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**Inflation Gap Persistence, Indeterminacy, and Monetary Policy**

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Empirical studies have documented that the persistence of the gap between inflation and its trend declined after the Volcker disinflation. Previous research into the source of the decline has offered competing views while sidestepping the possibility of equilibrium indeterminacy. This paper examines the source by estimating a medium-scale DSGE model using a Bayesian method that allows for indeterminacy. The estimated model shows that the Fed's change from a passive to an active policy response to the inflation gap or a decrease in firms' probability of price change can fully account for the decline in inflation gap persistence by ruling out indeterminacy that induces persistent dynamics of the economy.

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# 1 Introduction

Empirical studies have documented that the persistence of the inflation gap—the gap between inflation and its trend—declined after the Volcker disinflation in the early 1980s.<sup>1</sup> Previous research into the source of the decline has offered competing views. Some of the research attributes it to monetary policy. The persistence declined either because the Fed’s policy response to the inflation gap became more aggressive (Benati and Surico (2008); Carlstrom et al. (2009); Davig and Doh (2014)) or because its (implicit) inflation target became more stable (Cogley et al. (2010)). Other research points to factors outside the realm of monetary policy, such as a change in the relative importance of price markup shocks or a decrease in intrinsic inertia of inflation in Phillips curves.<sup>2</sup>

This paper examines the source of the decline in inflation gap persistence by estimating a medium-scale dynamic stochastic general equilibrium (henceforth DSGE) model for periods both before and after the Volcker disinflation of 1979–1982. Specifically, the model is estimated using a full-information Bayesian method that allows for the indeterminacy of equilibrium. Previous research into the source has sidestepped the possibility of indeterminacy by restricting the permissible range of values of model parameters within their determinacy region or using only post-1982 data in estimation, despite the evidence from estimated DSGE models that the US economy experienced indeterminacy before the Volcker disinflation.<sup>3</sup> Such limitations, however, may influence conclusions about the source of the decline in inflation gap persistence and its mechanism. Indeterminacy can substantially alter the propagation of shocks as pointed out by Lubik and Schorfheide (2004) and Fujiwara and Hirose (2014), thereby affecting inflation gap persistence in the pre-1979 period differently than determinacy and hence changes in the persistence between the pre-1979 and post-1982 periods.<sup>4</sup> In addition to the methodological advantage, our medium-scale DSGE model con-

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<sup>1</sup>See, e.g., Cogley and Sbordone (2008), Benati and Surico (2008), Cogley et al. (2010), and Ascari and Sbordone (2014).

<sup>2</sup>Carlstrom et al. (2009) and Davig and Doh (2014) highlight changes in the relative importance of shocks in addition to a more aggressive monetary policy response to the inflation gap. Fuhrer (2011) stresses the role of intrinsic inflation inertia in Phillips curves for inflation (gap) persistence.

<sup>3</sup>For the evidence, see, e.g., Lubik and Schorfheide (2004) and Hirose et al. (2020).

<sup>4</sup>Fujiwara and Hirose (2014) show that equilibrium indeterminacy can generate more persistent dynamics that improves the forecastability of inflation in a DSGE model, compared with the case of determinacy.

tains a multitude of frictions and shocks that are absent from the small-scale New Keynesian (henceforth NK) models used in previous research and make our model particularly suited to an empirical analysis of inflation gap persistence.

A noteworthy feature of the model is that intrinsic inertia of inflation emerges endogenously in the NK Phillips curve (henceforth NKPC). Each period, a fraction of the prices for individual goods remains unchanged in line with micro evidence, while the other prices are set given demand curves arising from a not necessarily constant elasticity of substitution (henceforth CES) aggregator of individual differentiated goods of the sort proposed by [Kimball \(1995\)](#) and developed by [Dotsey and King \(2005\)](#) and [Levin et al. \(2008\)](#), which includes the CES aggregator as a special case. The non-CES aggregator provides a parsimonious way of introducing variable (price) elasticity of demand and has been widely used as a source of strategic complementarity in firms' price-setting in the macroeconomic literature.<sup>5</sup> As shown by [Kurozumi and Van Zandweghe \(2019\)](#), the non-CES aggregator gives rise to lagged inflation in the NKPC under nonzero steady-state inflation and staggered price-setting, without relying on arbitrary ad hoc backward-looking price-setting behavior.<sup>6,7</sup>

The main results of the paper are twofold. First, the estimated model demonstrates a decline in inflation gap persistence after the Volcker disinflation, in line with the previous empirical studies, such as [Cogley and Sbordone \(2008\)](#), [Benati and Surico \(2008\)](#), and [Cogley et al. \(2010\)](#). This paper measures inflation gap persistence as its predictability, following [Cogley et al. \(2010\)](#). The predictability and the variability of the inflation gap declined from the pre-1979 to the post-1982 period in the estimated model.

Second, the estimated model shows that the Fed's change from a passive to an active policy response to the inflation gap or a decrease in firms' probability of price change can fully account for the decline in inflation gap persistence by ruling out indeterminacy that

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<sup>5</sup>See, e.g., [Eichenbaum and Fisher \(2007\)](#), [Smets and Wouters \(2007\)](#), and [Kurozumi and Van Zandweghe \(2016\)](#).

<sup>6</sup>Two popular assumptions for the presence of lagged inflation in NKPCs used in the literature are price indexation to past inflation ([Christiano et al. \(2005\)](#)) and backward-looking rule-of-thumb price-setters ([Galí and Gertler \(1999\)](#)). Such ad hoc formulations of intrinsic inertia of inflation may be more geared toward empirically capturing inflation (gap) persistence than providing a satisfactory theoretical explanation, because the assumptions are based on neither optimizing behavior nor micro evidence.

<sup>7</sup>Our model also features intrinsic inertia of wage inflation that emerges endogenously in the wage NKPC from a non-CES aggregator of individual differentiated labor under nonzero steady-state wage inflation and staggered wage-setting.

induces persistent dynamics of the economy.<sup>8</sup> To establish this result, the paper follows [Cogley et al. \(2010\)](#) to conduct a counterfactual exercise in which, starting with the pre-1979 estimates of all model parameters, the values of parameters that either pertain to sources of the decline considered in previous research or exhibit substantial changes between the pre-1979 and post-1982 periods are altered to their post-1982 estimates. This exercise indicates that the increase in the policy response to the inflation gap can fully explain the decline of inflation gap persistence in the estimated model, primarily by eliminating indeterminacy and to a lesser extent by dampening responses to shocks under determinacy. The exercise also suggests that the decrease in the probability of price change can provide a full account for the decline only through ruling out indeterminacy.

Other views on the source of the decline in inflation gap persistence that have been offered in previous research receive less support in our model estimated while allowing for indeterminacy. [Cogley et al. \(2010\)](#) point out that the decline in persistence was caused by a more stable inflation target (shock) in the Fed’s policy reaction function after the Volcker disinflation. [Carlstrom et al. \(2009\)](#) and [Davig and Doh \(2014\)](#) emphasize the secondary role of changes in nonpolicy shocks, including a price markup shock, in the decline, after accounting for the effect of a more aggressive policy response to the inflation gap. According to our counterfactual exercise, however, the changes in these shocks from the pre-1979 to the post-1982 estimates do not explain the decline in inflation gap persistence.<sup>9</sup> This is because such changes generate no shift from indeterminacy to determinacy in the model. Another view on the source of the decline in persistence is provided by [Fuhrer \(2011\)](#), who argues that lower intrinsic inertia of inflation in NKPCs is likely an important factor for the decline. This argument, however, fails to hold in our estimated model, since intrinsic inflation inertia in the NKPC increased from the pre-1979 to the post-1982 period.<sup>10</sup>

This paper also contributes to other strands of the literature. Our model (with the non-CES aggregators) is much less susceptible to indeterminacy of equilibrium than its counter-

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<sup>8</sup>A monetary policy response to the inflation gap is called active if it satisfies the Taylor principle that the nominal interest rate should be raised by more than the increase in inflation. Otherwise, it is called passive.

<sup>9</sup>The change in the inflation target shock can partly account for inflation gap predictability at longer forecasting horizons, although it cannot do so at a short forecasting horizon.

<sup>10</sup>The degree of intrinsic inflation inertia depends on our model’s structural parameters pertaining to firms’ price-setting behavior, and the increase in the inertia is mainly caused by the changes in firms’ probability of (no) price adjustment and the parameter governing the curvature of demand curves in the estimated model.

part in the case of CES aggregators. In the latter case, the model can induce indeterminacy even under low steady-state inflation, in line with the result of [Khan et al. \(2020\)](#). However, introducing the non-CES aggregators makes the model much less susceptible to indeterminacy, even at high rates of steady-state inflation.<sup>11</sup> In addition, incorporating differentiated labor along the lines of [Schmitt-Grohé and Uribe \(2006\)](#) instead of [Erceg et al. \(2000\)](#) plays a secondary role in making the model less susceptible to indeterminacy.

Our estimation results provide new evidence on the curvature of demand curves. The model (with the non-CES aggregators) is not observationally equivalent to its counterpart with CES aggregators, which implies the possibility of identifying the parameter governing the curvature.<sup>12</sup> Moreover, a Bayesian model comparison favors our model over the counterpart for both the pre-1979 and the post-1982 periods, thus supporting the presence of variable elasticity of demand. Our estimates of the curvature in demand curves are of the same order of magnitude as those of [Guerrieri et al. \(2010\)](#) based on US macroeconomic data, though they exceed the estimates of previous studies using European micro data, such as [Dossche et al. \(2010\)](#) and [Beck and Lein \(2020\)](#).

The remainder of the paper proceeds as follows. Section 2 presents a medium-scale DSGE model. Section 3 explains the estimation strategy and data. Section 4 presents and discusses the results of the empirical analysis. Section 5 concludes.

## 2 Model

This paper constructs a medium-scale DSGE model that extends those developed in the literature, including [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2007\)](#). A key feature of the model is that intrinsic inertia of inflation emerges endogenously in the NKPC from a non-CES aggregator of individual differentiated goods under nonzero steady-state inflation and staggered price-setting, without relying on the price indexation to past inflation rates assumed in the literature. Likewise, intrinsic inertia of wage inflation appears endogenously

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<sup>11</sup>Consistently, [Kurozumi and Van Zandweghe \(2016\)](#) point out that a Kimball-type non-CES aggregator of goods can largely prevent equilibrium indeterminacy induced by higher steady-state inflation in a small-scale DSGE model.

<sup>12</sup>As indicated by [Eichenbaum and Fisher \(2007\)](#), the parameters governing the curvature of demand curves and the degree of price rigidity are not separately identified in an NK model with price indexation, in which steady-state inflation has no influence on inflation dynamics.

in the wage NKPC from a non-CES aggregator of individual differentiated labor under nonzero steady-state wage inflation and staggered wage-setting.<sup>13</sup>

In the model there are a representative composite-good producer, firms that produce individual goods, a representative household with its members, a representative labor packer, a representative capital-service provider, and a monetary authority. The behavior of each economic agent is described in what follows.

## 2.1 Composite-good producer

A representative composite-good producer combines the outputs of a continuum of firms  $f \in [0, 1]$ , each of which produces an individual differentiated good  $X_t(f)$ .

As in [Kimball \(1995\)](#), the composite good  $X_t$  is produced by aggregating individual goods  $\{X_t(f)\}$  with

$$\int_0^1 F_p\left(\frac{X_t(f)}{X_t}\right) df = 1. \quad (1)$$

Following [Dotsey and King \(2005\)](#) and [Levin et al. \(2008\)](#), the function  $F_p(\cdot)$  is assumed to be of the form

$$F_p\left(\frac{X_t(f)}{X_t}\right) = \frac{\gamma_p}{(1 + \epsilon_p)(\gamma_p - 1)} \left[ (1 + \epsilon_p) \frac{X_t(f)}{X_t} - \epsilon_p \right]^{\frac{\gamma_p - 1}{\gamma_p}} + 1 - \frac{\gamma_p}{(1 + \epsilon_p)(\gamma_p - 1)},$$

where  $\gamma_p \equiv \theta_p(1 + \epsilon_p)$ . The parameter  $\epsilon_p$  governs the curvature (i.e., the price elasticity of the price elasticity) of the demand curve for each individual good, which is given by  $-\epsilon_p\theta_p$ . In the special case of  $\epsilon_p = 0$ , the Kimball-type aggregator (1) is reduced to the CES one  $X_t = \left[ \int_0^1 (X_t(f))^{\theta_p - 1} df \right]^{\theta_p / (\theta_p - 1)}$ , where  $\theta_p > 1$  represents the elasticity of substitution between individual differentiated goods. In the case of  $\epsilon_p < 0$ , strategic complementarity arises in firms' price-setting, as explained later. Both cases are considered in what follows.

The composite-good producer maximizes profit  $P_t X_t - \int_0^1 P_t(f) X_t(f) df$  subject to the aggregator (1), given the composite good's price  $P_t$  and the prices for individual goods  $\{P_t(f)\}$ . Combining the first-order conditions for profit maximization and the aggregator

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<sup>13</sup>Another feature is that each period a fraction of the prices for individual goods remains unchanged in line with micro evidence (e.g., [Klenow and Kryvtsov \(2008\)](#), [Nakamura and Steinsson \(2008\)](#), and [Nakamura et al. \(2018\)](#)), while the other prices are set by firms that face demand curves arising from the aggregator of goods. As a consequence, nonzero steady-state inflation influences inflation dynamics. An analogous feature holds for wage-setting.

(1) yields

$$\frac{X_t(f)}{X_t} = \frac{1}{1 + \epsilon_p} \left[ \left( \frac{P_t(f)}{P_t d_t} \right)^{-\gamma_p} + \epsilon_p \right], \quad (2)$$

$$d_t = \left[ \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{1-\gamma_p} df \right]^{\frac{1}{1-\gamma_p}}, \quad (3)$$

$$1 = \frac{1}{1 + \epsilon_p} d_t + \frac{\epsilon_p}{1 + \epsilon_p} e_t, \quad (4)$$

where  $d_t$  is the Lagrange multiplier on the aggregator (1) and

$$e_t \equiv \int_0^1 \frac{P_t(f)}{P_t} df. \quad (5)$$

The Lagrange multiplier  $d_t$  represents the real marginal cost of producing the composite good (or aggregating individual differentiated goods), and consists of the aggregate of the relative prices of individual goods that corresponds to the aggregator (1), as shown in (3). In the special case of  $\epsilon_p = 0$ , where the Kimball-type aggregator (1) becomes the CES one as noted above, eqs. (2)–(4) can be reduced to  $X_t(f)/X_t = (P_t(f)/P_t)^{-\theta_p}$ ,  $P_t = \left[ \int_0^1 (P_t(f))^{1-\theta_p} df \right]^{1/(1-\theta_p)}$ , and  $d_t = 1$ , respectively. The last equation indicates that the real marginal cost is constant in the case of the CES aggregator.<sup>14</sup>

Eq. (2) is the demand curve for each individual good  $X_t(f)$  and features a variable (price) elasticity of demand for the good given by  $\eta_t = \theta_p [1 + \epsilon_p - \epsilon_p (X_t(f)/X_t)^{-1}]$ . When  $\epsilon_p < 0$ , the elasticity  $\eta_t$  varies inversely with relative demand  $X_t(f)/X_t$ . That is, relative demand for each individual good becomes more price-elastic for a rise in the relative price of the good, whereas it becomes less price-elastic for a decline in the relative price. As is well understood, this feature induces strategic complementarity in firms' price-setting, since firms facing the increasing elasticity keep their products' relative prices near those of other firms (when they can adjust prices). By contrast, in the case of  $\epsilon_p = 0$ , the demand curve is reduced to  $X_t(f)/X_t = (P_t(f)/P_t)^{-\theta_p}$  and the elasticity of demand becomes constant, i.e.,  $\eta_t = \theta_p$ .

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<sup>14</sup>Note that if all firms share the same production technology (as assumed later) and all individual goods' prices are flexible (as supposed later in considering potential GDP), the prices are all identical and thus eqs. (3) and (4) imply that  $d_t = 1$  even in the case of the non-CES aggregator, i.e.,  $\epsilon_p < 0$ .

## 2.2 Firms

The model incorporates a roundabout production structure as in [Basu \(1995\)](#). Each firm  $f \in [0, 1]$  produces one kind of differentiated good  $X_t(f)$  using the production technology that is given by

$$X_t(f) = [(A_t l_t(f))^{1-\alpha} (K_t(f))^\alpha]^{1-\phi} (O_t(f))^\phi - \omega \Upsilon_t \quad (6)$$

if  $[(A_t l_t(f))^{1-\alpha} (K_t(f))^\alpha]^{1-\phi} (O_t(f))^\phi > \omega \Upsilon_t$ ; otherwise,  $X_t(f) = 0$ . Here  $l_t(f)$  and  $K_t(f)$  are firm  $f$ 's inputs of labor and capital,  $O_t(f)$  is its intermediate inputs in terms of the composite good,  $\omega \Upsilon_t$  is a fixed cost of production and is assumed to be identical to all firms,  $A_t$  is the level of neutral technology,  $\Upsilon_t$  is the level of composite technology given by  $\Upsilon_t \equiv A_t \Psi_t^{\alpha/(1-\alpha)}$ ,  $\Psi_t$  is the level of investment-specific technology explained later,  $\phi \in (0, 1)$  is the intermediate-input elasticity of production,  $\alpha \in (0, 1)$  is the parameter that governs the capital elasticity of production, and  $\omega$  is a nonnegative constant. We assume that the neutral technology level  $A_t$  is identical to all firms and follows the nonstationary stochastic process

$$\log A_t = \log g_A + \log A_{t-1} + z_{a,t}, \quad (7)$$

where  $g_A$  is the gross steady-state rate of neutral technological change and  $z_{a,t}$  is a neutral technology shock.

