Capital Heterogeneity and Investment Prices

How much are investment prices declining?

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Disclaimer: The views expressed here do not necessarily reflect those of the Federal Reserve Bank of Chicago or the Federal Reserve System.
Motivation

- Role of investment-specific technological change (ISTC)
  - Growth (e.g., Greenwood, Hercovitz and Krusell 1997)
  - Business cycles (e.g., Fisher 2006)
  - Labor Share (e.g., Karabarbounis and Neiman 2012)
  - Decline of $r^*$ (e.g., Sajedi and Thwaites 2016)
  - Evolution of big “ratios” (e.g., Philippon, Eggertsson, ..)

- ISTC measured using price of new investment goods
- But: huge heterogeneity in price trends - aggregation?
Heterogeneity in Equipment Price Trends

Log Investment Prices - Equipment Categories

- Computers
- CommEquipt
- Photocopy
- Office
- MedicalEquipt
- Autos
- ElectricalNEC
- OtherNonRes
- Electrical
- Nonmedical
- ServiceInd
- Trucks
- Furniture
- Metalworking
- GeneralInd
- Engines
- FabrMetals
- SpecialInd
- Railroad
- Railroad
- Ships
- Ships
- Agricultural
- Construction
- Construction
- Aircrafts
- Aircrafts
- MiningOil
- MiningOil
Flow- and Stock-weighted Prices

Log Price of Investment vs. Capital

I-weighted
K-weighted
Flow- and Stock-weighted Prices

Log Price of Investment vs. Capital vs. Rental

I-weighted
K-weighted
R-weighted
Outline

1. Simple framework
2. Role of ISTC for growth
3. Role of ISTC for big ratios
4. Role of ISTC for business cycles
5. Role of ISTC for labor share
6. Role of ISTC for $r^*$
Simple Framework
Simple Model

Utility function:

\[ U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} dt \]

Production function:

\[ Y_t = A_t L_t^{\alpha_L} K_{1t}^{\alpha_K} \dot{} K_{nt}^{\alpha_K} \]

Capital accumulation for each type:

\[ \dot{K}_{it} = I_{it} - \delta_i K_{it} \]

Resource constraint:

\[ Y_t = C_t + \sum_{i=1}^n p_{it} I_{it} \]

Exogenous: \( A_t, L_t, p_{it} \)
Equilibrium

Euler equation:

\[ \frac{\dot{C}_t}{C_t} = \frac{r_t - \rho}{\sigma}, \]

Perfect competition capital demand:

\[ \alpha K_i \frac{Y_t}{K_{it}} = R_{it}, \]

User cost equation:

\[ R_{it} = p_{it} \left( r_t + \delta_i - \frac{\dot{p}_{it}}{p_{it}} \right), \]

Combining:

\[ \frac{p_{it} K_{it}}{Y_t} = \frac{\alpha K_i}{r_t + \delta_i - g_{pi}}. \]
Balanced growth path

- Capital demand:

\[ \frac{p_{it}K_{it}}{Y_t} = \frac{\alpha_k}{r_t + \delta_i - g_p} \ implies \ g_K = g_Y - g_p, \]

- Production function:

\[ g_Y = g_A + \alpha_L g_L + \sum_{i=1}^{n} \alpha_K g_K, \]

- Substitute:

\[ g_Y - g_L = \frac{g_A - \alpha_K g_{pR}}{\alpha_L} \]

\[ g_{pR} \equiv \frac{\sum_{i=1}^{n} \alpha_K g_{pi}}{\sum_{i=1}^{n} \alpha_K} \]

\[ \implies \text{Aggregate invt prices using rental weights} \]
Rental-, Stock-, and Flow-weighted indices

General (Divisia) index for given shares $s$:

$$\frac{\dot{p}_t^s}{p_t^s} = \sum_{i=1}^{n} s_{it} \frac{\dot{p}_{it}}{p_{it}}$$

Flow-weighted: invt price index (NIPA) used in ISTC research:

$$s_{it}^l \propto p_{it} l_{it}$$

Stock-weighted: capital price index (FAT)

$$s_{it}^K \propto p_{it} K_{it}$$

Rental-weighted index:

$$s_{it}^R \propto R_{it} K_{it}$$
Rental-shares, Stock-shares, Flow-shares

Rental weights:

\[ s_{it}^R = \frac{R_{it} K_{it}}{\sum_{j=1}^{n} R_{jt} K_{jt}} \propto \alpha K_i \]

