Sequential Learning, Asset Allocation, and Bitcoin Returns

with James Yae

AFA 2022 Annual Meeting

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Structural Break in Correlation

Dynamic Correlation between Bitcoin and S&P500 returns: Pre- and Post-futures

- Correlations are time varying only after the inception of Bitcoin futures!

- How does it affect investors’ behavior who seek portfolio optimization?

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Sequential Learning, Asset Allocation, and Bitcoin Returns
**Bitcoin Demand Proxy**

**Optimal weight on Bitcoin** (in a BTC-stock market portfolio)

\[
w_{b,t} = \frac{\mu_t^* - \rho_t \sigma_t^*}{(\mu_t^* - \rho_t \sigma_t^*) + (\sigma_t^* - \rho_t \mu_t^*) \sigma_t^*} \quad \text{Max Sharpe Ratio!}
\]

- Conditional risk premium ratio \( \mu_t^* = \mu_{b,t} / \mu_{m,t} \)
- Conditional volatility ratio \( \sigma_t^* = \sigma_{b,t} / \sigma_{m,t} \)

**Bitcoin Demand Decomposition**

\[
\Delta w_{b,(t-1):t} \triangleq w_{b,t} - w_{b,t-1} = \Delta w_{b,(t-1):t}^{(cor)} + \Delta w_{b,(t-1):t}^{(vol)} + \Delta w_{b,(t-1):t}^{(mean)} + e_t
\]

DCC(1,1)-GARCH(1,1) for estimation

\[
w_{b,t}^{(mean)} \triangleq w_b(\mu_t^*, \overline{\sigma}^*, \overline{\rho}) \quad \text{investors only learn } \mu_t^*
\]

\[
w_{b,t}^{(cor)} \triangleq w_b(\mu_t^*, \overline{\sigma}^*, \rho_t) \quad \text{investors only learn } (\rho_t)
\]

\[
w_{b,t}^{(vol)} \triangleq w_b(\mu_t^*, \sigma_t^*, \overline{\rho}) \quad \text{investors only learn } (\sigma_t^*)
\]
Bitcoin Demand Proxy

\[ \text{Bitcoin demand variation} = \text{Due to Corr. change} + \text{Due to Vol. change} \]

Which effect is dominant?

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Daily Bitcoin Return Predictability

\[ r_{b,t+1} = b_0 + b_1 \Delta w_{b,(t-1):t}^{(cor)} + b_2 \Delta w_{b,(t-1):t}^{(vol)} + Z_t \gamma + \varepsilon_{t+1} \]

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Post-futures (12/18/2017 to 12/31/2020)</th>
<th>Post-futures before COVID-19 (12/18/2017 to 02/29/2020)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5)</td>
<td>(6) (7) (8) (9) (10)</td>
</tr>
<tr>
<td>\Delta w_{b,(t-1):t}^{(cor)}</td>
<td>0.51 (4.15) 0.47 (2.89) 0.47 (2.91) 0.47 (2.80)</td>
<td>0.50 (3.16) 0.48 (2.78) 0.47 (2.63) 0.48 (2.37)</td>
</tr>
<tr>
<td>\Delta w_{b,(t-1):t}^{(vol)}</td>
<td>0.06 (0.42) 0.03 (0.28) 0.04 (0.30) 0.03 (0.20)</td>
<td>-0.19 (-1.27) -0.23 (-1.66) -0.22 (-1.67) -0.20 (-1.56)</td>
</tr>
<tr>
<td>\beta_t</td>
<td>0.04 (0.22) 0.06 (0.31) 0.07 (0.31)</td>
<td>0.06 (0.28) 0.07 (0.32) 0.09 (0.40)</td>
</tr>
<tr>
<td>Volume_{b,t}</td>
<td>-0.58 (-2.59) -0.55 (-1.35) -0.55 (-1.39)</td>
<td>-0.74 (-3.52) -0.91 (-1.93) -0.87 (-1.78)</td>
</tr>
<tr>
<td>EPU_{b,t}</td>
<td>0.63 (1.71) 0.64 (1.68) 0.63 (1.76)</td>
<td>0.21 (0.98) 0.20 (0.93) 0.21 (0.96)</td>
</tr>
<tr>
<td>Controls</td>
<td>M          MB          MBL</td>
<td>M          MB          MBL</td>
</tr>
<tr>
<td>( R^2 ) (%)</td>
<td>1.15       0.01       3.94       3.99       4.00</td>
<td>1.17       0.16       4.04       4.14       4.38</td>
</tr>
<tr>
<td>( Adj.R^2 )</td>
<td>1.02       -0.12      2.02       2.71       2.47</td>
<td>0.99       -0.02      2.62       2.36       2.24</td>
</tr>
</tbody>
</table>

Data are at daily frequency. Newey-West t statistics are reported in parenthesis.

0.5% higher returns with 1 std demand increase

30 times higher est. uncertainty (noisy signal)
Equilibrium Model: Intuition

BTC Investors

Speculative Investors
$(\mu^*_t, \sigma^*_t, \bar{\rho})$

Non-speculative Investors
$(\mu^*, \sigma^*_t, \rho_t)$

“asynchronous portfolio rebalancing”
links increase in BTC demand due to $\rho_t$ (or decrease in $\rho_t$) to higher subsequent BTC returns!

$t$

$\text{SI}$ trade based on $\mu^*_t$ (or $P_{b,t}$)

$\text{NI}$ estimate $\rho_t$ (then set $w_{b,t}^{(cor)}$)

$\text{NI}$ buys BTC from $\text{SI}$

delay in order executions
asynchronous

trade based on $\mu^*_{t+1}$ (or $P_{b,t+1}$)

rebalance portfolio at $t+1$

$t+1$
1) **Bitcoin return predictability:**
Increase in daily Bitcoin demand change due to dynamic correlation predicts higher subsequent Bitcoin returns

2) **Rational asset allocation:**
The empirical pattern is consistent with investors' learning on time-varying correlations and practice on rational portfolio optimization

3) **Asynchronous portfolio rebalancing:**
We use an equilibrium model to explain how Bitcoin return predictability emerges from asynchronous portfolio rebalancing

The paper also explains:
- Why predictability from $\Delta w^{(cor)}_{b,(t-1):t}$ not $\Delta w^{(vol)}_{b,(t-1):t}$
- Is there out-of-sample predictability?
- other Bitcoin demand proxies?
- Does the evidence show up in other cryptos and other equity markets?
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