

GEKKO
DYNAMIC OPTIMIZATION

ACEMOGLU + CHERNOZHUKOV + WERNING
+ WHINSTON

OPTIMAL TARGETED LOCKDOWNS IN A MULTI-GROUP SIR MODEL

(MIT)

(MIT)

(MIT)

(MIT & SLOAN)

THIS PAPER

- ▶ **Policy analysis for COVID-19...**
- ▶ **Epi:** herd immunity, effect of mitigation, timing, etc.
- ▶ **Econ:** costs of lockdowns, optimal policy, etc.
- ▶ **COVID-19:** very asymmetric effects

Age Group	Mortality rate
20-49	0.001
50-64	0.01
65+	0.06

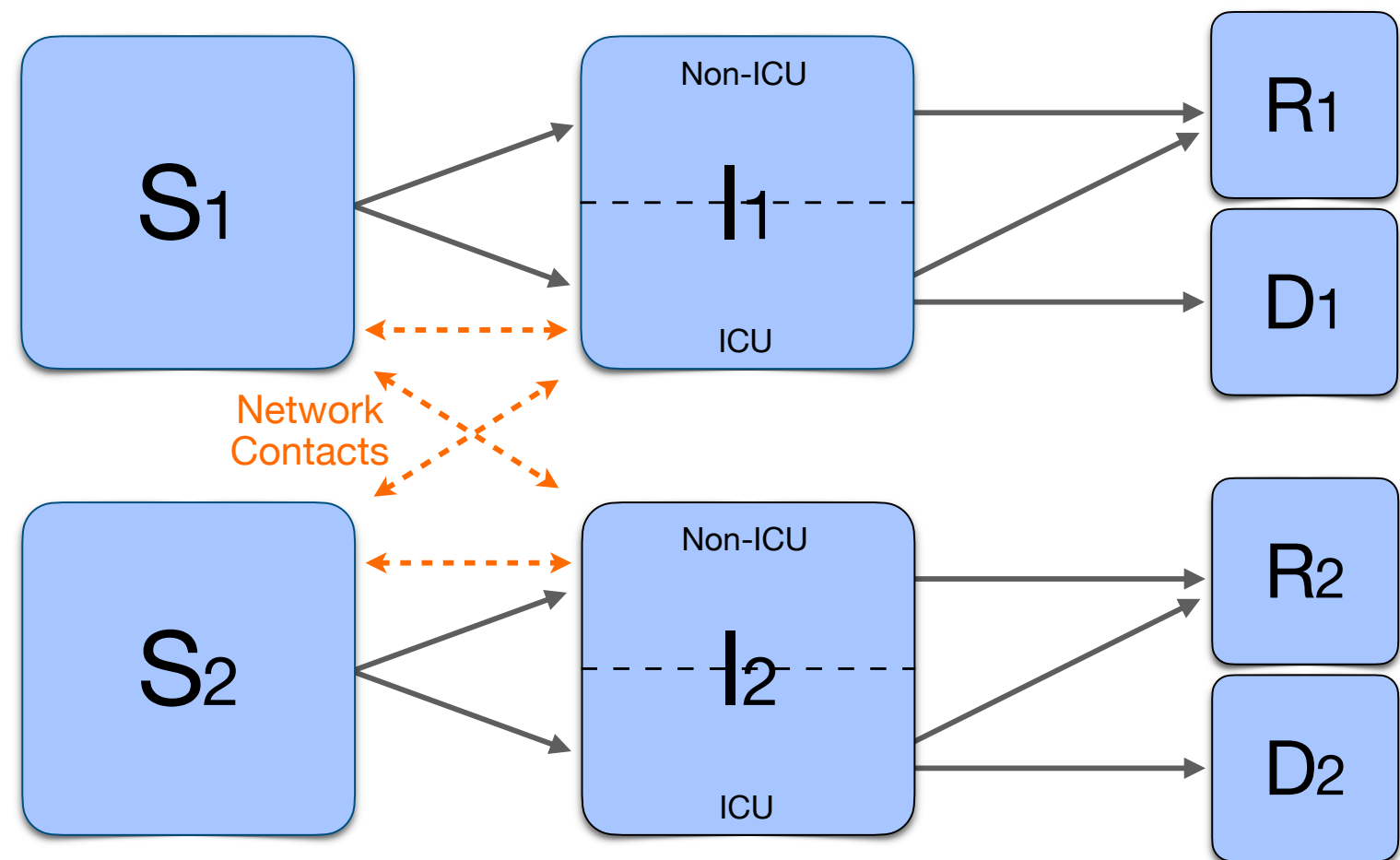
(Ferguson, 2020)

This paper: simple multi-group model
+
explore optimal policy implications

IMPORTANT CAVEATS

- ▶ We are obviously not epidemiologists, but we made our best effort to integrate Econ with Multi-Group SIR models as well as calibrate the model.
- ▶ Used Ferguson $R_0 = 2.4 \times .8$ (to reflect mask use etc).
- ▶ Not a literal policy prescription: The goal is to find areas of policy focus where welfare gains are the largest.
- ▶ Private behavior can help ease some implementational aspects.

