Price Destabilizing Speculation:
The Role of Strategic Limit Orders

Suman Banerjee \(^1\)  
Ravi Jagannathan \(^2\)  
Kai Wang \(^1\)

\(^1\)Stevens Institute of Technology  
\(^2\)Northwestern University

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Milton Friedman, Essays in Positive Economics (1953), p.175

“People who argue that speculation is generally destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on the average sell when the currency is low in price and buy when it is high.”
Hart and Kreps (JPE 1986): Speculators buy expecting to benefit from very high price next period due to rare but very severe shortage. Subsequently,

- if demand turns out to be high, their supply does not have much effect on the prevailing high price;
- if demand turns out to be low, they dump their inventories, thereby lowering the prices significantly;
- speculator has a destabilizing effect.

Speculators take prices as given, and still in equilibrium their actions have a destabilizing effect on prices.

There are many many more both theory and empirical papers...
We show that a large speculator, knows his actions affect prices, uses limit orders to profit from that knowledge.

- When the speculator buys, his demand lowers the market-clearing price.
- When the speculator sells, his supply increases the market-clearing price.

In the process, the speculator creates price volatility and profits from it!

Strategic sellers are strictly worse off, consumers are better off overall, but at times can be worse off.
We develop a two-period commodity-trading model.

We assume perfect contracting environment without any uncertainty and informational frictions.

The risk-free interest rate is assumed to be zero.

We assume a single tradable commodity – we call it “widgets.”

- We assume that widgets are bought/sold in forward markets one period ahead.
- There are no spot markets.

Agents

- Large number of small price-taking consumers
- Two identical large suppliers with market power: Strategic sellers A and B.
- One large speculator

In equilibrium, price clear the market, i.e., demand = supply in each period.
Types of equilibria

- The benchmark equilibria without the speculator
  - No price volatility; i.e., \( p_1 = p_2 = p^* \).
  - Strategic sellers earn equal profits in both periods; \( \pi_{j,1} = \pi_{j,2} = \pi^* \) where \( j = A, B \).

- The equilibria with the speculator but without the ability to dispose in period 2
  - Price is volatile but lower than benchmark price; i.e., \( p_1 < p_2 < p^* \).
  - Strategic sellers earn lower than the benchmark profits; \( \pi_{j,2} < \pi_{j,1} < \pi^* \), \( j = A, B \).
  - Consumers are better off relative the benchmark case.

- The equilibria with the speculator but with the ability to dispose in period 2
  - Price is volatile but level is lower than the benchmark price in period 1 and higher than the benchmark price in period 2; i.e., \( p_1 < p^* < p_2 \).
  - Strategic sellers earn lower than the benchmark profits; \( \pi_{j,1} < \pi^* < \pi_{j,2} \) and \( \pi_{j,1} + \pi_{j,2} < 2 \pi^* \) where \( j = A, B \).
  - Consumers are better off relative the benchmark case in period 1 but worse off in period 2.
The Timeline

Period 1
1. Forward contracts for delivery in period 2 are traded
2. Market clearing price is $p_1$

Period 2
1. Forward contracts entered in period 1 for delivery in period 2 are settled
2. Forward contracts for delivery in period 3 are traded
3. Market clearing price is $p_2$

Period 3
1. Forward contracts entered in period 2 for delivery in period 3 are settled
Benchmark Equilibrium: No Speculator Case

Standard Cournot model with (a) two strategic sellers, (b) price taking consumers, and (c) there is no linkage between the two periods.

- The aggregate demand of the price-taking atomistic consumers is \( p_t(Q_t) = a - b Q_t \), \( t = \{1, 2\} \), where \( Q_t \) is the aggregate demand.

- Two strategic sellers: \( \max q_A p(q_A + q_B)q_A \) and \( \max q_B p(q_A + q_B)q_B \). We normalize the production cost (c) of the sellers to zero.

- The best response functions (b.r.f.) of the two sellers: \( q_A(q_B) = \frac{a}{2b} - \frac{q_B}{2} \), and \( q_B(q_A) = \frac{a}{2b} - \frac{q_A}{2} \).

- We have the equilibrium supply of each seller: \( q^* = \frac{a}{3b} \), the market clearing price is \( p^* = \frac{a}{3} \), and each seller earns, \( \pi^* = \frac{a^2}{9b} \) in each period.
The Structure with Speculator

Speculator’s action links the two periods.

- The **speculator** posts a **demand** schedule in **period 1** and a **supply** schedule in **period 2**. He demands $q_S$ units in the forward market in period 1, stores with a fixed cost $c_S$ per unit, and sell in period 2.

- Each **strategic seller** takes $Q_t$ and rival strategic seller’s schedule as given, and chooses her own supply to **maximize** the sum of the two periods’ profits.

- The speculator takes the **behavior of the two sellers** as given, and chooses his own demand/supply schedule to maximize profits.

We consider **two** types of equilibrium:

- The speculator has to sell the **entire** acquired inventory, $q_S$ in period 2. We call this “without disposal” equilibrium.

