The State of Climate Finance

- To meet the internationally agreed climate objectives by 2030, an increase of at least 590% in annual climate finance is needed (CPI, 2021).
- While total climate investment has steadily increased over the past decade, flows have been slowing over the last few years however (CPI, 2021).
- The majority of climate finance 61% (US\$ 384 bn) was raised as debt. Equity investments came next with 33% of total investments, mostly directed towards energy systems (CPI, 2021).

Rationale and Main Objectives

A potential motive for the low investments in green assets, via equity markets, is the existence of an asymmetry with regards the availability and the value of information on the risk-return profiles of the green against fossil-fuel sectors.

Investors are more informed and have access to richer sources of information about mature industries than emerging ones. This can be reflected in the difference of costs of the underlying assets' private information.

- The paper aims at providing a theoretical framework to rationalize the behavior of decision makers with respect their investment decisions and how it can lead to asset allocation bias.
- We build upon the literature of investment under incomplete information, and employ a combined learning-investment portfolio model.

The ability to learn about a specific asset's returns and volatility reduces the uncertainty about that asset and encourages the investor to make more confident decisions.

A key characteristic of a combined learning-investment model is the role of the investor's attention in shaping her optimal investment rules.

Model

The financial market consists of one risk-free asset (bond) B_t and n = 2 risky assets P_t^i . The risky assets represent two different sectors: a dirty sector (i.e. *brown* asset) and a clean sector (i.e. green asset). The risk-free asset pays a constant interest rate r^{f} , and satisfies:

$$dB_t = B_t r^f dt$$

The risky assets are assumed to be correlated, and the price P_t of the risky asset i is approximated by the following stochastic differential equation.

$$dP_t^i \approx P_t^i \cdot \left[\nu_t^i dt + \sum_{j=1}^n \sigma_{P_{ij}}(t) \mathbb{Z}_{P_j,t} + d\zeta_t^i \right]$$

The instantaneous expected rate of return, ν_t , of asset i is unobservable and is goverened by a mean-reverting process. The investor has a-priori beliefs w.r.t the initial position of ν_0^i .

$$d\nu_t^i = \theta^i (\bar{\nu}^i - \nu_t^i) dt + \sqrt{1 - \xi_v^2 \sigma_\nu^i d\mathbb{Z}_{\nu,t} + \xi_\nu \sigma_\nu^i d\mathbb{Z}_{P,t}}$$

In addition to public information derived from changes in asset prices, investors process private information (obtained at a certain cost). The private signal S_t evolves as follow:

$$dS_t^i = \nu_t^i dt + \underline{\sigma}_S^i d\mathbb{Z}_{S,t}$$

The private signal flow, dS_t^i , is a function of a drift term that is equal to the fundamental value ν_t^i and a constant volatility parameters, σ_S^i .

Fossil Fuel Equity Bias

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Investor's Objective and Optimal Policies

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(4)

The investor chooses (conditional on her information set \tilde{I}_t) a strategy of consumption and attention/investment so to maximize her expected lifetime utility of consumption subject to her budget constraint dW_t .

$$U(c) = \mathbb{E}_t \left[\int_{s=t}^{\infty} e^{-\delta(s-t)} u(c_s) \, ds \, \middle| \, \tilde{I}_t \right] \tag{5}$$

Her decisions are based on the updated (Bayesian updating) posterior mean of the fundamental value $\hat{\nu}_t^i$ and variance of the belief $\hat{\sigma}^i(t)$, representing the uncertainty. The investor's wealth process $\{W_t\}$ satisfies a dynamic self-financing budget constraint:

$$dW_t = W_t \left[\sum_{i=1}^n \omega_t^i \frac{dP_t^i}{P_t^i} + \left(1 - \sum_{i=1}^n \omega_t^i \right) \frac{dB_t}{B_t} \right] - \left(c_t + \sum_{i=1}^n \underbrace{\tilde{K}\left(a_t^i\right)}_{\text{cost of private signal}} \right) dt \quad (6)$$

Proposition: For an investor withh CRRA utility and assuming a quadratic information cost function $\tilde{K}(a_t^i)$; then given the investor's inferences of the drift $\hat{\nu}_t^i$, the optimal consumption c_t^* , the optimal attention a_t^* , and the optimal investment rule ω_t^* of asset *i* are given by:

 $c_t^* = W/g$

$$\omega_{i,t}^{*} = \frac{1}{\gamma} \underbrace{\left(\frac{\left(\sigma_{\Lambda}^{j} \rho^{j} + \sigma_{P,j}^{2}\right) \left(\hat{\nu}_{t}^{i} - r^{f}\right) - \chi_{p} \left(\hat{\nu}_{t}^{j} - r^{f}\right)}{\prod_{i=1}^{n} (\sigma_{\Lambda}^{i} \rho^{i} + \sigma_{P,i}^{2}) - \chi_{p}^{2}} \right)} + \underbrace{\left(\frac{\left(\sigma_{\Lambda}^{j} \rho^{j} + \sigma_{P,j}^{2}\right) \left(\hat{\sigma}^{i}(t) + \xi_{\nu}^{i} \sigma_{\nu}^{i} \sigma_{P}^{j}\right)}{\prod_{i=1}^{n} (\sigma_{\Lambda}^{i} \rho^{i} + \sigma_{P,i}^{2}) - \chi_{p}^{2}} \frac{g_{W\nu^{i}}}{g} - \chi_{p} \frac{\left(\hat{\sigma}^{j}(t) + \xi_{\nu}^{j} \sigma_{\nu}^{j} \sigma_{P}^{j}\right)}{\prod_{i=1}^{n} (\sigma_{\Lambda}^{i} \rho^{i} + \sigma_{P,i}^{2}) - \chi_{p}^{2}} \frac{g_{W\nu^{j}}}{g} \right)}_{\text{hedging component}}$$

$$a_{i,t}^{*} = \hat{\sigma}_{i}^{2}(t) \frac{\gamma}{i} \left(\frac{1}{1-t} \left(\frac{1}{2}g_{\nu\nu_{i}} - \frac{1}{2}g_{\sigma_{i}}}\right)\right)$$

$$(9)$$

$$a_{i,t}^* = \hat{\sigma}_i^2(t) \frac{\gamma}{\kappa^i} \left(\frac{1}{1-\gamma} \left(\frac{1}{4} \frac{g_{\nu\nu_i}}{g} \right) \right)$$

for $i \neq j$. The value function g and its partial derivatives are determined numerically.

Cost of Information and Diversification

Two main factors drive attention to asset returns:

- The pure state risk aversion factor, which provides a measure of the degree of like or dislike by the investor with respect to variations in expected returns $\hat{\nu}_t$, itself determined by $\hat{\sigma}(t)$.
- The uncertainty factor measures the extent to which the investor likes or dislikes uncertainty $\hat{\sigma}(t)$ by itself.

Assuming a quadratic cost function:

- Optimal attention becomes an increasing quadratic function of uncertainty $\hat{\sigma}(t)$: The higher the uncertainty the higher the level of attention to reduce it.
- Optimal attention is reduced by the cost level, κ . The higher the κ the costlier it becomes to learn about a risky asset.

A high risk-averse CRRA investor will demand less of an asset as the costs of acquiring information about this asset increase. Asset allocation bias can thus be explained through information costs variations.



 $-\frac{1}{2}\frac{g}{g}$

To gain better insights and determine the quantitative effects of information cost on the investor's portfolio decision problem, we calibrate the model and run numerical simulations. The model is numerically solved by means of the projections method (least-square). The projection method is more accurate than the perturbabtion method, and suffers less from the curse of dimensionality than the linear programming technique.

Parameter Descriptions

Volatility of the asset price Volatility of the fundamen Correlation (asset price/fu Mean-reversion speed Long-term mean of funda

note: other common parameters include: $r^f = 0.02$ (ris-free rate); $\gamma = 3.2$ (risk-aversion parameter); $\rho = 0.98$ (discount factor); and $\xi = 0.3$ (asset-correlation)

Figures

(7)



Figure 1. Optimal attention

- the value function.
- Information cost will in this case have a greater impact on asset allocation.

Numerical Exercise

	Notations	F(ossil)	G(reen)
е	σ_p	0.09	0.11
ntal value	$\sigma_{ u}$	0.09	0.07
undamental value)	$\xi_ u$	-0.8	-0.6
	heta	0.04	0.05
imental value	$ar{ u}$	0.06	0.08

Table 1. Parameter values



Discussion

Increasing information cost of the green asset private signal as compared to the fossil-fuel based asset led to a noticeable decrease in attention as a function of uncertainty (shown in figure 1) and a slight decrease in asset allocation of the green asset (figure 2).

• In this model, optimal attention does not directly affect asset allocations, it is rather through

• If asset price movements dP_t^i are correlated with the private signal S_t . Then, reducing correlated errors will be directly reflected in the investor's optimal asset allocation decision.

Information acquisition can be considered in this model as another asset acquisition.

References

[2] Robert C. Merton. Optimum consumption and portfolio rules in a continuous-time model. Journal of Economic Theory, 3:373–413,

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