SIR MODEL + ECONOMIC COSTS



- ▶ “Lockdowns”
 - ▶ benefit: reduce interactions, lower infections
 - ▶ costs: lost output
- ▶ Optimal Control timing of lockdown

TWO OBJECTIVES

- Trade-off between deaths and economic losses
- Costs:
 - Lives lost = $\sum_j D_j(T)$
 - Economic losses: $\int_0^T \sum_j \psi_j(t) dt$
- Goal: Choosing policies to minimize overall losses:

$$\int_0^T \sum_j \Psi_j(t) dt + \chi \sum_j D_j(T).$$

- ▶ Or, Safety-Focus: Min EL st LL < .001
- ▶ Or, Econ-Focus: Min LL st EL < .10

ECON LOSS AND DYNAMIC STRUCTURE:

The economic losses for group j are given by

$$\begin{aligned}\Psi_j(t) = & (1 - \xi_j)w_j S_j(t)L_j(t) + \\ & + (1 - \xi_j)w_j E_j(t)(1 - \eta_k^E(1 - L_j(t))) + \\ & + (1 - \xi_j)w_j I_j(t)(1 - \eta_k^E \eta_k^I(1 - L_j(t))) + \\ & + (1 - \xi_j)w_j(1 - \kappa_j)R_j(t)L_j(t) + \\ & + w_j \Delta_j l_j \delta_j^d(t) I_j(t),\end{aligned}$$

$$\dot{E}_j = M_j(S, E, I, R, L; \alpha) \beta (1 - \theta_j L_j) S_j \sum_k \rho_{jk} \eta_k^E \eta_k^I (1 - \theta_k L_k) I_k - \gamma_j^E E_j$$

$$\dot{S}_j = -\dot{E}_j - \gamma_j^E E_j,$$

$$\dot{I}_j = \gamma_j^E E_k - \gamma_j^I I_j,$$

$$\dot{D}_j = \delta_j^d H_j,$$

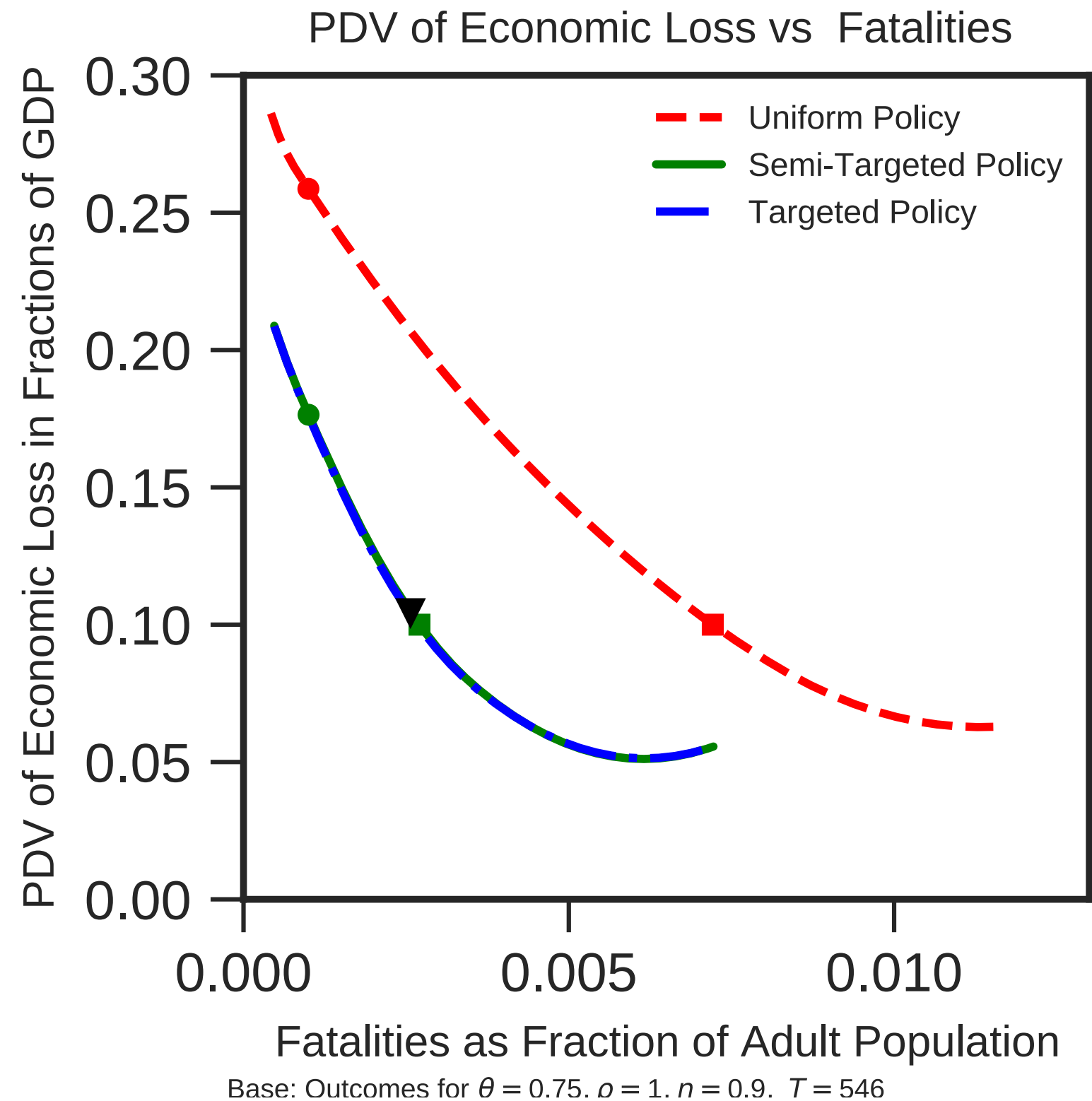
$$\dot{R}_j = \delta_j^r H_j + \gamma_j^I (I_j - H_j),$$