In the presence of economy-wide, perfectly competitive production-factor markets, each firm  $f$  minimizes the production cost  $P_t W_t l_t(f) + P_t R_{k,t} K_t(f) + P_t O_t(f)$  subject to the production technology (6), given the wage rate  $P_t W_t$ , the capital rental rate  $P_t R_{k,t}$ , and the composite good's price  $P_t$ . Combining the first-order conditions for cost minimization shows that all firms face the same real marginal cost of producing their products,

$$mc_t = \left[ \frac{W_t/A_t}{(1-\alpha)(1-\phi)} \right]^{(1-\alpha)(1-\phi)} \left[ \frac{R_{k,t}}{\alpha(1-\phi)} \right]^{\alpha(1-\phi)} \left( \frac{1}{\phi} \right)^\phi, \quad (8)$$

and choose identical ratios of the three production factors, so that

$$\frac{u_t K_{t-1}}{l_t} = \frac{\alpha}{(1-\alpha)} \frac{W_t}{R_{k,t}}, \quad (9)$$

$$\frac{O_t}{l_t} = \frac{\phi}{(1-\alpha)(1-\phi)} W_t, \quad (10)$$

where we use the factor market clearing conditions,  $u_t K_{t-1} = \int_0^1 K_t(f) df$ ,  $l_t = \int_0^1 l_t(f) df$ , and  $O_t = \int_0^1 O_t(f) df$ , where  $K_{t-1}$  is the stock of capital that the capital-service provider lends in period  $t$  by setting the utilization rate  $u_t$ ,  $l_t$  is labor input supplied by the labor packer, and  $O_t$  is intermediate inputs provided by the composite-good producer.

In the face of the demand curve (2) and the marginal cost (8), firms set their products' prices on a staggered basis as in Calvo (1983). In each period, a fraction  $\xi_p \in (0, 1)$  of firms keeps prices unchanged, while the remaining fraction  $1 - \xi_p$  sets the price  $P_t(f)$  so as to maximize the relevant profit

$$E_t \sum_{j=0}^{\infty} \xi_p^j M_{t,t+j} (P_t(f) - P_{t+j} mc_{t+j} \exp \tilde{z}_{p,t+j}) \frac{X_{t+j}}{1 + \epsilon_p} \left[ \left( \frac{P_t(f)}{P_{t+j} d_{t+j}} \right)^{-\gamma_p} + \epsilon_p \right],$$

where  $E_t$  denotes the expectation operator conditional on information available in period  $t$ ,  $M_{t,t+j}$  is the nominal stochastic discount factor between period  $t$  and period  $t + j$ , and  $\tilde{z}_{p,t}$  is a shock to the price markup and is assumed to be identical to all firms.

Using the equilibrium condition  $M_{t,t+j} = \beta^j (\Lambda_{t+j}/\Lambda_t) (P_t/P_{t+j})$ , where  $\beta \in (0, 1)$  is the subjective discount factor and  $\Lambda_t$  is the marginal utility of consumption, the first-order condition for profit maximization can be written as

$$\begin{aligned} & E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t} X_{t+j} \left( \frac{P_t^*/P_t}{d_{t+j}} \right)^{-\gamma_p} \prod_{k=1}^j \pi_{t+k}^{\gamma_p} \left( \frac{P_t^*}{P_t} \prod_{k=1}^j \pi_{t+k}^{-1} - \frac{\gamma_p}{\gamma_p - 1} mc_{t+j} \exp \tilde{z}_{p,t+j} \right) \\ & = E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t} X_{t+j} \frac{\epsilon_p}{\gamma_p - 1} \frac{P_t^*}{P_t} \prod_{k=1}^j \pi_{t+k}^{-1}, \end{aligned} \quad (11)$$

where  $P_t^*$  is the price set by firms that can adjust prices in period  $t$  and  $\pi_t \equiv P_t/P_{t-1}$  is the gross inflation rate of the composite good's price. Moreover, under staggered price-setting, eqs. (3) and (5) can be rewritten as, respectively,

$$d_t^{1-\gamma_p} = \xi_p \pi_t^{\gamma_p-1} d_{t-1}^{1-\gamma_p} + (1 - \xi_p) \left( \frac{P_t^*}{P_t} \right)^{1-\gamma_p}, \quad (12)$$

$$e_t = \xi_p \pi_t^{-1} e_{t-1} + (1 - \xi_p) \frac{P_t^*}{P_t}. \quad (13)$$

Eq. (12) describes a law of motion of the real marginal cost of aggregating individual goods

$d_t$ , which consists of the relative-price aggregate corresponding to the Kimball-type goods aggregator (1) as shown in (3), thus yielding the form (12) under staggered price-setting.

Aggregating the production technology (6) over firms and combining the resulting equation with the demand curve (2) leads to

$$X_t \Delta_t = [(A_t l_t)^{1-\alpha} (u_t K_{t-1})^\alpha]^{1-\phi} O_t^\phi - \omega \Upsilon_t, \quad (14)$$

where

$$\Delta_t \equiv \frac{s_t + \epsilon_p}{1 + \epsilon_p} \quad (15)$$

represents the relative price distortion and

$$s_t \equiv \int_0^1 \left( \frac{P_t(f)}{P_t d_t} \right)^{-\gamma_p} df,$$

which can be rewritten, under staggered price-setting, as

$$d_t^{-\gamma_p} s_t = \xi_p \pi_t^{\gamma_p} d_{t-1}^{-\gamma_p} s_{t-1} + (1 - \xi_p) \left( \frac{P_t^*}{P_t} \right)^{-\gamma_p}. \quad (16)$$

### 2.3 Household and labor packer

As in [Schmitt-Grohé and Uribe \(2006\)](#), there are a representative household with a continuum of members and a representative labor packer. The household's preferences over composite-good consumption  $\tilde{C}_t$  and labor effort  $\tilde{l}_t$  are represented as the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log \left( \tilde{C}_t - b C_{t-1} \right) \exp z_{c,t} - \frac{\tilde{l}_t^{1+\chi}}{1+\chi} \exp \tilde{z}_{w,t} \right], \quad (17)$$

where  $C_t$  is aggregate consumption,  $z_{c,t}$  and  $\tilde{z}_{w,t}$  are shocks to consumption and labor preferences,  $b \in [0, 1]$  is the degree of (external) habit persistence in consumption preferences, and  $\chi \geq 0$  is the inverse of the labor elasticity.

The representative labor packer supplies composite labor  $l_t$  to firms by differentiating labor services  $l_t(h)$ ,  $h \in [0, 1]$  subject to the resource constraint

$$\tilde{l}_t = \int_0^1 l_t(h) dh \quad (18)$$

and aggregating them with

$$\int_0^1 F_w \left( \frac{l_t(h)}{l_t} \right) dh = 1, \quad (19)$$

where the function  $F_w(\cdot)$  takes the same form as  $F_p(\cdot)$ , but with parameters  $\theta_w > 1$ ,  $\epsilon_w$ , and  $\gamma_w \equiv \theta_w(1 + \epsilon_w)$ . Like the goods aggregator, the Kimball-type aggregator (19) is reduced to the CES one in the special case of  $\epsilon_w = 0$ , while strategic complementarity in wage-setting is present in the case of  $\epsilon_w < 0$ . Both cases are considered in what follows.

The labor packer maximizes the profit  $P_t W_t l_t - \int_0^1 P_t W_t(h) l_t(h) dh$  subject to the Kimball-type aggregator (19), given the wage  $W_t$  for composite labor  $l_t$  and wages  $\{W_t(h)\}$  for individual labor services  $\{l_t(h)\}$ . Combining the first-order conditions for profit maximization and the aggregator (19) yields

$$\frac{l_t(h)}{l_t} = \frac{1}{1 + \epsilon_w} \left[ \left( \frac{P_t W_t(h)}{P_t W_t d_{w,t}} \right)^{-\gamma_w} + \epsilon_w \right], \quad (20)$$

$$d_{w,t} = \left[ \int_0^1 \left( \frac{P_t W_t(h)}{P_t W_t} \right)^{1-\gamma_w} dh \right]^{\frac{1}{1-\gamma_w}}, \quad (21)$$

$$1 = \frac{1}{1 + \epsilon_w} d_{w,t} + \frac{\epsilon_w}{1 + \epsilon_w} e_{w,t}, \quad (22)$$

where  $d_{w,t}$  is the Lagrange multiplier on the labor aggregator (19) and

$$e_{w,t} \equiv \int_0^1 \frac{P_t W_t(h)}{P_t W_t} dh. \quad (23)$$

The Lagrange multiplier  $d_{w,t}$  represents the real marginal cost of aggregating individual labor services and consists of the aggregate of their relative wages that corresponds to the Kimball-type labor aggregator (19), as shown in (21).

Given the labor resource constraint (18) and the labor demand curve (20), (nominal) wages are chosen on a Calvo-style staggered basis. In each period, a fraction  $\xi_w \in (0, 1)$  of wages remains unchanged, while the other fraction  $1 - \xi_w$  is chosen so as to maximize the relevant utility function

$$E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left[ -\frac{1}{1 + \chi} \left( \int_0^1 l_{t+j|t}(h) dh \right)^{1+\chi} \exp \tilde{z}_{w,t+j} + \Lambda_{t+j} \frac{P_t W_t(h)}{P_{t+j}} l_{t+j|t}(h) \right],$$

subject to the demand curve

$$l_{t+j|t}(h) = \frac{l_{t+j}}{1 + \epsilon_w} \left[ \left( \frac{P_t W_t(h)}{P_{t+j} W_{t+j} d_{w,t+j}} \right)^{-\gamma_w} + \epsilon_w \right].$$

The first-order condition for utility maximization with respect to the wage is given by

$$\begin{aligned} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t} l_{t+j} \left( \frac{W_t^* / W_t}{d_{w,t+j}} \right)^{-\gamma_w} \prod_{k=1}^j \pi_{w,t+k}^{\gamma_w} \left( \frac{W_t^*}{W_t} \prod_{k=1}^j \pi_{t+k}^{-1} - \frac{\gamma_w}{\gamma_w - 1} \frac{\tilde{l}_{t+j}^x \exp \tilde{z}_{w,t+j}}{\Lambda_{t+j} W_{t+j}} \prod_{k=1}^j \frac{W_{t+k}}{W_{t+k-1}} \right) \\ = E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t} l_{t+j} \frac{\epsilon_w}{\gamma_w - 1} \frac{W_t^*}{W_t} \prod_{k=1}^j \pi_{t+k}^{-1}, \end{aligned} \quad (24)$$

where  $W_t^*$  is the real wage that is determined in period  $t$  and

$$\pi_{w,t} \equiv \frac{P_t W_t}{P_{t-1} W_{t-1}} = \pi_t \frac{W_t}{W_{t-1}} \quad (25)$$

denotes wage inflation. Moreover, under staggered wage-setting, eqs. (21) and (23) can be rewritten as, respectively,

$$d_{w,t}^{1-\gamma_w} = \xi_w \pi_{w,t}^{\gamma_w-1} d_{w,t-1}^{1-\gamma_w} + (1 - \xi_w) \left( \frac{W_t^*}{W_t} \right)^{1-\gamma_w}, \quad (26)$$

$$e_{w,t} = \xi_w \pi_{w,t}^{-1} e_{w,t-1} + (1 - \xi_w) \frac{W_t^*}{W_t}. \quad (27)$$

Eq. (26) describes a law of motion of the real marginal cost of aggregating individual labor services  $d_{w,t}$ , which consists of the relative-wage aggregate that corresponds to the Kimball-type labor aggregator (19), as shown in (21).

Combining the labor resource constraint (18) with the labor demand curve (20) leads to

$$\tilde{l}_t = l_t \Delta_{w,t}, \quad (28)$$

where

$$\Delta_{w,t} \equiv \frac{s_{w,t} + \epsilon_w}{1 + \epsilon_w} \quad (29)$$

represents the relative wage distortion and

$$s_{w,t} \equiv \int_0^1 \left( \frac{P_t W_t(h)}{P_t W_t d_{w,t}} \right)^{-\gamma_w} dh,$$

which can be rewritten, under staggered wage-setting, as

$$d_{w,t}^{-\gamma_w} s_{w,t} = \xi_w \pi_{w,t}^{\gamma_w} d_{w,t-1}^{-\gamma_w} s_{w,t-1} + (1 - \xi_w) \left( \frac{W_t^*}{W_t} \right)^{-\gamma_w}. \quad (30)$$

We turn next to the household's consumption–saving problem. The household consumes the composite good  $\tilde{C}_t$  and purchases one-period bonds  $B_t$  so as to maximize the utility function (17) subject to the budget constraint

$$P_t \tilde{C}_t + B_t = \int_0^1 P_t W_t(h) l_t(h) dh + r_{t-1} B_{t-1} + T_t,$$

where  $r_t$  is the gross rate of interest on the bonds and is assumed to coincide with the monetary policy rate and  $T_t$  consists of lump-sum taxes and transfers as well as firms' profits received. Then, because the household's consumption  $\tilde{C}_t$  turns out to coincide with aggregate consumption  $C_t$ , the first-order condition for utility maximization with respect to consumption becomes

$$\Lambda_t = \frac{\exp z_{c,t}}{C_t - b C_{t-1}}, \quad (31)$$

and the first-order condition with respect to bond holdings is given by

$$1 = E_t \left( \frac{\beta \Lambda_{t+1} r_t}{\Lambda_t \pi_{t+1}} \right). \quad (32)$$

## 2.4 Capital-service provider

There is a representative capital-service provider who adjusts the utilization rate  $u_t$  on capital  $K_{t-1}$  to supply capital services  $u_t K_{t-1}$  to firms at the rental rate  $P_t R_{k,t}$ . After firms' production, capital is depreciated at the rate  $\delta(u_t)$ . As in [Greenwood et al. \(1988\)](#), a higher utilization rate is assumed to result in a higher depreciation rate. Specifically, the depreciation rate is of the form  $\delta(u_t) \equiv \delta + \delta_1 (u_t - 1) + (\delta_2/2) (u_t - 1)^2$  with  $\delta \in (0, 1)$ ,  $\delta_1 > 0$ , and  $\delta_2 > 0$ , following [Schmitt-Grohé and Uribe \(2012\)](#). Then, the capital-service

provider makes a capital investment  $I_t$  using the production technology that converts one unit of the composite good into  $\Psi_t$  units of capital, subject not only to an adjustment cost  $S((I_t/I_{t-1})/(g_\Upsilon g_\Psi)) \equiv (\zeta/2)[(I_t/I_{t-1})/(g_\Upsilon g_\Psi) - 1]^2$  with  $\zeta > 0$ , advocated by [Christiano et al. \(2005\)](#), but also to a shock to the marginal efficiency of investment (MEI),  $z_{i,t}$ , proposed by [Greenwood et al. \(1988\)](#). Thus,  $\Psi_t$  represents the level of investment-specific technology and is assumed to follow the nonstationary deterministic process

$$\log \Psi_t = \log g_\Psi + \log \Psi_{t-1}, \quad (33)$$

where  $g_\Psi$  is the (steady-state) gross rate of investment-specific technological change, so that the gross steady-state rate of the composite technological change  $\Upsilon_t/\Upsilon_{t-1}$  is given by  $g_\Upsilon = g_A g_\Psi^{\alpha/(1-\alpha)}$ .<sup>15</sup> Consequently, the capital-service provider chooses the utilization rate  $u_t$ , investment  $I_t$ , and capital  $K_t$  so as to maximize its profit

$$E_t \sum_{j=0}^{\infty} M_{t,t+j} \left( P_{t+j} R_{k,t+j} K_{t+j-1} - \frac{P_{t+j} I_{t+j}}{\Psi_{t+j}} \right)$$

subject to the capital accumulation equation

$$K_t = (1 - \delta(u_t)) K_{t-1} + \left[ 1 - S \left( \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) \right] I_t \exp z_{i,t}. \quad (34)$$

The first-order conditions for profit maximization with respect to  $u_t$ ,  $I_t$ , and  $K_t$  are given by

$$R_{k,t} = Q_t \delta'(u_t), \quad (35)$$

$$\begin{aligned} \frac{1}{\Psi_t} = & Q_t \left[ 1 - S \left( \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) - S' \left( \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right] \exp z_{i,t} \\ & + E_t \left[ \frac{\beta \Lambda_{t+1}}{\Lambda_t} Q_{t+1} S' \left( \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) \frac{(I_t/I_{t-1})^2}{g_\Upsilon g_\Psi} \exp z_{i,t+1} \right], \end{aligned} \quad (36)$$

$$1 = E_t \left[ \frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{R_{k,t+1} u_{t+1} + Q_{t+1} (1 - \delta(u_{t+1}))}{Q_t} \right], \quad (37)$$

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<sup>15</sup>[Justiniano et al. \(2011\)](#) find that MEI shocks are a much more important driver of business cycle fluctuations than investment-specific technology shocks, identified using the data on the relative price of investment. Thus our model abstracts from the latter shocks.

where  $Q_t$  is the real price of capital.

