# The Perils of Tracking r-Star

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# The natural rate of interest (r-star)

- Important benchmark for monetary policy

- Standard model

$$\tilde{Y}_t = -s \sum_{k=0}^{\infty} \mathbb{E}_t \left[ r_{t+k} - r^*_{t+k} \right],$$
$$\pi_t = \kappa \tilde{Y}_t + \beta \mathbb{E}_t \pi_{t+1},$$

- If $r_t = r_t^* \implies \tilde{Y}_t = 0 \implies \pi_t = 0 \qquad \text{(first best)}$

- Taylor (1993)

$$f = r^* + p^* + 1.5\left(p - p^*\right) + 0.5\left(y - y^*\right)$$

# Tracking r* in a bigger model: Barksy et al. (2014)

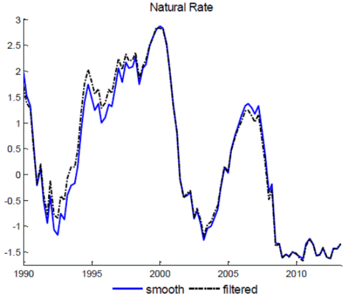



Figure 1: One-sided (filtered) and two-sided (smoothed) estimate of the natural rate.

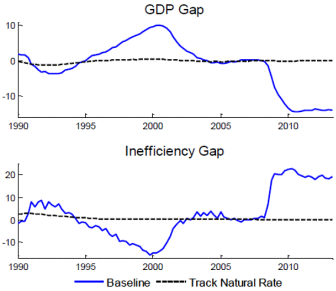


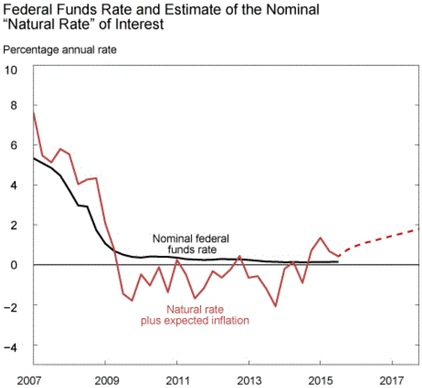

Figure 2: Output and Inefficienct Gaps under estimated interest rule and when track the natural rate.

$$\text{Fitted MP rule} \quad : \quad \hat{R}_t = \rho_R R_{t-1} + (1-\rho_R)\left(\phi_\pi \hat{\pi}_t^{avg} + \phi_Y \tilde{Y}_t^{avg}\right)$$

$$\text{r}^* \text{ tracking rule} \quad : \quad \hat{R}_t = \hat{r}_t^* + 1.0001\mathbb{E}_t \hat{\pi}_{t+1}$$

# Monetary policy in practice - 1

- Yellen (2015) "The New Normal Monetary Policy"
  - *"...the economy's underlying strength has been gradually improving, and the equilibrium real federal funds rate has been gradually rising. ... and as the equilibrium real funds rate continues to rise, it will accordingly be appropriate to raise the actual level of the real federal funds rate in tandem, all else being equal."*



# Monetary policy in practice -2

- Curdia et al. (2015)
  - $r^*$ tracking Taylor rule is a better description of monetary policy in US

- Model

$$
\begin{aligned}
x_t &= \mathbb{E}_t x_{t+1} - (1-\beta\eta)\left(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^*\right) \\
\hat{\pi}_t - \chi\hat{\pi}_{t-1} &= \kappa_H \left[\left(\tilde{Y}_t - \delta\tilde{Y}_{t-1}\right) - \beta\delta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \delta\tilde{Y}_t\right)\right] + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1} - \chi\hat{\pi}_t\right] \\
\hat{R}_t &= \rho_R R_{t-1} + (1-\rho_R)\left(\gamma\hat{r}_t^* + \phi_\pi\hat{\pi}_t + \phi_Y\tilde{Y}_t\right),
\end{aligned}
$$

where $x_t \equiv \left(\tilde{Y}_t - \eta\tilde{Y}_{t-1}\right) - \beta\eta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \eta\tilde{Y}_t\right)$.

- Bayesian estimation with $\gamma = 0$ and 1
  - Model fit comparison.

