Dynamic Privacy Choices

Shota Ichihashi

Bank of Canada

Econometric Society Winter Meeting
Jan 3, 2021

The views expressed are those of the author and do not necessarily reflect the views of the Bank of Canada.
This Paper

Consumer uses
Platform

Info about
consumer
Platform

monetizes info

Impact on consumer

Q: dynamics & long-run outcome
This Paper

Consumer uses
Platform

Impact on consumer
Q: dynamics & long-run outcome
This Paper

Consumer uses Platform

Info about consumer

Q: dynamics & long-run outcome
This Paper

Consumer uses Platform

Info about consumer

Platform monetizes info

Q: dynamics & long-run outcome
This Paper

Consumer uses Platform

Info about consumer

Impact on consumer

Platform monetizes info
This Paper

Consumer uses Platform

Impact on consumer

Q: dynamics & long-run outcome

Platform monetizes info

Info about consumer
Roadmap

1. Model
2. Results
3. Relaxing Commitment Assumption
4. Literature
Roadmap

1. Model
2. Results
3. Relaxing Commitment Assumption
4. Literature
Time $t = 1, 2, \ldots$
Model

Time $t = 1, 2, \ldots$

Consumer

Platform
Model

Time $t = 1, 2, \ldots$

Consumer

- Type $X \sim \mathcal{N}(0, \sigma^2_0)$, fixed, unobservable$^1$

Platform

\footnote{$^1$If privately observable, focus on a “pooling” equilibrium.}
Model

Time $t = 1, 2, \ldots$

Consumer

- Type $X \sim \mathcal{N}(0, \sigma_0^2)$, fixed, unobservable\(^1\)
- Choose an activity level $a_t \in A \subset \mathbb{R}_+$
- $A$ is finite, $\min A = 0$, and $\max A = a_{\text{max}} > 0$

Platform

\(^1\)If privately observable, focus on a “pooling” equilibrium.
Model

Time $t = 1, 2, \ldots$

Consumer

- Type $X \sim \mathcal{N}(0, \sigma_0^2)$, fixed, unobservable\(^1\)
- Choose an activity level $a_t \in A \subset \mathbb{R}_+$
- $A$ is finite, $\min A = 0$, and $\max A = a_{\text{max}} > 0$

Platform

- Privately observe a signal $X + \varepsilon_t$ with $\varepsilon_t \overset{\text{iid}}{\sim} \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$

\(^{1}\text{If privately observable, focus on a “pooling” equilibrium.}\)
Model

Time $t = 1, 2, \ldots$

Consumer

- Type $X \sim \mathcal{N}(0, \sigma^2_0)$, fixed, unobservable
- Choose an activity level $a_t \in A \subset \mathbb{R}_+$
- $A$ is finite, $\min A = 0$, and $\max A = a_{\text{max}} > 0$

Platform

- Privately observe a signal $X + \varepsilon_t$ with $\varepsilon_t \overset{iid}{\sim} \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t \right)$
- $\gamma_t$: level of privacy protection in $t$

---

1If privately observable, focus on a “pooling” equilibrium.
Platform Payoffs

Platform’s payoff in period $t$

$\sigma_0^2 - \sigma_t^2$
Platform Payoffs

Platform’s payoff in period $t$

$$\sigma_0^2 - \sigma_t^2$$

prior variance
Platform Payoffs

Platform’s payoff in period $t$

$$\sigma_0^2 - \sigma_t^2$$

prior variance

conditional variance of $X$

at the end of $t$
Platform Payoffs

Platform’s payoff in period $t$

$$\sigma_0^2 - \sigma_t^2$$

prior variance
conditional variance of $X$
at the end of $t$

- More info better
Platform Payoffs

Platform’s payoff in period $t$

\[ \sigma_0^2 - \sigma_t^2 \]

- Prior variance
- Conditional variance of $X$ at the end of $t$

- More info better

- Increasing in $(a_1, \ldots, a_t)$ and decreasing in $(\gamma_1, \ldots, \gamma_t)$
Platform Payoffs