EVOLUTION

OF (S, I, D, R) by Group

IN SEIR STRUCTURE

FRONTIER

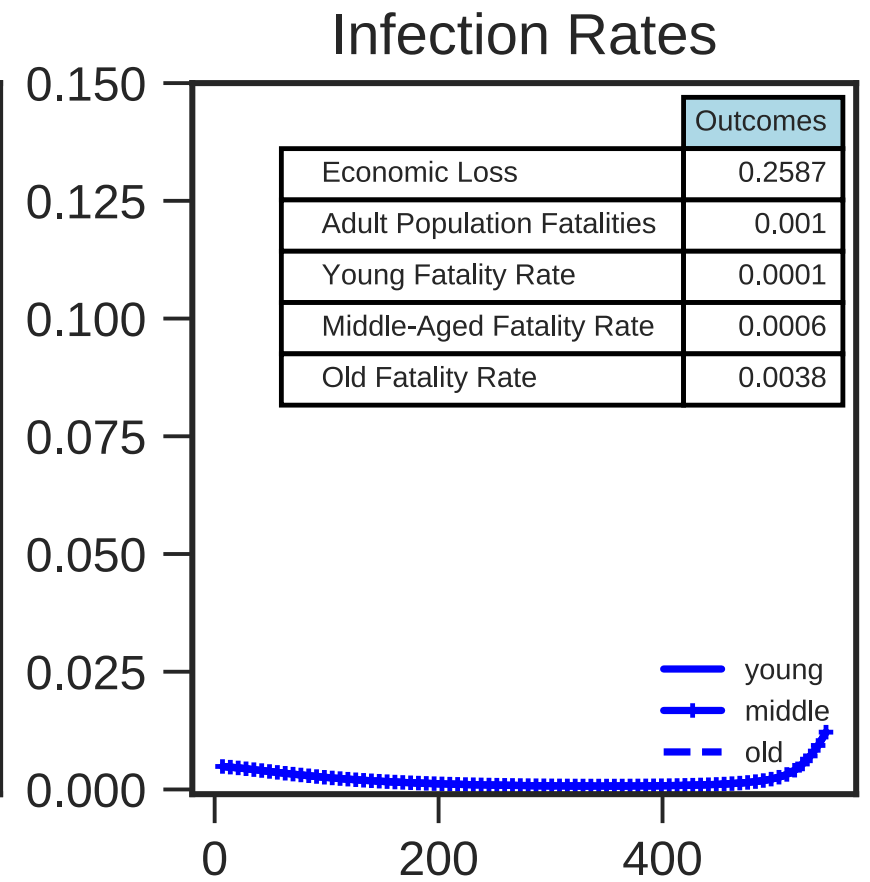
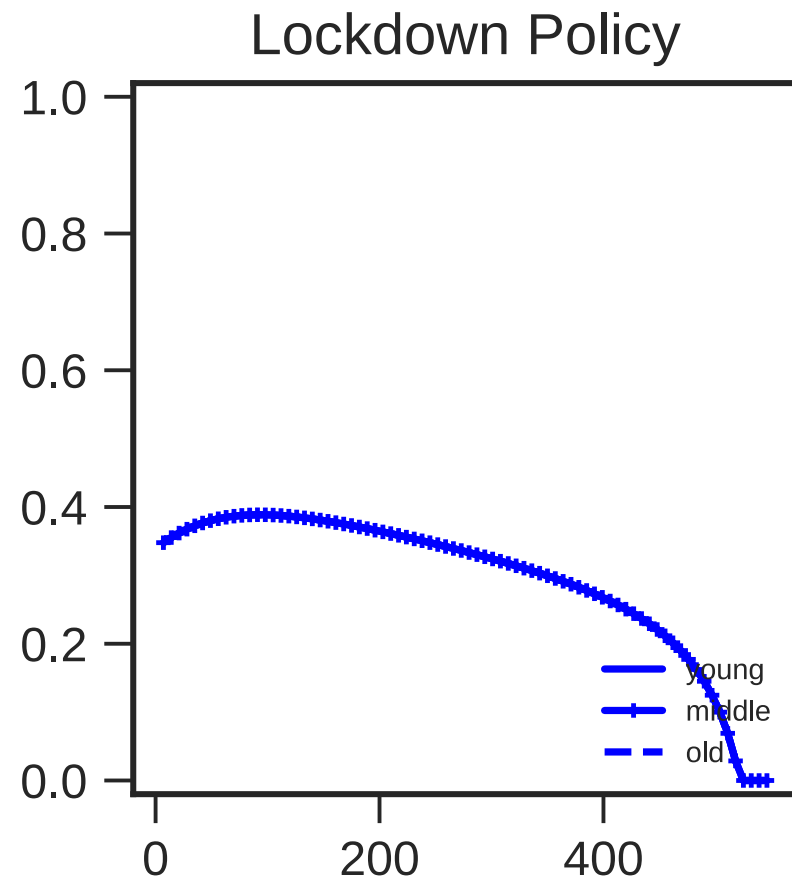