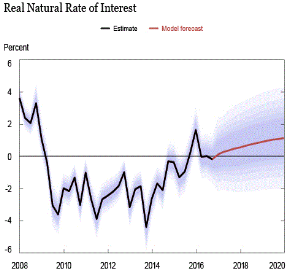
# Debates

The existing debates focus mostly on **implementation** issues

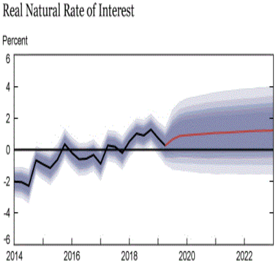
- Model and shock specific (Taylor and Wieland 2016)
- Imprecise real time estimates (Hamilton et al. 2016; Beyer and Wieland 2017)
- Tracking *wrong* $r^*$ could generate
  - unnecessary interest rate volatility
  - macro instability
  - welfare loss

# NY Fed Model Forecast

Feb. 2017




Sep. 2019

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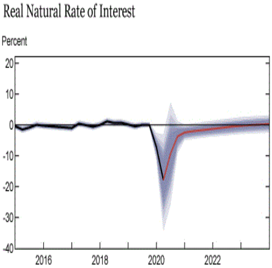


# This paper: perils of tracking r-star

- Tracking even *correct* $r^*$ can lead to macro instability and lower welfare (full information environment)

- $\pi$ may not be completely insulated from fiscal condition (the state of government indebtedness)

- *Fiscal* channel:

$$\text{nominal public debt} \uparrow \Longrightarrow \pi \uparrow$$

- Fiscal channel operates if
  - *<u>insufficient</u> tax increase in response to debt increases*

# Mechanisms

- Suppose
  (i) a shock increases $r^*$
  (ii) Fed tracks $r^*$

- **Conventional** AD channel
  - $r^* \uparrow \implies R \uparrow \implies$ stabilizes $\tilde{Y}$ and $\pi$ as usual

- **Fiscal** channel
  - $r^* \uparrow \implies R \uparrow \implies$ interest payment $\uparrow \implies$ public debt $\uparrow \implies \pi \uparrow \implies \tilde{Y} \uparrow$

- Two countervailing channels

# This paper: additional point

- Perils of tracking $r^*$ with interest rate smoothing.

$$\hat{R}_t = \rho_R R_{t-1} + (1-\rho_R)\left(\hat{r}_t^* + \phi_\pi \hat{\pi}_t + \phi_Y \tilde{Y}_t\right)$$

- This result has nothing to do with the fiscal channel.

- Subsequent movements in $\hat{R}_t$ can be quite different from $\hat{r}_t^*$.

# Literature

- R-star estimates
  - Laubach and Williams (2003), Justiniano and Primiceri (2012), Lubik and Matthes (2015), Holston et al. (2017), Edge et al. (2008)
- Monetary & fiscal policy interactions
  - Sims (1994), Woodford (1994), Leeper (1990), Loyo (2000), and Cochrane (2001), Bhattarai et al. (2014)

# Prototype NK model

- Augmented with the fiscal policy bloc
- System of $\left\{\tilde{Y}_t, \hat{\pi}_t, \hat{R}_t, \hat{\tau}_t, \hat{b}_t\right\}$:

$$
\begin{aligned}
\tilde{Y}_t &= \mathbb{E}_t \tilde{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1}\right) + \hat{r}_t^*, \\
\hat{\pi}_t &= \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}, \\
\hat{R}_t &= \gamma \hat{r}_t^* + \phi \hat{\pi}_t, \qquad 0 \leq \gamma \leq 1 \\
\hat{\tau}_t &= \psi \hat{b}_{t-1}, \\
\hat{b}_t &= \beta^{-1} \hat{b}_{t-1} - \beta^{-1} \bar{b} \hat{\pi}_t - \beta^{-1} \hat{\tau}_t + \bar{b} \hat{R}_t.
\end{aligned}
$$

