Keeping the Agents in the Dark: Private Disclosures in Competing Mechanisms

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January 4, 2021

Introduction

We study competing-mechanism games in which:

1) principals compete through mechanisms to deal with several agents;

2) agents have exogenous private information and negotiate with all principals.

Applications

Competing auctions: McAfee 1993, Peters and Severinov 1997, Virag 2010.

Competitive search: Moen 1997, Guerrieri, Shimer, and Wright 2010, Auster and Gottardi 2018.

Financial and insurance markets: Biais, Martimort, and Rochet 2000, Attar, Mariotti, and Salanié 2011, 2014, 2019.

Lobbying: Prat and Rustichini 2003.

Two takeaways from the literature

Relevant mechanisms: A principal can let agents communicate their endogenous *market information* to her in addition to their exogenous private information.

$\Rightarrow$ Epstein and Peters 1999 identify universal message spaces allowing them to formulate a *revelation principle*.

Equilibrium characterization: Allowing agents to communicate their endogenous market information spectacularly enlarges the set of equilibrium allocations if message spaces are rich enough.

$\Rightarrow$ This allows Yamashita 2010 and Peters and Troncoso-Valverde 2013 to establish *folk theorems*: any IC allocation that yields each principal a payoff above an appropriate bound can be supported in equilibrium.

Communication in competing-mechanism games

These results obtain under a standard communication protocol:

1. principals post *public* mechanisms;

2. agents send *private* messages to each principal.

We evaluate the role of *private signals* sent by the principals to the agents before the agents send them messages.

By sending private signals to the agents, a principal can make them differently informed about her decision rule.

This paper

1. Private signals sent by the principals allow one to support equilibria that cannot be supported via the universal mechanisms of Epstein and Peters 1999.

2. There exist equilibrium allocations in Yamashita 2010 and Peters and Troncoso-Valverde 2013 that cannot be supported if principals can send private signals.

3. We aim at providing a canonical framework to incorporate two-sided private communication in competing-mechanism games. In contrast with mechanism design with a single principal, multiple rounds of communication may be needed.

This talk

We focus on the first question in a pure incomplete-information setting in which agents take no payoff-relevant actions.

We construct an equilibrium in which principals post mechanisms with signals, such that the corresponding allocation cannot be supported using the universal message spaces of Epstein and Peters 1999.

By sending signals to the agents before they send messages, a principal is able to *correlate* their messages to the other principals with her own decisions in a way that cannot be replicated with standard mechanisms.

Players, decisions, and information

Two principals, P1 and P2, and two agents, A1 and A2.

The decision sets are

$X_1 \equiv \{x_{11}, x_{12}, x_{13}, x_{14}\}$ for P1 and $X_2 \equiv \{x_{21}, x_{22}\}$ for P2.

Each agent can be of two types, with

$\Omega^1 = \Omega^2 \equiv \{\omega_L, \omega_H\}$.

Types are perfectly correlated,

$\mathbf{P}[(\omega_L, \omega_L)] = 1 - \mathbf{P}[(\omega_H, \omega_H)] \in (0, 1)$.

Payoffs

P2's, A1's, and A2's payoffs are

	x_{21}	x_{22}
x_{11}	$(\zeta, 4, 4.5)$	$(\zeta, 4.5, 5)$
x_{12}	$(\zeta, 2, 5)$	$(\zeta, 5, 6)$
x_{13}	$(10, 3, 3)$	$(\zeta, 5.5, 3.5)$
x_{14}	$(\zeta, 1, 3.5)$	$(10, 7.5, 7.5)$

In state (ω_L, ω_L)

	x_{21}	x_{22}
x_{11}	$(\zeta, 1, 3)$	$(10, 7.5, 5)$
x_{12}	$(10, 3, 6)$	$(\zeta, 5.5, 6)$
x_{13}	$(\zeta, 8, 8)$	$(\zeta, 4.5, 7)$
x_{14}	$(\zeta, 9, 3)$	$(\zeta, 3, 0)$

In state (ω_H, ω_H)

$\zeta \ll 0$ is a large loss for P2.

P1's payoff is constant throughout the game.

An informal description of the game G^{SM}

1. P1 and P2 publicly post mechanisms with signals.

2. P1's and P2's mechanisms send private signals to A1 and A2.

3. Given their types, A1 and A2 send messages to P1 and P2.

4. P1's and P2's decisions are implemented and payoffs accrue.

Principals' mechanisms and agents' strategies

Let S_j^i be the set of signals from Pj to Ai, and M_j^i be the set of messages from Ai to Pj.

A mechanism with signals for Pj is a pair $\gamma_j \equiv (\sigma_j, \phi_j)$ such that

$$\sigma_j \in \Delta(S_j^1 \times S_j^2), \quad \phi_j : S_j^1 \times S_j^2 \times M_j^1 \times M_j^2 \to \Delta(X_j).$$

A standard mechanism has $S_j^1 = S_j^2 \equiv \{\emptyset\}$.

