < \alpha_p(Y_2)$ then (2) will be violated when setting $\omega = (\theta_1, \gamma_3)$ and $\omega' = (\theta_2, \gamma_2)$ as $v(\theta_1, \gamma_3) = 1$ and $v(\theta_2, \gamma_2) = 3$ and $\alpha_p(Y_2) - \alpha_p(Y_1) > 0$ in this case.

The above example and Equation (2) demonstrates some more general conditions for incentive compatibility under any prior $p \in \Delta^+(\Omega)$ which we summarize in the following lemma.

Lemma 1. *Let $\mathcal{M}$ be an incentive compatible mechanism under the admissible prior p. Then,*

1.) If $\alpha_p(Y(\tilde{\omega})) < \alpha_p(Y(\tilde{\omega}'))$ then $v(\omega) \geq v(\omega')$ for all $\omega \in Y(\tilde{\omega})$ and all $\omega' \in Y(\tilde{\omega}')$.

2.) If $\alpha_p(Y(\omega)) > \alpha_p(Y(\omega'))$ then $T(\omega') > T(\omega)$.

3.) If $\alpha_p(Y(\omega)) = \alpha_p(Y(\omega'))$ then $T(\omega) = T(\omega')$.

4.) If there exists $\omega, \omega' \in Y(\tilde{\omega})$ and $\omega'' \in Y(\tilde{\omega}'')$ such that $v(\omega) > v(\omega'') > v(\omega')$ then $\alpha_p(Y(\tilde{\omega})) = \alpha_p(Y(\tilde{\omega}''))$.

Proof. See Appendix Section 5.2 □

Lemma 1.1 states that if the investor values learning that the manager's type is in $Y(\tilde{\omega})$ more than learning their type is in $Y(\tilde{\omega}')$ then all types in $Y(\tilde{\omega})$ must value their firm more than all types in $Y(\tilde{\omega}')$. This condition was demonstrated in the previous example and can be directly observed from Equation (2). Note, that this is a form of single crossing condition where by the types that receive the best financing terms (i.e. lowest α) must value those terms more than the types who receive worse financing terms. Lemma 1.2 states that in order to obtain separation, the types that receive better financing terms must also pay a higher transfer (i.e. must send a more costly signal) in order to achieve separation while Lemma 1.3 states that types that receive the same financing terms must pay the same transfer. Finally, Lemma 1.4 states that the investor cannot value the pool $Y(\tilde{\omega})$ more than the pool $Y(\tilde{\omega}')$ if there are two types in $Y(\tilde{\omega})$ that value their firm both strictly more and strictly less than any type in $Y(\tilde{\omega}')$.

Given the perceived difficulty of managers to form objective beliefs regarding the investor's higher order beliefs we now seek to characterize robust disclosure mechanisms. Unfortunately, our results are negative, stating that no informative mechanism that is robust to higher order beliefs exists. Denote by $\mathcal{M}_0$ the uninformative (status quo) mechanism that sets $Y(\omega) = \Omega$ and $T(\omega) = 0$ for all $\omega \in \Omega$. This leads to our main result.

Proposition 2. *Any robust mechanism $\mathcal{M}$ is entirely uninformative and therefore (outcome) equivalent to $\mathcal{M}_0$.*

Proof. See Appendix Subsection 5.3. □

The proof proceeds in a number of steps. We first use Lemma 1 to characterize some additional necessary conditions that are required for incentive compatibility for any prior p and then show how robustness to all priors $p \in \Delta^+(\Omega)$ will eventually violate one of these conditions when the mechanism is informative. To prove this latter point, we show that for any informative mechanism satisfying incentive compatibility for a particular p, there exists another p' under which that mechanism is no longer incentive compatible.

3.1 Sketch of the Proof of Proposition 2

In lieu of demonstrating each step of the Proof of Proposition 2 we will illustrate these results with our simple Example 3.1. Ultimately, we will show that robust incentive compatibility requires that $Y(\omega) = \Omega$ for all $\omega \in \Omega := \{\theta_1, \theta_2, \theta_3\} \times \{\gamma_1, \gamma_2, \gamma_3\}$ in which case any robust mechanism is entirely uninformative.

To start, we consider the pool $Y(\theta_1, \gamma_3)$ and note that it must be the case that there exists $j \in \{1, ..., n\}$ such that $(\theta_j, \gamma) \in Y(\theta_1, \gamma_3)$ for all $\gamma \in \{\gamma_1, \gamma_2, \gamma_3\}$. To see why this is the case consider the following example pool $Y(\theta_1, \gamma_3)$ given by the shaded blocks of Figure 2 (i.e. all types with a shaded block are in the pool $Y(\theta_1, \gamma_3)$).

$v(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	3	2	1
θ_2	2	3	2
θ_3	1	2	3

$\tilde{p}(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	ϵ	ϵ	$q_1 - 2\epsilon$
θ_2	$q_1 - 2\epsilon$	ϵ	ϵ
θ_3	ϵ	$q_3 - 2\epsilon$	ϵ

Figure 2: An example pool (represented by the shaded entries) and prior distribution $\tilde{p} \in \Delta^+(\Omega)$.

Note that for each j there exists γ such that $(\theta_j, \gamma) \notin Y(\theta_1, \gamma_3)$. Next, consider the prior distribution $\tilde{p}$ shown in the right grid of Figure 2. For ϵ sufficiently small it can be checked that $\tilde{p}$ is admissible and therefore $\tilde{p} \in \Delta^+(\Omega)$. Next, we can see that under the prior $\tilde{p}$, as $\epsilon \to 0$ then the investor's valuation $\mathbb{E}_{\tilde{p}}[v(\theta, \gamma_1)|Y(\theta_1, \gamma_3)] \to v(\theta_1, \gamma_1)$. Further, for any pool $Y' \neq Y(\theta_1, \gamma_3)$ as

$\epsilon \to 0$ the investor's valuation $\mathbb{E}_{\tilde{p}}[v(\theta, \gamma_1)|Y'] \to \tilde{v} \leq v(\theta_2, \gamma_1) < v(\theta_1, \gamma_1)$.[10] Hence, there exists $\bar{\epsilon}$ such that whenever $\epsilon < \bar{\epsilon}$ it must be the case that $\alpha_{\tilde{p}}(Y(\theta_1, \gamma_3)) < \alpha_{\tilde{p}}(Y')$ for all $Y' \neq Y(\theta_1, \gamma_3)$. Yet we know that $(\theta_3, \gamma_3) \notin Y(\theta_1, \gamma_3)$ and $v(\theta_3, \gamma_3) > v(\theta_1, \gamma_3)$ which violates Lemma 1.1. In the same spirit, we show in the proof of Proposition 2 that we can find a prior $\tilde{p}$ that violates incentive compatibility if there does not exist j such that $(\theta_j, \gamma) \in Y(\theta_1, \gamma_3)$ for all $\gamma \in \Gamma$.