Platform’s payoff in period $t$

$$\sigma_0^2 - \sigma_t^2$$

- prior variance
- conditional variance of $X$ at the end of $t$

- More info better

- Increasing in $(a_1, \ldots, a_t)$ and decreasing in $(\gamma_1, \ldots, \gamma_t)$

- Discount future payoffs
Consumer Payoffs

Consumer payoff in period $t$

$$u(a_t) - v \cdot (\sigma_0^2 - \sigma_t^2)$$
Consumer Payoffs

Consumer payoff in period $t$

$$u(a_t) - \nu \cdot \left( \sigma_0^2 - \sigma_t^2 \right)$$

$u(\cdot)$ increasing
Consumer Payoffs

Consumer payoff in period $t$

$$u(a_t) - v \cdot \left( \sigma_0^2 - \sigma_t^2 \right)$$

$u(\cdot)$ increasing

privacy cost
Consumer Payoffs

Consumer payoff in period $t$

\[ u(a_t) - v \cdot \left( \sigma_0^2 - \sigma_t^2 \right) \]

- $u(\cdot)$ increasing
- Privacy cost
- Value of privacy $\in \mathbb{R}_+$
Consumer Payoffs

Consumer payoff in period $t$

$u(a_t) - v \cdot \left( \sigma_0^2 - \sigma_t^2 \right)$

privacy cost

$u(\cdot)$ increasing

value of privacy $\in \mathbb{R}_+$

Discount future payoffs
Timing

1. Platform chooses a privacy policy \((\gamma_1, \gamma_2, \ldots ) \in \mathbb{R}_+^\infty\)
   
   \(\text{Signal } X + \varepsilon_t \text{ with } \varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)\)

2. Consumer chooses \(a_1, a_2, \ldots\)

Solution: SPE
Roadmap

1. Model

2. Results

3. Relaxing Commitment Assumption

4. Literature
1. Platform chooses a privacy policy \((\gamma_1, \gamma_2, \ldots)\)
   - Signal \(X + \varepsilon_t\) with \(\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)\)

2. Consumer chooses \(a_1, a_2, \ldots\)
Timing

1. Platform chooses a privacy policy \((\gamma_1, \gamma_2, \ldots)\)

   - Signal \(X + \varepsilon_t\) with \(\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)\)

2. Consumer chooses \(a_1, a_2, \ldots\)
Flow Payoffs

Consumer’s period-\( t \) payoff

\[
  u(a_t) - \nu \left( \sigma_0^2 - \sigma_t^2 \right)
\]
Flow Payoffs

Consumer’s period-\(t\) payoff

\[
\begin{align*}
    u(a_t) - v \left( \sigma_0^2 - \sigma_t^2 \right) \\
    = u(a_t) - v \left( \sigma_0^2 - \frac{1}{\sigma_{t-1}^2} + \frac{1}{a_t + \gamma_t} \right)
\end{align*}
\]
Consumer’s period-\(t\) payoff

\[
u(a_t) - \nu (\sigma_0^2 - \sigma_t^2)
\]

\[
= u(a_t) - \nu \left( \sigma_0^2 - \frac{1}{\sigma_{t-1}^2} + \frac{1}{a_t + \gamma_t} \right)
\]

\[:= C_t\]
Marginal Privacy Cost

\[ \frac{\partial C_t}{\partial a_t} \text{ is decreasing in } \gamma_t \text{ and increasing in } \sigma_t^2 - 1. \]

▶ Less privacy (lower \( \sigma_t^2 - 1 \)) \( \rightarrow \) Lower marginal cost
▶ Lower payoff \( \leftrightarrow \) Higher incentive to raise \( a_t \)
Lemma

\[ \frac{\partial C_t}{\partial a_t} \text{ is decreasing in } \gamma_t \text{ and increasing in } \sigma^2_{t-1}. \]
Lemma

