The Perils of Tracking r-Star

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The natural rate of interest (r-star)

- Important benchmark for monetary policy

- Standard model

\[ \tilde{Y}_t = -s \sum_{k=0}^{\infty} E_t [r_{t+k} - r^*_{t+k}], \]

\[ \pi_t = \kappa \tilde{Y}_t + \beta E_t \pi_{t+1}, \]

- If \( r_t = r^*_t \) \( \implies \) \( \tilde{Y}_t = 0 \) \( \implies \) \( \pi_t = 0 \) (first best)

- Taylor (1993)

\[ f = r^* + p^* + 1.5 (p - p^*) + 0.5 (y - y^*) \]
Tracking \( r^* \) in a bigger model: Barksy et al. (2014)

\[
\hat{R}_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \phi_{\pi} \hat{\pi}_{t}^{avg} + \phi_Y \tilde{Y}_{t}^{avg} \right)
\]

\( r^* \) tracking rule : 
\[
\hat{R}_t = \hat{r}_t^* + 1.0001 E_t \hat{\pi}_{t+1}
\]

**Figure 1**: One-sided (filtered) and two-sided (smoothed) estimate of the natural rate.

**Figure 2**: Output and Inefficiency Gaps under estimated interest rule and when track the natural rate.
Monetary policy in practice - 1

  
  “... the economy’s underlying strength has been gradually improving, and the equilibrium real federal funds rate has been gradually rising. ... and as the equilibrium real funds rate continues to rise, it will accordingly be appropriate to raise the actual level of the real federal funds rate in tandem, all else being equal.”
Monetary policy in practice -2

- Curdia et al. (2015)
  - $r^*$ tracking Taylor rule is a better description of monetary policy in US

- Model

\[
\begin{align*}
x_t &= E_t x_{t+1} - (1 - \beta \eta) \left( \hat{R}_t - E_t \hat{\pi}_{t+1} - \hat{r}^*_t \right) \\
\hat{\pi}_t - \chi \hat{\pi}_{t-1} &= \kappa_H \left[ ( \hat{Y}_t - \delta \hat{Y}_{t-1} ) - \beta \delta E_t ( \hat{Y}_{t+1} - \delta \hat{Y}_t ) \right] + \beta E_t [ \hat{\pi}_{t+1} - \chi \hat{\pi}_t ] \\
\hat{R}_t &= \rho_R R_{t-1} + (1 - \rho_R) ( \gamma \hat{r}^*_t + \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t ),
\end{align*}
\]

where $x_t \equiv ( \hat{Y}_t - \eta \hat{Y}_{t-1} ) - \beta \eta E_t ( \hat{Y}_{t+1} - \eta \hat{Y}_t )$.

- Bayesian estimation with $\gamma = 0$ and 1
  - Model fit comparison.
Debates

The existing debates focus mostly on **implementation** issues

- Model and shock specific (Taylor and Wieland 2016)

- Imprecise real time estimates (Hamilton et al. 2016; Beyer and Wieland 2017)

- Tracking *wrong* $r^*$ could generate
  - unnecessary interest rate volatility
  - macro instability
  - welfare loss
This paper: perils of tracking r-star

- Tracking even *correct* \( r^* \) can lead to macro instability and lower welfare (full information environment)

- \( \pi \) may not be completely insulated from fiscal condition (the state of government indebtedness)

- *Fiscal channel:*

\[
\text{nominal public debt} \uparrow \implies \pi \uparrow
\]

- Fiscal channel operates if
  - *insufficient tax increase in response to debt increases*
Mechanisms

- Suppose
  
  (i) a shock increases $r^*$
  (ii) Fed tracks $r^*$

- **Conventional AD channel**
  
  - $r^* \uparrow \implies R \uparrow \implies$ stabilizes $\tilde{Y}$ and $\pi$ as usual

- **Fiscal channel**
  
  - $r^* \uparrow \implies R \uparrow \implies$ interest payment $\uparrow \implies$ public debt $\uparrow \implies \pi \uparrow \implies \tilde{Y} \uparrow$

- Two countervailing channels
This paper: additional point

- Perils of tracking \( r^* \) with interest rate smoothing.

\[
\hat{R}_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \hat{r}_t^* + \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t \right)
\]

- This result has nothing to do with the fiscal channel.