$v(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	3	2	**1**
θ_2	**2**	**3**	**2**
θ_3	1	2	3

$v(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	3	**2**	**1**
θ_2	**2**	**3**	**2**
θ_3	1	**2**	3

Figure 3: An example pool (represented by the shaded entries) and application of Lemma 1.4.

Next, suppose that the above condition holds for $j = 2$ so that $(\theta_2, \gamma) \in Y(\theta_1, \gamma_3)$ for all $\gamma \in \{\gamma_1, \gamma_2, \gamma_3\}$ as illustrated in the example pool given by the shaded entries in the left hand grid of Figure 3. Then, Lemma 1.4 tell us that incentive compatibility requires $(\theta_1, \gamma_2), (\theta_3, \gamma_2) \in Y(\theta_1, \gamma_3)$ due to the fact that $(\theta_1, \gamma_3), (\theta_2, \gamma_2) \in Y(\theta_1, \gamma_3)$ and $v(\theta_1, \gamma_3) = 1 < v(\theta_1, \gamma_2) = 2 = v(\theta_3, \gamma_2) < v(\theta_2, \gamma_2)$. Therefore, we have shown that robust incentive compatibility requires $Y(\theta_1, \gamma_3) \supset \Omega \backslash \{(\theta_1, \gamma_2), (\theta_3, \gamma_1), (\theta_3, \gamma_3)\}$ which is illustrated by the right hand grid of Figure 3.

$v(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	3	**2**	**1**
θ_2	**2**	**3**	**2**
θ_3	1	**2**	3

$\tilde{p}(\theta,\gamma)$	γ_1	γ_2	γ_3
θ_1	ϵ	**$q_1 - 2\epsilon$**	**ϵ**
θ_2	**$q_2 - 2\epsilon$**	**ϵ**	**ϵ**
θ_3	ϵ	**ϵ**	$q_3 - 2\epsilon$

Figure 4: An example pool (represented by the shaded entries) and application of Lemma 1.4.

Next, we will show that robust incentive compatibility requires $(\theta_3, \gamma_3) \in Y(\theta_1, \gamma_3)$. In order to do so, consider the prior $\tilde{p}$ of Figure 4. Under this prior, there exists $\bar{\epsilon}$ such that $\epsilon < \bar{\epsilon}$ implies $\alpha_{\tilde{p}}(Y(\theta_1, \gamma_3)) < \alpha_{\tilde{p}}(Y(\theta_3, \gamma_3))$, regardless of the additional elements of $Y(\theta_3, \gamma_3)$ (as long as Y

[10] This is due to the fact that as $\epsilon \to 0$ under $\tilde{p}$, the probability that the type is θ_1 conditional on the signal Y_1 goes to zero.

still represents a partition). Thus, $\alpha_{\tilde{p}}(Y(\theta_1, \gamma_3)) < \alpha_{\tilde{p}}(Y(\theta_3, \gamma_3))$ combined with $v(\theta_1, \gamma_3) < v(\theta_3, \gamma_3)$ violates Lemma 1.1 and therefore it must be the case that $(\theta_3, \gamma_3) \in Y(\theta_1, \gamma_3)$.

$v(\theta, \gamma)$	γ_1	γ_2	γ_3
θ_1	3	2	1
θ_2	2	3	2
θ_3	1	2	3

$\tilde{p}_2(\theta, \gamma)$	γ_1	γ_2	γ_3
θ_1	$q_1 - 2\epsilon$	ϵ	ϵ
θ_2	ϵ	$q_2 - 2\epsilon$	ϵ
θ_3	ϵ	ϵ	$q_3 - 2\epsilon$

$\tilde{p}_1(\theta, \gamma)$	γ_1	γ_2	γ_3
θ_1	$\frac{1}{2}q_1 - \epsilon$	$\frac{1}{2}q_1$	ϵ
θ_2	ϵ	$q_2 - 2\epsilon$	ϵ
θ_3	ϵ	ϵ	$q_3 - 2\epsilon$

Figure 5: An example pool (represented by the shaded entries) and application of Lemma 1.4.

As our final step, we must show that $(\theta_1, \gamma_1), (\theta_3, \gamma_1) \in Y(\theta_1, \gamma_3)$. The step proceeds identically for both (θ_1, γ_1) and (θ_3, γ_1) so we will only present it for (θ_1, γ_1). In particular, note that if $(\theta_1, \gamma_1) \notin Y(\theta_1, \gamma_3)$ then using the fact that $(\theta_3, \gamma_3) \in Y(\theta_1, \gamma_3)$ and $v(\theta_1, \gamma_1) = v(\theta_3, \gamma_3)$ implies that incentive compatibility (1) for each type requires

$$(v(\theta_1, \gamma_1) - T(\omega)) \cdot (1 - \alpha_p(Y(\theta_1, \gamma_1))) = (v(\theta_3, \gamma_3) - T(\omega')) \cdot (1 - \alpha_p(Y(\theta_3, \gamma_3)))$$

for all $p \in \Delta^+(\Omega)$. Thus, given that transfers T cannot depend on the prior p when seeking robustness, it must be the case that $\frac{1-\alpha_p(Y(\theta_1,\gamma_1))}{1-\alpha_p(Y(\theta_3,\gamma_3))}$ is a constant function of p. The final step of the proof then consists of designing two priors $\tilde{p}_1$ and $\tilde{p}_2$ such that $\frac{1-\alpha_{\tilde{p}_1}(Y(\theta_1,\gamma_1))}{1-\alpha_{\tilde{p}_1}(Y(\theta_3,\gamma_3))} = \frac{1-\alpha_{\tilde{p}_2}(Y(\theta_1,\gamma_1))}{1-\alpha_{\tilde{p}_2}(Y(\theta_3,\gamma_3))}$ implies that $\alpha_p(Y(\theta_1, \gamma_1)) = \alpha_p(Y(\theta_3, \gamma_3))$ for all $p \in \Delta^+(\Omega)$ which implies that the mechanism is outcome equivalent to one whereby $(\theta_1, \gamma_1) \in Y(\theta_3, \gamma_3)$.