\frac{\partial C_t}{\partial a_t} is decreasing in \gamma_t and increasing in \sigma_{t-1}^2.
Lemma

$$\frac{\partial C_t}{\partial a_t} \text{ is decreasing in } \gamma_t \text{ and increasing in } \sigma^2_{t-1}.$$ 

- Less privacy (lower $\sigma^2_{t-1}$) $\rightarrow$ Lower marginal cost
Lemma

\[ \frac{\partial C_t}{\partial a_t} \text{ is decreasing in } \gamma_t \text{ and increasing in } \sigma_{t-1}^2. \]

- Less privacy (lower \( \sigma_{t-1}^2 \)) \( \rightarrow \) Lower marginal cost
- Lower payoff \( \leftrightarrow \) Higher incentive to raise \( a_t \)
Equilibrium

1. Platform chooses a privacy policy $(\gamma_1, \gamma_2, \ldots)$
   - Signal $X + \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0, \frac{1}{a_t} + \gamma)$

2. Consumer solves

$$\max_{(a_t)_{t=1}^\infty} \sum_{t=1}^\infty \delta_c^{t-1} \left[ u(a_t) - v \left( \sigma_0^2 - \sigma_t^2(a_t, \gamma^t) \right) \right].$$
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{\text{max}} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)

▶ Early: high MC → high \( \gamma_t \) to encourage activity
▶ Learning becomes easier over time
▶ No value of stopping data collection
▶ E.g., \( \gamma_t = \infty \) after some period?
▶ Committing to erode privacy → higher activity today
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)
2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

For any \( v \) and discount factors, in any equilibrium:

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \text{ s.t. } \forall v \geq v^*, \gamma_t^* > 0 \text{ for } t = 1, \ldots, \tau \)

- Early: high MC \( \to \) high \( \gamma_t \) to encourage activity
Equilibrium

Reminder: \( u(a) - v \cdot (\sigma_0^2 - \sigma_t^2) \)

Theorem

*For any \( v \) and discount factors, in any equilibrium:*

1. \( \sigma_t^2 \to 0 \) and \( a_t^* \to a_{max} \) as \( t \to \infty \)

2. \( \forall \tau \in \mathbb{N}, \exists v^* > 0 \) s.t. \( \forall v \geq v^*, \gamma_t^* > 0 \) for \( t = 1, \ldots, \tau \)

- Early: high MC \( \to \) high \( \gamma_t \) to encourage activity
- Learning becomes easier over time
Equilibrium

Reminder: $u(a) - v \cdot (\sigma_0^2 - \sigma_t^2)$

Theorem

For any $v$ and discount factors, in any equilibrium:

1. $\sigma_t^2 \to 0$ and $a_t^* \to a_{\text{max}}$ as $t \to \infty$

2. $\forall \tau \in \mathbb{N}, \exists v^* > 0 \text{ s.t. } \forall v \geq v^*, \gamma_t^* > 0 \text{ for } t = 1, \ldots, \tau$

- Early: high MC $\to$ high $\gamma_t$ to encourage activity
- Learning becomes easier over time
- No value of stopping data collection
  - E.g., $\gamma_t = \infty$ after some period?
  - Committing to erode privacy $\to$ higher activity today
Generalization

Platform’s payoff is strictly increasing in \((a_t, -\sigma_t^2)\)
Generalization

Platform’s payoff is strictly increasing in \((a_t, -\sigma_t^2)\)

Proposition

A sufficiently patient platform induces the long-run privacy loss:

\[ \lim_{\delta_p \to 1} \lim_{t \to \infty} \sigma_t^2 = 0 \quad \text{and} \quad \lim_{\delta_p \to 1} \lim_{t \to \infty} a_t^* = a_{max}. \]
Platform’s payoff is strictly increasing in \((a_t, -\sigma_t^2)\)

**Proposition**

A sufficiently patient platform induces the long-run privacy loss:

\[
\lim_{\delta_p \to 1} \lim_{t \to \infty} \sigma_t^2 = 0 \quad \text{and} \quad \lim_{\delta_p \to 1} \lim_{t \to \infty} a_t^* = a_{max}.
\]