Stock weights:

\[ s_{it}^K = \frac{p_{it} K_{it}}{\sum_{j=1}^{n} p_{jt} K_{jt}} \propto \frac{\alpha K_i}{r_t + \delta_i - g_{p_i}} \]

Investment weights on the BGP:

\[ s_{it}^I = \frac{p_{it} l_{it}}{\sum_{j=1}^{n} p_{jt} l_{jt}} \propto \frac{\alpha K_i}{g_{Y_t} + \delta_i - g_{p_i}} \]

These shares are \textbf{very} different!
I-share and K-share

Flow Shares

Stock Shares

NonResEquipt
NonResStruct
ResStruct
NonResIPP
Relation between shares along BGP

On the balanced growth path:

\[ s^R_i = \frac{s_I}{\alpha_K} s^I_i + \left( 1 - \frac{s_I}{\alpha_K} \right) s^K_i \]

where:

- \( s_I \) is the investment share of output
- \( \alpha_K \) is aggregate capital share

Hence relation between price indices:

\[ g^R_{p_i} = \frac{s_I}{\alpha_K} g^I_{p_i} + \left( 1 - \frac{s_I}{\alpha_K} \right) g^K_{p_i} \]

\[ \implies \text{Can infer } g^R_{p_i} \text{ from observables} \]
Contribution of ISTC to Growth
Data

- Fixed Asset Tables: All private fixed assets
- Disaggregation in 57 categories
- Ex.: Invt::NonRes Equipt::Info Processing::Computers
- We use the BEA deflators (not Gordon-Violante-Cummins)
Flow- and Stock-weighted Prices

Log Price of Investment vs. Capital vs. Rental

I-weighted
K-weighted
R-weighted
Contribution of ISTC to growth

- GHK: “ISTC contributes 58% to growth”
- Our approach (similar to theirs)
  1. Observe $\alpha_L, \alpha_K, g_{pR}, g_Y - g_L$
  2. Infer TFP $g_A$ from:

$$g_Y - g_L = \frac{g_A - \alpha_K g_{pR}}{\alpha_L}$$

3. Calculate counterfactual growth if $g_{pR} = 0$
4. What if use $g_{pI}$ instead of $g_{pR}$
Smaller ISTC contribution with R-weighting

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Iw:ITC</th>
<th>Iw:TFP</th>
<th>Rw:ITC</th>
<th>Rw:TFP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970-2017</td>
<td>1.19</td>
<td>0.52</td>
<td>0.66</td>
<td>0.21</td>
<td>0.98</td>
</tr>
<tr>
<td>(%)</td>
<td>100.00</td>
<td>43.80</td>
<td>55.91</td>
<td>17.46</td>
<td>82.37</td>
</tr>
</tbody>
</table>

Avg. growth of Y/L and contributions of ISTC and TFP using either I-w or R-w to infer ISTC.
Contributions to Growth: I-w (GHK)

Growth of Y/L, contrib of Invt Prices and TFP (using I-w)
Centered rolling window 15 years
Growth of Y/L, contrib of Inv prices and TFP (using R-w)

Centered rolling window 15 years

- Y/L
- Contrib ISTC
- Contrib TFP
ISTC and the Big Ratios
Aggregation

Result: along the BGP,

\[
\frac{l}{K} = g_Y + \delta^K - g_p^K
\]

\[
\Pi = \frac{r + \delta^K - g_p^K}{r + \delta^K - g_p^K}
\]

\[
\frac{K}{Y} = \frac{\alpha_K}{r + \delta^K - g_p^K}
\]

where

\[ l = \sum p_i l_i, \quad K = \sum p_i K_i, \quad \Pi = \sum R_i K_i \]

are the (current-cost nominal) aggregates.

\[\implies\] To calibrate one-capital model, use **stock-weighted** \( \delta \) and price growth.
Application: the decline of investment

Slope=-0.039; SE=0.008
Application: the decline of investment

\[ \frac{I}{K} - \delta^K = g_Y - g_{p^K} \]

