Dynamic Privacy Choices

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Bank of Canada

Econometric Society Winter Meeting
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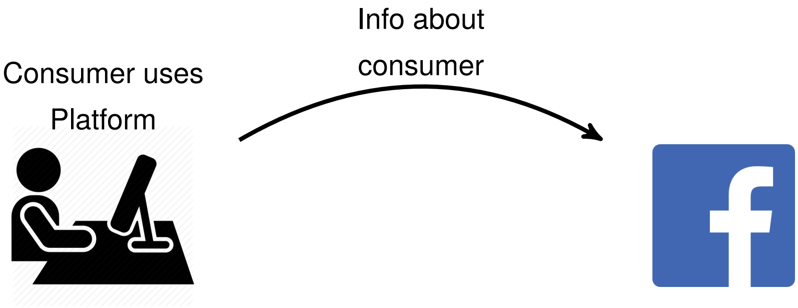
Motivation

This Paper

This Paper

Consumer uses
Platform

This Paper



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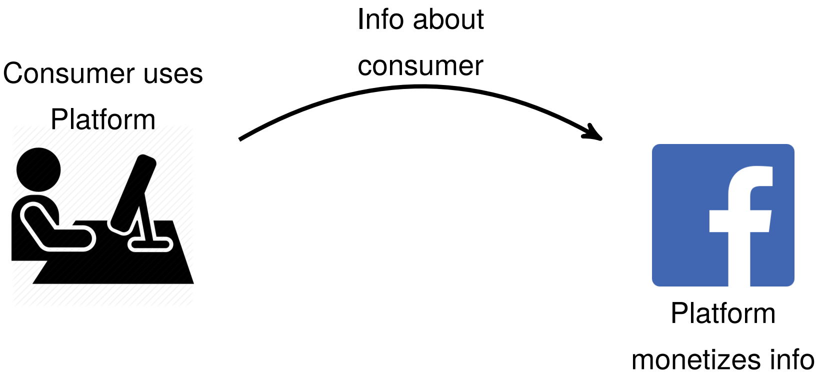
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Info about
consumer
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monetizes info

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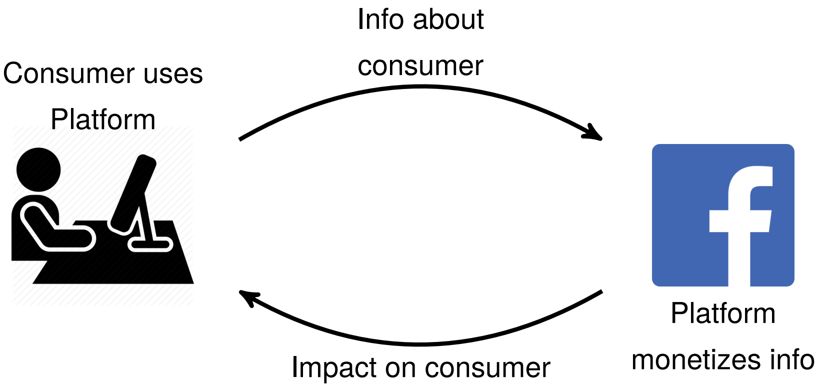
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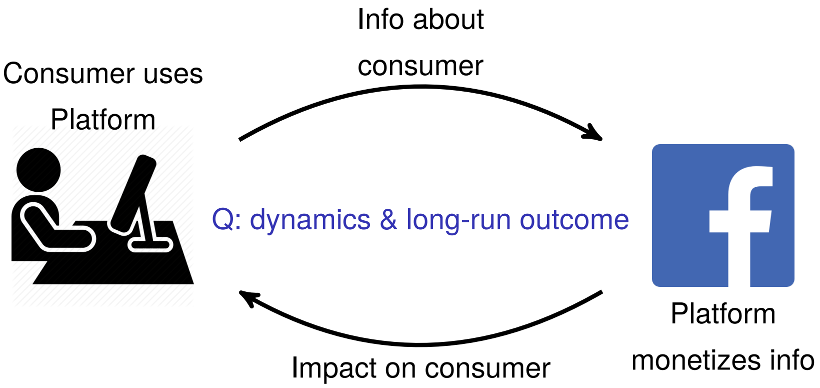
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Roadmap