- High activity if high \(\gamma_t\) or low \(\sigma_t^2\)
- Activity-driven platforms benefit from collecting data
Implications

1. Privacy paradox (cf. Acquisti et al. 2016)

2. Rational addiction (Becker and Murphy, 1988)
Implications

1. Privacy paradox (cf. Acquisti et al. 2016)
Implications

1. Privacy paradox (cf. Acquisti et al. 2016)

2. Rational addiction (Becker and Murphy, 1988)
Roadmap

1. Model

2. Results

3. Relaxing Commitment Assumption

4. Extension: Competition

5. Literature
Relaxing Commitment Assumption

Platform with “one-period commitment”

- Platform P sets $\gamma_t$
- Platform C sets $a_t$

- Time periods:
  - $t - 1$
  - Period $t$
  - $t + 1$
Relaxing Commitment Assumption

Platform with “one-period commitment”

Assumption

\[ A = \{ 0, a_{\text{max}} \} \]

\[ P \text{ sets } \gamma_t \quad C \text{ sets } a_t \]

\[ t - 1 \quad \text{period } t \quad t + 1 \]
Proposition (informal)

There is a "consumer-worst" eqm such that:

1. The outcome is the same as long-run commitment.
2. Platform strategy is greedy.

If $\sigma_0^2$ is small, the eqm is unique.
Consumer-Best Outcome

Proposition

If $\delta_C \geq \frac{1}{2}$ and $\sigma^2_0$ is large, there is an eqm in which the platform sets $\gamma_t = \infty$ and the consumer chooses $a_t = a_{max}$ in all periods.
Consumer-Best Outcome

Proposition

If $\delta_C \geq \frac{1}{2}$ and $\sigma_0^2$ is large, there is an eqm in which the platform sets $\gamma_t = \infty$ and the consumer chooses $a_t = a_{\text{max}}$ in all periods.
Consumer-Best Outcome

Proposition

If \( \delta_C \geq \frac{1}{2} \) and \( \sigma_0^2 \) is large, there is an eqm in which the platform sets \( \gamma_t = \infty \) and the consumer chooses \( a_t = a_{\text{max}} \) in all periods.
Consumer-Best Outcome

Proposition

If $\delta_C \geq \frac{1}{2}$ and $\sigma_0^2$ is large, there is an eqm in which the platform sets $\gamma_t = \infty$ and the consumer chooses $a_t = a_{max}$ in all periods.

\begin{align*}
\sigma_t^2 & \\
\gamma_t & < \infty \\
\text{and } a_t & > 0
\end{align*}
Consumer-Best Outcome

Proposition

If $\delta_C \geq \frac{1}{2}$ and $\sigma_0^2$ is large, there is an eqm in which the platform sets $\gamma_t = \infty$ and the consumer chooses $a_t = a_{max}$ in all periods.
Introducing a New Digital Product

Two firms

- **Existing** firm with a low $\sigma_0^2$ (e.g., data from other services)
- **New** firm with a high $\sigma_0^2$

Which firm has higher willingness to launch a new digital service?

**New** firm faces a higher marginal value of info

But, platform-worst eqm → only the **existing** firm can collect info

Inefficiency: Data go to a firm that already has a lot of data
Literature (not exhaustive!)

Platform data collection: Acemoglu et al. (2019); Bergemann et al. (2019); Choi et al. (2018); Garratt and van Oordt (2019)

Competition with data: Cornière and Taylor (2020); Prufer and Schottmüller (2017); Hagiu and Wright (2020)

Switching cost, barrier to entry: Farrell and Shapiro (1988); Klemperer (1995); Fudenberg and Tirole (2000)

Recap

- A dynamic model of a platform collecting consumer data
- Key: decreasing marginal privacy cost
- Long-run privacy loss with high activity level
- Weaker commitment: optimistic belief prevents data collection
- Data-driven advantage due to lower MC of privacy loss