<table>
<thead>
<tr>
<th></th>
<th>Net I/K</th>
<th>Contrib ( g_Y )</th>
<th>Contrib ( g_{p^K} )</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-2004</td>
<td>2.39</td>
<td>2.51</td>
<td>-0.21</td>
<td>0.10</td>
</tr>
<tr>
<td>2003-2017</td>
<td>1.51</td>
<td>1.58</td>
<td>-0.37</td>
<td>0.30</td>
</tr>
<tr>
<td>Change</td>
<td>-0.89</td>
<td>-0.93</td>
<td>-0.16</td>
<td>0.20</td>
</tr>
<tr>
<td>Change If use PI</td>
<td>-0.89</td>
<td>-0.93</td>
<td>-0.69</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Application: the stability of MPK (despite low Rf)

- Slope = 0.04; SE = 0.009
Application: the stability of MPK (despite low Rf)

\[
\frac{\Pi}{K} = r + \delta^K - g_{pK}
\]

We use this equation to infer \( r \)

<table>
<thead>
<tr>
<th></th>
<th>( \Pi/K )</th>
<th>Contrib ( \delta )</th>
<th>Contrib ( g_{pK} )</th>
<th>Contrib ( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-2004</td>
<td>15.61</td>
<td>5.69</td>
<td>-0.21</td>
<td>10.14</td>
</tr>
<tr>
<td>2003-2017</td>
<td>15.46</td>
<td>5.68</td>
<td>-0.37</td>
<td>10.15</td>
</tr>
<tr>
<td>Change</td>
<td>-0.15</td>
<td>-0.01</td>
<td>-0.16</td>
<td>0.02</td>
</tr>
<tr>
<td>Change If use PI</td>
<td>-0.15</td>
<td>-0.01</td>
<td>-0.69</td>
<td>0.55</td>
</tr>
</tbody>
</table>
ISTC and Business Cycles
Transitional Dynamics (w elastic labor)

\[
\begin{align*}
\max_{C_t, I_{it}, K_{it}} U &= \int_0^\infty e^{-\rho t} \frac{C_t^{1-\sigma}}{1-\sigma} v(L_t) dt \\
\text{s.t. : } & \\
\dot{K}_{it} &= I_{it} - \delta_i K_{it} \\
Y_t &= C_t + \sum_{i=1}^N P_{it} I_{it} \\
Y_t &= A_t L_t^{\alpha L} K_{1t}^{\alpha K_1} \ldots K_{nt}^{\alpha K_n}
\end{align*}
\]

for given \((K_{i0})\), and \((A_t, P_{it})\)
Consider a small, permanent, unexpected shock to vector $p_{i0}$.
Then, the full path of aggregates $(Y_t, L_t, C_t, I_t)_{t \geq 0}$ (in deviation from BGP) depends only on:

$$\hat{p} = (1 - s_I)\hat{p}^K + s_I\hat{p}^I$$

where $s_I$ is the aggregate investment share

Intuition:

- State variable = total capital relative to BGP
- The shock shifts BGP to a parallel path
- Shock also shifts total capital at $t = 0$
- Overall effect on deviation depends only on its effect on state variable at $t = 0$
Result

KW

\[
\begin{array}{cccc}
0 & 10 & 20 & 30 \\
-0.6 & -0.4 & -0.2 & 0 \\
\end{array}
\]

C

\[
\begin{array}{cccc}
0 & 10 & 20 & 30 \\
-0.25 & -0.2 & -0.15 & -0.1 & -0.05 & 0 \\
\end{array}
\]

Y

\[
\begin{array}{cccc}
0 & 10 & 20 & 30 \\
-0.15 & -0.1 & -0.05 & 0 \\
\end{array}
\]

r

\[
\begin{array}{cccc}
3 & 2 & 1 & 0 \\
0 & 10 & 20 & 30 \\
Bps \\
\end{array}
\]

Benchmark

Shock-w

Time

31
Run Fisher-style VAR:

- 3 variables: dlog(Invt Price), dlog(Y/L), log(L/Pop)
- Long-run restrictions to identify ISTC shock, TFP shock
- Quarterly data, 4 lags, 1982IV-2019IV
- Now only 14 categories of goods (e.g. info processing)
Price indices

BFI price indices

- our index
- flow

[Graph showing BFI price indices with years from 1985 to 2020 and index values ranging from -8 to 10]
VAR comparison

![Graphs showing the levels of I price, Y, and N over quarters.](image-url)
Variance Decomposition BFI

Share of variance of hours due to ISTC / TFP / demand

Investment-weighted

Rental-weighted
ISTC and the Labor share
Labor Share

• If EOS $K/L \sigma \neq 1$, chg invt prices affect labor share

• Model extension:

$$Y = \left( b_K K^{\frac{\sigma-1}{\sigma}} + b_L L^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

$$K = K_1^{\gamma K_1} \ldots K_n^{\gamma K_n}.$$ 

• Note: nonstationary shares if ISTC

• Consider a permanent small shock to vector $p_i$.