Under the roundabout production structure, the composite-good market clearing condition is given by

$$X_t - O_t = Y_t = C_t + \frac{I_t}{\Psi_t} + G_t, \quad (38)$$

where  $Y_t$  is value added or GDP and  $G_t$  is government spending. The latter is determined by

$$G_t = Y_t \frac{g}{y} \exp z_{g,t}, \quad (39)$$

where  $g/y$  is the steady-state GDP ratio of government spending and  $z_{g,t}$  is a government spending shock.

## 2.5 Monetary authority

We assume that the monetary authority follows a rule of the sort proposed by [Taylor \(1993\)](#) and adjusts the policy rate in response to the inflation gap, the GDP gap, and GDP growth in the presence of policy-rate smoothing:

$$\begin{aligned} \log r_t = & \phi_r \log r_{t-1} + (1 - \phi_r) \left\{ \log r + \phi_\pi [\log \pi_t - \log (\pi \exp z_{\pi,t})] + \phi_y \left( \log \frac{Y_t}{y \Upsilon_t} - \log \frac{Y_t^n}{y^n \Upsilon_t} \right) \right. \\ & \left. + \phi_{gy} \left( \log \frac{Y_t}{Y_{t-1}} - \log g\Upsilon \right) \right\} + \varepsilon_{r,t}. \end{aligned} \quad (40)$$

Here  $r \geq 1$  is the gross steady-state policy rate,  $\pi \exp z_{\pi,t}$  represents the monetary authority's time-varying inflation target,  $\pi$  is gross steady-state inflation,  $z_{\pi,t}$  is a shock to the target,  $Y_t^n$  denotes potential GDP that would be obtained under flexible prices and wages in the absence of price and wage markup shocks (i.e.,  $\tilde{z}_{p,t}$ ,  $\tilde{z}_{w,t}$ ),  $y$  and  $y^n$  are the steady-state levels of detrended GDP and its potential, and  $\varepsilon_{r,t}$  is a shock to the policy rate. The coefficient  $\phi_r \in [0, 1)$  measures the degree of policy-rate smoothing and  $\phi_\pi$ ,  $\phi_y$ ,  $\phi_{gy}$  are the degrees of policy responses to the inflation gap, the GDP gap, and GDP growth.

## 2.6 Equilibrium conditions

In the model the equilibrium conditions consist of (4), (8)–(16), (22), (24)–(32), (34)–(39), and (40), along with the neutral and investment-specific technological change processes (7)

and (33), the processes of the eight (fundamental) shocks (i.e.,  $z_{j,t}$ ,  $j \in \{a, c, i, g, \pi\}$ ;  $\tilde{z}_{j,t}$ ,  $j \in \{p, w\}$ ;  $\varepsilon_{r,t}$ ), and the equilibrium conditions that would be obtained under flexible prices and wages in the absence of price and wage markup shocks. These conditions are detrended by the composite technology level  $\Upsilon_t$  and the investment-specific technology level  $\Psi_t$  and then log-linearized for estimating parameters of the model.

To ensure that the steady state of the detrended equilibrium conditions of the model is well defined, we assume that the following conditions are satisfied:

$$\xi_p \max(\pi^{\gamma_p}, \pi^{\gamma_p-1}, \pi^{-1}) < 1, \quad \xi_w \max(\pi_w^{\gamma_w}, \pi_w^{\gamma_w-1}, \pi_w^{-1}) < 1, \quad (41)$$

where  $\pi_w (= \pi g \Upsilon)$  is the steady-state rate of wage inflation  $\pi_{w,t}$ . These conditions are rewritten as  $\xi_p \max(\pi^{\theta_p}, \pi^{\theta_p-1}) < 1$  and  $\xi_w \max(\pi_w^{\theta_w}, \pi_w^{\theta_w-1}) < 1$  in the special case of CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ .

## 2.7 Price and wage New Keynesian Phillips curves

This subsection demonstrates the model's key feature that intrinsic inertia of price and wage inflation emerges endogenously in the price and wage NKPCs from the non-CES aggregators of goods and labor under the nonzero steady-state inflation of prices and wages and staggered price- and wage-setting, without relying on price and wage indexation to past rates of price and wage inflation assumed in previous studies.

As for intrinsic inertia of price inflation, detrending and log-linearizing the equilibrium conditions (11) and (12) under the assumptions (41) and combining the resulting equations lead to the price NKPC

$$\hat{\pi}_t = \beta \pi E_t \hat{\pi}_{t+1} + \kappa_p \hat{m} c_t + \kappa_{p\lambda} (\hat{\lambda}_t + \hat{x}_t) + \kappa_{pd} \hat{d}_t + \beta \pi E_t \hat{d}_{t+1} + \hat{d}_{t-1} + \varphi_t + \psi_t + z_{p,t}, \quad (42)$$

where lower case variables denote the detrended variables of their corresponding upper case ones, hatted variables denote percentage deviations from steady-state values,  $z_{p,t} \equiv \kappa_p \tilde{z}_{p,t}$  is the price markup shock, and  $\varphi_t$  and  $\psi_t$  are auxiliary variables that are additional drivers of

inflation under nonzero steady-state inflation and satisfy

$$\varphi_t = \beta \xi_p \pi^{\gamma_p - 1} E_t \varphi_{t+1} + \kappa_{p\varphi} \left[ (\gamma_p - 1) E_t \hat{\pi}_{t+1} + (1 - \beta \xi_p \pi^{\gamma_p - 1}) (E_t \hat{\lambda}_{t+1} + E_t \hat{x}_{t+1} + \gamma_p E_t \hat{d}_{t+1}) \right], \quad (43)$$

$$\psi_t = \beta \xi_p \pi^{-1} E_t \psi_{t+1} + \kappa_{p\psi} \left[ E_t \hat{\pi}_{t+1} - (1 - \beta \xi_p \pi^{-1}) (E_t \hat{\lambda}_{t+1} + E_t \hat{x}_{t+1}) \right]. \quad (44)$$

In addition, from (4) and (13), it follows that the law of motion of the real marginal cost of aggregating individual goods is given by

$$\hat{d}_t = \rho_{pd} \hat{d}_{t-1} + \kappa_{ped} \hat{\pi}_t. \quad (45)$$

The composite coefficients  $\kappa_p$ ,  $\kappa_{p\lambda}$ ,  $\kappa_{pd}$ ,  $\kappa_{p\varphi}$ ,  $\kappa_{p\psi}$ ,  $\rho_{pd}$ , and  $\kappa_{ped}$  in (42)–(45) consist of the model's structural parameters, as shown in Appendix A.

The presence of the real marginal cost  $\hat{d}_t$  is a novel feature of the price NKPC (42). While the CES aggregator of goods (i.e.,  $\epsilon_p = 0$ ) keeps a unit real marginal cost (i.e.,  $d_t = 1$ ), the Kimball-type non-CES aggregator (1) (i.e.,  $\epsilon_p < 0$ ) leads the cost  $\hat{d}_t$  to vary under nonzero steady-state inflation and staggered price-setting. Then, the real marginal cost  $\hat{d}_t$  is a source of intrinsic inertia in inflation. To see this, eq. (45) implies that the real marginal cost is determined by current and past inflation rates:  $\hat{d}_t = \kappa_{ped} \sum_{j=0}^{\infty} \rho_{pd}^j \hat{\pi}_{t-j}$ . Combining this and the price NKPC (42) leads to

$$\hat{\pi}_t = b_{p\epsilon 1} \sum_{j=1}^{\infty} \rho_{pd}^{j-1} \hat{\pi}_{t-j} + b_{p2} E_t \hat{\pi}_{t+1} + b_{p3} \left[ \kappa_p \hat{m} c_t + \kappa_{p\lambda} (\hat{\lambda}_t + \hat{x}_t) + \varphi_t + \psi_t + z_{p,t} \right], \quad (46)$$

where  $b_{p\epsilon 1} \equiv \kappa_{ped} b_{p3} [1 + \rho_{pd} (\kappa_{pd} + \beta \pi \rho_{pd})]$ ,  $b_{p2} \equiv \beta \pi b_{p3} (1 + \kappa_{ped})$ , and  $b_{p3} \equiv 1 / [1 - \kappa_{ped} (\kappa_{pd} + \beta \pi \rho_{pd})]$ . This shows that intrinsic inertia of inflation appears endogenously in the price NKPC of our model, without relying on ad hoc backward-looking price-setting behavior. The degree of intrinsic inflation inertia can be summarized as the sum of the coefficients on lagged inflation rates,  $\lambda_{p\epsilon} \equiv b_{p\epsilon 1} \sum_{j=1}^{\infty} \rho_{pd}^{j-1} = b_{p\epsilon 1} / (1 - \rho_{pd})$ , which depends on the model's parameters, including steady-state inflation  $\pi$ , the probability of no price change  $\xi_p$ , and the goods demand curves' curvature parameter  $\epsilon_p$ .<sup>16</sup>

<sup>16</sup>Note that intrinsic inertia of inflation is absent from (46) under flexible prices (i.e.,  $\xi_p = 0$ ), zero steady-

Likewise, intrinsic inertia of wage inflation emerges endogenously in the wage NKPC. From the equilibrium conditions (24) and (26), the wage NKPC can be derived as

$$\begin{aligned}\hat{\pi}_{w,t} = & \beta\pi_w E_t \hat{\pi}_{w,t+1} + \kappa_w \left[ \chi \left( \hat{l}_t + \hat{\Delta}_{w,t} \right) - \hat{\lambda}_t - \hat{w}_t \right] + \kappa_{w\lambda} \left( \hat{\lambda}_t + \hat{l}_t \right) + \kappa_{wd} \hat{d}_{w,t} + \beta\pi_w E_t \hat{d}_{w,t+1} \\ & + \hat{d}_{w,t-1} + \varphi_{w,t} + \psi_{w,t} + z_{w,t},\end{aligned}\quad (47)$$

where  $z_{w,t} \equiv \kappa_w \tilde{z}_{w,t}$  is the wage markup shock and  $\varphi_{w,t}$  and  $\psi_{w,t}$  are auxiliary variables that are additional drivers of wage inflation under nonzero steady-state wage inflation and satisfy

$$\begin{aligned}\varphi_{w,t} = & \beta\xi_w \pi_w^{\gamma_w - 1} E_t \varphi_{w,t+1} \\ & + \kappa_{w\varphi} \left[ \gamma_w E_t \hat{\pi}_{w,t+1} - E_t \hat{\pi}_{t+1} - E_t z_{a,t+1} + (1 - \beta\xi_w \pi_w^{\gamma_w - 1}) \left( E_t \hat{\lambda}_{t+1} + E_t \hat{l}_{t+1} + \gamma_w E_t \hat{d}_{w,t+1} \right) \right],\end{aligned}\quad (48)$$

$$\psi_{w,t} = \beta\xi_w \pi_w^{-1} E_t \psi_{w,t+1} + \kappa_{w\epsilon\psi} \left[ E_t \hat{\pi}_{t+1} + E_t z_{a,t+1} - (1 - \beta\xi_w \pi_w^{-1}) \left( E_t \hat{\lambda}_{t+1} + E_t \hat{l}_{t+1} \right) \right]. \quad (49)$$

Here, it is worth noting that the relative wage distortion  $\hat{\Delta}_{w,t}$  has an influence on wage inflation dynamics.<sup>17</sup>

From (22) and (27), it follows that the law of motion of the real marginal cost of aggregating individual labor is given by

$$\hat{d}_{w,t} = \rho_{wd} \hat{d}_{w,t-1} + \kappa_{wed} \hat{\pi}_{w,t}. \quad (50)$$

The composite coefficients  $\kappa_w$ ,  $\kappa_{w\lambda}$ ,  $\kappa_{wd}$ ,  $\kappa_{w\varphi}$ ,  $\kappa_{w\epsilon\psi}$ ,  $\rho_{wd}$ , and  $\kappa_{wed}$  in (47)–(50) are presented in Appendix A. Moreover, eq. (50) implies that the real marginal cost  $\hat{d}_{w,t}$  is determined by current and past rates of wage inflation:  $\hat{d}_{w,t} = \kappa_{wed} \sum_{j=0}^{\infty} \rho_{wd}^j \hat{\pi}_{w,t-j}$ . Combining this and the

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state inflation (i.e.,  $\pi = 1$ ), or the CES aggregator of goods (i.e.,  $\epsilon_p = 0$ ).

<sup>17</sup>The relative wage distortion is present in the wage NKPC (47) because our paper follows [Schmitt-Grohé and Uribe \(2006\)](#) (henceforth SU) to introduce differentiated labor and thus the distortion is an explicit cost. In Appendix B, this approach is compared with that of [Erceg et al. \(2000\)](#) (henceforth EHL) in terms of ensuring the determinacy of equilibrium. The latter approach differs from the former in that relative wage distortion is an implicit cost of labor disutility and therefore it is absent from the wage NKPC. A drawback of that approach is that CES aggregators of differentiated goods and labor make the model susceptible to indeterminacy even under low steady-state inflation, as indicated by [Khan et al. \(2020\)](#). The appendix shows that our model (with the SU specification of labor disutility) is much less susceptible to indeterminacy than that with the EHL specification, which is also difficult to log-linearize for any arbitrary value of the labor elasticity in the presence of the Kimball-type non-CES aggregator of differentiated labor.

wage NKPC (47) leads to

$$\begin{aligned} \hat{\pi}_{w,t} = & b_{w\epsilon 1} \sum_{j=1}^{\infty} \rho_{wd}^{j-1} \hat{\pi}_{w,t-j} + b_{w2} E_t \hat{\pi}_{w,t+1} \\ & + b_{w3} \left\{ \kappa_w \left[ \chi \left( \hat{l}_t + \hat{\Delta}_{w,t} \right) - \left( \hat{\lambda}_t + \hat{w}_t \right) \right] + \kappa_w \lambda \left( \hat{\lambda}_t + \hat{l}_t \right) + \varphi_{w,t} + \psi_{w,t} + z_{w,t} \right\}, \end{aligned} \quad (51)$$

where  $b_{w\epsilon 1} \equiv \kappa_{wed} b_{w3} [1 + \rho_{wd}(\kappa_{wd} + \beta \pi_w \rho_{wd})]$ ,  $b_{w2} \equiv \beta \pi_w b_{w3} (1 + \kappa_{wed})$ , and  $b_{w3} \equiv 1/[1 - \kappa_{wed}(\kappa_{wd} + \beta \pi_w \rho_{wd})]$ . Therefore, intrinsic inertia of wage inflation also appears endogenously in the wage NKPC of our model, without relying on ad hoc backward-looking wage-setting behavior. The degree of intrinsic inertia in wage inflation can also be summarized as the sum of the coefficients on past rates of wage inflation,  $\lambda_{w\epsilon} \equiv b_{w\epsilon 1}/(1 - \rho_{wd})$ , which depends on the model's parameters, including steady-state wage inflation  $\pi_w$ , the probability of no (nominal) wage change  $\xi_w$ , and the labor demand curves' curvature parameter  $\epsilon_w$ .<sup>18</sup>

## 2.8 Shock processes

Before proceeding to the model estimation, the stochastic processes of the eight (fundamental) shocks must be specified. It is assumed that the monetary policy rate shock  $\varepsilon_{r,t}$  is a normally distributed i.i.d. shock, i.e.,  $\varepsilon_{r,t} \sim \text{i.i.d. } N(0, \sigma_r^2)$  and that the other seven shocks  $z_{j,t}$ ,  $j \in \{a, c, i, g, p, w, \pi\}$  follow the respective stationary first-order autoregressive processes

$$z_{j,t} = \rho_j z_{j,t-1} + \varepsilon_{j,t} \quad (52)$$

with the persistence parameter  $\rho_j \in [0, 1)$  and the shock innovation  $\varepsilon_{j,t} \sim \text{i.i.d. } N(0, \sigma_j^2)$ .

## 3 Estimation Strategy and Data

This section describes the strategy and data for estimating the model presented in the preceding section. The model is estimated using a full-information Bayesian approach based on [Lubik and Schorfheide \(2004\)](#). Specifically, the model's likelihood function is constructed not only for the determinacy region of its parameter space but also for the indeterminacy region.

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<sup>18</sup>Note that intrinsic inertia of wage inflation is absent from (51) under flexible wages (i.e.,  $\xi_w = 0$ ), zero steady-state wage inflation (i.e.,  $\pi_w = 1$ ), or the CES aggregator of labor (i.e.,  $\epsilon_w = 0$ ).

