The State of Climate Finance

- To meet the internationally agreed climate objectives by 2030, an increase of at least 590% in annual climate finance is needed (CPI, 2021).
- While total climate investment has steadily increased over the past decade, flows have been slowing over the last few years however (CPI, 2021).
- The majority of climate finance – 61% (US$ 384 bn) – was raised as debt. Equity investments came next with 33% of total investments, mostly directed towards energy systems (CPI, 2021).

Rationale and Main Objectives

A potential motive for the low investments in green assets, via equity markets, is the existence of an asymmetry with regards the availability and the value of information on the risk-return profiles of the green against fossil-fuel sectors.

Investors are more informed and have access to richer sources of information about mature industries than emerging ones. This can be reflected in the differences of costs of the underlying assets' private information.

- The paper aims at providing a theoretical framework to rationalize the behavior of decision makers with respect to their investment decisions and how it can lead to asset allocation bias.
- We build upon the literature of investment under incomplete information, and employ a combined learning-investment portfolio model.

The ability to learn about a specific asset's returns and volatility reduces the uncertainty about that asset and encourages the investor to make more confident decisions.

A key characteristic of a combined learning-investment model is the role of the investor's attention in shaping her optimal investment rules.

Model

The financial market consists of one risk-free asset (bond) B_t and 2 risky assets P^g_t and P^f_t. The risky assets represent two different sectors: a dirty sector (i.e., brown asset) and a clean sector (i.e., green asset). The risk-free asset pays a constant interest rate r_f, and satisfies:

$$d B_t = r_f B_t dt$$  \[1\]

The risky assets are assumed to be correlated, and the price of the risky asset s is approximated by the following stochastic differential equation:

$$d S_t = \mu S_t dt + \sigma S_t dW_t$$  \[2\]

The instantaneous expected rate of return, \(\mu_t\), of asset s is unobservable and is governed by a mean-reverting process. The investor has a prior belief \(\theta_0\), the initial position of \(\epsilon_0\).

$$d \theta_t = \theta_0 - \frac{\theta_t}{\kappa} dt + \sqrt{2} \epsilon_t dZ_t \quad \epsilon_t \sim N(0, 1)$$  \[3\]

In addition to public information derived from changes in asset prices, investors process private information (obtained at a certain cost). The private signal \(S_t\) evolves as follows:

$$d S_t = \theta_t dt + \sigma S_t d\epsilon_t$$  \[4\]

The private signal flow, \(d S_t\), is a function of a drift term that is equal to the fundamental value \(\mu_t\) and a constant volatility parameters, \(\sigma^2\).

Cost of Information and Diversification

Two main factors drive attention to asset returns:

- The pure state risk aversion factor, which provides a measure of the degree of like or dislike by the investor with respect to variations in expected returns \(\Delta r_t\) itself determined by \(\theta\).
- The uncertainty factor measures the extent to which the investor likes or dislikes uncertainty \(\Delta \theta_t\) by itself.

Assuming a quadratic cost function:

- Optimal attention becomes an increasing quadratic function of uncertainty \(\Delta \theta_t\). The higher the uncertainty the higher the level of attention to reduce it.
- Optimal attention is reduced by the cost level, \(c\). The higher the \(c\), the costlier it becomes to learn about a risky asset.

A high-risk-averse CRRA investor will demand less of an asset as the costs of acquiring information about this asset increase. Asset allocation bias can thus be explained through information costs variations.

Investor's Objective and Optimal Policies

The investor chooses (conditional on her information set \(\mathcal{I}\)) a strategy of consumption and attention/investment so to maximize her expected lifetime utility of consumption subject to her budget constraint \(\mathcal{B}_t\).

$$U(c) = E_{\mathcal{F}_0} \left[ \int_0^\infty e^{-r_t u_t} (c_t + \lambda \Delta r_t) dt \right]$$  \[5\]

Her decisions are based on the updated (Bayesian updating) posterior mean of the fundamental value \(\bar{\mu}\) and variance of the belief \(\bar{\theta}\), representing the uncertainty. The investor's wealth process \(\mathcal{W}_t\) satisfies a dynamic self-financing budget constraint:

$$d W_t = W_t \left( 1 + \frac{1}{2} \sum_{i=1}^2 \left[ \hat{\rho}_{i} \left( S_{i,t} + \hat{\sigma}_{i} d\epsilon_{i,t} \right) \right] \right) dt$$  \[6\]

Proposition: For an investor with CRRA utility and assuming a quadratic information cost function \(\bar{K}(\bar{\theta}_t)\), then given the investor's inferences of the drift \(\bar{\mu}_t\), the optimal consumption \(\bar{c}_t\), the optimal attention \(\bar{\omega}_t\), and the optimal investment rule \(\bar{\nu}_t\) of asset s are given by:

$$\bar{\nu}_t = W_t$$  \[7\]

$$\bar{\omega}_t = \frac{\mu_t - \theta_t}{\sigma^2}$$  \[8\]

$$\bar{\sigma}^2 = \frac{\mu_t^2}{\kappa/(1 - \theta_t/\kappa)}$$  \[9\]

for \(i \neq j\). The value function \(g\) and its partial derivatives are determined numerically.

Cost of Information and Diversification

To gain better insights and determine the quantitative effects of information cost on the investor's portfolio decision problem, we calibrate the model and run numerical simulations. The model is numerically solved by means of the projection method (least-square). The projection method is more accurate than the perturbation method, and suffers less from the curse of dimensionality than the linear programming technique.

Numerical Exercise

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### References