The two priors $\tilde{p}_1$ and $\tilde{p}_2$ of Figure 5 generate this property for ϵ sufficiently small whereby setting $\frac{1-\alpha_{\tilde{p}_1}(Y(\theta_1,\gamma_1))}{1-\alpha_{\tilde{p}_1}(Y(\theta_3,\gamma_3))} = \frac{1-\alpha_{\tilde{p}_2}(Y(\theta_1,\gamma_1))}{1-\alpha_{\tilde{p}_2}(Y(\theta_3,\gamma_3))}$ will require $\alpha_{\tilde{p}_1}(Y(\theta_1, \gamma_1)) = \alpha_{\tilde{p}_1}(Y(\theta_3, \gamma_3))$ implying that $\frac{1-\alpha_p(Y(\theta_1,\gamma_1))}{1-\alpha_p(Y(\theta_3,\gamma_3))} = 1$ for all $p \in \Delta(\Omega)$ which can only be the case if $\alpha_p(Y(\theta_1, \gamma_1)) = \alpha_p(Y(\theta_3, \gamma_3))$ for all $p \in \Delta^+(\Omega)$.[11] Finally, this last condition implies by Lemma 1.3 that any incentive compatible mechanism is outcome equivalent to a mechanism whereby $(\theta_1, \gamma_1) \in Y(\theta_3, \gamma_3)$. The proof proceeds identically to show $(\theta_3, \gamma_1) \in Y(\theta_3, \gamma_3)$, in which case we have shown that it must be

[11] This is demonstrated explicitly in the proof of Proposition A.1 in the appendix.

the case that any robust incentive compatible mechanism is outcome equivalent to one that sets $Y(\omega) = \Omega$ for all $\omega \in \Omega$. Thus, any robust incentive compatible mechanism is uninformative.

We have just outlined intuition of the general of Proposition 2. While there are slightly more cases in the general proof, the logic is more or less identitical to the above example.

4 Conclusion

In this paper we demonstrate the difficulties generated by relaxing the homogeneous beliefs assumption in the setting of dynamic corporate signaling. We first show that any informative signaling scheme will generate equity prices that are dependent on the investor's perceived prior distribution of managerial beliefs. In this case, incentive compatibility of any signaling scheme must require the manager to have precise beliefs regarding the investor's higher order beliefs. Given the difficulty of forming objective beliefs regarding other agents higher order beliefs, we then ask whether it is possible to design an incentive compatible signaling scheme that is robust to the higher order beliefs of the investor. Unfortunately, we show that any robust mechanism must necessarily be uninformative. This implies that informative corporate signaling must rely on common higher order beliefs between managers and their perspective investors. Therefore, unless one can be sure that such common beliefs exist then corporate signaling will only be effective if the information that is being revealed is entirely verifiable. Yet, when information is verifiable then there is no need to incur costs to signal firm characteristics, narrowing the scope for costly signaling to play a role in corporate disclosure.

5 Appendix

5.1 Proof of Proposition 1

Proof. Consider any mechanism such that $p(\theta|Y(\omega))$ is only a function of q for all $\theta \in \Theta$ and all $\omega \in \Omega$. This implies that if for some $\omega \in \Omega$ it is the case that $(\theta, \gamma) \in Y(\omega)$, then $(\theta, \gamma') \in Y(\omega)$ for all $\gamma' \in \Gamma$. Now, suppose by contradiction that there exists ω, ω' such that $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$ under $\mathcal{M}_0$ and assume without loss that $(\theta, \gamma) \in Y(\omega)$. Then, by Lemma 1.1, it must be the case that $v(\theta, \gamma) \geq v(\theta', \gamma')$ for all $(\theta, \gamma) \in Y(\omega)$ and all $(\theta', \gamma') \in Y(\omega')$. Again, without loss assume that $(\theta_k, \gamma_k) \in Y(\omega)$ and $(\theta_j, \gamma_j) \in Y(\omega')$ for $k \neq j$. For a fixed prior $p \in \Delta(\Omega)$, incentive compatibility requires

$$(v(\omega) - T(\omega)) \cdot (1 - \alpha_p(Y(\omega))) \geq (v(\omega) - T(\omega')) \cdot (1 - \alpha_p(Y(\omega')))$$

and

$$(v(\omega') - T(\omega')) \cdot (1 - \alpha_p(Y(\omega'))) \geq (v(\omega') - T(\omega)) \cdot (1 - \alpha_p(Y(\omega)))$$

which after rearranging yields

$$v(\omega)(\alpha_p(\omega') - \alpha_p(\omega)) \geq v(\omega')(\alpha_p(\omega') - \alpha_p(\omega)) \tag{3}$$

for all $\omega \in Y(\omega)$ and $\omega' \in Y(\omega')$. Now, $(\theta_k, \gamma_k) \in Y(\omega)$ implies $(\theta_k, \gamma_j) \in Y(\omega)$ for all $j \neq k$, and by construction $v(\theta_j, \gamma_j) > v(\theta_k, \gamma_j)$. Therefore incentive compatibility requires $\alpha(\omega) < \alpha(\omega')$. Yet, $(\theta_k, \gamma_j) \in Y(\omega)$, $(\theta_j, \gamma_j) \in Y(\omega')$, and $v(\theta_j, \gamma_j) > v(\theta_k, \gamma_j)$, which violates (3) when $\alpha(\omega) < \alpha(\omega')$. □

5.2 Proof of Lemma 1

Proof. To prove 1, note that if there exists $p \in \Delta(\Omega)$ with full support such that $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$ then, as in the proof of Proposition 1 incentive compatibility requires

$$v(\omega)(\alpha_p(\omega') - \alpha_p(\omega)) \geq v(\omega')(\alpha_p(\omega') - \alpha_p(\omega)) \tag{4}$$

and therefore it must be the case that $v(\theta, \gamma) > v(\theta', \gamma')$ for all $(\theta, \gamma) \in Y(\omega)$ and all $(\theta', \gamma') \in Y(\omega')$ as otherwise the mechanism is not incentive compatible under beliefs p and therefore cannot be robust incentive compatible.

To prove 2, note that if $\alpha_p(\omega) > \alpha_p(\omega')$ but $T(\omega) > T(\omega')$ then

$$(v(\omega) - T(\omega))(1 - \alpha_p(\omega)) < (v(\omega) - T(\omega'))(1 - \alpha_p(\omega'))$$

and therefore the mechanism is not incentive compatible.