While [Lubik and Schorfheide \(2004\)](#) conduct model estimation for each region separately, our paper estimates the model over both determinacy and indeterminacy regions in one step by adopting a sequential Monte Carlo (henceforth SMC) algorithm, as is done by [Hirose et al. \(2020\)](#). The SMC algorithm can deal with discontinuity in the likelihood function at the boundary of each region and help us find the entire posterior distribution of parameters. The section begins by presenting solutions to linear rational expectations models, then explains how Bayesian inferences over both determinacy and indeterminacy regions of the model parameter space are made with the SMC algorithm, and lastly describes the data and prior distributions used in the model estimation.

### 3.1 Rational expectations solutions under indeterminacy

[Lubik and Schorfheide \(2003\)](#) derive a full set of solutions to linear rational expectations models under indeterminacy of the form

$$\mathbf{s}_t = \Phi_1^I(\boldsymbol{\theta})\mathbf{s}_{t-1} + \Phi_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})\boldsymbol{\varepsilon}_t + \Phi_\zeta^I(\boldsymbol{\theta})\zeta_t, \quad (53)$$

where  $\Phi_1^I(\boldsymbol{\theta})$ ,  $\Phi_\varepsilon^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})$ , and  $\Phi_\zeta^I(\boldsymbol{\theta})$  are coefficient matrices that depend on the vector of model parameters  $\boldsymbol{\theta}$  and an arbitrary matrix  $\tilde{\mathbf{M}}$ ;  $\mathbf{s}_t$  is a vector of endogenous variables;  $\boldsymbol{\varepsilon}_t = [\varepsilon_{a,t}, \varepsilon_{c,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{p,t}, \varepsilon_{w,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}]'$  is a vector of fundamental shocks; and  $\zeta_t \sim \text{i.i.d. } N(0, \sigma_s^2)$  is a reduced-form sunspot shock, which is a nonfundamental disturbance.<sup>19</sup> There are two features in the solution (53) under indeterminacy. First, the equilibrium dynamics are driven not only by the fundamental shocks  $\boldsymbol{\varepsilon}_t$  but also by the sunspot shock  $\zeta_t$ . Second, the solution is not unique due to the presence of the arbitrary matrix  $\tilde{\mathbf{M}}$ . More concisely, the solution (53) can be expressed as

$$\mathbf{s}_t = \Phi_1^I(\boldsymbol{\theta})\mathbf{s}_{t-1} + \Phi_{\varepsilon\zeta}^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})\tilde{\boldsymbol{\varepsilon}}_t, \quad (54)$$

where  $\Phi_{\varepsilon\zeta}^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})$  is a coefficient matrix and  $\tilde{\boldsymbol{\varepsilon}}_t = [\varepsilon_{a,t}, \varepsilon_{c,t}, \varepsilon_{i,t}, \varepsilon_{g,t}, \varepsilon_{p,t}, \varepsilon_{w,t}, \varepsilon_{\pi,t}, \varepsilon_{r,t}, \zeta_t]'$  is a stacked vector of the fundamental and sunspot shocks.

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<sup>19</sup>[Lubik and Schorfheide \(2003\)](#) originally express the last term in (53) as  $\Phi_\zeta^I(\boldsymbol{\theta}, \mathbf{M}_\zeta)\boldsymbol{\zeta}_t$ , where  $\mathbf{M}_\zeta$  is an arbitrary matrix and  $\boldsymbol{\zeta}_t$  is a vector of sunspot shocks. For identification, [Lubik and Schorfheide \(2004\)](#) impose the normalization on  $\mathbf{M}_\zeta$  with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a “reduced-form sunspot shock” in that it contains beliefs associated with all the expectational variables.

In the case of determinacy, the solution is of the form

$$\mathbf{s}_t = \Phi_1^D(\boldsymbol{\theta}) \mathbf{s}_{t-1} + \Phi_\varepsilon^D(\boldsymbol{\theta}) \boldsymbol{\varepsilon}_t, \quad (55)$$

where  $\Phi_1^D(\boldsymbol{\theta})$  and  $\Phi_\varepsilon^D(\boldsymbol{\theta})$  are coefficient matrices that depend only on model parameters  $\boldsymbol{\theta}$ . Thus, neither the sunspot shock  $\zeta_t$  nor the arbitrary matrix  $\tilde{\mathbf{M}}$  is present and the solution is uniquely determined.

Under indeterminacy, the matrix  $\tilde{\mathbf{M}}$  must be pinned down to specify the law of motion of the endogenous variables  $\mathbf{s}_t$ . Following [Lubik and Schorfheide \(2004\)](#), we can estimate the components of  $\tilde{\mathbf{M}}$  along with other structural parameters in the model. The prior distribution for  $\tilde{\mathbf{M}}$  is set so that it is centered around the matrix  $\mathbf{M}^*(\boldsymbol{\theta})$  given in a particular solution. That is,  $\tilde{\mathbf{M}}$  is replaced with  $\mathbf{M}^*(\boldsymbol{\theta}) + \mathbf{M}$ , and the components of  $\mathbf{M}$  are estimated with prior mean zero. As proposed by [Lubik and Schorfheide \(2004\)](#), the matrix  $\mathbf{M}^*(\boldsymbol{\theta})$  is selected so that the contemporaneous impulse responses of endogenous variables to fundamental shocks, i.e.,  $\partial \mathbf{s}_t / \partial \boldsymbol{\varepsilon}_t$ , are continuous at the boundary between determinacy and indeterminacy regions of the parameter space. In the estimation procedure, for each draw of  $\boldsymbol{\theta}$ ,  $\mathbf{M}^*(\boldsymbol{\theta})$  is constructed using a numerical routine developed by [Hirose et al. \(2020\)](#).

### 3.2 Bayesian estimation with a sequential Monte Carlo algorithm

The model is estimated using a full-information Bayesian approach in which the likelihood function is constructed over both determinacy and indeterminacy regions of the parameter space. Following [Lubik and Schorfheide \(2004\)](#), the likelihood function for a sample of observations  $\mathbf{Y}^T = [\mathbf{Y}_1, \dots, \mathbf{Y}_T]'$  is given by

$$p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}) = \mathbf{1}\{\boldsymbol{\theta} \in \Theta^D\} p^D(\mathbf{Y}^T | \boldsymbol{\theta}) + \mathbf{1}\{\boldsymbol{\theta} \in \Theta^I\} p^I(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M}),$$

where  $\Theta^D$  and  $\Theta^I$  are the determinacy and indeterminacy regions of the parameter space;  $\mathbf{1}\{\boldsymbol{\theta} \in \Theta^i\}$ ,  $i \in \{D, I\}$  is the indicator function that is equal to one if  $\boldsymbol{\theta} \in \Theta^i$  and zero otherwise; and  $p^D(\mathbf{Y}^T | \boldsymbol{\theta})$  and  $p^I(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})$  are the likelihood functions of the state-space models that consist of observation equations and either the determinacy solution (55) or the indeterminacy solution (54). Then, by Bayes' theorem, updating a prior distribution

$p(\boldsymbol{\theta}, \mathbf{M})$  with the sample observations  $\mathbf{Y}^T$  leads to the posterior distribution

$$p(\boldsymbol{\theta}, \mathbf{M} | \mathbf{Y}^T) = \frac{p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})}{p(\mathbf{Y}^T)} = \frac{p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})}{\int p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})p(\boldsymbol{\theta}, \mathbf{M})d\boldsymbol{\theta}d\mathbf{M}}.$$

To approximate the posterior distribution, we adopt the generic SMC algorithm with likelihood tempering described in [Herbst and Schorfheide \(2014\)](#) and [Herbst and Schorfheide \(2015\)](#). In the algorithm, a sequence of tempered posteriors is defined as

$$\varpi_n(\boldsymbol{\theta}) = \frac{[p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M})}{\int [p(\mathbf{Y}^T | \boldsymbol{\theta}, \mathbf{M})]^{\tau_n} p(\boldsymbol{\theta}, \mathbf{M}) d\boldsymbol{\theta} d\mathbf{M}}, \quad n = 0, \dots, N_\tau,$$

where  $N_\tau$  denotes the number of stages and is set at  $N_\tau = 200$ . The tempering schedule  $\{\tau_n\}_{n=0}^{N_\tau}$  is determined by  $\tau_n = (n/N_\tau)^\mu$ , where  $\mu$  is a parameter that controls the shape of the tempering schedule and is set at  $\mu = 2$ , following [Herbst and Schorfheide \(2014\)](#) and [Herbst and Schorfheide \(2015\)](#). The SMC algorithm generates parameter draws  $\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}$  and associated importance weights  $w_n^{(i)}$ —which are called particles—from the sequence of posteriors  $\{\varpi_n\}_{n=1}^{N_\tau}$ ; that is, at each stage,  $\varpi_n(\boldsymbol{\theta})$  is represented by a swarm of particles  $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$ , where  $N$  denotes the number of particles. In the subsequent estimation, the algorithm uses  $N = 10,000$  particles. For  $n = 0, \dots, N_\tau$ , the algorithm sequentially updates the swarm of particles  $\{\boldsymbol{\theta}_n^{(i)}, \mathbf{M}_n^{(i)}, w_n^{(i)}\}_{i=1}^N$  through importance sampling.<sup>20</sup>

Posterior inferences on estimated parameters are made based on the particles  $\{\boldsymbol{\theta}_{N_\tau}^{(i)}, \mathbf{M}_{N_\tau}^{(i)}, w_{N_\tau}^{(i)}\}_{i=1}^N$  from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(\mathbf{Y}^T) = \prod_{n=1}^{N_\tau} \left( \frac{1}{N} \sum_{i=1}^N \tilde{w}_n^{(i)} w_{n-1}^{(i)} \right),$$

where  $\tilde{w}_n^{(i)}$  is the incremental weight defined as  $\tilde{w}_n^{(i)} = [p(\mathbf{Y}^T | \boldsymbol{\theta}_{n-1}^{(i)}, \mathbf{M}_{n-1}^{(i)})]^{\tau_n - \tau_{n-1}}$ . The posterior probability of equilibrium determinacy can be calculated as<sup>21</sup>

$$\mathbb{P}\{\boldsymbol{\theta} \in \Theta^D | \mathbf{Y}^T\} = \frac{1}{N} \sum_{i=1}^N \mathbf{1}\{\boldsymbol{\theta}_{N_\tau}^{(i)} \in \Theta^D\}.$$

<sup>20</sup>This process includes one step of a single-block random walk Metropolis-Hastings algorithm.

<sup>21</sup>Likewise, the prior probability of equilibrium determinacy can be computed using prior draws.

### 3.3 Data

The model is estimated using seven quarterly US time series, on  $\log \pi_t$ ,  $\log r_t$ ,  $\Delta \log Y_t$ ,  $\Delta \log C_t$ ,  $\Delta \log I_t$ ,  $\Delta \log W_t$ , and  $\log l_t$ . The data on these observables are the same as those of [Smets and Wouters \(2007\)](#), except that our paper utilizes the data on per-capita real investment growth because of the presence of investment-specific technological change.<sup>22</sup> Thus the observation equations that relate the data to the corresponding variables in the model are given by

$$\begin{bmatrix} 100 \log \pi_t \\ 100 \log r_t \\ 100 \Delta \log Y_t \\ 100 \Delta \log C_t \\ 100 \Delta \log I_t \\ 100 \Delta \log W_t \\ 100 \log l_t \end{bmatrix} = \begin{bmatrix} \bar{\pi} \\ \bar{r} \\ \overline{g_Y} \\ \overline{g_C} \\ \overline{g_I} + \overline{g_\Psi} \\ \overline{g_W} \\ \bar{l} \end{bmatrix} + \begin{bmatrix} \hat{\pi}_t \\ \hat{r}_t \\ \hat{y}_t - \hat{y}_{t-1} + z_{a,t} \\ \hat{c}_t - \hat{c}_{t-1} + z_{a,t} \\ \hat{i}_t - \hat{i}_{t-1} + z_{a,t} \\ \hat{w}_t - \hat{w}_{t-1} + z_{a,t} \\ \hat{l}_t \end{bmatrix},$$

where  $\bar{\pi} \equiv 100(\pi - 1)$ ,  $\bar{r} \equiv 100(r - 1)$ ,  $\overline{g_Y} \equiv 100(g_Y - 1)$ ,  $\overline{g_\Psi} \equiv 100(g_\Psi - 1)$ , and  $\bar{l} \equiv 100 \log l$ .

To examine whether and how inflation gap persistence changed after the Volcker disinflation of 1979:Q3–1982:Q3, the estimation is conducted for two periods: the pre-1979 period from 1966:Q1 to 1979:Q2 and the post-1982 period from 1982:Q4 to 2008:Q4.<sup>23</sup>

### 3.4 Fixed parameters and prior distributions

Before the model estimation, we fix the values of two parameters. The (quarterly) depreciation rate of capital is fixed at  $\delta = 0.1/4 = 0.025$ , which is a standard value used in the macroeconomic literature. The steady-state GDP ratio of government spending is fixed at its average over the whole sample period 1966:Q1–2008:Q4, that is,  $g/y = 0.194$ .

All the other parameters of the model are estimated.<sup>24</sup> Their prior distributions are

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<sup>22</sup>[Smets and Wouters \(2007\)](#) employ the data on the growth rate of nominal investment deflated by the GDP deflator in the absence of investment-specific technological change.

<sup>23</sup>Because the post-1982 period ends before the nominal interest rate reached its effective lower bound, the nonlinearity arising from the lower bound is not a critical issue for our estimation strategy.

<sup>24</sup>As for the subjective discount factor  $\beta$ , the steady-state condition  $\beta = \pi g_Y / r$  is used in estimation.

Table 1: Prior distributions for estimated parameters of the quarterly model

Parameter		Distribution	Mean	St. dev.
$\bar{\pi}$	Steady-state inflation rate	Normal	0.969	0.500
$\bar{r}$	Steady-state nominal interest rate	Gamma	1.597	0.150
$\overline{g_Y}$	Steady-state GDP growth rate	Normal	0.381	0.100
$\overline{g_\Psi}$	Steady-state rate of investment-specific technological change	Normal	0.137	0.150
$\bar{l}$	Normalized log steady-state labor	Normal	0.000	0.050
$b$	Consumption habit persistence	Beta	0.700	0.100
$\chi$	Inverse of labor elasticity	Gamma	2.000	0.750
$\alpha$	Parameter governing capital elasticity of production	Beta	0.300	0.025
$\phi$	Intermediate-input elasticity of production	Beta	0.500	0.050
$\delta_2/\delta_1$	Parameter governing capital utilization adjustment costs	Gamma	0.750	0.100
$\zeta$	Elasticity of investment adjustment costs	Gamma	4.000	1.500
$\theta_p - 1$	Parameter governing elasticity of goods substitution	Gamma	9.000	1.500
$\theta_w - 1$	Parameter governing elasticity of labor substitution	Gamma	9.000	1.500
$-\epsilon_p$	Parameter governing curvature of goods demand curves	Gamma	3.000	1.000
$-\epsilon_w$	Parameter governing curvature of labor demand curves	Gamma	3.000	1.000
$\xi_p$	Probability of no price change	Beta	0.500	0.100
$\xi_w$	Probability of no (nominal) wage change	Beta	0.500	0.100
$\phi_r$	Monetary policy-rate smoothing	Beta	0.750	0.100
$\phi_\pi$	Monetary policy response to inflation gap	Gamma	1.100	0.500
$\phi_y$	Monetary policy response to GDP gap	Gamma	0.125	0.050
$\phi_{gy}$	Monetary policy response to GDP growth	Gamma	0.125	0.050
$\rho_\pi$	Persistence of inflation target shock $z_{\pi,t}$	Beta	0.900	0.100
$\rho_j$	Persistence of other AR(1) shocks $z_{j,t}$ , $j \in \{a, c, i, g, p, w\}$	Beta	0.500	0.200
$\sigma_\pi$	St. dev. of inflation target shock innovation $\varepsilon_{\pi,t}$	Inv. gamma	0.139	0.099
$\sigma_j$	St. dev. of other shock innovations $\varepsilon_{j,t}$ , $j \in \{a, c, i, g, p, w, r, s\}$	Inv. gamma	0.627	0.328
$M_j$	Each fundamental shock's coefficient under indeterminacy	Normal	0.000	1.000

Notes: Given the two fixed parameter values of  $\delta = 0.025$  and  $g/y = 0.194$ , the prior distributions presented here lead to the prior probability of equilibrium determinacy of 0.708. Inverse gamma distributions are of the form  $p(\sigma|\nu, \tau) \propto \sigma^{-\nu-1} e^{-\nu\tau^2/2\sigma^2}$ , where  $\nu = 3$ ,  $\tau = 0.1$  for  $\sigma_\pi$  and  $\nu = 4$ ,  $\tau = 0.5$  for  $\sigma_j$ ,  $j \in \{a, c, i, g, p, w, r, s\}$ .

presented in Table 1. The prior mean of the steady-state (quarterly) rates of inflation, nominal interest, GDP growth, and investment-specific technological change (i.e.,  $\bar{\pi}$ ,  $\bar{r}$ ,  $\bar{g}_Y$ ,  $\bar{g}_\Psi$ ) is set at the respective averages of the inflation rate of the GDP deflator, the federal funds rate, the growth rate of real GDP per capita, and the price of investment relative to the GDP deflator over the whole sample period 1966:Q1–2008:Q4. The prior mean of the steady-state labor  $\bar{l}$  is set at zero because the labor data are demeaned for normalization. The prior distributions for the structural parameters are chosen following previous studies. Those for consumption habit persistence  $b$ , the inverse of the labor elasticity  $\chi$ , the parameter governing the capital elasticity of production  $\alpha$ , the elasticity of investment adjustment costs  $\zeta$ , the probabilities of no price change and no wage change  $\xi_p, \xi_w$ , the monetary policy-rate smoothing  $\phi_r$ , and the policy responses to the GDP gap and GDP growth  $\phi_y, \phi_{\Delta y}$  are set based on [Smets and Wouters \(2007\)](#). The prior mean of the intermediate-input elasticity of production  $\phi$ , the parameter governing capital utilization adjustment costs  $\delta_2/\delta_1$ , and the policy response to the inflation gap  $\phi_\pi$  is chosen from [Basu \(1995\)](#), [Schmitt-Grohé and Uribe \(2012\)](#), and [Lubik and Schorfheide \(2004\)](#), respectively. The priors for the other structural parameters—the parameters governing the elasticities of goods and labor substitution  $\theta_p, \theta_w$  and those governing the curvature of goods and labor demand curves  $\epsilon_p, \epsilon_w$ —are centered at the values used in [Kurozumi and Van Zandweghe \(2019\)](#).