Large gains for Semi-Targeting

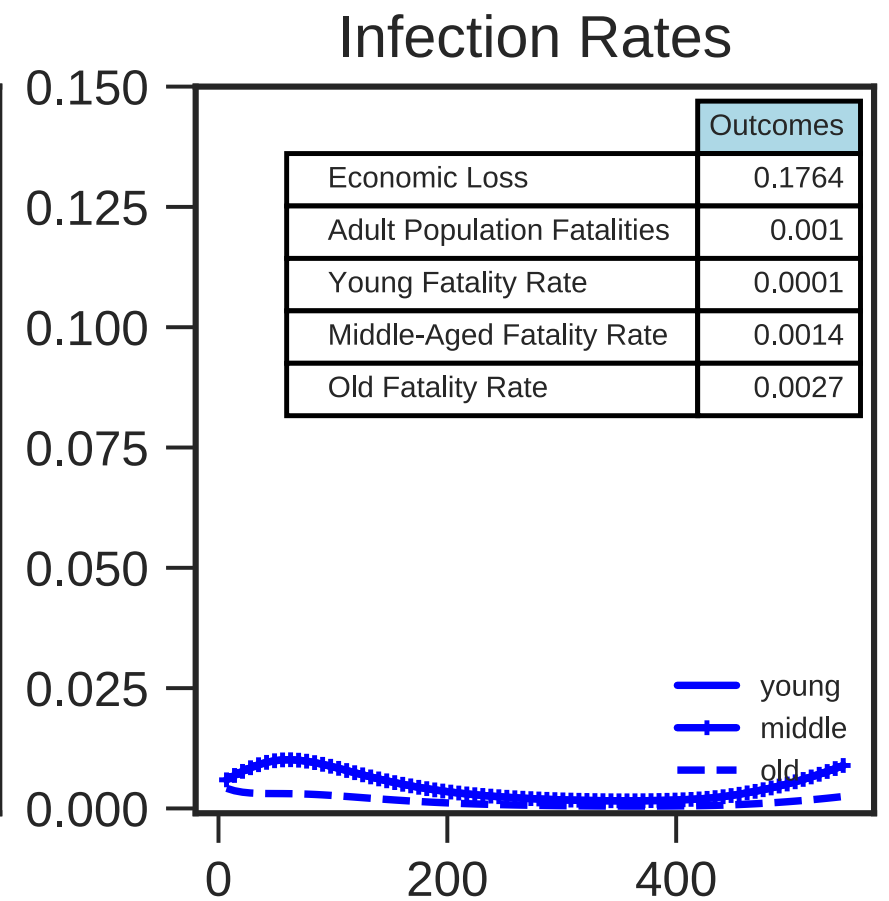
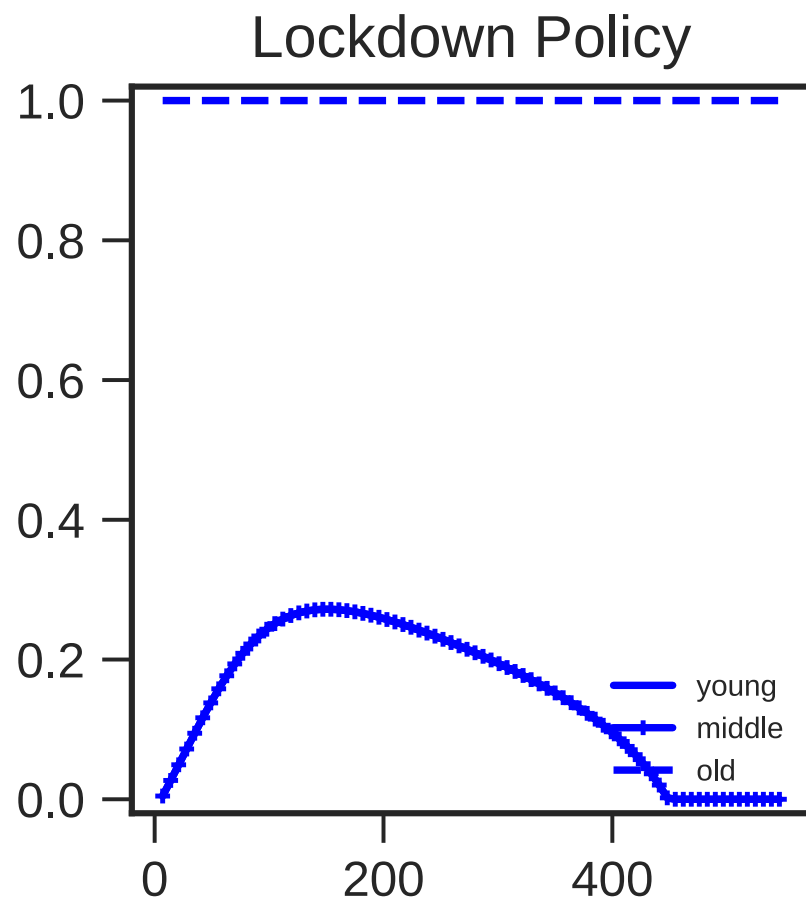
Small gains for Full-Targeting