To prove 3, note that if $\alpha_p(\omega) = \alpha_p(\omega')$ but $T(\omega) > T(\omega')$ then

$$(v(\omega) - T(\omega))(1 - \alpha_p(\omega)) < (v(\omega) - T(\omega'))(1 - \alpha_p(\omega'))$$

and therefore the mechanism is not incentive compatible for type ω. Similarly, if $\alpha_p(\omega) = \alpha_p(\omega')$ but $T(\omega) < T(\omega')$ then

$$(v(\omega') - T(\omega'))(1 - \alpha_p(\omega')) < (v(\omega') - T(\omega))(1 - \alpha_p(\omega))$$

and therefore the mechanism is not incentive compatible for type ω'.

To prove 4, we use the same logic whereby incentive compatibility requires (4) to hold for any fixed prior p. Therefore, if there exists $(\theta, \gamma), (\theta', \gamma') \in Y(\omega)$ and $(\theta'', \gamma'') \in Y(\omega')$ such that $v(\theta, \gamma) > v(\theta'', \gamma'') > v(\theta', \gamma')$ then any incentive compatible mechanism must satisfy $\alpha_p(Y(\omega)) = \alpha_p(Y(\omega'))$.

□

5.3 Proof of Proposition 2

We will first present a number of preliminary results used to proved the main result.

Lemma 2. *Suppose that* $(\theta_k, \gamma_j) \in Y(\omega)$ *for all* $j = 1, ..., n$. *Then,* $\Omega \backslash (\{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\}) \subset Y(\omega)$.

Proof. Suppose that there exists k such that $(\theta_k, \gamma_j) \in Y(\omega)$ for all $j = 1, ..., n$. We will first show that this implies that for all $l = 1, ..., n$ there exists $j \neq l$ such that $(\theta_j, \gamma_l) \in Y(\omega)$. Namely, denoting by $h \in \operatorname{argmin}_j v(\theta_k, \gamma_j)$, then Lemma 1 tells us that for any (θ, γ) such that $v(\theta_k, \gamma_k) > v(\theta_l, \gamma_j) > v(\theta_h, \gamma_k)$ it must be the case that $(\theta_l, \gamma_j) \in Y(\omega)$. Therefore, for all $0 < |l-j| < |k-h|$ it must be the case that $(\theta_l, \gamma_j) \in Y(\omega)$. When $n \geq 4$, this implies $|k-h| \geq 2$, and therefore given that for all l there exists j such that $0 < |l-j| < 2$ it must be the case that for all $l = 1, ..., n$ there exists $j \neq l$ such that $(\theta_j, \gamma_l) \in Y(\omega)$.

Next, we will show that this latter condition implies that $(\theta_1, \gamma_n) \in Y(\omega)$. Namely, if not, then consider the prior such that $p(\theta_1, \gamma_n) = q_1 - (n-1)\epsilon$ and $p(\theta_l, \gamma_j) = q_l - (n-1)\epsilon$ for each $l > 1$, and $j \neq l$ such that $(\theta_l, \gamma_j) \in Y(\omega)$ and $p(\theta, \gamma) = \epsilon$ for all other (θ, γ). Under this prior, we know that $\alpha_p(Y(\theta_1, \gamma_n)) \to \frac{k}{v(\theta_1, \gamma_1) - T(Y(\theta_1, \gamma_n))}$ and $\alpha_p(Y(\omega)) \to \tilde{\alpha} \geq \frac{k}{v(\theta_2, \gamma_1) - T(\omega)}$ as $\epsilon \to 0$. Therefore, there exists $\bar{\epsilon}$ such that whenever $\epsilon < \bar{\epsilon}$ then $\alpha_p(Y(\theta_1, \gamma_n)) > \alpha_p(Y(\omega))$ only if $T(\theta_1, \gamma_n) > T(\omega)$ contradicting 1.4. Thus, it must be the case that $\epsilon < \bar{\epsilon}$ implies $\alpha_p(Y(\theta_1, \gamma_n)) < \alpha_p(Y(\omega))$ and given that $v(\theta_k, \gamma_k) > v(\theta_1, \gamma_n)$, this implies that the mechanism is not incentive compatible as it violatesLemma 1.1. Hence, $(\theta_1, \gamma_n) \in Y(\omega)$ which by the above logic implies that if $(\theta, \gamma) \notin Y(\omega)$ then it must be the case that either $v(\theta, \gamma) = v(\theta_k, \gamma_k)$ or $v(\theta, \gamma) = v(\theta_1, \gamma_n)$. Finally, note that $v(\theta, \gamma) = v(\theta_k, \gamma_k)$ implies $(\theta, \gamma) \in \{(\theta_l, \gamma_l) : l = 1, ..., n\}$. Further, $(\theta, \gamma) \notin Y(\omega)$ and $v(\theta, \gamma) = v(\theta_1, \gamma_n)$ implies $(\theta, \gamma) = (\theta_n, \gamma_1)$. Therefore, $(\theta, \gamma) \in \{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_1, \gamma_n)\}$ which implies $\Omega \backslash (\{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\}) \subset Y(\omega)$. □

We will now present another intermediate result that will be used to prove the main proposition.

Lemma 3. *If there exists $\omega \in \Omega$ such that $\{(\theta_k, \gamma_k) : k = 1, ..., n\} \subset Y(\omega)$, then $\Omega \backslash \{(\theta_n, \gamma_1)\} \subset Y(\omega)$.*

Proof. First note that if $\{(\theta_k, \gamma_k) : k = 1...n\} \subset Y(\omega)$, then for any prior p it must be the case that $\alpha_p(Y(\omega)) \leq \alpha_p(Y(\omega'))$ for all $\omega' \neq \omega$. This is due to the fact that if $\{(\theta_k, \gamma_k) : k = 1...n\} \subset Y(\omega)$ and $\alpha_p(Y(\omega)) > \alpha_p(Y(\omega'))$ then the mechanism cannot be incentive compatible as $(\theta_k, \gamma_j) \in Y(\omega')$ implies that $k \neq j$ and therefore $v(\theta_k, \gamma_j) < v(\theta_k, \gamma_k)$, thus violating 1.1.