A communication strategy for Ai is a mapping

$$\lambda^i : \Gamma_1 \times \Gamma_2 \times S_1^i \times S_2^i \times \Omega^i \to \Delta(M^i).$$

Equilibrium

The strategy profile $(\tilde{\gamma}_1, \tilde{\gamma}_2, \lambda^1, \lambda^2)$ is a PBE of G^{SM} if

1. For each $\gamma \equiv (\gamma_1, \gamma_2)$, $(\lambda^1(\gamma), \lambda^2(\gamma))$ is a BNE in the subgame γ played by A1 and A2.

2. Given the continuation equilibrium $\lambda \equiv (\lambda^1, \lambda^2)$, $\tilde{\gamma} \equiv (\tilde{\gamma}_1, \tilde{\gamma}_2)$ is a NE of the game played by P1 and P2

The main result

Suppose that $M_j^i \equiv \Omega^i$ for all i and j, that $S_1^1 = S_1^2 = S_2^2 \equiv \{\emptyset\}$, and that $S_2^1 \equiv \{1, 2\}$. Thus P2 can send a (binary) signal to A1.

Proposition *For each* $\alpha \in (\frac{2}{5}, \frac{1}{2})$,

$$z(\omega_L, \omega_L) \equiv \alpha\delta_{(x_{13}, x_{21})} + (1 - \alpha)\delta_{(x_{14}, x_{22})}$$

$$z(\omega_H, \omega_H) \equiv \alpha\delta_{(x_{12}, x_{21})} + (1 - \alpha)\delta_{(x_{11}, x_{22})}$$

is a PBE outcome of G^{SM}.

Implications for Epstein and Peters 1999

Only cells with a $*$ can obtain on the equilibrium path:

	x_{21}	x_{22}
x_{11}	$(\zeta, 4, 4.5)$	$(\zeta, 4.5, 5)$
x_{12}	$(\zeta, 2, 5)$	$(\zeta, 5, 6)$
x_{13}	$(10, 3, 3)^*$	$(\zeta, 5.5, 3.5)$
x_{14}	$(\zeta, 1, 3.5)$	$(10, 7.5, 7.5)^*$

In state (ω_L, ω_L)

	x_{21}	x_{22}
x_{11}	$(\zeta, 1, 3)$	$(10, 7.5, 5)^*$
x_{12}	$(10, 3, 6)^*$	$(\zeta, 5.5, 6)$
x_{13}	$(\zeta, 8, 8)$	$(\zeta, 4.5, 7)$
x_{14}	$(\zeta, 9, 3)$	$(\zeta, 3, 0)$

In state (ω_H, ω_H)

It is impossible to reproduce this outcome via an arbitrary pair of standard mechanisms, even if A1 and A2 play mixed strategies.

More generally, for arbitrary message spaces, there is no PBE of a game G^M without signals that implements this outcome.

Equilibrium mechanisms

Let P1 commit to the deterministic mechanism

$$\gamma_1^*(\omega_L, \omega_L) \equiv \delta_{x_{11}}, \quad \gamma_1^*(\omega_H, \omega_L) \equiv \delta_{x_{12}},$$

$$\gamma_1^*(\omega_L, \omega_H) \equiv \delta_{x_{13}}, \quad \gamma_1^*(\omega_H, \omega_H) \equiv \delta_{x_{14}},$$

and consider the following mechanism γ_2^* with signals for P2:

1. with probability α, P2 privately communicates 1 to A1 and commits to take the decision x_{21} for every profile of received messages;

2. with probability $1-\alpha$, P2 privately communicates 2 to A1 and commits to take the decision x_{22} for every profile of received messages.

In state (ω_L, ω_L)

Upon receiving 1 from P2, A1's payoffs are

	ω_L	ω_H
ω_L *	4	3
ω_H	2	1

Upon receiving 2 from P2, A1's payoffs are

	ω_L	ω_H
ω_L	4.5	5.5
ω_H *	5	7.5

A2, anticipating this unique best response of A1, strictly prefers to report ω_H to P1 if $\alpha < \frac{1}{2}$.

In state (ω_H, ω_H)

Upon receiving 1 from P2, A1's payoffs are

	ω_L	ω_H
ω_L	1	8
ω_H *	3	9

Upon receiving 2 from P2, A1's payoffs are

	ω_L	ω_H
ω_L *	7.5	4.5
ω_H	5.5	3

A2, anticipating this unique best response of A1, strictly prefers to report ω_L to P1 if $\alpha > \frac{2}{5}$.

Wrapping up

1. There exists a unique BNE in the subgame (γ_1^*, γ_2^*) played by A1 and A2;

2. this BNE, together with the mechanisms (γ_1^*, γ_2^*), implements the required outcome;

3. if P1 posts γ_1^*, P2 can obtain her maximal payoff of 10 by posting γ_2^*;

4. since P1's payoff is constant throughout the game, she may as well post γ_1^*.

Conclusion

This example shows that the classical Epstein and Peters 1999 construction of universal mechanisms may fail to capture some relevant set of equilibrium allocations.

In the paper, we conversely show that folk theorems in the spirit of Yamashita 2010 and Peters and Troncoso-Valverde 2013 may generate equilibria that are not robust to private signals.

This raises the question of the canonical model for two-sided private communication in competing-mechanism games. This is still work in progress...