1. Model
2. Results
3. Relaxing Commitment Assumption
4. Literature

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Model

Time $t = 1, 2, \ldots$

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Consumer

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Consumer

- Type $X \sim \mathcal{N}(0, \sigma_0^2)$, fixed, unobservable[1]

Platform

[1] If privately observable, focus on a "pooling" equilibrium.

Model

Time $t = 1, 2, \dots$

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- A is finite, $\min A = 0$, and $\max A = a_{max} > 0$

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- Privately observe a signal $X + \varepsilon_t$ with $\varepsilon_t \overset{iid}{\sim} \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$

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- γ_t: level of privacy protection in t

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Platform Payoffs

Platform's payoff in period t

$$\sigma_0^2 - \sigma_t^2$$

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conditional variance of X at the end of t

- More info better
- Increasing in $(a_1, \ldots, a_t)$ and decreasing in $(\gamma_1, \ldots, \gamma_t)$
- Discount future payoffs

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Discount future payoffs

Timing

1. Platform chooses a privacy policy $(\gamma_1, \gamma_2, \dots) \in \mathbb{R}_+^\infty$
   - Signal $X + \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}\left(0, \frac{1}{a_t} + \gamma_t\right)$
2. Consumer chooses a_1, a_2,...

Solution: SPE

Roadmap

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2. Results
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- Less privacy (lower σ^2_{t-1}) $\rightarrow$ Lower marginal cost
- Lower payoff $\leftrightarrow$ Higher incentive to raise a_t

Equilibrium

1. Platform chooses a privacy policy $(\gamma_1, \gamma_2, \dots)$
   - Signal $X + \varepsilon_t$ with $\varepsilon_t \sim \mathcal{N}(0, \frac{1}{a_t} + \gamma)$
2. Consumer solves

$$\max_{(a_t)_{t=1}^{\infty}} \sum_{t=1}^{\infty} \delta_C^{t-1} \left[u(a_t) - v\left(\sigma_0^2 - \sigma_t^2(\boldsymbol{a}^t, \boldsymbol{\gamma}^t)\right)\right].$$

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Reminder: $u(a) - v \cdot (\sigma_0^2 - \sigma_t^2)$

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Theorem

For any v and discount factors, in any equilibrium:

1. $\sigma_t^2 \to 0$ *and* $a_t^* \to a_{max}$ *as* $t \to \infty$
2. $\forall \tau \in \mathbb{N}$, $\exists v^* > 0$ *s.t.* $\forall v \geq v^*$, $\gamma_t^* > 0$ *for* $t = 1, \ldots, \tau$

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- Early: high MC $\to$ high γ_t to encourage activity
- Learning becomes easier over time
- No value of stopping data collection
  - E.g., $\gamma_t = \infty$ after some period?
  - Committing to erode privacy $\to$ higher activity today

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Platform's payoff is strictly increasing in $(a_t, -\sigma_t^2)$

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Proposition

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$$\lim_{\delta_P \to 1} \lim_{t \to \infty} \sigma_t^2 = 0 \quad \textit{and} \quad \lim_{\delta_P \to 1} \lim_{t \to \infty} a_t^* = a_{max}.$$

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- High activity if high γ_t or low σ_t^2
- Activity-driven platforms benefit from collecting data

Implications

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2. Rational addiction (Becker and Murphy, 1988)

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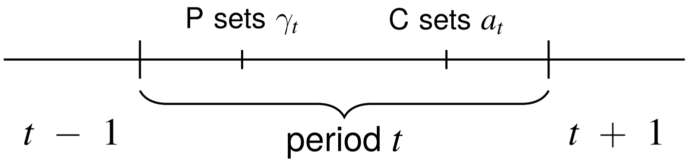
Relaxing Commitment Assumption

Platform with "one-period commitment"



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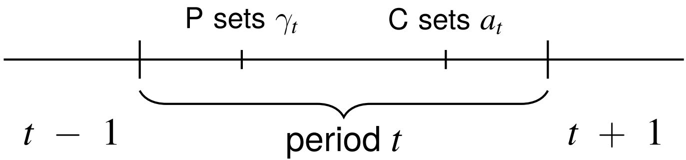

Assumption

Binary activity level: $A = \{0, a_{max}\}$.