Now, suppose that there exists ω' such that $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$ (otherwise $Y(\omega) = \Omega$ and our claim is proven). Then, there must exist k such that $(\theta_k, \gamma_j) \in Y(\omega)$ for all $j = 1, ..., n$. To see why this is the case, suppose that for all k there exists j such that $(\theta_k, \gamma_j) \notin Y(\omega)$. Next, consider the prior p such that $p(\theta_k, \gamma_j) = q_k - (n-1) \cdot \epsilon$ for $k = 1...n-1$ and j such that $(\theta_k, \gamma_j) \notin Y(\omega)$, $p(\theta_n, \gamma_n) = q_n - (n-1) \cdot \epsilon$, and $p(\theta, \gamma) = \epsilon$ for all other (θ, γ). Under this prior, $Pr_p((\theta_n, \gamma_n) | \omega \in Y(\omega)) \to 1$ as $\epsilon \to 0$ and therefore if $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$ for all $\epsilon > 0$ then it must be the case that $Y(\omega') \subset \{(\theta_n, \gamma_l) : l = 1, ..., n-1\}$.

Case 1: $Y(\omega'') = Y(\omega)$ for all $\omega'' \notin \{\omega, \omega'\}$. In this case, we note that $Y(\omega') \subset \{(\theta_n, \gamma_l) : l = 1, ..., n-1\}$ plus $Y(\omega'') = Y(\omega)$ for all $\omega'' \notin \{\omega, \omega'\}$ then implies that $(\theta_1, \gamma_n) \in Y(\omega)$. We will show that this implies that it must be the case that $Y(\omega') = \{(\theta_n, \gamma_1)\}$. Namely, we know that $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$ for all p, and therefore if there exists $l \in \{2, ..., n-1\}$ such that $(\theta_n, \gamma_l) \in Y(\omega')$, then this violates incentive compatibility as $(\theta_1, \gamma_n) \in Y(\omega)$, $(\theta_n, \gamma_l) \in Y(\omega')$, $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega'))$, but $v(\theta_n, \gamma_l) > v(\theta_1, \gamma_n)$, contradicting Lemma 1.1. Finally, in this case, if $Y(\omega') = \{(\theta_n, \gamma_1)\}$ then it must be the case that $Y(\omega) = \Omega \backslash \{(\theta_n, \gamma_1)\}$ and our claim holds.

Case 2: There exists $\omega'' \notin Y(\omega')$ such that $Y(\omega'') \neq Y(\omega)$. Note that if $\omega'' \notin Y(\omega')$ then it must be the case that there exists $k' < n$ such that $(\theta_{k'}, \gamma_l) \in Y(\omega'')$ for some $l \neq k'$ (otherwise $\alpha_p(Y(\omega'')) = \alpha_p(Y(\omega'))$ for all $p \in \Delta^+(\Omega)$ contradicting $\omega'' \notin Y(\omega')$). Further, by Lemma 2, we know that either the claim of this proof holds, or for all $k = 1, ..., n$ there exists j such that $(\theta_k, \gamma_j) \notin Y(\omega)$.

Then, consider the prior $p(\theta_{k'}, \gamma_l) = q_{k'} - (n-1) \cdot \epsilon$, $p(\theta_k, \gamma_j) = q_k - (n-1) \cdot \epsilon$ for all $k \notin \{k', n\}$ and j such that $(\theta_k, \gamma_j) \notin Y(\omega)$, $p(\theta_n, \gamma_n) = q_n - (n-1) \cdot \epsilon$, and $q(\theta, \gamma) = \epsilon$ for all other (θ, γ). Under this prior, there exist $\bar{\epsilon} > 0$ such that $\epsilon < \bar{\epsilon}$ implies $\alpha_p(Y(\omega)) < \alpha_p(Y(\omega''))$ only if $T(\omega) > T(\omega'')$ contradicting Lemma 1.2. Therefore, if the mechanism is incentive compatible then it requires $\alpha_p(Y(\omega)) > \alpha_p(Y(\omega''))$ which presents a contradiction as $(\theta_k, \gamma_k) \in Y(\omega)$, $(\theta_{k'}, \gamma_l) \in Y(\omega'')$, and $k' \neq l$ by construction implies $v(\theta_k, \gamma_k) > v(\theta_{k'}, \gamma_l)$ violating Lemma 1.1.

Finally, note that after ruling out Case 2, we know that we must be in Case 1 whereby $Y(\omega) =$

$\Omega\backslash\{(\theta_n,\gamma_1)\}$ and $Y(\omega')=\{(\theta_n,\gamma_1)\}$ and therefore our claim holds.

□

Lemma 4. *Let $\mathcal{M}$ be a robust mechanism. Then, there exists ω such that*

$$\Omega\backslash(\{(\theta_k,\gamma_k):k=1,...,n\}\cup\{(\theta_n,\gamma_1\})\subset Y(\omega)$$

Therefore, $\omega'\neq\omega$ and $\omega'\notin Y(\omega)$ implies that either $Y(\omega')\subset\{(\theta_1,\gamma_1),...,(\theta_k,\gamma_k)\}$ or $Y(\omega')=\{(\theta_1,\gamma_n)\}$.

Proof. In order to prove the above claim, consider $Y(\theta_1,\gamma_1)$. If $\{(\theta_k,\gamma_k):k=1,...,n\}\subset Y(\theta_1,\gamma_1)$ then by Lemma 3 it must be the case that $\Omega\backslash\{(\theta_n,\gamma_1\}\subset Y(\theta_1,\gamma_1)$ and therefore our claim holds.

If instead, $\{(\theta_k,\gamma_k):k=1,...,n\}\not\subset Y(\theta_1,\gamma_1)$ then there exists $k\leq n$ such that $(\theta_k,\gamma_k)\notin Y(\theta_1,\gamma_1)$. In proceeding we will let $\hat{k}$ be the smallest integer such that $(\theta_{\hat{k}},\gamma_{\hat{k}})\notin Y(\theta_1,\gamma_1)$ and will proceed with two cases.

Case 1: $\alpha_p(Y(\theta_1,\gamma_1))<\alpha_p(Y(\theta_{\hat{k}},\gamma_{\hat{k}}))$ for all $p\in\Delta_q^+(\Omega)$. Note that in this case $Y(\theta_1,\gamma_1)\subset\{(\theta_k,\gamma_k):k=1,...,n\}$. Namely, if there exists $(\theta_k,\gamma_j)\in Y(\omega)$ for some $k\neq j$ then $\alpha_p(Y(\theta_1,\gamma_1))<\alpha_p(Y(\theta_{\hat{k}},\gamma_{\hat{k}}))$ will violate Lemma 1.1 as $(\theta_{\hat{k}},\gamma_{\hat{k}})\in Y(\theta_{\hat{k}},\gamma_{\hat{k}})$ and $v(\theta_{\hat{k}},\gamma_{\hat{k}})>v(\theta_k,\gamma_j)$.

