Research question
How to measure the strength of the interdependence of inflation uncertainty?
• We estimate inflation uncertainty by ex post forecast errors and the interdependence of uncertainty by a probability model.
• We show a potential endogeneity bias of the probability model estimates and propose a new empirical framework exploiting heteroskedasticity in the data.

Measuring inflation uncertainty
Inflation uncertainty is measured by ex post forecast errors from a bivariate VAR-BEKK GARCH model using inflation of the UK and the euro area (Jan 1997–March 2016).

\[ U_{t,h} = \frac{\sigma_{ht}^2}{\sigma_{ht}^2 + \sigma_{ht}^2} (\hat{r}_{t,h} - \hat{r}_{t,h}) \]  

(1)

where \( \sigma_{ht}^2 \) and \( \sigma_{ht}^2 \) are the variance-covariance matrix of \( \hat{r}_t \) and \( \hat{r}_h \).

Measuring independence by a probability model: Part I

Conditional density function (copulas)

\[ \gamma = \arg \max_{\gamma} \sum_{t=1}^{n} \left( \mathbb{E}(F_1(U_1, \gamma), F_2(U_2, \beta)) \right) \]  

(4)

where \( \gamma \) is copula parameter; \( \hat{r}_1, \hat{r}_2 \) are marginal densities estimated by the simulated minimum distance.

The pdff of Frank copula

\[ \mathbb{E}(X_1, X_2; \gamma) = \frac{1}{(1+\gamma)^2} \sum_{n=0}^{\gamma-1} \frac{1}{n+1} \sum_{m=0}^{\gamma-\gamma n} \frac{1}{m+1} \]  

(5)

Endogenous model of interdependence

To illustrate a potential bias, an endogenous model of interdependence is assumed as in [2].

\[ A \hat{U} = \beta \hat{C}_2 \]  

(6)

where \( \beta \) and \( \beta \) are the factors capturing interdependence of uncertainty.

Reduced form → a potential bias if endogeneity is not properly addressed in the estimation.

\[ U_1 = 1 - \left( \alpha \sigma_1^2 + \epsilon \right) \]  

(7)

\[ \Omega = \frac{1}{1-\alpha} \left[ \sigma_1^2 \sigma_2^2 + \alpha \sigma_1^2 \sigma_2^2 + \beta \sigma_2^2 \right] \]  

(8)

Measuring independence by identification through heteroskedasticity

The variance-covariance matrix of the structural model:

\[ \text{VAR}(A) = \begin{pmatrix} \beta \sigma_1^2 + \alpha \sigma_2^2 & \alpha \sigma_1 \sigma_2 \beta \sigma_2 + \epsilon \sigma_2^2 \ 
\alpha \sigma_1 \sigma_2 \beta \sigma_2 + \epsilon \sigma_2^2 & \beta \sigma_2^2 + \epsilon \end{pmatrix} \]  

(9)

Optimization problem: the off-diagonal terms in Equation (11) need to be equal to zero.

\[ \min \{ \alpha, \beta \} : V_1^T V_2, C_2\beta = - \beta V_1^T + \alpha \sigma_2^2 + \epsilon \]  

(10)

Impose additional assumptions: the parameters in A are stable over time; heteroskedasticity.

Define two regimes: \( RH \) if \( \theta > \text{median}(\theta) \) and \( RL \), otherwise. \( \theta \) is computed using different sample periods (\( \alpha \)), forecast horizons (\( \beta \)), and rolling windows (\( \epsilon \)).

\[ \theta = \frac{1}{h} \sum_{t=1}^{h} \ln(p, \theta, \epsilon) \]  

(11)

where \( \theta \in [1, 3] \) with \( 1 \): pre-crisis period, \( 2 \): the Global Financial Crisis period, \( 3 \): post-crisis period, \( h = 1, 2, \ldots 24 \) and \( \epsilon = 12 \).

The minimum distance estimates of \( \alpha \) and \( \beta \) using \( V_1, V_2 \) and \( C_2 \) for each regime.

Endogenous model of interdependence

To illustrate a potential bias, an endogenous model of interdependence is assumed as in [2].

\[ A \hat{U} = \hat{C}_2 \]  

(6)

where \( A = (\sqrt{\text{V}^2(\hat{r}_1, \hat{r}_2)} / 2)^{-1} \). \( \hat{C}_2 \) is the variance-covariance matrix of \( [U_1; U_2] \).

Reduced form → a potential bias if endogeneity is not properly addressed in the estimation.

\[ U_1 = 1 - \left( \alpha \sigma_1^2 + \epsilon \right) \]  

(7)

\[ \Omega = \frac{1}{1-\alpha} \left[ \sigma_1^2 \sigma_2^2 + \alpha \sigma_1^2 \sigma_2^2 + \beta \sigma_2^2 \right] \]  

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The minimum distance estimates of \( \alpha \) and \( \beta \) using \( V_1, V_2 \) and \( C_2 \) for each regime.

Conclusions

- Probability model: The simultaneous spillover of inflation uncertainty is stronger for uncertainty about distant future than near future.
- Endogenous model: The strength of the propagation of inflation uncertainty intensifies during the GFS period while the interdependence dampens during the post-crisis period.

References