Regarding the shock parameters, the prior mean of the inflation target shock persistence parameter  $\rho_\pi$  is set equal to 0.9, while that of the other AR(1) shock persistence parameters  $\rho_j, j \in \{a, c, i, g, p, w\}$  is chosen at 0.5. The high prior mean of the inflation target shock persistence is based on previous studies, including [Cogley et al. \(2010\)](#). The prior distribution for the standard deviations of the inflation target shock innovation  $\sigma_\pi$  is the inverse gamma distribution with mean of 0.139 and standard deviation of 0.099, whereas those for the standard deviations of the other shock innovations  $\sigma_j, i \in \{a, c, i, g, p, w, r, s\}$  are inverse gamma distributions with mean of 0.627 and standard deviation of 0.328. For the components  $M_j, j \in \{a, c, i, g, p, w, \pi, r\}$  of the arbitrary matrix  $\mathbf{M}$  in the indeterminacy solution, we use the same priors as those in [Lubik and Schorfheide \(2004\)](#), i.e., normal distributions with mean zero and standard deviation of one.

## 4 Results of Empirical Analysis

The empirical analysis of the estimated model proceeds in two steps. First, we present and discuss the model’s estimation results. Second, we demonstrate a decline in inflation gap persistence from the pre-1979 to the post-1982 period in the estimated model, and then address the paper’s main questions of what the source of the decline was and how the source decreased the persistence.

### 4.1 Model estimation results

This subsection presents and discusses the model’s estimation results. Table 2 reports the posterior estimates of parameters of the model for the pre-1979 and post-1982 samples. The table exhibits three key features.

First, the US economy was likely in the indeterminacy region of the model’s parameter space during the pre-1979 period, while the economy likely entered the determinacy region during the post-1982 period. The last line of Table 2 displays the posterior probability of equilibrium determinacy  $\mathbb{P}\{\boldsymbol{\theta} \in \Theta^D | \mathbf{Y}^T\}$  in each estimation period. The probability of determinacy is zero in the pre-1979 period, whereas it is one in the post-1982 period. This feature corresponds with the results in the previous literature such as [Lubik and Schorfheide \(2004\)](#) and [Hirose et al. \(2020\)](#).

Second, the posterior estimates of the six parameters  $\phi_\pi$ ,  $\xi_p$ ,  $\zeta$ ,  $\rho_\pi$ ,  $\rho_g$ , and  $\sigma_a$  changed substantially between the pre-1979 and post-1982 periods as each parameter’s 90 percent highest posterior density (henceforth HPD) intervals in the two periods are disjoint. At the posterior mean, the monetary policy response to the inflation gap  $\phi_\pi$  increased almost threefold from 0.57 to 1.59, indicating the Fed’s change from a passive to an active policy response to the gap, in line with the literature, such as [Clarida et al. \(2000\)](#) and [Lubik and Schorfheide \(2004\)](#). Firms’ probability of no price change  $\xi_p$  also rose from 0.47 to 0.79, and the implied average duration of price changes— $3/(1 - \xi_p)$  months—extended from 5.7 months to 14.6 months. In addition, the elasticity of investment adjustment costs  $\zeta$  more than doubled from 1.53 to 3.65, eliciting more sluggish investment dynamics. Alongside these changes in the structural parameters of the model, the inflation target shock persistence  $\rho_\pi$  increased from 0.85 to 0.98, and the variance of the shock  $z_{\pi,t}$ — $\text{var}(z_{\pi,t}) = \sigma_\pi^2/(1 - \rho_\pi^2)$ —

Table 2: Posterior estimates of parameters of the model

Parameter	Pre-1979		Post-1982	
	Mean	90% interval	Mean	90% interval
$\bar{\pi}$	1.089	[0.850, 1.338]	0.806	[0.609, 1.001]
$\bar{r}$	1.547	[1.372, 1.694]	1.518	[1.358, 1.700]
$\overline{g\Upsilon}$	0.346	[0.240, 0.465]	0.411	[0.302, 0.507]
$\overline{g\Psi}$	0.191	[0.087, 0.298]	0.297	[0.194, 0.416]
$\bar{l}$	0.032	[-0.031, 0.090]	-0.006	[-0.090, 0.064]
$b$	0.648	[0.564, 0.730]	0.594	[0.500, 0.692]
$\chi$	0.998	[0.395, 1.543]	1.610	[0.855, 2.345]
$\alpha$	0.235	[0.208, 0.261]	0.221	[0.197, 0.248]
$\phi$	0.489	[0.433, 0.548]	0.497	[0.428, 0.566]
$\delta_2/\delta_1$	0.772	[0.637, 0.908]	0.751	[0.609, 0.890]
$\zeta$	1.533	[0.984, 2.042]	3.645	[2.421, 4.720]
$\theta_p - 1$	10.521	[8.635, 12.304]	9.298	[7.477, 11.303]
$\theta_w - 1$	8.946	[6.901, 10.834]	7.943	[5.823, 9.959]
$-\epsilon_p$	3.262	[2.171, 4.266]	4.772	[3.448, 6.058]
$-\epsilon_w$	2.474	[1.395, 3.553]	2.235	[1.170, 3.156]
$\xi_p$	0.466	[0.375, 0.548]	0.790	[0.741, 0.839]
$\xi_w$	0.622	[0.513, 0.721]	0.636	[0.499, 0.804]
$\phi_r$	0.724	[0.654, 0.795]	0.809	[0.759, 0.866]
$\phi_\pi$	0.566	[0.358, 0.759]	1.591	[1.049, 2.053]
$\phi_y$	0.086	[0.042, 0.133]	0.065	[0.024, 0.104]
$\phi_{gy}$	0.113	[0.054, 0.170]	0.204	[0.114, 0.299]
$\rho_\pi$	0.854	[0.782, 0.932]	0.981	[0.961, 1.000]
$\rho_a$	0.199	[0.079, 0.322]	0.097	[0.023, 0.175]
$\rho_c$	0.526	[0.257, 0.772]	0.857	[0.752, 0.975]
$\rho_i$	0.509	[0.218, 0.841]	0.707	[0.446, 0.928]
$\rho_g$	0.852	[0.776, 0.929]	0.966	[0.942, 0.989]
$\rho_p$	0.521	[0.375, 0.656]	0.307	[0.158, 0.457]
$\rho_w$	0.673	[0.516, 0.828]	0.784	[0.507, 0.960]
$\sigma_\pi$	0.146	[0.052, 0.247]	0.099	[0.062, 0.139]
$\sigma_a$	1.089	[0.927, 1.235]	0.778	[0.680, 0.885]
$\sigma_c$	1.770	[1.351, 2.179]	1.453	[1.068, 1.834]
$\sigma_i$	0.720	[0.284, 1.125]	1.862	[1.103, 2.600]
$\sigma_g$	2.609	[2.298, 2.922]	2.085	[1.846, 2.324]
$\sigma_p$	0.425	[0.322, 0.529]	0.389	[0.281, 0.491]
$\sigma_w$	0.787	[0.576, 0.995]	0.790	[0.548, 1.026]
$\sigma_r$	0.268	[0.225, 0.311]	0.210	[0.179, 0.239]
$\sigma_s$	0.522	[0.325, 0.722]	-	-
$M_\pi$	0.280	[-0.984, 1.382]	-	-
$M_a$	0.171	[-0.156, 0.469]	-	-
$M_c$	-0.171	[-0.361, 0.028]	-	-
$M_i$	0.236	[-0.538, 1.176]	-	-
$M_g$	-0.303	[-0.403, -0.200]	-	-
$M_p$	-0.402	[-1.149, 0.363]	-	-
$M_w$	-1.036	[-1.605, -0.431]	-	-
$M_r$	1.418	[0.364, 2.410]	-	-
$\log p(\mathbf{Y}^T)$		-410.427		-592.146
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D   \mathbf{Y}^T\}$		0.000		1.000

Notes: This table reports the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table,  $\log p(\mathbf{Y}^T)$  represents the SMC-based approximation of log marginal data density and  $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\}$  denotes the posterior probability of equilibrium determinacy.

substantially rose from 0.09 to 1.68, despite a decline in the posterior mean of the shock innovation’s variance  $\sigma_\pi^2$ .<sup>25</sup> The government spending shock persistence  $\rho_g$  also increased from 0.85 to 0.97, whereas the standard deviation of the neutral technology shock innovation  $\sigma_a$  decreased from 1.09 to 0.78.

Last but not least, the estimation results empirically support the presence of variable elasticity of demand for goods and labor. To establish this, we conduct a Bayesian model comparison by comparing the marginal data densities between the baseline model and its counterpart with CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ . The second to last line of Table 2 presents the model’s log marginal data density  $\log p(\mathbf{Y}^T)$ , which is  $-410.43$  for the pre-1979 period and  $-592.15$  for the post-1982 period. These values are greater than their counterparts in the case of CES aggregators,  $-417.86$  for the pre-1979 period and  $-654.44$  for the post-1982 period, as reported in Table 5 of Appendix C.<sup>26</sup> This indicates that the model (with the non-CES aggregators) empirically outperforms its counterpart with CES aggregators during both the pre-1979 and the post-1982 periods, thus demonstrating the presence of curvature in demand curves for goods and labor.

Our estimates provide new evidence on the curvature of goods and labor demand curves. As pointed out by Eichenbaum and Fisher (2007), the parameter governing the curvature of goods demand curves and other parameters pertaining to price-setting behavior are not separately identified in an NK model with price indexation, because such a model with the non-CES aggregator of goods is observationally equivalent to its counterpart in the case of the CES aggregator. This leads most studies using NK models with the non-CES aggregator and price indexation to calibrate the curvature parameter when estimating the models. By contrast, our model—where there is no price indexation—shows that nonzero values of the goods demand curves’ curvature parameter  $\epsilon_p$  introduce past inflation rates  $\hat{\pi}_{t-j}, j \geq 1$  in the NKPC (46), as discussed in Section 2.7. Likewise, nonzero values of the labor demand curves’ curvature parameter  $\epsilon_w$  incorporate past wage inflation rates  $\hat{\pi}_{w,t-j}, j \geq 1$  in the wage NKPC

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<sup>25</sup>In a small-scale DSGE model estimated within its equilibrium determinacy region, Cogley et al. (2010) also report a decline in the posterior median of the inflation target shock innovation’s variance from the pre-1979 to the post-1982 period, which implies a concurrent decline in the posterior median of the shock variance because the shock persistence parameter is assumed to be constant over time.

<sup>26</sup>Appendix C presents and discusses the estimation results in the case of CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ .

(51). Therefore, the curvature parameters should be identified. Indeed, we conducted the identification analysis proposed by [Iskrev \(2010\)](#) and confirmed that all estimated parameters of the model are (locally) identified. In our estimated model, the curvature of goods demand curves is  $-\epsilon_p\theta_p = 37.5$  at the posterior mean and its 90 percent HPD interval is  $[24.8, 50.4]$  in the pre-1979 period. These changed to 49.1 and  $[33.5, 64.2]$  in the post-1982 period. The estimates are of the same order of magnitude as those of [Guerrieri et al. \(2010\)](#), who estimate an open-economy NK model using international trade data for the US, though they exceed those of [Dossche et al. \(2010\)](#) and [Beck and Lein \(2020\)](#), who estimate a non-CES aggregator of goods using European scanner price data.<sup>27</sup> As for the labor demand curves, the posterior mean and the 90 percent HPD interval of the curvature in our estimated model are  $-\epsilon_w\theta_w = 24.3$  and  $[14.4, 33.8]$  in the pre-1979 period, and these changed to 19.8 and  $[10.6, 28.6]$  in the post-1982 period.

The curvature parameters of goods and labor demand curves generate intrinsic inertia of price and wage inflation in the NKPCs (46) and (51), as mentioned above. The intrinsic price inflation inertia measured as  $\lambda_{pe}$  is 0.13 at the posterior mean and its 90 percent HPD interval is  $[0.08, 0.17]$  in the pre-1979 period. These increased to 0.25 and  $[0.19, 0.30]$  in the post-1982 period, mainly reflecting the changes in the probability of no price change  $\xi_p$  and the curvature parameter  $\epsilon_p$ . The posterior mean and the 90 percent HPD interval of the intrinsic wage inflation inertia measured as  $\lambda_{we}$  are 0.17 and  $[0.11, 0.23]$  in the pre-1979 period, and these changed to 0.12 and  $[0.08, 0.17]$  in the post-1982 period. Therefore, nonnegligible degrees of intrinsic inertia of price and wage inflation are present in our estimated model during both the pre-1979 and the post-1982 periods.

## 4.2 Why did inflation gap persistence decline?

A number of empirical studies have documented a decline in inflation gap persistence in the US after the early 1980s.<sup>28</sup> [Cogley and Sbordone \(2008\)](#) use a vector autoregressive

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<sup>27</sup>[Guerrieri et al. \(2010\)](#) obtain two estimates of goods demand curves' curvature of 16.7 and 65.9 for two calibrations of the parameter governing the substitution elasticity of 6 and 11, respectively. Using Italian data and a calibration of the substitution elasticity parameter of 6, [Riggi and Santoro \(2015\)](#) estimate an NK model and obtain an estimate of the parameter governing the curvature of goods demand curves of  $-2.0$  for the period 1999–2012, implying an estimate of curvature of 12.0.

<sup>28</sup>These studies are part of a large empirical literature on inflation persistence. For a review of the literature on inflation persistence, see [Fuhrer \(2011\)](#) and the references cited therein.

(henceforth VAR) model with time-varying coefficients and stochastic volatility, and indicate that the autocorrelation of the inflation gap declined after the Volcker disinflation in the early 1980s. Employing the same VAR methodology, [Cogley et al. \(2010\)](#) and [Benati and Surico \(2008\)](#) show that inflation gap persistence rose during the Great Inflation of the 1970s and fell after the Volcker disinflation. The former study uses predictability as a measure of persistence because more persistent shocks make time series more predictable, while the latter documents parallel declines in persistence, computed using spectral analysis, as well as predictability.

Our paper follows [Cogley et al. \(2010\)](#) and measures inflation gap persistence as its predictability and demonstrates a decline in inflation gap persistence from the pre-1979 to the post-1982 period in the estimated model. The predictability of the inflation gap is measured as the fraction of total variation in the inflation gap at a forecasting horizon  $h$  that is not due to future shocks. That is,

$$R_h^2 \approx 1 - \frac{\mathbf{e}_\pi \left[ \sum_{j=0}^{h-1} (\Phi^j) \Sigma (\Phi^j)' \right] \mathbf{e}_\pi'}{\mathbf{e}_\pi \left[ \sum_{j=0}^{\infty} (\Phi^j) \Sigma (\Phi^j)' \right] \mathbf{e}_\pi'}, \quad (56)$$

where

$$[\Phi, \Sigma] = \begin{cases} \left[ \Phi_1^D(\boldsymbol{\theta}), (\Phi_\varepsilon^D(\boldsymbol{\theta})) \text{var}(\varepsilon_t) (\Phi_\varepsilon^D(\boldsymbol{\theta}))' \right] & \text{if } \boldsymbol{\theta} \in \Theta^D \\ \left[ \Phi_1^I(\boldsymbol{\theta}), (\Phi_{\varepsilon\zeta}^I(\boldsymbol{\theta}, \tilde{\mathbf{M}})) \text{var}(\tilde{\varepsilon}_t) (\Phi_{\varepsilon\zeta}^I(\boldsymbol{\theta}, \tilde{\mathbf{M}}))' \right] & \text{if } \boldsymbol{\theta} \in \Theta^I \end{cases}$$

and  $\mathbf{e}_\pi$  is a selector vector. [Table 3](#) reports the variability of the inflation gap and its predictability at forecasting horizons of one, four, and eight quarters in the model with the posterior mean estimates of parameters for the pre-1979 and post-1982 periods. The estimated model shows declines in the variability and the predictability of the inflation gap from the pre-1979 to the post-1982 period. Moreover, the decline in the predictability is larger for the longer forecasting horizon.