Now, consider the type (θ_1,γ_n). If $(\theta_1,\gamma_n)\in Y(\theta_{\hat{k}},\gamma_{\hat{k}})$, then it must be the case that for all $j=1,...,n$ there exists k such that $(\theta_k,\gamma_j)\in Y(\theta_{\hat{k}},\gamma_{\hat{k}})$ as otherwise the prior $p(\theta_1,\gamma_n)=q_1-(n-1)\cdot\epsilon$, $p(\theta_k,\gamma_j)=q_k-(n-1)\cdot\epsilon$ for all $k\neq 1$ and j such that $(\theta_k,\gamma_j)\notin Y(\theta_{\hat{k}},\gamma_{\hat{k}})$, and $p(\theta,\gamma)=\epsilon$ for all other (θ,γ) will ensure that $\alpha_p(Y(\theta_1,\gamma_1))\geq\alpha_p(Y(\theta_{\hat{k}},\gamma_{\hat{k}}))$, a contradiction. Thus, if $(\theta_1,\gamma_n)\in Y(\theta_{\hat{k}},\gamma_{\hat{k}})$ then by Lemma 2 it must be the case that $\Omega\backslash(\{(\theta_l,\gamma_l):l=1,...,n\}\cup\{(\theta_1,\gamma_n)\})\subset Y(\theta_{\hat{k}},\gamma_{\hat{k}})$ and we have proven our claim.

If instead $(\theta_1,\gamma_n)\notin Y(\theta_{\hat{k}},\gamma_{\hat{k}})$, then we claim that there exists $k\geq\hat{k}$ such that $(\theta_k,\gamma_j)\in Y(\theta_1,\gamma_n)$ for all $j=1,...,n$. Namely, if this is not the case, then using the prior $p(\theta_1,\gamma_n)=q_1-(n-1)\cdot\epsilon$, $p(\theta_l,\gamma_l)=q_l-(n-1)\cdot\epsilon$ for all $l<\hat{k}$, and $p(\theta_k,\gamma_j)=q_k-(n-1)\cdot\epsilon$ for all

$k \geq \hat{k}$ and j such that $(\theta_k, \gamma_j) \notin Y(\theta_1, \gamma_n)$, and $p(\theta, \gamma) = \epsilon$ for all other (θ, γ) implies that there exists $\bar{\epsilon} > 0$ such that $\epsilon < \bar{\epsilon}$ implies $\alpha_p(Y(\theta_1, \gamma_n)) < \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$ which violates Lemma 1.1 given that $v(\theta_1, \gamma_n) < v(\theta_{\hat{k}}, \gamma_{\hat{k}})$. Finally, if there exists $k \geq \hat{k}$ such that $(\theta_k, \gamma_j) \in Y(\theta_1, \gamma_n)$ for all $j = 1, ..., n$ then by Lemma 2 it must be the case that $\Omega \backslash (\{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\}) \subset Y(\theta_1, \gamma_n)$.

Finally, note that $\Omega \backslash (\{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\}) \subset Y(\theta_1, \gamma_n)$ and $(\theta_k, \gamma_k) \in Y(\theta_1, \gamma_n)$ implies that $Y(\theta_{\hat{k}}, \gamma_{\hat{k}}) \subset \{(\theta_l, \gamma_l) : l = 1, ..., n\}$. To see why this is the case, note that if there exists $(\theta_{k'}, \gamma_{j'}) \in Y(\theta_{\hat{k}}, \gamma_{\hat{k}})$ such that $k' \neq j'$, then $(\theta_k, \gamma_k) \in Y(\theta_1, \gamma_n)$ implies $\alpha(Y(\theta_1, \gamma_n)) \leq \alpha(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$ otherwise the contrapositive of 1.1 is violated. Yet, $(\theta_{\hat{k}}, \gamma_{\hat{k}}) \in Y(\theta_{\hat{k}}, \gamma_{\hat{k}})$ and $(\theta_1, \gamma_n) \in Y(\theta_1, \gamma_n)$ implies $\alpha_p(Y(\theta_1, \gamma_n)) \geq \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$ and therefore $\alpha_p(Y(\theta_1, \gamma_n)) = \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$ contradicting the fact that it was assumed above that $(\theta_{\hat{k}}, \gamma_{\hat{k}}) \notin Y(\theta_1, \gamma_n)$. Hence, we have proven our main claim for this case.

Case 2: $\alpha_p(Y(\theta_1, \gamma_1)) > \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$ for all $p \in \Delta_q^+(\Omega)$. By the same logic above, this implies that $Y(\theta_{\hat{k}}, \gamma_{\hat{k}}) \subset \{(\theta_l, \gamma_l) : l = 1, ..., n\}$. In this case, given that $\hat{k}$ is the smallest integer such that $(\theta_{\hat{k}}, \gamma_{\hat{k}}) \notin Y(\theta_1, \gamma_1)$, then it must be the case that there exists $k > \hat{k}$ such that $(\theta_k, \gamma_j) \in Y(\theta_1, \gamma_1)$ for all $j = 1, ..., n$. Namely, if this is not the case, then consider the prior p satisfying $p(\theta_k, \gamma_k) = q_k - (n-1) \cdot \epsilon$ for all $k \leq \hat{k}$, $p(\theta_{k'}, \gamma_j) = q_{k'} - (n-1) \cdot \epsilon$ for all $k' > \hat{k}$ and j such that $(\theta_{k'}, \gamma_j) \notin Y(\theta_1, \gamma_1)$, and $p(\theta, \gamma) = \epsilon$ for all other (θ, γ). Under this prior, there exists $\bar{\epsilon} > 0$ such that $\epsilon < \bar{\epsilon}$ implies $\alpha_p(Y(\theta_1, \gamma_1)) \leq \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$, contradicting the main assumption of this case. Therefore, by Lemma 2 it must be the case that $\Omega \backslash (\{(\theta_l, \gamma_l) : l = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\}) \subset Y(\theta_1, \gamma_1)$ and thus $Y(\theta_{\hat{k}}, \gamma_{\hat{k}}) \subset \{(\theta_l, \gamma_l : l = 1, ..., n\} \cup \{\theta_n, \gamma_1\}$. Finally, note that this implies $Y(\theta_{\hat{k}}, \gamma_{\hat{k}}) \subset \{(\theta_l, \gamma_l) : l = 1, ..., n\}$ as $\alpha_p(Y(\theta_1, \gamma_1)) > \alpha_p(Y(\theta_{\hat{k}}, \gamma_{\hat{k}}))$, $(\theta_1, \gamma_n) \in Y(\theta_{\hat{k}}, \gamma_{\hat{k}})$, and $(\theta_1, \gamma_1) \in Y(\theta_1, \gamma_1)$ implies the mechanism is not incentive compatible given that $v(\theta_1, \gamma_1) > v(\theta_1, \gamma_n)$. □

Proposition A.1. *Any mechanism that is robust incentive compatible is completely uninformative so that for all $p \in \Delta_q^+(\Omega)$ it must be the case that $Pr_p(\theta | Y(\omega)) = q(\theta)$.*

Proof. We will prove this result in three steps.