# Prototype NK model

- Substitute out the policy instruments
- System of $\left\{\tilde{Y}_t, \hat{\pi}_t, \hat{b}_t\right\}$:

$$\tilde{Y}_t = \mathbb{E}_t \tilde{Y}_{t+1} - \phi\hat{\pi}_t + \mathbb{E}_t \hat{\pi}_{t+1} + \underbrace{(1-\gamma)\,\hat{r}^*_t}_{\textbf{aggregate demand channel}},$$

$$\hat{\pi}_t = \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1},$$

$$\hat{b}_t = \beta^{-1}(1-\psi)\hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right)\hat{\pi}_t + \underbrace{\bar{b}\gamma\hat{r}^*_t}_{\textbf{fiscal channel}}.$$

# Case 1 (monetary regime)

- Conventional case
- $\psi$ is sufficiently large: fiscal bloc and channel irrelevant.

$$\tilde{Y}_t = \mathbb{E}_t \tilde{Y}_{t+1} - \phi\hat{\pi}_t + \mathbb{E}_t \hat{\pi}_{t+1} + (1-\gamma)\,\hat{r}^*_t$$
$$\hat{\pi}_t = \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}$$
$$\hat{b}_t = \underbrace{\beta^{-1}(1-\psi)}_{<1} \hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right)\hat{\pi}_t + \bar{b}\gamma\hat{r}^*_t$$

- $\gamma = 1$ (i.e. "full tracking") $\Longrightarrow \hat{\pi}_t = \tilde{Y}_t = 0$

$$\hat{\pi}_t = \frac{\kappa(1-\gamma)}{\kappa\phi + 1}\hat{r}^*_t, \qquad \tilde{Y}_t = \frac{(1-\gamma)}{\kappa\phi+1}\hat{r}^*_t,$$

# Case 2 (fiscal regime)

- $\psi$ is small

$$\tilde{Y}_t = \mathbb{E}_t \tilde{Y}_{t+1} - \phi \hat{\pi}_t + \mathbb{E}_t \hat{\pi}_{t+1} + (1-\gamma)\,\hat{r}_t^*$$
$$\hat{\pi}_t = \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1}$$
$$\hat{b}_t = \underbrace{\beta^{-1}(1-\psi)}_{>1} \hat{b}_{t-1} - \bar{b}\left(\beta^{-1} - \phi\right)\hat{\pi}_t + \bar{b}\gamma \hat{r}_t^*$$

- Debt ($\hat{b}_t$) won't be stabilized by fiscal policy only, and $\hat{\pi}_t$ needs to rise.
- For that to happen, $\phi$ should be sufficiently small
  - Accommodating (or "passive") monetary policy

# Proposition 1

*When the fiscal channel is operative (i.e. $\psi \in (-\infty, \bar{\psi})$ and $\phi \in [0, 1)$), the solution for debt, inflation, and the output gap is given by*

$$\hat{b}_t = \Theta(\gamma)\,\hat{r}^*_t + \Omega_b \hat{b}_{t-1} = \Theta(\gamma) \sum_{k=0}^{\infty} \Omega_b^k \hat{r}^*_{t-k},$$

$$\hat{\pi}_t = \Gamma(\gamma)\,\hat{r}^*_t + \Omega_\pi \hat{b}_{t-1} = \Gamma(\gamma)\,\hat{r}^*_t + \Omega_\pi \Theta(\gamma) \sum_{k=1}^{\infty} \Omega_b^{k-1} \hat{r}^*_{t-k},$$

$$\tilde{Y}_t = \Lambda(\gamma)\,\hat{r}^*_t + \Omega_Y \hat{b}_{t-1} = \Lambda(\gamma)\,\hat{r}^*_t + \Omega_Y \Theta(\gamma) \sum_{k=1}^{\infty} \Omega_b^{k-1} \hat{r}^*_{t-k},$$

*where the coefficients are composites of the structural parameters. Moreover,*

1. $\Omega_b, \Omega_\pi$ *and* $\Omega_Y$ *are independent of how the central bank responds to the natural rate* ($\gamma$)*; they are all positive.*
2. $\Theta$, $\Gamma$, *and* $\Lambda$ *are linear functions of* $\gamma$, *conditional on other structural parameters;* $\Gamma$*and* $\Lambda$ *are positive* $\forall \gamma \geq 0$.