- Subsequent movements in \( \hat{R}_t \) can be quite different from \( \hat{r}_t^* \).
Literature

- R-star estimates

- Monetary & fiscal policy interactions
Prototype NK model

- Augmented with the fiscal policy bloc

- System of \( \{ \tilde{Y}_t, \hat{\pi}_t, \hat{R}_t, \hat{\tau}_t, \hat{b}_t \} \):

\[
\tilde{Y}_t = E_t \tilde{Y}_{t+1} - (\hat{R}_t - E_t \hat{\pi}_{t+1}) + \hat{r}_t^* , \\
\hat{\pi}_t = \kappa \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1} , \\
\hat{R}_t = \gamma \hat{r}_t^* + \phi \hat{\pi}_t , \quad 0 \leq \gamma \leq 1 \\
\hat{\tau}_t = \psi \hat{b}_{t-1} , \\
\hat{b}_t = \beta^{-1} \hat{b}_{t-1} - \beta^{-1} \bar{b}\hat{\pi}_t - \beta^{-1} \hat{\tau}_t + \bar{b}\hat{R}_t .
\]
Prototype NK model

- Substitute out the policy instruments

- System of \( \{ \tilde{Y}_t, \hat{\pi}_t, \hat{b}_t \} \):

\[
\tilde{Y}_t = E_t \tilde{Y}_{t+1} - \phi \hat{\pi}_t + E_t \hat{\pi}_{t+1} + \left(1 - \gamma \right) \hat{r}_t^* ,
\]

aggregate demand channel

\[
\hat{\pi}_t = \kappa \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1} ,
\]

\[
\hat{b}_t = \beta^{-1} (1 - \psi) \hat{b}_{t-1} - \bar{b} \left( \beta^{-1} - \phi \right) \hat{\pi}_t + \bar{b} \gamma \hat{r}_t^* .
\]

fiscal channel

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Case 1 (monetary regime)

- Conventional case

  - $\psi$ is sufficiently large: fiscal bloc and channel irrelevant.

    \[
    \begin{align*}
    \tilde{Y}_t &= E_t \tilde{Y}_{t+1} - \phi \hat{\pi}_t + E_t \hat{\pi}_{t+1} + (1 - \gamma) \hat{r}_t^* \\
    \hat{\pi}_t &= \kappa \tilde{Y}_t + \beta E_t \hat{\pi}_{t+1} \\
    \hat{b}_t &= \beta^{-1} (1 - \psi) \hat{b}_{t-1} - \bar{b} \left( \beta^{-1} - \phi \right) \hat{\pi}_t + \bar{b} \gamma \hat{r}_t^* \\
    \end{align*}
    \]

  - $\gamma = 1$ (i.e. "full tracking") $\implies \hat{\pi}_t = \tilde{Y}_t = 0$

    \[
    \begin{align*}
    \hat{\pi}_t &= \frac{\kappa (1 - \gamma)}{\kappa \phi + 1} \hat{r}_t^* , \quad \tilde{Y}_t = \frac{(1 - \gamma)}{\kappa \phi + 1} \hat{r}_t^* , \\
    \end{align*}
    \]
Case 2 (fiscal regime)

- $\psi$ is small

\[
\begin{align*}
\tilde{Y}_t &= \mathbb{E}_t \tilde{Y}_{t+1} - \phi \hat{\pi}_t + \mathbb{E}_t \hat{\pi}_{t+1} + (1 - \gamma) \hat{r}_t^* \\
\hat{\pi}_t &= \kappa \tilde{Y}_t + \beta \mathbb{E}_t \hat{\pi}_{t+1} \\
\hat{b}_t &= \beta^{-1}(1 - \psi) \hat{b}_{t-1} - \bar{b} \left( \beta^{-1} - \phi \right) \hat{\pi}_t + \bar{b} \gamma \hat{r}_t^*
\end{align*}
\]