Step 1: $(\theta_n, \gamma_n) \in Y(\omega)$ implies $Y(\omega) \not\subset \{(\theta_k, \gamma_k) : k = 1, ..., n\}$. To see why this is the case, suppose $Y(\omega) \subset \{(\theta_k, \gamma_k) : k = 1, ..., n\}$ and $(\theta_n, \gamma_n) \in Y(\omega)$. Denote by ω' any type such that $Y(\omega') \supset \Omega \backslash (\{(\theta_k, \gamma_k) : k = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\})$. Then, $(\theta_1, \gamma_n) \in Y(\omega')$ and $(\theta_n, \gamma_n) \in Y(\omega)$ implies $\alpha_p(Y(\omega')) > \alpha_p(Y(\omega))$ for all $p \in \Delta^+(\Omega)$ as otherwise this would contradict 1.1. Next, consider the prior p such that $p(\theta_1, \gamma_n) = q_1 - (n-1) \cdot \epsilon$, $p(\theta_k, \gamma_j) = q_k - (n-1) \cdot \epsilon$ for each $k = 2, ..., n-1$ and $j \neq k$ such that $(\theta_k, \gamma_j) \in Y(\omega')$, $p(\theta_n, \gamma_n) = q_n - (n-1) \cdot \epsilon$, and $p(\theta, \gamma) = \epsilon$ for all other (θ, γ). Then, there exists $\bar{\epsilon} > 0$ such that $\epsilon < \bar{\epsilon}$ implies $\alpha_p(Y(\omega')) < \alpha_p(Y(\omega))$, a contradiction.

Therefore, by Lemma 4 we know that it must be the case that $Y(\theta_n, \gamma_n) \supset \Omega \backslash (\{(\theta_k, \gamma_k) : k = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\})$. We will use this result to prove the next step.

Step 2: $\{(\theta_k, \gamma_k) : k = 1, ..., n\} \subset Y(\theta_n, \gamma_n)$. In order to prove this claim, suppose that there exists $(\theta_k, \gamma_k) \notin Y(\theta_n, \gamma_n)$ for some $k < n$. Further, using the fact that $v(\theta_k, \gamma_k) = v(\theta_n, \gamma_n)$, incentive compatibility between (θ_k, γ_k) and (θ_n, γ_n) for any fixed prior $p \in \Delta_q^+(\Omega)$ requires

$$(v(\theta_k, \gamma_k) - T(Y(\theta_k, \gamma_k)))(1 - \alpha_p(Y(\theta_k, \gamma_k))) = (v(\theta_k, \gamma_k) - T(Y(\theta_n, \gamma_n)))(1 - \alpha_p(Y(\theta_n, \gamma_n)))$$

and therefore it must be the case that $\frac{1-\alpha_p(\theta_n, \gamma_n)}{1-\alpha_p(\theta_k, \gamma_k)}$ is a constant function of p. We will now show that this latter condition requires $\alpha_p(\theta_k, \gamma_k) = \alpha_p(\theta_n, \gamma_n)$ for all $p \in \Delta^+(\Omega)$ and therefore $(\theta_k, \gamma_k) \in Y(\theta_n, \gamma_n)$. In order to do so, consider the prior $p \in \Delta(\Omega)$ such that $p(\theta_l, \gamma_j) = q_k - \epsilon \cdot (n-1)$ for each $l \neq \{k, n\}$ and j such that $(\theta_l, \gamma_j) \in Y(\theta_n, \gamma_n)$ (guaranteed to exist given that $Y(\theta_n, \gamma_n) \supset \Omega \backslash (\{(\theta_k, \gamma_k) : k = 1, ..., n\} \cup \{(\theta_n, \gamma_1)\})$), $p(\theta_k, \gamma_k) = q_k - \epsilon \cdot (n-1)$, $p(\theta_n, \gamma_n) = q_n - \epsilon \cdot (n-1)$, and $p(\theta, \gamma) = \epsilon$ for all other $(\theta, \gamma) \in \Omega$. Note that under this prior, as $\epsilon \to 0$ then

$$\mathbb{E}_p[v(\theta, \gamma_1) | Y(\theta_k, \gamma_k)] \to v(\theta_k, \gamma_1)$$

Similarly, as $\epsilon \to 0$ then

$$\mathbb{E}_p[v(\theta, \gamma_1)|Y(\theta_n, \gamma_n)] \to \frac{1}{1-q_k} \sum_{j \neq k} q_j \cdot v(\theta_j, \gamma_1)$$

Next consider the following prior $p' \in \Delta^+(\Omega)$ such that $p'(\theta_l, \gamma_j) = q_k - \epsilon \cdot (n-1)$ for each $l \neq \{k, n\}$ and j such that $(\theta_l, \gamma_j) \in Y(\theta_n, \gamma_n)$, $p'(\theta_k, \gamma_k) = \frac{q_k}{2} - \epsilon \cdot (n-1)$, $p'(\theta_k, \gamma_j) = \frac{q_k}{2}$ for some $j \neq k$ such that $(\theta_k, \gamma_j) \in Y(\theta_n, \gamma_n)$, $p'(\theta_n, \gamma_n) = q_n - \epsilon \cdot (n-1)$, and $p'(\theta, \gamma) = \epsilon$ for all other $(\theta, \gamma) \in \Omega$. In this case, as $\epsilon \to 0$

$$\mathbb{E}_{p'}[v(\theta, \gamma_1)|Y(\theta_k, \gamma_k)] \to v(\theta_k, \gamma_1)$$

while

$$\mathbb{E}_{p'}[v(\theta, \gamma_1)|Y(\theta_n, \gamma_n)] \to \frac{1}{1-\frac{q_k}{2}} \sum_{j \neq k} q_j \cdot v(\theta_j, \gamma_1)$$