BENEFIT OF SEMI-TARGETING

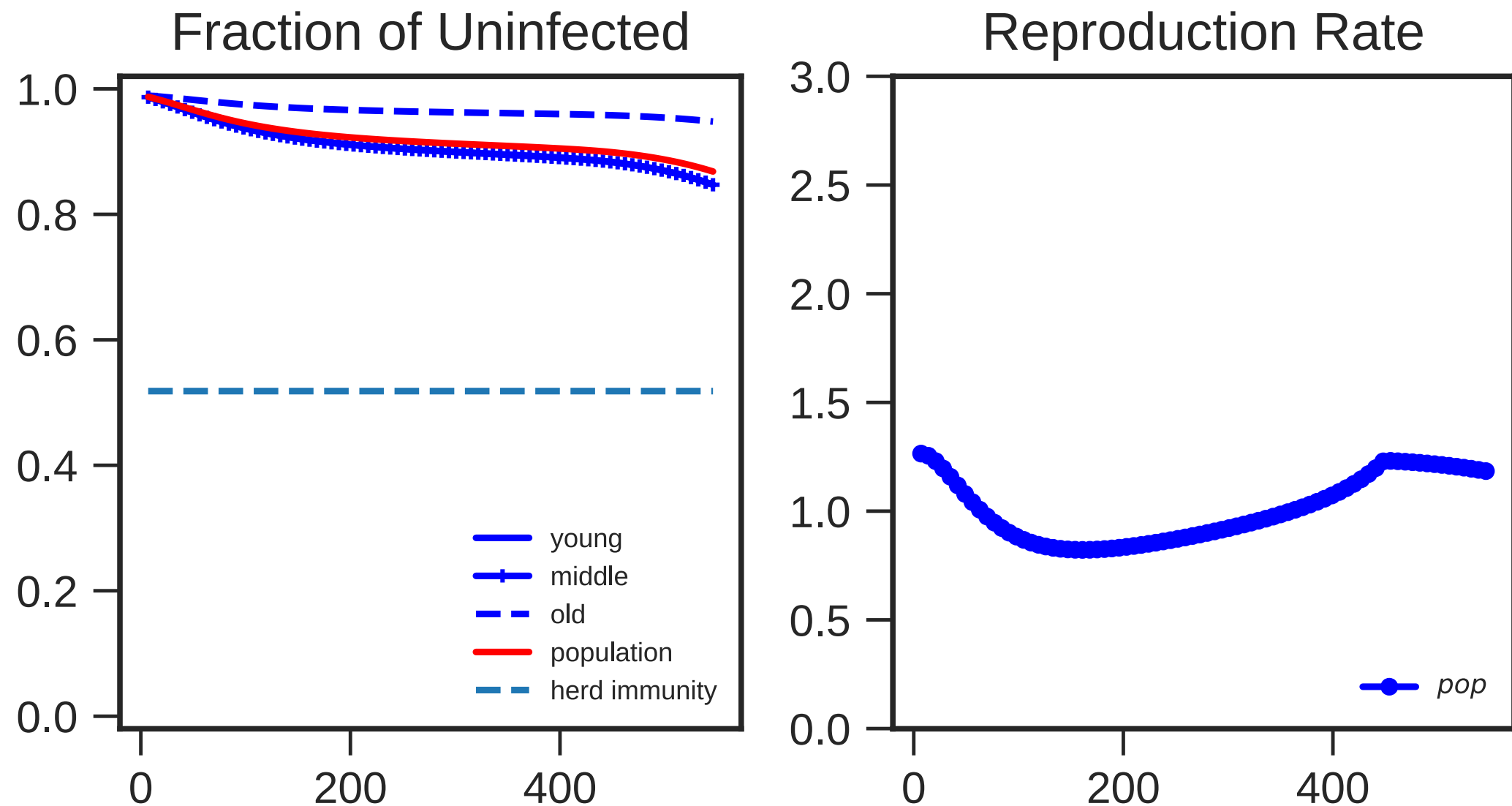
Uniform Policy



Semi-Targeted Policy



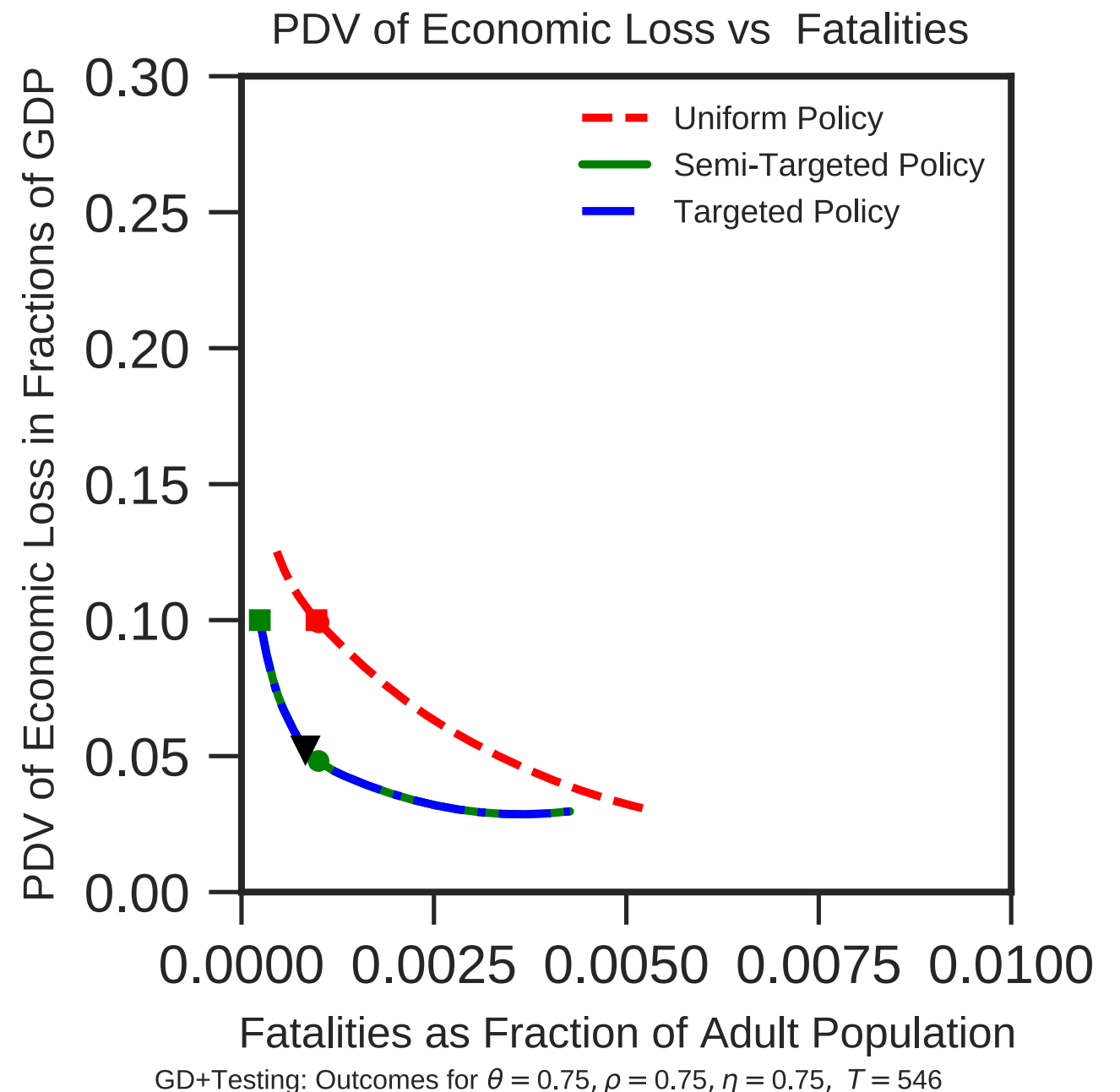
HERD IMMUNITY IS NOT THE GOAL



Base: Safety Focused Optimal SemiTargeted Policy for $\theta = 0.75$ $\eta = .9$ $\rho = 1.0$