We now address the paper's main question of why inflation gap persistence declined. Previous studies of the source of the decline in US inflation (gap) persistence have offered competing views.<sup>29</sup> The views differ on the role of monetary policy versus nonpolicy factors.

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<sup>29</sup>Some papers assume a constant rate of trend inflation, which implies that the persistence of inflation and that of the inflation gap coincide.

Table 3: Inflation gap variability and predictability in the estimated model

	$\text{std}(\hat{\pi}_t - z_{\pi,t})$	$R_1^2$	$R_4^2$	$R_8^2$
Pre-1979 period	1.215	0.904	0.777	0.548
Post-1982 period	0.320	0.692	0.440	0.262
Percent change	-74	-23	-43	-52

Note: The statistics are calculated using the posterior mean estimates of parameters reported in Table 2.

Regarding the latter, some estimated NKPCs that rely on ad hoc assumptions for intrinsic inflation inertia—for example, the NKPC of [Smets and Wouters \(2007\)](#)—show that the inertia decreased after the early 1980s, and [Fuhrer \(2011\)](#) argues that lower intrinsic inertia of inflation is likely an important factor for the decline in inflation (gap) persistence, using a calibrated small-scale NK model.<sup>30</sup> This argument, however, fails to hold in our estimated medium-scale model, because intrinsic inflation inertia  $\lambda_{pe}$  in the NKPC (46) increased from the pre-1979 to the post-1982 period, as noted in the preceding subsection. Other research stresses the role of price markup shocks in the decline of inflation (gap) persistence because of changes in either their autocorrelation ([Fuhrer \(2011\)](#)) or their relative importance ([Carlstrom et al. \(2009\)](#), [Davig and Doh \(2014\)](#)).

Even among the views emphasizing monetary policy factors, there are differences in which aspects of monetary policy explain the decline in inflation gap persistence. [Carlstrom et al. \(2009\)](#) show that a stronger policy response to (the) inflation (gap) can reduce inflation (gap) persistence in a calibrated small-scale NK model. In addition, [Davig and Doh \(2014\)](#) estimate a Markov-switching NK model within its determinacy region for the period 1953–2006, and single out changes in the policy response to the inflation gap as the primary factor behind the rise and fall of inflation gap persistence.<sup>31</sup> By contrast, [Cogley et al. \(2010\)](#) estimate a small-scale DSGE model within its determinacy region for both the pre-1979 and the post-1982 periods, and find that a more aggressive policy response to the inflation gap played only a secondary role in the post-1982 decline in inflation gap persistence, next to the decreased variance of the inflation target (shock), which is the primary source in

<sup>30</sup>[Galí and Gertler \(1999\)](#) obtain ambiguous results on whether intrinsic inertia of inflation in their estimated NKPC increased or decreased after the early 1980s.

<sup>31</sup>[Benati and Surico \(2008\)](#) estimate a small-scale NK model using post-1982 data and show that a weaker policy response to the inflation gap would induce a substantial increase in inflation gap persistence.

their estimated model. Relatedly, [Cogley and Sbordone \(2008\)](#) indicate that time variation in the long-run component of inflation attributed to shifts in monetary policy provides a better account of inflation gap persistence than intrinsic inertia of inflation in their estimated NKPC.

These previous studies, however, have sidestepped the possibility of equilibrium indeterminacy, even though the existing literature, including [Lubik and Schorfheide \(2004\)](#) and [Hirose et al. \(2020\)](#), estimates small-scale DSGE models while allowing for indeterminacy and shows that the US economy experienced indeterminacy in the pre-1979 period. As [Lubik and Schorfheide \(2004\)](#) and [Fujiwara and Hirose \(2014\)](#) indicate, indeterminacy can substantially alter the dynamic properties of models and the propagation of shocks. Therefore, it can affect inflation gap persistence in the pre-1979 period differently than determinacy and can hence affect changes in the persistence between the pre-1979 and post-1982 periods. Thus we investigate the source of the decline in inflation gap persistence by utilizing our medium-scale DSGE model estimated over both the determinacy and indeterminacy regions of its parameter space. Specifically, we follow [Cogley et al. \(2010\)](#) to conduct counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of the parameters in question are altered to their post-1982 posterior mean. Table 4 presents the results of the exercises. The second to fifth columns of the table report the percentage changes in the inflation gap variability and predictability when the values of model parameters indicated in the first column are changed:

$$100 \times \frac{\text{counterfactual change}}{\text{total change}}.$$

The last column shows whether the change in the values of the model parameters generates a shift to determinacy from indeterminacy induced at the pre-1979 posterior mean of all estimated model parameters.

The first line of Table 4 presents the contribution of the changes in all estimated model parameters pertaining to monetary policy— $\phi_r$ ,  $\phi_\pi$ ,  $\phi_y$ ,  $\phi_{gy}$ ,  $\pi$ ,  $\rho_\pi$ ,  $\sigma_\pi$ , and  $\sigma_r$ —to their post-1982 posterior mean. These changes in monetary policy parameters can explain almost the entire declines in inflation gap predictability as well as about half of the decline in inflation gap variability. As noted in the preceding subsection, the estimates of the two policy

Table 4: Counterfactual exercises based on the estimated model: Accounting for the decline in inflation gap variability and predictability

Model parameter	$\text{std}(\hat{\pi}_t - z_{\pi,t})$	$R_1^2$	$R_4^2$	$R_8^2$	determinacy
1. Post-1982 policy	52	120	114	91	yes
2. Post-1982 $\phi_\pi$	78	228	212	188	yes
3. Post-1982 $\rho_\pi, \sigma_\pi$	-104	-8	34	54	no
4. Post-1982 policy excl. $\phi_\pi, \rho_\pi, \sigma_\pi$	24	32	30	42	no
5. Post-1982 nonpolicy	88	52	105	139	yes
6. Post-1982 $\xi_p$	86	108	181	178	yes
7. Post-1982 $\zeta$	-4	-2	-3	-6	no
8. Post-1982 nonpolicy shocks	-2	-8	4	-15	no
9. Post-1982 $\rho_g$	4	1	-2	-6	no
10. Post-1982 $\sigma_a$	2	1	1	0	no

Notes: The table shows the percentage changes in inflation gap variability and predictability in counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of parameters indicated in the first column are changed to their post-1982 posterior mean reported in Table 2. “Post-1982 policy” refers to the post-1982 posterior mean of  $\phi_r, \phi_\pi, \phi_y, \phi_{gy}, \pi, \rho_\pi, \sigma_\pi,$  and  $\sigma_r$ , while “Post-1982 nonpolicy” refers to the post-1982 posterior mean of the other estimated parameters.

parameters  $\phi_\pi$  and  $\rho_\pi$  changed substantially between the pre-1979 and post-1982 periods. The second line shows the effects of changing only the policy response to the inflation gap  $\phi_\pi$ . The change from the passive policy response in the pre-1979 period to the active one in the post-1982 period alone can account for more than the entire declines in the predictability. The third line indicates that the post-1982 increase in the variance of the inflation target shock  $z_{\pi,t}$ , induced by the concurrent changes in  $\rho_\pi$  and  $\sigma_\pi$  noted above, cannot explain the decline in inflation gap predictability at the short forecasting horizon, though it can provide a partial account of the declines at the longer forecasting horizons. In addition, the fourth line points out that changing the other estimated policy parameters— $\phi_r, \phi_y, \phi_{gy}, \pi,$  and  $\sigma_r$ —to their post-1982 posterior mean can explain relatively minor portions of the declines.

Changes in estimated nonpolicy parameters of the model can also account for much of the declines in inflation gap variability and predictability, as reported on the fifth line of Table 4. The changes from the pre-1979 to the post-1982 posterior mean of all estimated nonpolicy parameters decrease not only inflation gap variability by almost the entire post-1982 decline but also its predictability by about half of the decline at the one-quarter forecasting horizon and more than the entire declines at the four- and eight-quarters forecasting horizons. As

noted in the preceding subsection, the estimates of the four nonpolicy parameters  $\xi_p$ ,  $\zeta$ ,  $\rho_g$ , and  $\sigma_a$  changed substantially between the pre-1979 and post-1982 periods. Line 6 of the table indicates that the rise in firms' probability of no price change  $\xi_p$  alone can explain more than the entire declines in the predictability. By contrast, the increase in the elasticity of investment adjustment costs  $\zeta$  is of little consequence for the predictability, as reported on line 7. In addition, the rise in the government spending shock persistence  $\rho_g$  and the decrease in the standard deviation of the neutral technology shock innovation  $\sigma_a$  make little contribution to the predictability, and moreover, even the changes in estimated parameters governing the processes of all nonpolicy shocks—all shocks except for  $z_{\pi,t}$  and  $\varepsilon_{r,t}$ —make negligible contributions to the predictability, as seen on lines 8–10.

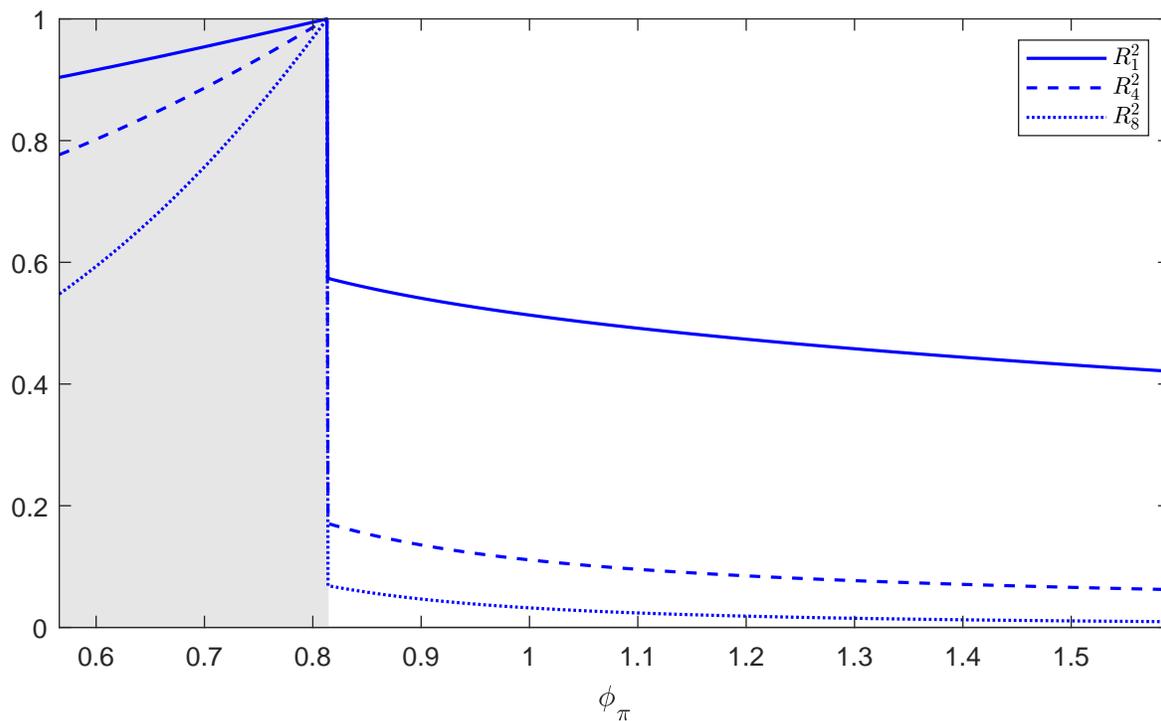
### 4.3 How did inflation gap persistence decline?

In the preceding subsection, the counterfactual exercises based on the estimated model have shown that the Fed's change from a passive to an active policy response to the inflation gap or a rise in firms' probability of no price change can fully account for the decline in inflation gap persistence. This subsection addresses the other main question of the paper: How did these two factors decrease inflation gap persistence?

The last column of Table 4 reveals one answer to the question. A shift to determinacy of equilibrium does matter for the decline in inflation gap persistence. [Fujiwara and Hirose \(2014\)](#) demonstrate that indeterminacy can generate more persistent dynamics that improve the forecastability of inflation in a DSGE model, compared with the case of determinacy. Technically speaking, equilibrium dynamics under indeterminacy can be more persistent than those under determinacy because fewer AR roots (i.e., eigenvalues) in the indeterminacy solution to the model are being suppressed.

Figure 1 illustrates how the three measures of inflation gap predictability  $R_j^2$ ,  $j = 1, 4, 8$  change with the monetary policy response to the inflation gap  $\phi_\pi$ , keeping the other estimated model parameters fixed at their posterior mean in the pre-1979 period reported in Table 2. The increase in  $\phi_\pi$  from the pre-1979 posterior mean of 0.57 to the post-1982 posterior mean of 1.59 leads to declines in all the three measures ( $R_1^2, R_4^2, R_8^2$ ) from (0.90, 0.78, 0.55) to (0.42, 0.06, 0.01). The decline in inflation gap predictability can be decomposed into the

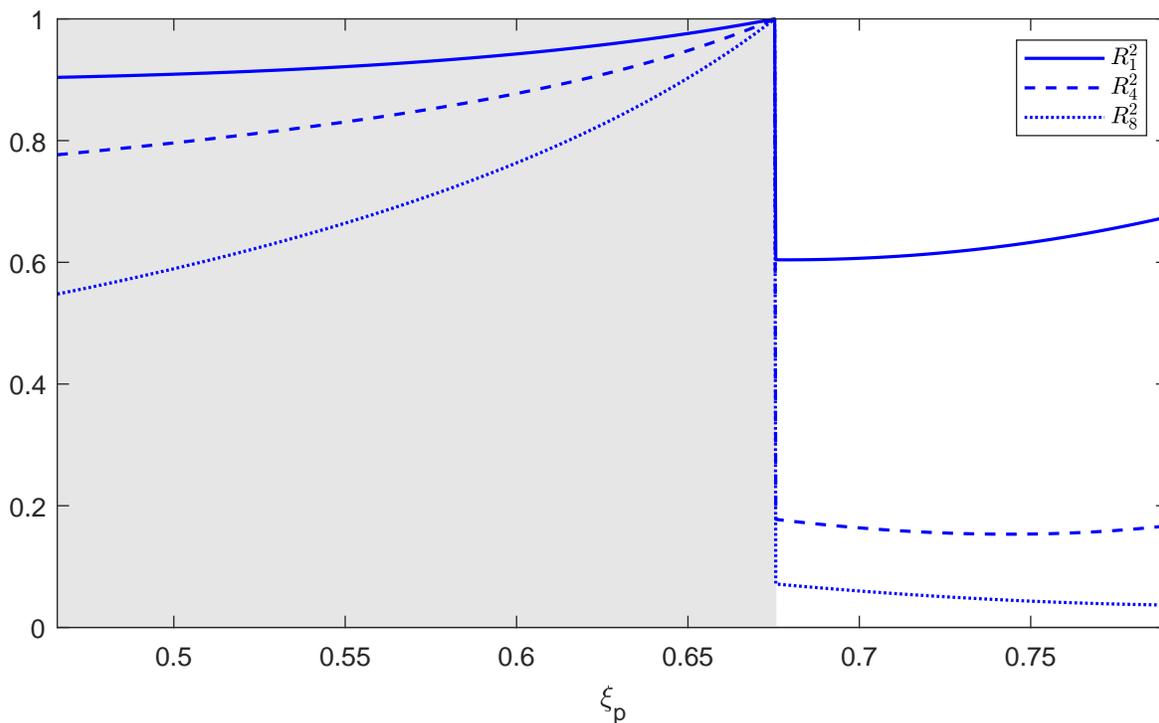
Figure 1: Inflation gap predictability and the monetary policy response to the inflation gap.



Notes: The figure illustrates how the three measures of inflation gap predictability  $R_j^2$ ,  $j = 1, 4, 8$  change with the monetary policy response to the inflation gap  $\phi_\pi$ , keeping the other estimated parameters fixed at their pre-1979 posterior mean reported in Table 2. The white and gray areas display regions in which each value of  $\phi_\pi$  generates equilibrium determinacy and indeterminacy, respectively.

effect of increasing  $\phi_\pi$  from its pre-1979 posterior mean to the smallest value ensuring determinacy of equilibrium, which is  $\phi_\pi = 0.81$ , and the effect of further increasing the parameter to its post-1982 posterior mean. The former increase in  $\phi_\pi$  (from 0.57 to 0.81) decreases the three measures ( $R_1^2, R_4^2, R_8^2$ ) from (0.90, 0.78, 0.55) to (0.57, 0.17, 0.07) or by about 70 to 90 percent of the total declines. Within the indeterminacy region of the model's parameter space (gray area), a stronger policy response to the inflation gap increases inflation gap predictability, such that  $\phi_\pi = 0.80$  generates very high inflation gap predictability of  $(R_1^2, R_4^2, R_8^2) = (0.99, 0.99, 0.99)$ . Because larger values of  $\phi_\pi$  do not reduce inflation gap predictability under indeterminacy, the decline in the predictability resulting from the increase to  $\phi_\pi = 0.81$  is entirely due to ruling out indeterminacy. The remaining portion of the declines in the predictability—between about 10 and 30 percent—then reflects the dampening effect of a more aggressive policy response to the inflation gap within the determinacy region

Figure 2: Inflation gap predictability and firms' probability of no price change.