# Proposition 2

*When the fiscal channel is operative, at* $\gamma = 0$, $\Theta(\gamma) < 0$, $\Gamma(\gamma) > 0$, *and* $\Lambda(\gamma) > 0$. *Moreover,*

$$\Theta'(\gamma) > 0 \quad \text{for } \psi \in (-\infty, \bar{\psi}) \quad \text{and} \quad \phi \in [0, 1),$$
$$\Gamma'(\gamma) > 0 \quad \text{for } \psi \in \left(-\infty, \bar{\psi}^*\right) \quad \text{and} \quad \phi \in [0, 1),$$

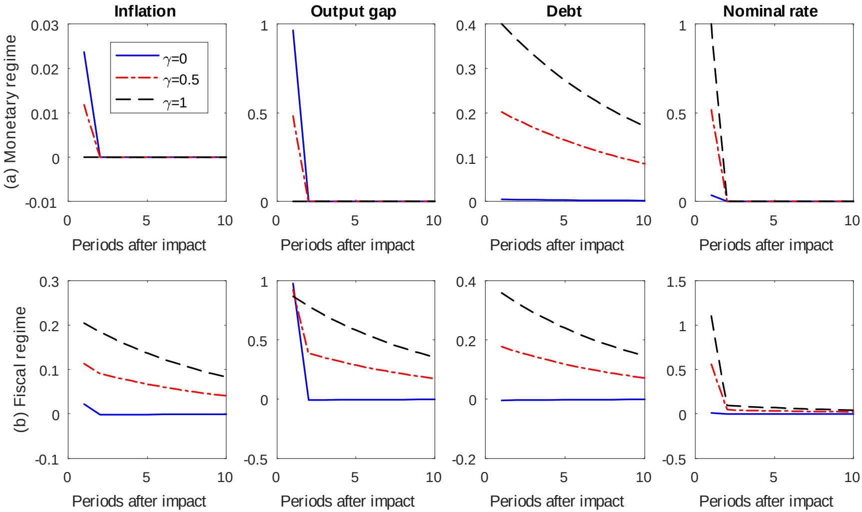
*where* $0 < \bar{\psi}^* \leq \bar{\psi}$ *is a reduced-form parameter.*

# Parameterization

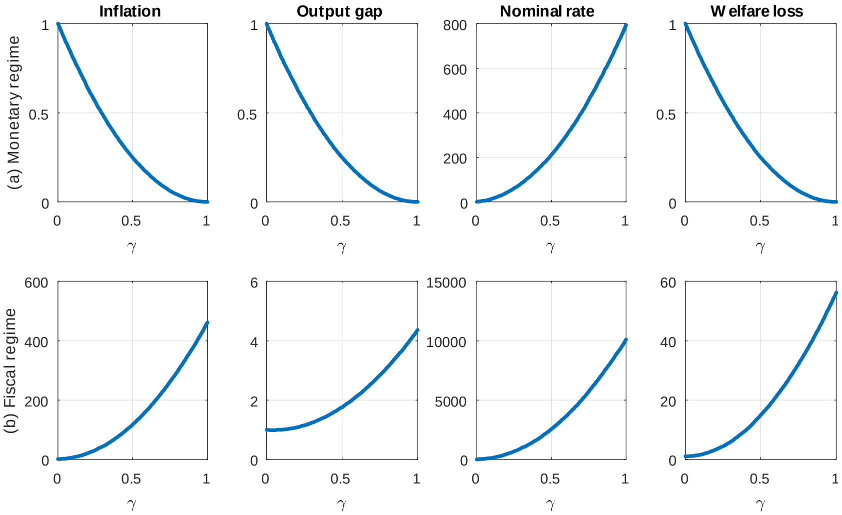
Table: Parameter values used in the numerical analysis

| Parameter | Description | Value | Note |
|---|---|---|---|
| $\beta$ | Discount factor | 0.99 | Long-run interest rate |
| $\kappa$ | Phillips curve slope | 0.0245 | Underlying parameter values |
| $\bar{b}$ | Steady-state debt-GDP | 0.4 | U.S. data |
| $\phi$ | Inflation coefficient in MP | 1.5 | Monetary regime |
| | | 0.5 | Fiscal regime |
| $\psi$ | Debt coefficient in FP | 0.1 | Monetary regime |
| | | 0 | Fiscal regime |