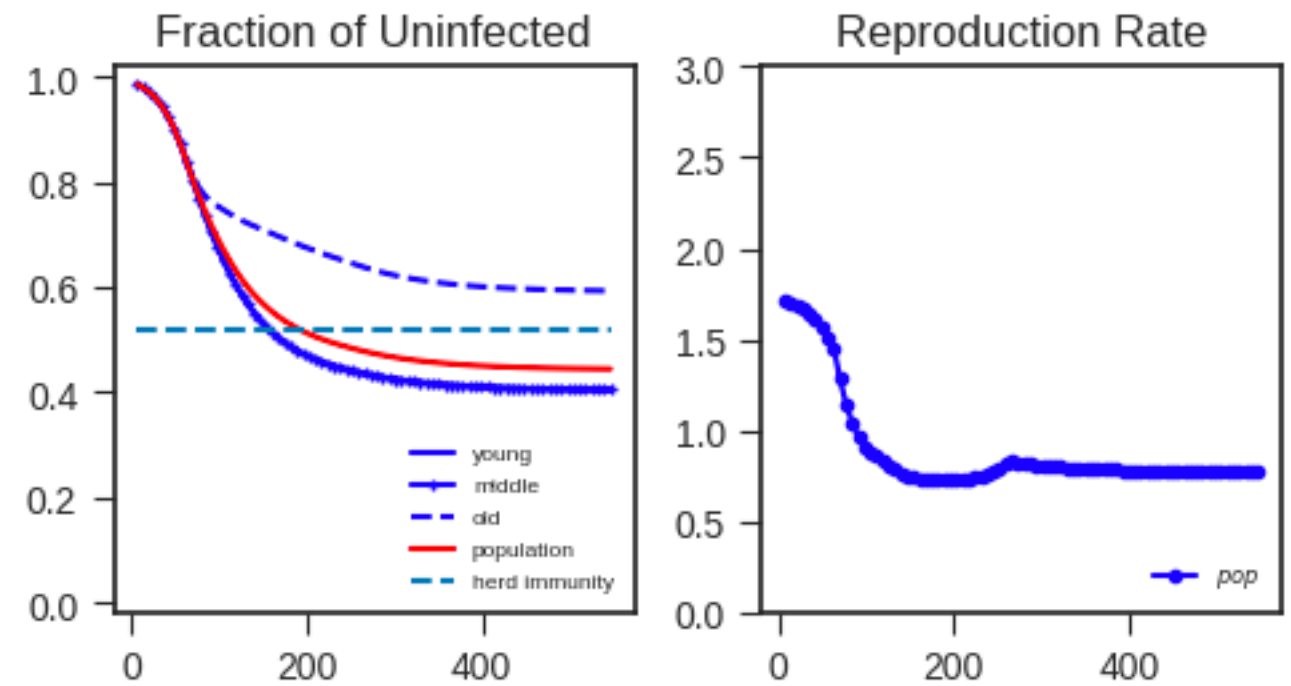
FINDINGS

- ▶ Large gains from targeted policy
- ▶ Most gains from simple semi-targeted policies: treat 65+ group differentially
- ▶ Non-zero lockdown for young
- ▶ Testing is important
- ▶ COMBO (test+dist+targ)