Hence, if $\frac{1-\alpha_p(\theta_n,\gamma_n)}{1-\alpha_p(\theta_k,\gamma_k)}$ is a constant function of p then it must be the case that $\frac{1-\alpha_p(\theta_n,\gamma_n)}{1-\alpha_p(\theta_k,\gamma_k)} = \frac{1-\alpha_{p'}(\theta_n,\gamma_n)}{1-\alpha_{p'}(\theta_k,\gamma_k)}$ and therefore after substituting, as $\epsilon \to 0$ it must be the case that for any transfers $T(\theta_k, \gamma_k)$ and $T(\theta_n, \gamma_n)$ it must be the case that

$$\mathbb{E}_{p'}[v(\theta, \gamma_1)|Y(\theta_k, \gamma_k)] - \mathbb{E}_p[v(\theta, \gamma_1)|Y(\theta_k, \gamma_k)] \to 0$$

which implies that

$$\frac{1}{1-q_k} \sum_{j \neq k} q_j \cdot v(\theta_j, \gamma_1) - v(\theta_k, \gamma_1) \to 0$$

as $\epsilon \to 0$. Finally, note that whenever this is the case then it must be the case that $\frac{1-\alpha_p(\theta_n,\gamma_n)}{1-\alpha_p(\theta_k,\gamma_k)} \to 1$ as $\epsilon \to 0$. Yet, $\frac{1-\alpha_p(\theta_n,\gamma_n)}{1-\alpha_p(\theta_k,\gamma_k)}$ must be a constant function of p and therefore if the mechanism satisfies robust incentive compatibility then $\frac{1-\alpha_p(\theta_n,\gamma_n)}{1-\alpha_p(\theta_k,\gamma_k)} = 1$ for all ϵ small enough to ensure $p \in \Delta^+(\Omega)$. Yet, this implies that $\frac{1-\alpha_{\tilde{p}}(\theta_n,\gamma_n)}{1-\alpha_{\tilde{p}}(\theta_k,\gamma_k)} = 1$ for all $\tilde{p} \in \Delta^+(\Omega)$ which can only be the case if $\alpha_{\tilde{p}}(\theta_n, \gamma_n) = \alpha_{\tilde{p}}(\theta_k, \gamma_k)$ for all $\tilde{p} \in \Delta^+(\Omega)$, and we have proven our claim and therefore it must be the case that

$\{(\theta_k, \gamma_k) : k = 1, ..., n\} \subset Y(\theta_n, \gamma_n)$.

What we have just shown is that $\Omega \backslash \{(\theta_n, \gamma_1)\} \subset Y(\theta_n, \gamma_n)$. Thus, in order to prove the proposition we must show that robust incentive compatibility requires $(\theta_n, \gamma_1) \in Y(\theta_n, \gamma_n)$ which is our final step.

<u>**Step 3:** $(\theta_n, \gamma_1) \in Y(\theta_n, \gamma_n)$</u>. The proof here is identical to the previous step. Namely, suppose that $(\theta_n, \gamma_1) \notin Y(\theta_n, \gamma_n)$. Then we know that $(\theta_1, \gamma_n) \in Y(\theta_n, \gamma_n)$ and $v(\theta_1, \gamma_n) = v(\theta_n, \gamma_1)$. Hence, it must be the case that $\frac{1-\alpha_p(\theta_n,\gamma_1)}{1-\alpha_p(\theta_n,\gamma_n)}$ is a constant function of p. Now, consider the prior $p \in \Delta^+(\Omega)$ such that $p(\theta_k, \gamma_k) = q_k - \epsilon \cdot (n-1)$ for each $k = 1, ..., n-1$, $p(\theta_n, \gamma_1) = q_n - \epsilon \cdot (n-1)$, and $p(\theta, \gamma) = \epsilon$ for all other $(\theta, \gamma) \in \Omega$. Then, under p

$$\mathbb{E}_p[v(\theta, \gamma_1) | Y(\theta_n, \gamma_1)] = v(\theta_n, \gamma_1)$$

and

$$\mathbb{E}_p[v(\theta, \gamma_1) | Y(\theta_n, \gamma_n)] = \frac{1}{1 - q_n} \sum_{j \neq n} q_j \cdot v(\theta_j, \gamma_1)$$

Next, consider the prior $p' \in \Delta^+(\Omega)$ such that $p'(\theta_k, \gamma_k) = q_k - \epsilon \cdot (n-1)$ for each $k = 1, ..., n-1$, $p'(\theta_n, \gamma_1) = \frac{q_n}{2} - \epsilon \cdot (n-1)$, and $p'(\theta, \gamma) = \epsilon$ for all other $(\theta, \gamma) \in \Omega$. Then again,

$$\mathbb{E}_{p'}[v(\theta, \gamma_1) | Y(\theta_n, \gamma_1)] = v(\theta_n, \gamma_1)$$

and

$$\mathbb{E}_{p'}[v(\theta, \gamma_1) | Y(\theta_n, \gamma_n)] = \frac{1}{1 - \frac{q_n}{2}} \sum_{j \neq n} q_j \cdot v(\theta_j, \gamma_1)$$

Finally, $\frac{1-\alpha_p(\theta_n,\gamma_1)}{1-\alpha_p(\theta_n,\gamma_n)} = \frac{1-\alpha_{p'}(\theta_n,\gamma_1)}{1-\alpha_{p'}(\theta_n,\gamma_n)}$ implies that for any transfers $T(\theta_n, \gamma_1)$ and $T(\theta_n, \gamma_n)$ it must be the case that

$$\frac{1}{1 - q_n} \sum_{j \neq n} q_j \cdot v(\theta_j, \gamma_1) = v(\theta_n, \gamma_1)$$

in which case $\frac{1-\alpha_p(\theta_n,\gamma_1)}{1-\alpha_p(\theta_n,\gamma_n)} = 1$ and therefore robust incentive compatibility requires $\frac{1-\alpha_{\tilde{p}}(\theta_n,\gamma_1)}{1-\alpha_{\tilde{p}}(\theta_n,\gamma_n)} = 1$

for all $\tilde{p} \in \Delta^+(\Omega)$ which requires that $\alpha_p(\theta_n, \gamma_1) = \alpha_p(\theta_n, \gamma_n)$ for all $p \in \Delta^+(\Omega)$ which implies $Y(\theta_n, \gamma_1) = Y(\theta_n, \gamma_n) = \Omega$. □

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