- The speculator has the ability to **dispose** part of the acquired inventory and **sell a fraction**, $\alpha q_S$, where $\alpha < 1$, in period 2. We call this “with disposal” equilibrium.
Equilibrium Without Disposal

We solve backwards: period 2 first, and then period 1.

Period 2 equilibrium:

- The speculator supplies his inventory $q_S$ which he bought in period 1.
- The sellers maximize their period 2 payoff:

\[
\max_{q_A} p(q_A + q_B + q_S)q_A, \quad \text{and} \quad \max_{q_B} p(q_A + q_B + q_S)q_B
\]

where \( p(q_A + q_B + q_S) = a - b(q_A + q_B + q_S) \).

- The best response functions of sellers:

\[
q_A(q_B) = \frac{a}{2b} - \frac{q_B}{2} - \frac{q_S}{2}, \quad \text{and} \quad q_B(q_A) = \frac{a}{2b} - \frac{q_A}{2} - \frac{q_S}{2}.
\]

- The equilibrium supply of each seller, market clearing price and resulting profits

\[
q_2(q_S) = q^* - \frac{q_S}{3}, \quad p_2(q_S) = p^* - \frac{b q_S}{3}, \quad \pi_2(q_S) = \frac{(a - b q_S)^2}{9b}.
\]
Each seller supplies \((\frac{a}{3b} - \frac{qs}{3})\) in equilibrium. Given each seller supplies \(\frac{a}{3b}\) in the benchmark case, each seller reduces supply by \(\frac{qs}{3}\).

The speculator sells all his entire inventory \(qs\). Hence, the total supply increases by \(\frac{qs}{3}\) which causes the clearing price to drop by \(\frac{b}{3}qs\).

The arrow from the gray dot to the black dot shows the shift of the benchmark to the new equilibrium.

Since the strategic sellers sell less and equilibrium price is lower relative to the benchmark case, they earn lower profits. The consumers are strictly better-off in period 2.
Period 1 Equilibrium Without Disposal: The speculator’s strategy

The speculator uses a limit order to buy:

\[ q_S(p_1) = \begin{cases} 
0 & \text{for } p_1 > p_S \\
[0, q_S] & \text{for } p_1 = p_S \\
q_S & \text{for } p_1 < p_S 
\end{cases} \]

where \( p_1 \) is the limit buy-price, and \( q_S \) is the limit quantity.

The market price in period 1:

\[ p_1(Q_1; q_S, p_S) = \begin{cases} 
a - b Q_1 & \text{for } Q_1 < \frac{a-p_S}{b} \\
p_S & \text{for } Q_1 \in \left[\frac{a-p_S}{b}, \frac{a-p_S}{b} + q_S\right] \\
a - b(Q_1 - q_S) & \text{for } Q_1 > \frac{a-p_S}{b} + q_S, 
\end{cases} \]

where \( Q_1 = q_{A1} + q_{B1} \) is the aggregate supply of the sellers in period 1.
The Strategic Sellers’ and the Speculator’s Objective

- The strategic sellers maximize their profit in period 1 while taking into account: a) rivals supply decisions; b) the speculator’s demand in period 1; and c) the effect of current supply decision on the supply in period 2:

\[
\max_{q_{A1}} \pi_{A1}(q_{A1} + q_{B1}; q_{S}, p_{S}) + \pi_{2}(q_{S}(p_{1}(q_{A1} + q_{B1}; q_{S}, p_{S}))), \\
\max_{q_{B1}} \pi_{B1}(q_{A1} + q_{B1}; q_{S}, p_{S}) + \pi_{2}(q_{S}(p_{1}(q_{A1} + q_{B1}; q_{S}, p_{S}))).
\]

- The speculator maximizes his round-trade profit subject to two constraints:
  - The round-trade profit must be nonnegative (the participation constraint):
    \[
    \max_{q_{S}, p_{S}} q_{S}(p_{2}(q_{S}(p_{1}(q_{A1} + q_{B1}; q_{S}, p_{S}))) - p_{1}(q_{A1} + q_{B1}; q_{S}, p_{S}) - c_{S})
    \]
  - It is the interest of both strategic sellers to meet the speculator’s demand in full (the incentive compatibility constraint).
Lemma 1

Let $p^*$ denote the benchmark equilibrium price; $q_\text{S}$ denote the limit quantity in the limit order of the speculator; and $a$ and $b$ are the demand and sensitivity parameters respectively. Then, for any given $q_\text{S}$, there exists a lowest limit buy price

$$p_\text{S}(q_\text{S}) = p^* - \frac{\sqrt{b \cdot q_\text{S}(4a + 13b \cdot q_\text{S})} - 3b \cdot q_\text{S}}{6},$$

which guarantees that it is the period 1 clearing price and the speculator’s demand $q_\text{S}$ is fully supplied.

Note that the lowest limit buy-price of the speculator, $p_\text{S}(q_\text{S})$, is always strictly smaller than the benchmark price $p^*$ when $q_\text{S} > 0$. 
Why \( p_S(q_S) \) is an equilibrium price?

- \( p_S(q_S) \) provides one strategic seller enough **incentive** to produce for the speculator *when the other strategic seller sticks to the benchmark quantity*.
- Also, \( p_S(q_S) \) ensure that **unilateral deviation** from higher than benchmark supply is unprofitable.