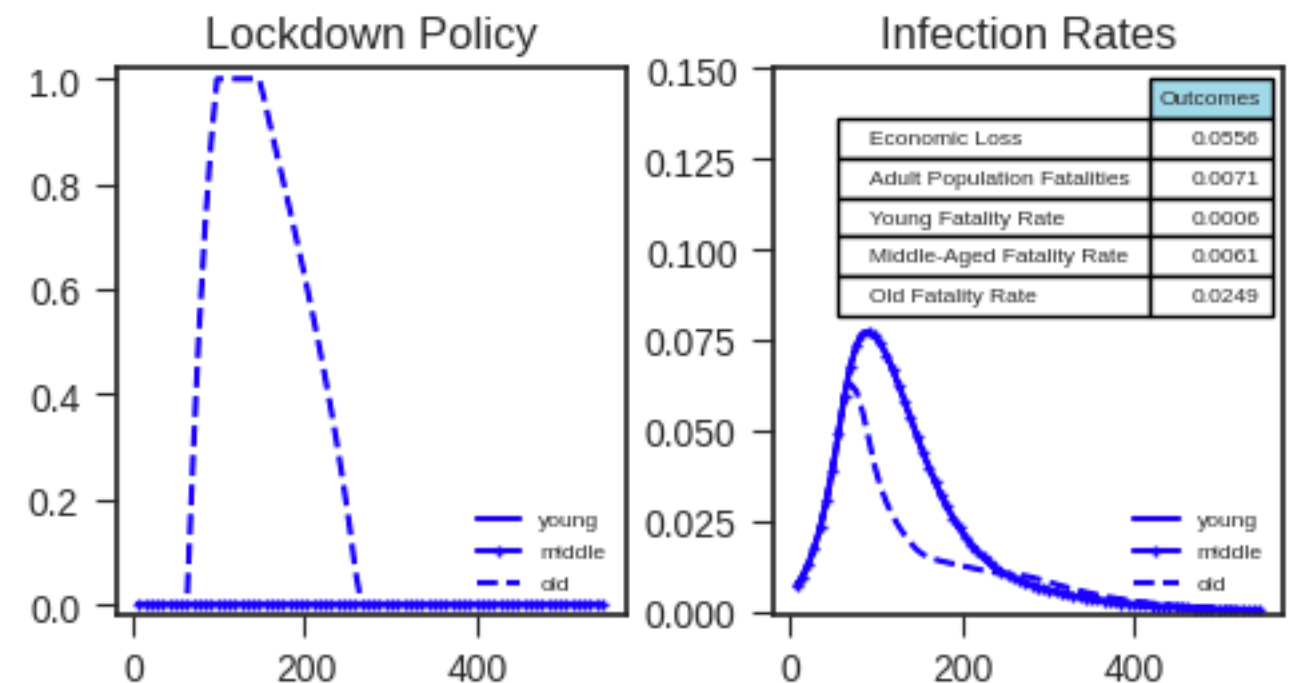


FINDINGS: “OBVIOUS” THINGS THAT DIDN’T WORK

- ▶ **“Quasi-SWEDISH”**– mild shield the old, let everyone go free and attain herd immunity.
 - ▶ Sub-optimal policy. **Even herd immunity is attained sub optimally.** (Total Infected > Herd Immunity Threshold).
- ▶ **“OBVIOUS”**: Treat <45 and 45-65 very differently. Wrong again. Small gains only.



Swedish: Fixed χ Optimal SemiTargeted Policy for $\theta = 0.75$ $\eta = .9$ $\rho = 1.0$



Swedish: Fixed χ Optimal SemiTargeted Policy for $\theta = 0.75$ $\eta = .9$ $\rho = 1.0$

COMPUTATION

Optimal Dynamic Control

$$\min J(L(\cdot), X(\cdot)) = \int_0^T \ell(L(t), X(t), t) dt$$

where $L(\cdot)$ is vector of control functions; $X(\cdot)$ is a vector of state functions; t is time from 0 up to $T \leq \infty$, $\ell(\cdot, \cdot, \cdot)$ is incremental loss, such that

$\dot{X}(t) = f(X(t), L(t), t)$		state evolution
$h(X(t), L(t), t) \leq 0$		path constraints
$e[X(0), L(0)] \leq 0$		initial constraints

COMPUTATION:

- ▶ “Indirect solution” methods:
 - ▶ Dynamic Programming: Value Function Iterations.
 - ▶ Indirect methods: solve a dynamic system – Hamiltonian – via first-order conditions subject to boundary conditions.
- ▶ “Direct solution” method: which is widely *used in large scale industrial engineering applications*. Write as a large Nonlinear Programming Problem with flexibly parameterized – nonparametric – policy and state functions and optimize.