- Debt ($\hat{b}_t$) won’t be stabilized by fiscal policy only, and $\hat{\pi}_t$ needs to rise.
- For that to happen, $\phi$ should be sufficiently small
  - Accommodating (or “passive”) monetary policy
Proposition 1

When the fiscal channel is operative (i.e. $\psi \in (-\infty, \bar{\psi})$ and $\phi \in [0, 1]$), the solution for debt, inflation, and the output gap is given by

\[
\hat{b}_t = \Theta(\gamma) \hat{r}_t^* + \Omega_b \hat{b}_{t-1} = \Theta(\gamma) \sum_{k=0}^{\infty} \Omega_b^k \hat{r}_{t-k},
\]

\[
\hat{\pi}_t = \Gamma(\gamma) \hat{r}_t^* + \Omega_{\pi} \hat{b}_{t-1} = \Gamma(\gamma) \hat{r}_t^* + \Omega_{\pi} \Theta(\gamma) \sum_{k=1}^{\infty} \Omega_b^{k-1} \hat{r}_{t-k},
\]

\[
\tilde{Y}_t = \Lambda(\gamma) \hat{r}_t^* + \Omega_Y \hat{b}_{t-1} = \Lambda(\gamma) \hat{r}_t^* + \Omega_Y \Theta(\gamma) \sum_{k=1}^{\infty} \Omega_b^{k-1} \hat{r}_{t-k},
\]

where the coefficients are composites of the structural parameters. Moreover,

1. $\Omega_b, \Omega_\pi$ and $\Omega_Y$ are independent of how the central bank responds to the natural rate ($\gamma$); they are all positive.

2. $\Theta, \Gamma,$ and $\Lambda$ are linear functions of $\gamma$, conditional on other structural parameters; $\Gamma$ and $\Lambda$ are positive $\forall \gamma \geq 0$. 

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Proposition 2

When the fiscal channel is operative, at $\gamma = 0$, $\Theta(\gamma) < 0$, $\Gamma(\gamma) > 0$, and $\Lambda(\gamma) > 0$. Moreover,

$$
\Theta'(\gamma) > 0 \quad \text{for } \psi \in (\infty, \bar{\psi}) \quad \text{and} \quad \phi \in [0, 1), \\
\Gamma'(\gamma) > 0 \quad \text{for } \psi \in (\infty, \bar{\psi}^*) \quad \text{and} \quad \phi \in [0, 1),
$$

where $0 < \bar{\psi}^* \leq \bar{\psi}$ is a reduced-form parameter.
## Parameterization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
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<td>Long-run interest rate</td>
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<td>$\kappa$</td>
<td>Phillips curve slope</td>
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<td>Underlying parameter values</td>
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<td>$\phi$</td>
<td>Inflation coefficient in MP</td>
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<td>Monetary regime</td>
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<td></td>
<td></td>
<td>0.5</td>
<td>Fiscal regime</td>
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<tr>
<td>$\psi$</td>
<td>Debt coefficient in FP</td>
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<td>Monetary regime</td>
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<td>0</td>
<td>Fiscal regime</td>
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</table>
Impulse responses to iid $r^*$ shock

(a) Monetary regime

- **Inflation**
  - $\gamma = 0$
  - $\gamma = 0.5$
  - $\gamma = 1$

(b) Fiscal regime

- **Output gap**
- **Debt**
- **Nominal rate**
Volatility and welfare loss

(a) Monetary regime

- Inflation
- Output gap
- Nominal rate
- Welfare loss

(b) Fiscal regime

- Inflation
- Output gap
- Nominal rate
- Welfare loss

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Quantitative model

- Main points robustly hold in a quantitative model with propagation mechanisms
  - habit formation in consumption
  - price indexation or rule-of-thumb price setters
  - interest rate smoothing
  - tax smoothing