# Impulse responses to iid r* shock



# Volatility and welfare loss



# Quantitative model

- Main points robustly hold in a quantitative model with propagation mechanisms
  - habit formation in consumption
  - price indexation or rule-of-thumb price setters
  - interest rate smoothing
  - tax smoothing

- By the way, the quantitative model with interest rate smoothing may fit the data better, as favored by Curdia et al (2015), but is not desirable as interest rate smoothing + tracking r* can propagate transitory shocks to r* over time

# Quantitative model

$$x_t = \mathbb{E}_t x_{t+1} - (1-\beta\eta)\left(\hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^*\right)$$

$$\hat{\pi}_t - \chi\hat{\pi}_{t-1} = \kappa_H \left[\left(\tilde{Y}_t - \delta\tilde{Y}_{t-1}\right) - \beta\delta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \delta\tilde{Y}_t\right)\right] + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1} - \chi\hat{\pi}_t\right]$$

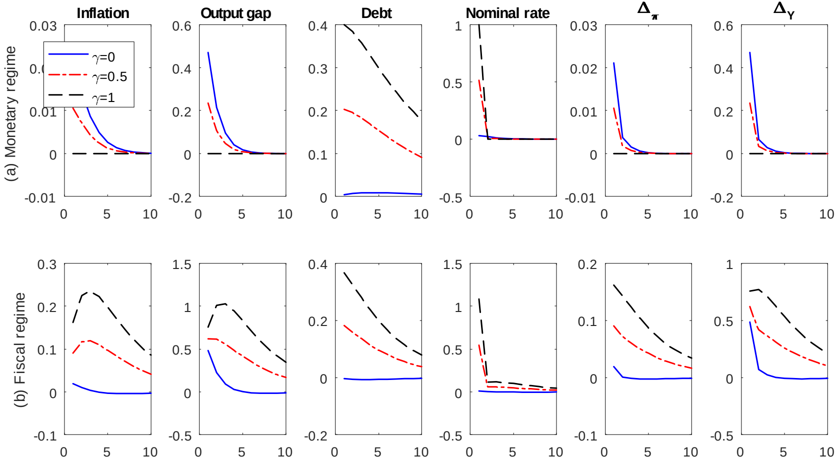
$$\hat{R}_t = \rho_R R_{t-1} + (1-\rho_R)\left(\gamma\hat{r}_t^* + \phi_\pi\hat{\pi}_t + \phi_Y\tilde{Y}_t\right),$$

$$\hat{\tau}_t = \rho_\tau\hat{\tau}_{t-1} + (1-\rho_\tau)\left(\psi_b\hat{b}_{t-1} + \psi_Y\tilde{Y}_t\right),$$

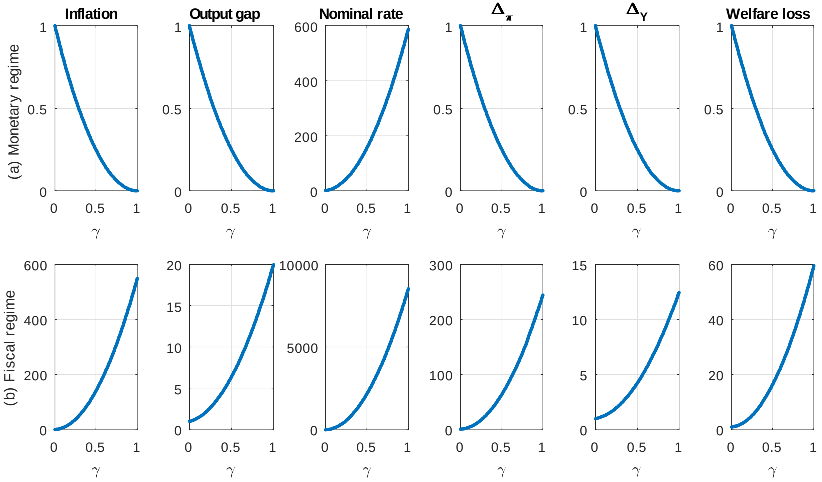
$$\hat{b}_t = \beta^{-1}\hat{b}_{t-1} - \beta^{-1}\bar{b}\hat{\pi}_t - \beta^{-1}\hat{\tau}_t + \bar{b}\hat{R}_t,$$

where $x_t \equiv \left(\tilde{Y}_t - \eta\tilde{Y}_{t-1}\right) - \beta\eta\mathbb{E}_t\left(\tilde{Y}_{t+1} - \eta\tilde{Y}_t\right).$