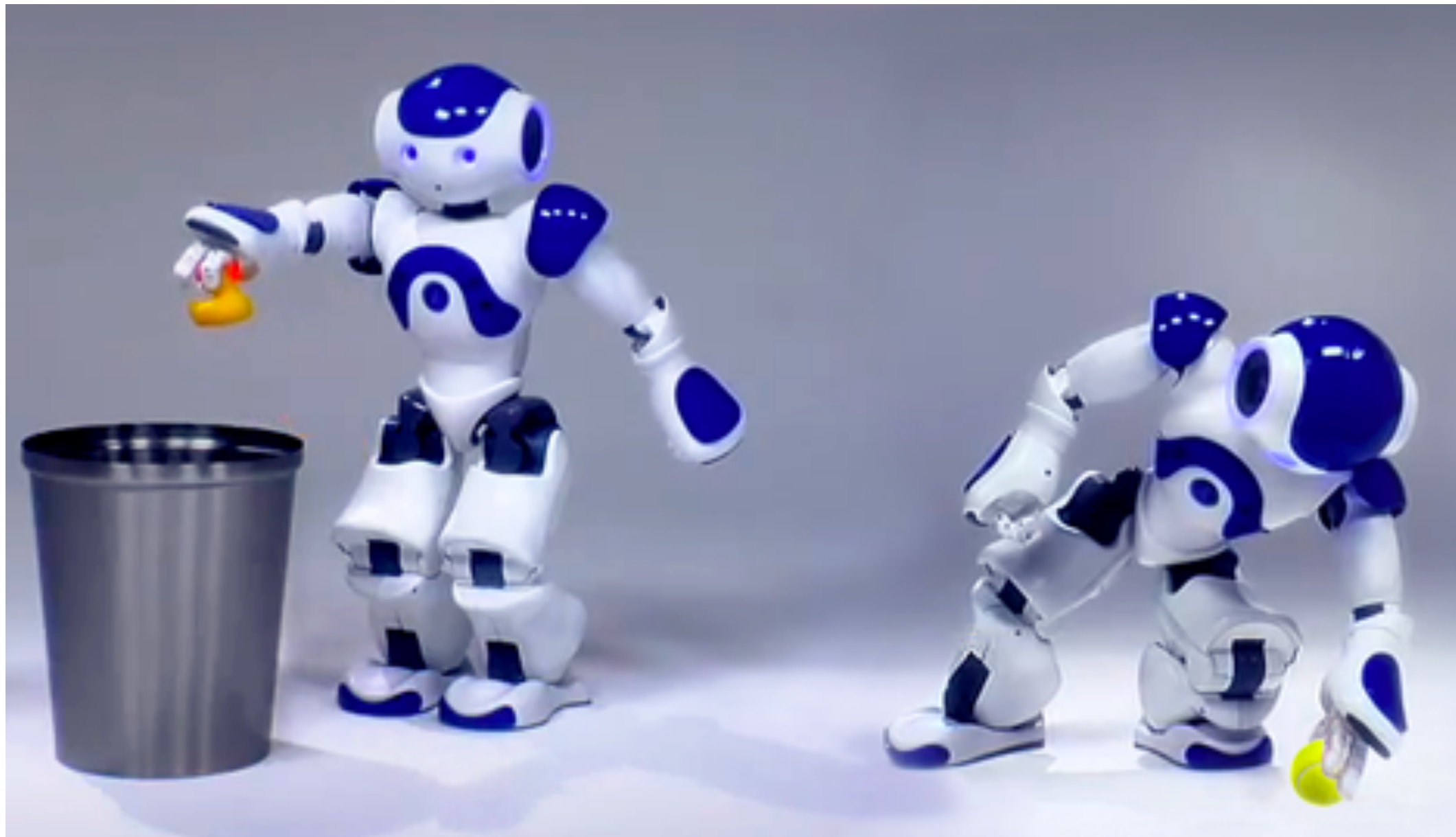
See Wikipedia.



GEKKO
DYNAMIC OPTIMIZATION

COMPUTATION: DIRECT SOLUTION

USED IN MOTION ROBOTICS





COMPUTATION: DIRECT SOLUTION

$$\min_Z F(Z)$$

over discretized (over time) control and policy variables

$$Z = \{L_j(t_k), X_m(t_k) : 1 \leq j \leq J, 1 \leq m \leq M, 1 \leq k \leq K\} \in \mathbb{R}^{JKM}.$$

subject to equality and inequality constraints:

$$g(Z) \leq 0, h(Z) = 0 \quad (L \text{ constraints})$$

- ▶ Size of the problem: $L + JKM$ is several thousands.
 - ▶ IOPT: Interior Point Optimal Nonlinear Programming
 - ▶ GEKKO interface: takes Optimal Control Problem, reformulates, passes to IPOPT.

CONNECTIONS TO “AI”

“AI” is a collection of methods, developed mostly by engineering and CS community:

- ▶ nonparametric (extremely high-dimensional) parameterization of policy P ;
beautiful art in many applications
- ▶ “train” policy by minimizing objective function $F(P)$ using closed-form gradients;
- ▶ over-parameterization and adding noise to gradients helps optimization.
- ▶ gives good practical solutions, though “global” optima are not guaranteed.

THANK YOU

PYTHON CODE IN THE CLOUD (Google Colab):

[https://colab.research.google.com/drive/
16zhss0-NzNbXn9C_MMP4lqzdVA2N-xxT?
usp=sharing](https://colab.research.google.com/drive/16zhss0-NzNbXn9C_MMP4lqzdVA2N-xxT?usp=sharing)

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[https://apmonitor.com/wiki/index.php/Main/
GekkoPythonOptimization](https://apmonitor.com/wiki/index.php/Main/GekkoPythonOptimization)