- By the way, the quantitative model with interest rate smoothing may fit the data better, as favored by Curdia et al (2015), but is not desirable as interest rate smoothing + tracking $r^*$ can propagate transitory shocks to $r^*$ over time
Quantitative model

\[ x_t = \mathbb{E}_t x_{t+1} - (1 - \beta \eta) \left( \hat{R}_t - \mathbb{E}_t \hat{\pi}_{t+1} - \hat{r}_t^* \right) \]

\[ \hat{\pi}_t - \chi \hat{\pi}_{t-1} = \kappa_H \left[ (\tilde{Y}_t - \delta \tilde{Y}_{t-1}) - \beta \delta \mathbb{E}_t (\tilde{Y}_{t+1} - \delta \tilde{Y}_t) \right] + \beta \mathbb{E}_t [\hat{\pi}_{t+1} - \chi \hat{\pi}_t] \]

\[ \hat{R}_t = \rho_R R_{t-1} + (1 - \rho_R) \left( \gamma \hat{r}_t^* + \phi_\pi \hat{\pi}_t + \phi_Y \tilde{Y}_t \right), \]

\[ \hat{\tau}_t = \rho_\tau \hat{\tau}_{t-1} + (1 - \rho_\tau) \left( \psi_b \hat{b}_{t-1} + \psi_Y \tilde{Y}_t \right), \]

\[ \hat{b}_t = \beta^{-1} \hat{b}_{t-1} - \beta^{-1} \bar{b} \hat{\pi}_t - \beta^{-1} \hat{\tau}_t + \bar{b} \hat{R}_t, \]

where \( x_t \equiv (\tilde{Y}_t - \eta \tilde{Y}_{t-1}) - \beta \eta \mathbb{E}_t (\tilde{Y}_{t+1} - \eta \tilde{Y}_t) \).
IRFs – without interest smoothing

(a) Monetary regime
Inflation
Output gap
Debt
Nominal rate

(b) Fiscal regime

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Volatility and welfare loss – without interest smoothing

(a) Monetary regime

(b) Fiscal regime
IRFs – with interest smoothing

(a) Monetary regime
- Inflation
- Output gap
- Debt
- Nominal rate

(b) Fiscal regime

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Volatility and welfare loss – with interest smoothing

(a) Monetary regime
- Inflation
- Output gap
- Nominal rate
- ∆ₚ
- ∆ₚ
- Welfare loss

(b) Fiscal regime
- Inflation
- Output gap
- Nominal rate
- ∆ₚ
- ∆ₚ
- Welfare loss

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Relevant going forward? (CBO projection)

Net interest on US government debt

- **Net Interest on Debt**
  - Actual
  - Projected

Billions of Dollars

Year

Relevant going forward?

Components of US government outlays

- Social Security
- Major Health Care Programs
- Other Mandatory Spending
- Discretionary Spending
- Net Interest

Percentage of GDP
Relevant going forward?

Primary deficit and net interest

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Relevant going forward?

US government debt under alternate interest rate scenarios

- **Extended Baseline**
- **Interest Rates That Are 1 Percentage Point Higher**
- **Interest Rates That Are 1 Percentage Point Lower**

**Years:** 2004, 2009, 2014, 2019, 2024, 2029, 2034, 2039, 2044, 2049

**Percentage of GDP:**
- Range: 0 to 200

**轴:**
- **横轴:** Year
- **纵轴:** Percentage of GDP
Summary

- There exists a *fiscal limit* to a monetary policy that incorporates $r$-star targeting.

- Tracking (even the *correct*) $r$-star will be desirable only if the public expects a sufficient tax increase in response to government debt increases.
  - Condition uncertain to hold in future in the U.S., based on CBO projections.

- Cautionary note on the policy recommendation that the Federal Reserve should track $r$-star going forward.