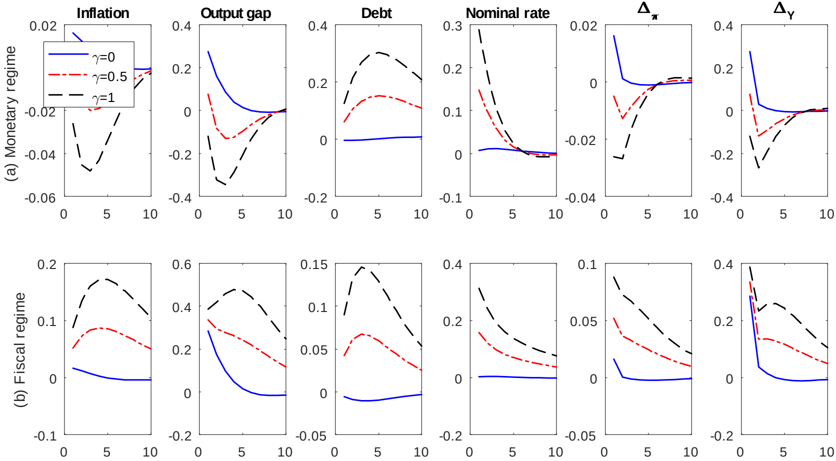
# IRFs – without interest smoothing



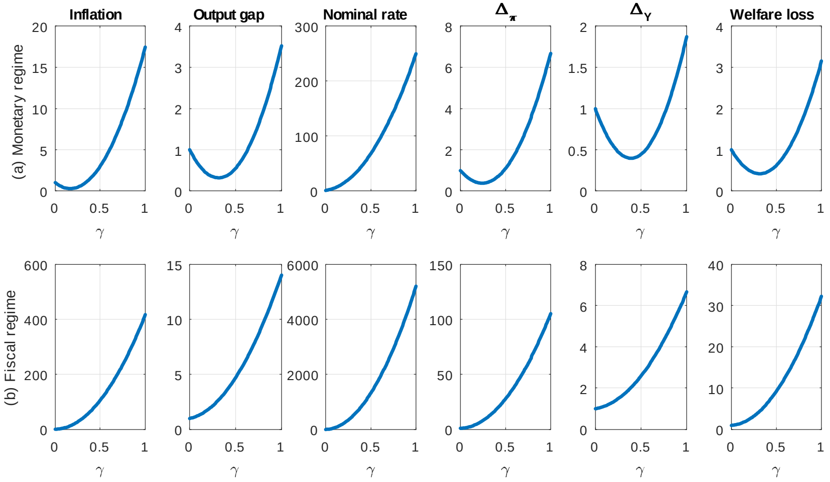
# Volatility and welfare loss – without interest smoothing