Notes: The figure illustrates how the three measures of inflation gap predictability  $R_j^2$ ,  $j = 1, 4, 8$  vary with firms' probability of no price change  $\xi_p$ , keeping the other estimated parameters fixed at their pre-1979 posterior mean reported in Table 2. The white and gray areas display regions in which each value of  $\xi_p$  generates equilibrium determinacy and indeterminacy, respectively.

(white area).

In short, the Fed's change from a passive to an active policy response to the inflation gap has a powerful effect on the decline of inflation gap persistence in our estimated model because it works through two channels, primarily by ruling out indeterminacy of equilibrium that induces persistent dynamics and thus raises inflation gap predictability, and, to a lesser extent, by dampening responses to shocks and hence the predictability under determinacy. Note that only the second channel is highlighted by previous studies that emphasize the role of the policy response to the inflation gap in the decline of inflation gap persistence, such as Benati and Surico (2008), Carlstrom et al. (2009), and Davig and Doh (2014).

We turn next to the effect of the rise in firms' probability of no price change  $\xi_p$  on the decline in inflation gap persistence. Figure 2 displays how the three measures of inflation gap predictability change with the probability  $\xi_p$ . Unlike the monetary policy response to the

inflation gap, increasing  $\xi_p$  from the pre-1979 posterior mean of 0.47 to the smallest value guaranteeing determinacy, which is  $\xi_p = 0.68$ , has almost the entire effect of the change in the probability  $\xi_p$  on the decline of inflation gap predictability.<sup>32</sup> The increase of  $\xi_p$  (from 0.47 to 0.68) decreases the three measures  $(R_1^2, R_4^2, R_8^2)$  from  $(0.90, 0.78, 0.55)$  to  $(0.60, 0.18, 0.07)$ . Within the indeterminacy region of the model’s parameter space (gray area), larger values of  $\xi_p$  increase inflation gap predictability. Therefore, the decline in the predictability resulting from the increase to  $\xi_p = 0.68$  is entirely due to eliminating indeterminacy. Within the determinacy region (white area), increasing  $\xi_p$  to the post-1982 posterior mean of 0.79 leads to slightly different changes among the three measures of inflation gap predictability: the increase slightly raises the one-quarter-ahead measure, while it slightly reduces the eight-quarters-ahead measure.

In short, the rise in firms’ probability of no price change also has a substantial effect on the decline in inflation gap persistence in our estimated model, but it works only through ruling out indeterminacy.

#### 4.4 Revisiting the policy implications of raising the inflation target

The analysis so far has related the historical decline in inflation gap persistence to the Fed’s change from a passive to an active policy response to the inflation gap. This in turn raises the question: How would other prominent possible changes in monetary policy affect inflation gap persistence? One prominent possible change to be investigated is an increase in the Fed’s inflation target. [Blanchard et al. \(2010\)](#), [Ball \(2013\)](#), and [Krugman \(2014\)](#) propose raising the inflation target in light of the effective lower bound (henceforth ELB) on nominal interest rates, from its current annualized rate of 2 percent to 4 percent or even higher. Recent research considers the implications of such a higher inflation target for welfare ([Ascari et al. \(2018\)](#)), equilibrium determinacy ([Khan et al. \(2020\)](#)), and inflation (gap) persistence ([Kurozumi and Van Zandweghe \(2019\)](#)). Because our model relates inflation dynamics to

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<sup>32</sup>One may wonder why higher values of firms’ probability of no price change can guarantee determinacy of equilibrium even when the monetary policy response to the inflation gap is passive at the pre-1979 posterior mean. This is because the pre-1979 posterior mean of the policy response to the GDP gap is positive, which causes higher values of the probability to lower the smallest value of the policy response to the inflation gap that brings about determinacy. This result is consistent with [Woodford \(2003\)](#), who considers the case of zero steady-state inflation, and [Kurozumi and Van Zandweghe \(2016\)](#), who employ a small-scale DSGE model with the Kimball-type aggregator of goods.

the steady-state inflation rate, it can provide another perspective on the debate about raising the inflation target.

Increasing the inflation target does not threaten a return to indeterminacy of equilibrium in our model. Indeed, even extremely high steady-state inflation (up to the rate of 49.6 percent annually) ensures determinacy along with the post-1982 posterior mean of the other estimated parameters of the model. This result is in stark contrast to the conclusion of [Khan et al. \(2020\)](#), who employ the CES aggregators of goods and labor and the specification of labor disutility as in [Erceg et al. \(2000\)](#) in a calibrated medium-scale DSGE model and find that even low steady-state inflation can induce indeterminacy unless the policy response to the inflation gap is very strong. In addition to the distinct specification of labor disutility in our model, the non-CES aggregators make the model much less susceptible to indeterminacy by leading higher steady-state inflation to enlarge the determinacy region of model parameters, unlike the CES aggregators, which cause higher steady-state inflation to shrink the determinacy region (see [Appendix B](#)). The data used in the model estimation favor our model (with the non-CES aggregators), which indicates that indeterminacy would not be a serious concern raised by a higher inflation target.

Even without an increase in inflation gap persistence caused by indeterminacy, a higher inflation target could still affect inflation dynamics. Indeed, in our model the degree of intrinsic inflation inertia depends on the steady-state inflation rate, among other parameters. To quantify the effect of higher steady-state inflation on inflation gap persistence, we consider a counterfactual increase in the steady-state inflation rate from 2 to 4 percent annually, keeping the other estimated model parameters fixed at their post-1982 posterior mean. Such an increase would nearly double the degree of intrinsic inflation inertia  $\lambda_{pe}$  from 0.16 to 0.30 and raise the three measures of inflation gap predictability by 33 to 47 percent.<sup>33</sup> These model predictions suggest some caution regarding calls to raise the Fed’s inflation target. While a higher inflation target would make the inflation gap more predictable, it could also make stabilizing the inflation gap more challenging for monetary policymakers as transitory

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<sup>33</sup>On the other hand, stabilizing the inflation target would reduce inflation gap persistence according to our estimated model. Setting the variance of the inflation target shock at zero and keeping the remaining estimated parameters fixed at their post-1982 posterior mean imply declines in the three measures of inflation gap predictability by 15 to 78 percent.

shocks would have more persistent effects on the gap.<sup>34</sup>

## 5 Concluding Remarks

This paper has examined the source of the decline in inflation gap persistence after the Volcker disinflation in the early 1980s. Previous research into the source has offered competing views, whose validity has been investigated only under determinacy of equilibrium. Our paper has analyzed the source by estimating a medium-scale DSGE model over the entire parameter space, allowing for both determinacy and indeterminacy. The estimated model has shown that the Fed’s change from a passive to an active policy response to the inflation gap can fully explain the decline in inflation gap persistence, primarily by ruling out indeterminacy and, to a lesser extent, by dampening responses to shocks under determinacy. A decrease in firms’ probability of price change can also provide a full account of the decline only through eliminating indeterminacy.

The paper treats the monetary policy response to the inflation gap and the probability of price change as independent sources of the decline in inflation gap persistence. In the literature there have been studies that consider the possibility that firms’ probability of price change responds to the stance of monetary policy, for example, a shift in the inflation target (see, e.g., [Ball et al. \(1988\)](#) and [Kurozumi \(2016\)](#)). Along the lines of the studies, [Kimura and Kurozumi \(2010\)](#) demonstrate that a more aggressive policy response to the inflation gap makes firms less likely to reset prices. Introducing such endogenous nominal price rigidity in our model might attribute the decline in inflation gap persistence only to the change from a passive to an active policy response to the inflation gap. This would be a fruitful agenda for future research.

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<sup>34</sup>As noted above, the debate on raising the Fed’s inflation target stems from the presence of the ELB, which presents a challenge to stabilizing the inflation gap around the current inflation target of two percent. In this context, the Fed recently adopted a flexible average inflation targeting (henceforth FAIT) strategy as described in its Statement on Longer-Run Goals and Monetary Policy, which is available on the Federal Reserve Board’s website at [https://www.federalreserve.gov/monetarypolicy/files/FOMC\\_LongerRunGoals.pdf](https://www.federalreserve.gov/monetarypolicy/files/FOMC_LongerRunGoals.pdf). We leave an analysis of inflation gap persistence in the presence of the ELB and the FAIT strategy for future research.

## Appendix

### A Composite Coefficients in Price and Wage New Keynesian Phillips Curves

The composite coefficients in the price NKPC (42)–(45) are given by

$$\begin{aligned} \kappa_p &\equiv \frac{(1 - \xi_p \pi^{\gamma_p - 1})(1 - \beta \xi_p \pi^{\gamma_p})}{\xi_p \pi^{\gamma_p - 1} [1 - \epsilon_{p2} \gamma_p / (\gamma_p - 1 - \epsilon_{p2})]}, & \kappa_{p\lambda} &\equiv \kappa_p - \tilde{\kappa}_{p\lambda} + \frac{\epsilon_{p2}(1 - \xi_p \pi^{\gamma_p - 1})(1 - \beta \xi_p \pi^{\gamma_p})}{\xi_p \pi^{\gamma_p - 1} [\gamma_p - 1 - \epsilon_{p2}(1 + \gamma_p)]}, \\ \kappa_{pd} &\equiv \gamma_p(\kappa_p - \tilde{\kappa}_{p\lambda}) - \beta \xi_p \pi^{\gamma_p} - \frac{1}{\xi_p \pi^{\gamma_p - 1}}, & \kappa_{p\varphi} &\equiv \frac{\beta(\pi - 1)(1 - \xi_p \pi^{\gamma_p - 1})}{1 - \epsilon_{p2}(1 + \gamma_p) / (\gamma_p - 1)}, \\ \kappa_{p\epsilon\psi} &\equiv \frac{\epsilon_{p2} \beta (\pi^{1 + \gamma_p} - 1)(1 - \xi_p \pi^{\gamma_p - 1})}{\pi^{\gamma_p} [\gamma_p - 1 - \epsilon_{p2}(1 + \gamma_p)]}, & \rho_{pd} &\equiv \frac{\xi_p \pi^{-1} (1 + \epsilon_{p1} \pi^{\gamma_p})}{1 + \epsilon_{p1}}, & \kappa_{p\epsilon d} &\equiv -\frac{\epsilon_{p1} \xi_p \pi^{-1} (\pi^{\gamma_p} - 1)}{(1 + \epsilon_{p1})(1 - \xi_p \pi^{-1})}, \end{aligned}$$

where

$$\epsilon_{p1} \equiv \epsilon_p \left( \frac{1 - \xi_p \pi^{\gamma_p - 1}}{1 - \xi_p} \right)^{\frac{\gamma_p}{1 - \gamma_p}}, \quad \epsilon_{p2} \equiv \epsilon_{p1} \frac{1 - \beta \xi_p \pi^{\gamma_p - 1}}{1 - \beta \xi_p \pi^{-1}}, \quad \tilde{\kappa}_{p\lambda} \equiv \frac{(1 - \xi_p \pi^{\gamma_p - 1})(1 - \beta \xi_p \pi^{\gamma_p - 1})}{\xi_p \pi^{\gamma_p - 1} [1 - \epsilon_{p2}(1 + \gamma_p) / (\gamma_p - 1)]}.$$

Likewise, the composite coefficients in the wage NKPC (47)–(50) are given by

$$\begin{aligned} \kappa_w &\equiv \frac{(1 - \xi_w \pi_w^{\gamma_w - 1})(1 - \beta \xi_w \pi_w^{\gamma_w})}{\xi_w \pi_w^{\gamma_w - 1} [1 - \epsilon_{w2} \gamma_w / (\gamma_w - 1 - \epsilon_{w2})]}, & \kappa_{w\lambda} &\equiv \kappa_w - \tilde{\kappa}_{w\lambda} + \frac{\epsilon_{w2}(1 - \xi_w \pi_w^{\gamma_w - 1})(1 - \beta \xi_w \pi_w^{\gamma_w})}{\xi_w \pi_w^{\gamma_w - 1} [\gamma_w - 1 - \epsilon_{w2}(1 + \gamma_w)]}, \\ \kappa_{wd} &\equiv \gamma_w(\kappa_w - \tilde{\kappa}_{w\lambda}) - \beta \xi_w \pi_w^{\gamma_w} - \frac{1}{\xi_w \pi_w^{\gamma_w - 1}}, & \kappa_{w\varphi} &\equiv \frac{\beta(\pi_w - 1)(1 - \xi_w \pi_w^{\gamma_w - 1})}{1 - \epsilon_{w2}(1 + \gamma_w) / (\gamma_w - 1)}, \\ \kappa_{w\epsilon\psi} &\equiv \frac{\epsilon_{w2} \beta (\pi_w^{1 + \gamma_w} - 1)(1 - \xi_w \pi_w^{\gamma_w - 1})}{\pi_w^{\gamma_w} [\gamma_w - 1 - \epsilon_{w2}(1 + \gamma_w)]}, & \rho_{wd} &\equiv \frac{\xi_w \pi_w^{-1} (1 + \epsilon_{w1} \pi_w^{\gamma_w})}{1 + \epsilon_{w1}}, & \kappa_{w\epsilon d} &\equiv -\frac{\epsilon_{w1} \xi_w \pi_w^{-1} (\pi_w^{\gamma_w} - 1)}{(1 + \epsilon_{w1})(1 - \xi_w \pi_w^{-1})}, \end{aligned}$$

where

$$\epsilon_{w1} \equiv \epsilon_w \left( \frac{1 - \xi_w \pi_w^{\gamma_w - 1}}{1 - \xi_w} \right)^{\frac{\gamma_w}{1 - \gamma_w}}, \quad \epsilon_{w2} \equiv \epsilon_{w1} \frac{1 - \beta \xi_w \pi_w^{\gamma_w - 1}}{1 - \beta \xi_w \pi_w^{-1}}, \quad \tilde{\kappa}_{w\lambda} \equiv \frac{(1 - \xi_w \pi_w^{\gamma_w - 1})(1 - \beta \xi_w \pi_w^{\gamma_w - 1})}{\xi_w \pi_w^{\gamma_w - 1} [1 - \epsilon_{w2}(1 + \gamma_w) / (\gamma_w - 1)]}.$$

### B Comparison of Labor Disutility Specifications in Terms of Equilibrium Determinacy

This appendix compares our specification of labor disutility—which follows [Schmitt-Grohé and Uribe \(2006\)](#) (henceforth SU)—with that of [Erceg et al. \(2000\)](#) (henceforth EHL) in terms of guaranteeing determinacy of equilibrium.