Consumer-Worst Outcome

Full characterization in the paper

Proposition (informal)

There is a "consumer-worst" eqm such that:

1. *The outcome is the same as long-run commitment.*
2. *Platform strategy is greedy.*

If σ_0^2 is small, the eqm is unique.

Consumer-Best Outcome

Proposition

If $\delta_C \geq \frac{1}{2}$ and σ_0^2 is large, there is an eqm in which the platform sets $\gamma_t = \infty$ and the consumer chooses $a_t = a_{max}$ in all periods.

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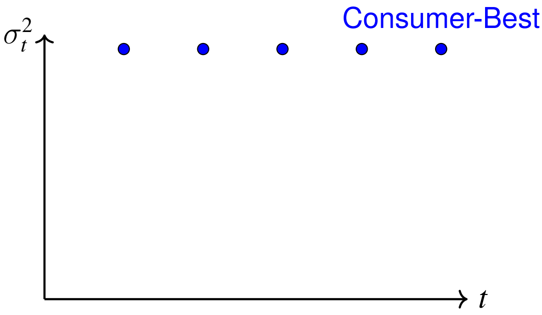

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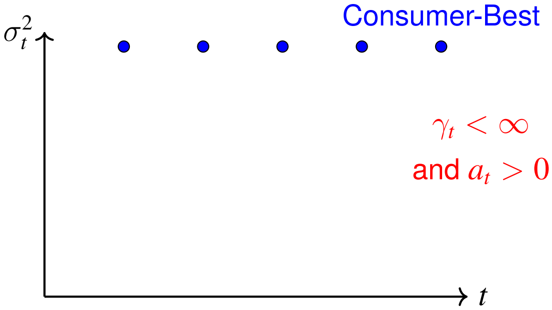

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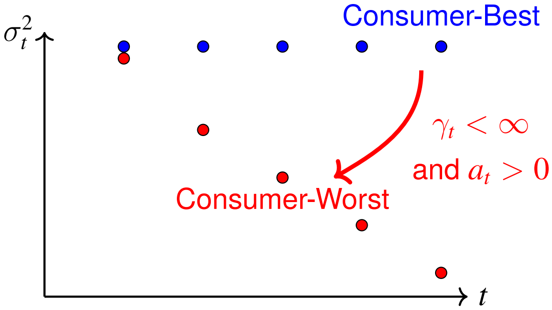

Introducing a New Digital Product

Two firms

- Existing firm with a low σ_0^2 (e.g., data from other services)
- New firm with a high σ_0^2

Which firm has higher willingness to launch a new digital service?

New firm faces a higher marginal value of info

But, platform-worst eqm $\rightarrow$ only the existing firm can collect info

Inefficiency: Data go to a firm that already has a lot of data

Literature (not exhaustive!)

Platform data collection: Acemoglu et al. (2019); Bergemann et al. (2019); Choi et al. (2018); Garratt and van Oordt (2019)

Competition with data: Cornière and Taylor (2020); Prufer and Schottmüller (2017); Hagiu and Wright (2020)

Switching cost, barrier to entry: Farrell and Shapiro (1988); Klemperer (1995); Fudenberg and Tirole (2000)

Signal-jamming: Holmstrom (1999)

Recap

- A dynamic model of a platform collecting consumer data
- Key: decreasing marginal privacy cost
- Long-run privacy loss with high activity level
- Weaker commitment: optimistic belief prevents data collection
- Data-driven advantage due to lower MC of privacy loss