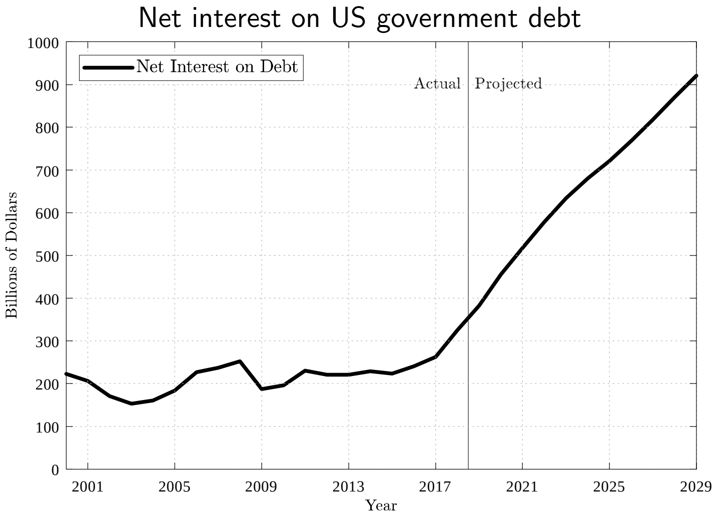
# IRFs – with interest smoothing



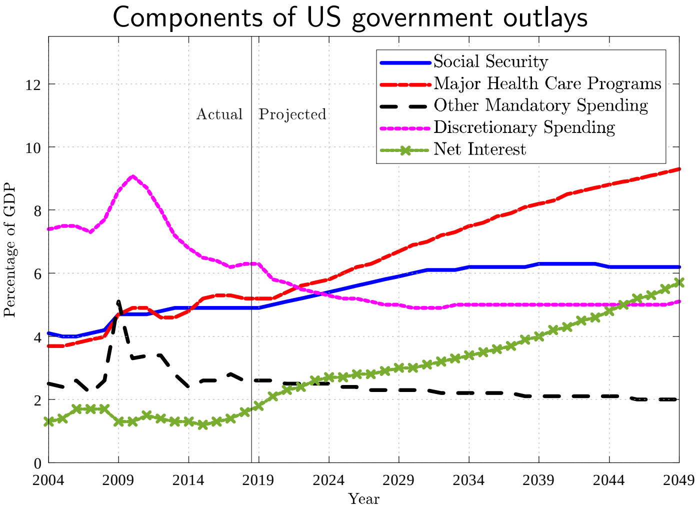
# Volatility and welfare loss – with interest smoothing



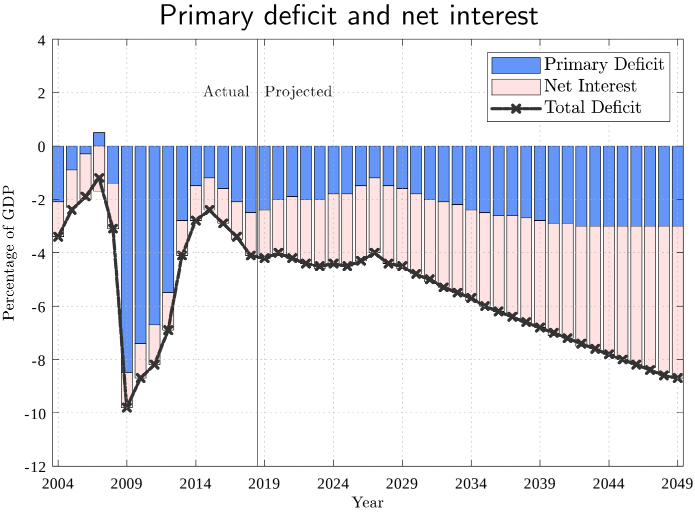
# Relevant going forward? (CBO projection)



# Relevant going forward?

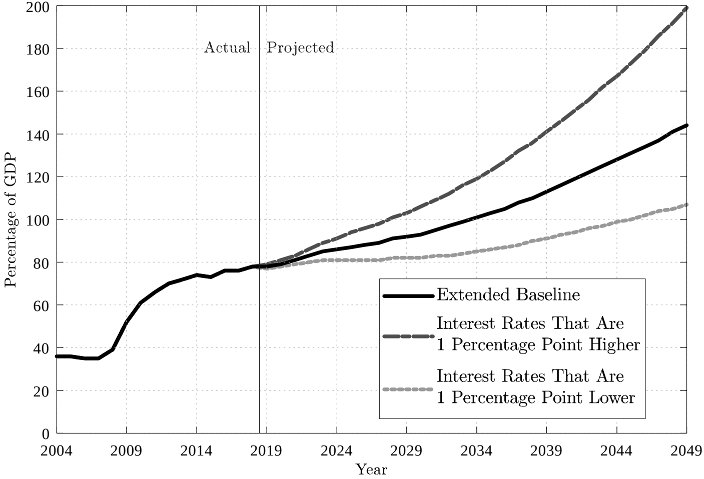


# Relevant going forward?



# Relevant going forward?

US government debt under alternate interest rate scenarios



# Summary

- There exists a *fiscal limit* to a monetary policy that incorporates *r*-star targeting.
- Tracking (even the *correct*) *r*-star will be desirable only if the public expects a sufficient tax increase in response to government debt increases
  - Condition uncertain to hold in future in the U.S., based on CBO projections.
- Cautionary note on the policy recommendation that the Federal Reserve should track *r*-star going forward.