A numerical example:

<table>
<thead>
<tr>
<th>Seller A</th>
<th>Benchmark</th>
<th>Supply More</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^*(=1800) )</td>
<td>( \pi^*(=1800) )</td>
<td>( \pi_A(\geq 1800), \pi_B(=1294) )</td>
</tr>
<tr>
<td>( \pi_A(\geq 1800) )</td>
<td>( \pi_B(=1294) )</td>
<td>( \pi_A^<em>(S)(=1547), \pi_B^</em>(S)(=1547) )</td>
</tr>
</tbody>
</table>

We assume \( a = 90, b = 1 \), and \( q_S = 15 \). When one seller supplies more, the other seller’s sticks to benchmark quantity, she earns \( 30 \times 22.3 + 25 \times 25 = 1294 \) as profit. The profit of the unilateral deviating seller is \( 52.7 \times 22.3 + 25 \times 25 \geq 1800 \).
Equilibrium Without Disposal: The Speculator’s Profit

Proposition 1: When the speculator’s storage cost is lower than a per unit threshold cost, \( \bar{c}_S \), then there exists an equilibrium where the market clearing prices are different in period 1 and period 2 and the speculator earns positive profit. The threshold of the storage cost is

\[
\bar{c}_S = \frac{5 - 2\sqrt{3}}{39} a \approx 0.04 a.
\]

When \( c_S > \bar{c}_S \), then the speculator does not enter the market and the market clearing prices are same as the benchmark price, \( p^* \) in both periods.

After the speculator enters the commodity market

- The speculator buys 13.70 units and atomistic consumers buy 7.47 additional units \( \Rightarrow 21.17/60 = 35.28\% \) jump in demand in period 1;
- yet, the clearing price in period 1 drops from 30 to 22.3, which is a \( 24.9\% \) fall relative to the benchmark clearing price \( \Rightarrow \text{The speculator buys and lowers the market clearing price.} \)
The Resulting Price Spread and the Speculator’s Profit

Figure: The figure shows the price spread between period 2 and period 1 in the equilibrium without disposal (on the left panel) and the round-trade profit $\pi_S$ of the speculator (on the right panel) given different value of $q_S$ with parameters $a = 90$, $b = 1$, and $c_S = 1$. In both figures, we use the limit price that satisfies the incentive compatibility constraints of the producers as the clearing price in period 1. The price spread reaches its maximum 3.54 when $q_S = 6.14$, while the speculator earns highest profit 25.94 when $q_S = 13.70$. 
The speculator uses a combination of market order and stop-loss order to supply in period 2.

- The speculator disposes part of the acquired inventory and sells only a fraction in period 2.
  - The resulting market clearing price in period 2 is higher than the benchmark price and the market clearing price in period 1 is lower than the period 1’s market clearing price without disposal.
  - Hence, the price spread is even greater!
- The consumers are better-off in period 1 and worse-off in period 2.
- Both strategic sellers earn lower aggregate profits relative to benchmark profits.
- The speculator earns more with disposal relative to without disposal.
We assume that the speculator’s inventory is $q_{S1}$. He submits a market order to supply $\alpha q_{S1}$, $\alpha \in [0, 1]$, and a stop-loss order to supply $(1 - \alpha) q_{S1}$ when the clearing price in period 2 is below $p_{S2}$ and zero otherwise. Hence, the speculator’s supply schedule given as below.

$$q_{S2}(p_2) = \begin{cases} \alpha q_{S1} & \text{for } p_2 \geq p_{S2} \\ q_{S1} & \text{for } p_2 < p_{S2} \end{cases}$$

Given the speculator’s supply, the market-clearing price in period 2 with free disposal is:

$$p_2(Q_2; \alpha, \cdot) = \begin{cases} a - b(Q_2 + q_{S1}) & \text{for } Q_2 < \frac{a - p_{S2}}{b} - \alpha q_{S1} \\ a - b(Q_2 + \alpha q_{S1}) & \text{for } Q_2 \geq \frac{a - p_{S2}}{b} - \alpha q_{S1}. \end{cases}$$
The spread between period-2 price and period-1 price and the speculator’s net profit (on the right panel) as a function of $\alpha \in [0.55, 1.0]$. We assume $a = 90$, $b = 1$, and $q_{S1} = 15$.

- The speculator’s profit is maximized when $\alpha = 0.88$ (optimal disposal). When $\alpha = 0.625$, the speculator’s profit with disposal is same as without disposal.
- The gray dots and black dots represent the price spread (left panel) without disposal and with optimal disposal respectively. The gray dots and black dots represent the speculator’s profits (right panel) without disposal and with optimal disposal respectively.
A large speculator with access to storage facility can influence the behavior of all other rational participants in such a way that he obtains a lower price while buying and a higher price while selling, and profiting from it.

- The presence of the speculator introduces price volatility. Both strategic sellers are made worse off in both periods.

We consider two cases: one with free disposal and one where the entire inventory has to be sold in the market.

- We show that the degree of destabilization is higher when the speculator has access to free disposal technology.