In the approach of EHL, the representative household has a continuum of members  $h \in [0, 1]$ , each of whom supplies a differentiated labor service  $l_t(h)$ , and derives disutility from

the labor services. Consequently, the household's preferences are described as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log \left( \tilde{C}_t - b C_{t-1} \right) \exp z_{c,t} - \int_0^1 \frac{(l_t(h))^{1+\chi}}{1+\chi} \exp \tilde{z}_{w,t} dh \right].$$

Then, under staggered wage-setting, the fraction  $1 - \xi_w$  of (nominal) wages is set so as to maximize the relevant utility function

$$E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left[ -\frac{(l_{t+j|t}(h))^{1+\chi}}{1+\chi} \exp \tilde{z}_{w,t+j} + \Lambda_{t+j} \frac{P_t W_t(h)}{P_{t+j}} l_{t+j|t}(h) \right]$$

subject to the same demand curve as in Section 2.3. Consequently, the first-order condition for utility maximization with respect to the wage is given by

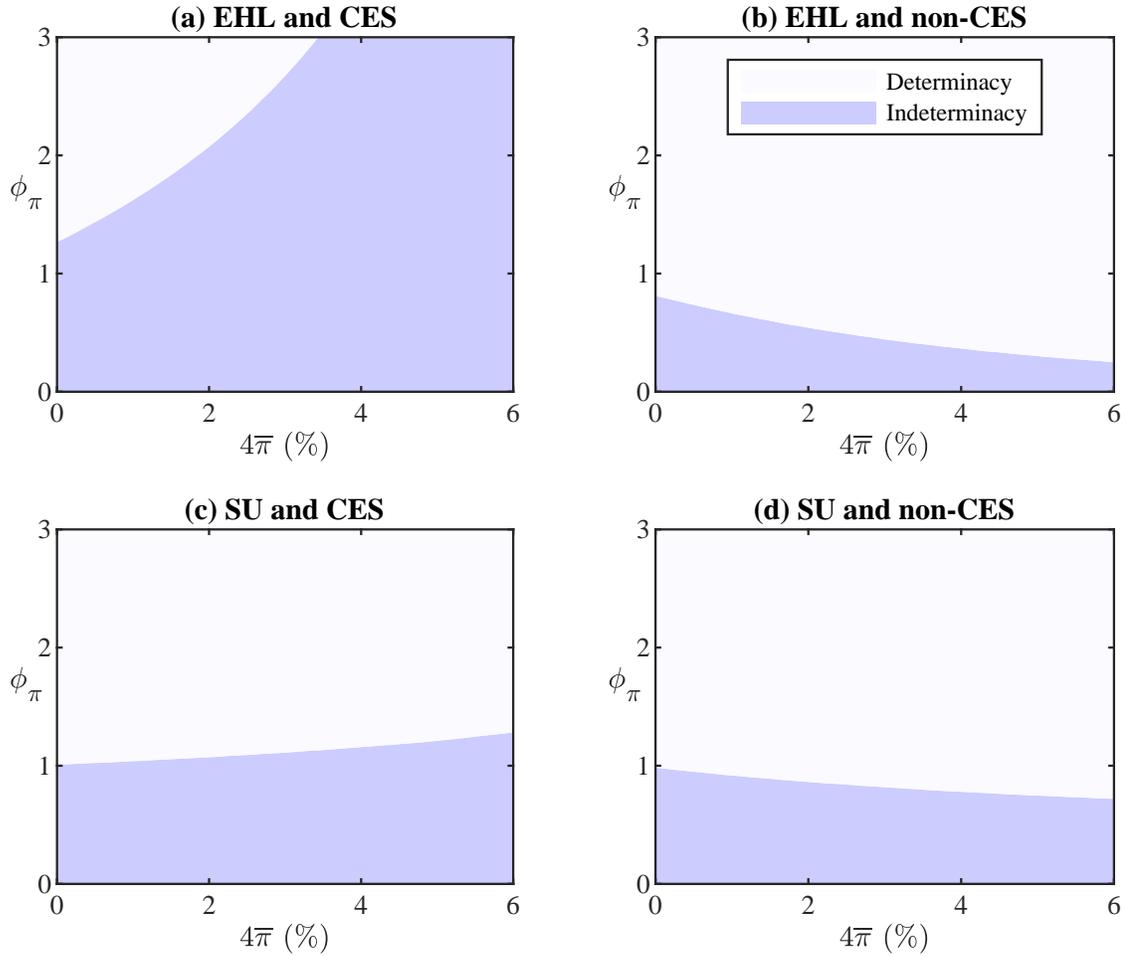
$$\begin{aligned} E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t} l_{t+j} \left( \frac{W_t^*/W_t}{d_{w,t+j}} \right)^{-\gamma_w} \prod_{k=1}^j \pi_{w,t+k}^{\gamma_w} \left( \frac{W_t^*}{W_t} \prod_{k=1}^j \pi_{t+k}^{-1} - \frac{\gamma_w}{\gamma_w - 1} \left\{ \frac{l_{t+j}}{1 + \epsilon_w} \left[ \left( \frac{W_t^*/W_t}{d_{w,t+j}} \right)^{-\gamma_w} \right. \right. \right. \\ \left. \left. \left. \times \prod_{k=1}^j \pi_{w,t+k}^{\gamma_w} + \epsilon_w \right] \right\}^{\chi} \frac{\exp \tilde{z}_{w,t+j}}{\Lambda_{t+j} W_{t+j}} \prod_{k=1}^j \frac{W_{t+k}}{W_{t+k-1}} \right) = E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t} l_{t+j} \frac{\epsilon_w}{\gamma_w - 1} \frac{W_t^*}{W_t} \prod_{k=1}^j \pi_{t+k}^{-1}. \end{aligned} \quad (57)$$

This condition is the counterpart of our model's four equilibrium conditions (24), (28), (29), and (30), which implies that relative wage distortion has no influence on wage inflation dynamics if labor disutility follows the EHL specification. The other equilibrium conditions are the same as those of our model by assuming complete contingent-claims markets for consumption.

We now compare the SU specification of labor disutility with that of EHL in terms of ensuring determinacy, using the two fixed parameter values of  $\delta = 0.025$  and  $g/y = 0.194$  and the prior mean of the other parameters presented in Table 1. Figure 3 illustrates the effects of the different specifications of labor disutility and aggregators of goods and labor on the determinacy region of the model's parameter space. In each panel the horizontal and vertical axes represent the annualized steady-state inflation rate  $4\bar{\pi}$  and the monetary policy response to the inflation gap  $\phi_{\pi}$ , respectively.

The two top panels show the model with the EHL specification of labor disutility. In the top left panel, the case of CES aggregators (i.e.,  $\epsilon_p = \epsilon_w = 0$ ) is presented. In this case higher steady-state inflation sharply increases the minimum policy response to the inflation

Figure 3: Effects of different specifications of labor disutility and aggregators of goods and labor on equilibrium determinacy region of the parameter space



Notes: For the annualized steady-state inflation rate ( $4\bar{\pi}$ ) and the monetary policy response to the inflation gap  $\phi_\pi$ , each panel of the figure displays equilibrium determinacy and indeterminacy regions of the parameter space, using the two fixed parameter values of  $\delta = 0.025$  and  $g/y = 0.194$  and the prior mean of the other parameters reported in Table 1. Panels (a) and (b) show the cases of the CES and the Kimball-type non-CES aggregators of goods and labor, respectively, in the model with the [Erceg et al. \(2000\)](#) (EHL) specification of labor disutility, while panels (c) and (d) present those in our model (with the [Schmitt-Grohé and Uribe \(2006\)](#) (SU) specification).

gap that is required for determinacy, as indicated by [Khan et al. \(2020\)](#). For example, the minimum response is  $\phi_\pi = 2.07$  and  $\phi_\pi = 3.50$  at the annualized steady-state inflation rate of 2 and 4 percent, respectively. However, once the aggregators become the Kimball-type non-CES ones (i.e.,  $\epsilon_p = \epsilon_w = -3$ ), higher steady-state inflation decreases the minimum policy response, as seen in the top right panel. In particular, the Taylor principle (i.e.,  $\phi_\pi > 1$ ) then serves as a sufficient condition for determinacy in the panel.

Turning to our model (with the SU specification of labor disutility), the two bottom panels illustrate the cases of the CES and the Kimball-type non-CES aggregators, respectively. The qualitative properties of the effects of the aggregators on the determinacy region are the same as in the model with the EHL specification. That is, higher steady-state inflation increases the minimum policy response to the inflation gap required for determinacy in the case of the CES aggregators, whereas it decreases the minimum policy response in the case of the non-CES aggregators. Yet quantitatively, these effects are much more mild in our model. This result indicates that our model is much less susceptible to indeterminacy induced by high steady-state inflation than the model with the EHL specification.

## C Comparison with the Case of CES Aggregators

This appendix compares the roles of the non-CES aggregators of goods and labor employed in our model and the widely used CES aggregators in terms of model estimation results, the effect on the decline in inflation gap persistence, and the source of the decline. Before proceeding, it is worth mentioning that our model (with the non-CES aggregators) empirically outperforms its counterpart with CES aggregators during both the pre-1979 and the post-1982 periods, as noted in [Section 4.1](#).

[Table 5](#) reports the posterior estimates of model parameters for the pre-1979 and post-1982 samples in the case of CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ . The last line of the table displays the posterior probability of equilibrium determinacy  $\mathbb{P}\{\boldsymbol{\theta} \in \Theta^D | \mathbf{Y}^T\}$  in the two periods, and shows the same result as that obtained in our estimated model (with the non-CES aggregators): the US economy was likely in the indeterminacy region of the model's parameter space during the pre-1979 period, while the economy likely entered the determinacy region during the post-1982 period. The table also indicates that the posterior

Table 5: Posterior estimates of model parameters in the case of CES aggregators:  $\epsilon_p = \epsilon_w = 0$ 

Parameter	Pre-1979		Post-1982	
	Mean	90% interval	Mean	90% interval
$\bar{\pi}$	0.906	[0.680, 1.129]	0.653	[0.389, 0.910]
$\bar{r}$	1.482	[1.331, 1.643]	1.395	[1.184, 1.609]
$\overline{g\Upsilon}$	0.421	[0.325, 0.513]	0.427	[0.322, 0.522]
$\overline{g\overline{\Psi}}$	0.312	[0.226, 0.397]	0.377	[0.291, 0.461]
$\bar{l}$	-0.073	[-0.147, -0.013]	-0.020	[-0.094, 0.052]
$b$	0.606	[0.533, 0.680]	0.623	[0.530, 0.723]
$\chi$	0.957	[0.493, 1.401]	2.524	[1.716, 3.336]
$\alpha$	0.234	[0.208, 0.257]	0.235	[0.206, 0.263]
$\phi$	0.442	[0.392, 0.492]	0.486	[0.427, 0.542]
$\delta_2/\delta_1$	0.872	[0.752, 1.007]	0.788	[0.653, 0.917]
$\zeta$	1.197	[0.804, 1.575]	2.174	[1.313, 2.917]
$\theta_p - 1$	9.826	[8.207, 11.473]	9.651	[7.804, 11.586]
$\theta_w - 1$	9.526	[7.688, 11.310]	9.787	[7.273, 12.413]
$\xi_p$	0.453	[0.385, 0.519]	0.603	[0.555, 0.650]
$\xi_w$	0.744	[0.691, 0.799]	0.515	[0.430, 0.608]
$\phi_r$	0.796	[0.739, 0.864]	0.793	[0.746, 0.838]
$\phi_\pi$	0.415	[0.190, 0.614]	2.626	[2.103, 3.112]
$\phi_y$	0.102	[0.060, 0.145]	0.022	[0.007, 0.036]
$\phi_{gy}$	0.141	[0.072, 0.214]	0.137	[0.057, 0.199]
$\rho_\pi$	0.946	[0.894, 1.000]	0.976	[0.956, 0.999]
$\rho_a$	0.099	[0.024, 0.170]	0.040	[0.006, 0.072]
$\rho_c$	0.507	[0.291, 0.734]	0.741	[0.569, 0.927]
$\rho_i$	0.587	[0.350, 0.819]	0.758	[0.675, 0.849]
$\rho_g$	0.848	[0.775, 0.923]	0.964	[0.941, 0.989]
$\rho_p$	0.628	[0.515, 0.746]	0.973	[0.947, 0.997]
$\rho_w$	0.624	[0.486, 0.769]	0.964	[0.940, 0.990]
$\sigma_\pi$	0.114	[0.059, 0.165]	0.068	[0.043, 0.090]
$\sigma_a$	1.135	[0.968, 1.299]	0.820	[0.702, 0.942]
$\sigma_c$	1.357	[1.057, 1.636]	1.685	[1.162, 2.230]
$\sigma_i$	0.640	[0.302, 0.994]	1.721	[1.229, 2.225]
$\sigma_g$	2.617	[2.339, 2.922]	2.037	[1.825, 2.225]
$\sigma_p$	0.450	[0.326, 0.568]	0.184	[0.138, 0.223]
$\sigma_w$	0.386	[0.302, 0.466]	0.960	[0.551, 1.307]
$\sigma_r$	0.283	[0.233, 0.330]	0.208	[0.171, 0.245]
$\sigma_s$	0.536	[0.374, 0.705]	-	-
$M_\pi$	0.399	[-0.941, 1.676]	-	-
$M_a$	0.254	[0.057, 0.476]	-	-
$M_c$	-0.235	[-0.395, -0.068]	-	-
$M_i$	-0.124	[-0.704, 0.385]	-	-
$M_g$	-0.171	[-0.261, -0.085]	-	-
$M_p$	-0.256	[-0.766, 0.250]	-	-
$M_w$	-0.360	[-1.081, 0.285]	-	-
$M_r$	0.789	[-0.087, 1.659]	-	-
$\log p(\mathbf{Y}^T)$		-417.861		-654.435
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D   \mathbf{Y}^T\}$		0.000		1.000

Notes: This table reports the posterior mean and 90 percent HPD intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table,  $\log p(\mathbf{Y}^T)$  represents the SMC-based approximation of log marginal data density and  $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\}$  denotes the posterior probability of equilibrium determinacy.

estimates of 13 parameters— $\chi$ ,  $\xi_p$ ,  $\xi_w$ ,  $\phi_\pi$ ,  $\phi_y$ ,  $\rho_g$ ,  $\rho_p$ ,  $\rho_w$ ,  $\sigma_a$ ,  $\sigma_i$ ,  $\sigma_g$ ,  $\sigma_p$ , and  $\sigma_w$ —changed substantially in that each parameter’s 90 percent HPD intervals in the two periods are disjoint. Among the changes in the parameters, the following three are worth noting. First, the monetary policy response to the inflation gap  $\phi_\pi$  exhibited an even larger increase than in our estimated model, and demonstrates a change from a passive to an active policy response in line with the result of our model. Second, the policy response to the GDP gap  $\phi_y$  fell substantially. Third, the probability of no price change  $\xi_p$  displayed a smaller increase than in our estimated model, whereas the probability of no wage change  $\xi_w$  decreased.

Based on the posterior mean estimates of parameters for the pre-1979 and post-1982 periods, Table 6 presents the variability of the inflation gap and its predictability at forecasting horizons of one, four, and eight quarters in the case of CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ . This case also exhibits the decline in inflation gap persistence, although the decline in the eight-quarters-ahead measure of the predictability is much smaller than in our estimated model.

Table 6: Inflation gap variability and predictability in the case of CES aggregators

	$std(\hat{\pi}_t - z_{\pi,t})$	$R_1^2$	$R_4^2$	$R_8^2$
Pre-1979 period	1.050	0.817	0.489	0.189
Post-1982 period	0.385	0.512	0.222	0.179
Percent change	-63	-37	-55	-5

Note: The statistics are calculated using the posterior mean estimates of parameters in the case of CES aggregators (i.e.,  $\epsilon_p = \epsilon_w = 0$ ) reported in Table 5.

Table 7 presents the results of counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of parameters indicated in the first column of the table are altered to their post-1982 posterior mean in the case of CES aggregators of goods and labor, i.e.,  $\epsilon_p = \epsilon_w = 0$ . In this case, the changes in all estimated monetary policy parameters— $\phi_r$ ,  $\phi_\pi$ ,  $\phi_y$ ,  $\phi_{gy}$ ,  $\pi$ ,  $\rho_\pi$ ,  $\sigma_\pi$ , and  $\sigma_r$ —to their post-1982 posterior mean can explain more than the entire declines in inflation gap predictability as shown on the first line of the table, and moreover, most of the contribution of the changes in monetary policy parameters is attributed to the change from a passive to an active policy response to the inflation gap—which generates a shift to determinacy—as indicated on the second line. These results are consistent with those obtained with our estimated model. One difference

from the result with our model is that the decrease in the policy response to the GDP gap  $\phi_y$  plays a secondary role in the contribution of the changes in policy parameters as seen on the third to fourth lines. The changes in the other policy parameters— $\phi_r$ ,  $\phi_{gy}$ ,  $\pi$ ,  $\rho_\pi$ ,  $\sigma_\pi$ , and  $\sigma_r$ —do not contribute meaningfully to the decline in inflation gap predictability. As for the changes in nonpolicy parameters, they can also account for the decline in the predictability as displayed on the fifth line. In addition, lines 6 and 7 demonstrate that most of the contribution of the changes in nonpolicy parameters is attributed to the decrease in the probability of no wage change  $\xi_w$  but not to the increase in the probability of no price change  $\xi_p$ , which contrasts sharply with the results obtained with our estimated model. Notably, the decrease in  $\xi_w$  reduces inflation gap predictability even though it does not bring about a shift to determinacy. The changes in the other nonpolicy parameters, including the parameters governing processes of all nonpolicy shocks (line 8), do not contribute to the decline in the predictability.

Table 7: Counterfactual exercises in the case of CES aggregators: Accounting for the decline in inflation gap variability and predictability

Model parameter	$\text{std}(\hat{\pi}_t - z_{\pi,t})$	$R_1^2$	$R_4^2$	$R_8^2$	determinacy
1. Post-1982 policy	72	173	138	1,121	yes
2. Post-1982 $\phi_\pi$	43	131	116	711	yes
3. Post-1982 $\phi_y$	9	56	68	467	no
4. Post-1982 $\phi_\pi, \phi_y$	65	170	149	1,474	yes
5. Post-1982 nonpolicy	-348	85	146	1,290	no
6. Post-1982 $\xi_p$	-11	-1	-13	-17	no
7. Post-1982 $\xi_w$	-34	89	127	1,196	no
8. Post-1982 nonpolicy shocks	-1,220	-25	-78	-3,408	no

Notes: The table shows the percentage changes in inflation gap variability and predictability in counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of parameters indicated in the first column are changed to their post-1982 posterior mean reported in Table 5. “Post-1982 policy” refers to the post-1982 posterior mean of  $\phi_r$ ,  $\phi_\pi$ ,  $\phi_y$ ,  $\phi_{gy}$ ,  $\pi$ ,  $\rho_\pi$ ,  $\sigma_\pi$ , and  $\sigma_r$ , while “Post-1982 nonpolicy” refers to the post-1982 posterior mean of the other estimated parameters.

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