• Then change in gross labor share is:

$$(\sigma - 1) \alpha_K \hat{p}^R$$

$\implies$ Relevant price for labor share is **R-weighted**
Implied change in labor share since 1970 given observed prices changes and assumed EOS:

<table>
<thead>
<tr>
<th>σ</th>
<th>lw</th>
<th>rw</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>-0.17</td>
<td>-0.07</td>
</tr>
<tr>
<td>1.25</td>
<td>-0.09</td>
<td>-0.03</td>
</tr>
<tr>
<td>0.75</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>0.5</td>
<td>0.17</td>
<td>0.07</td>
</tr>
</tbody>
</table>
ISTC and the decline of $r^*$
Decline of $r^*$

- Lower investment price may reduce eqm interest rate
- Model extension: upward-sloping savings $W_tL_tS(r_t)$
  - e.g., OLG or Aiyagari
  - Otherwise, $r^*$ pinned down by preferences
- Equilibrium in asset market:
  $$\sum_{i=1}^{n} p_{it}K_{it} = W_tL_tS(r_t)$$
- Consider a permanent small shock to vector $p_{i0}$.
- Then change in $r^*$ is $\zeta \hat{p}^R$
- Correct aggregation for $r^*$ is **R-weighted**
Conclusion
Conclusion

- Methodology: appropriate aggregation depends on question at hand! I-w, K-w, R-w, Stock-w ...

- Simple calculations illustrate this can matter

- In progress: relax some simplifying assumptions (BGP, perfect competition, Cobb-Douglas, ...)


Backup
Rolling windows: Price Growth

I-w, K-w and R-w Price Growth
Centered rolling window 15 years

-2 -1.5 -1 -0.5 0 0.5 1
I-w, K-w and R-w Price Growth
Centered rolling window 15 years
Iw Kw Rw
Contributions to Equipment Deflator

Average contrib. to change in rel. equipt price (I-weighted)

-4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 % per year

Capital Type

-4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 % per year

Capital Type
Contributions to Investment Deflator

Avg contrib. to chg in rel. invt price (I-weighted)

% per year

Capital Type

Computers, Comm Equip, Software, Custom Eqmt, Prepack, Software, Custom Software, Medical Eqmt, Ownaccount Software, Autos, R&D, business, Photocopy, Trucks, TV, Office, Other...

Other Capital Type

Other NonRes Struct, Other Resid Struct, Fabr Metals, Mining, Struct, Transport, Struct, Ships, Fun, Struct, Special Ind, Religious Struct, Hotel, Struct, Health, Offices, Police, Comm, Oil, Gas, Struct, Perm Housing, Broker, Comm, Oil, Gas, Struct, Perm Housing, Other NonRes Struct, Other...

% per year

-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

Comparison with BLS
Comparison with BLS
Contributions to I-w E&I price

Average contrib. to change in rel. E&I price (I-weighted)

Capital Type

% per year

-1.2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

Increasing contributions to the E&I price index by I-weighted average.
Log Investment Prices - Equipment Categories

- InfoProces
- Transportation
- Other Equipment
- Industrial


Logarithmic scale on the y-axis.
Prices
Prices

![Log Investment Prices](chart1)

![Log Capital Prices](chart2)
Proof

\[ R_i K_i = (r + \delta_i - g_{p_i}) P_i K_i \]
\[ = (r - g_Y) P_i K_i + (g_Y + \delta_i - g_{p_i}) P_i K_i \]
\[ = (r - g_Y) P_i K_i + P_i I_i \]
\[ \sum R_i K_i = \alpha_K Y = (r - g_Y) K + I \]

\[ s_i^R = \frac{R_i K_i}{\sum R_j K_j} \]
\[ = \frac{(r - g_Y) P_i K_i + P_i I_i}{(r - g_Y) K + I} \]
\[ = \frac{P_i K_i}{K} \left(1 - \frac{s_I}{\alpha_K} \right) + \frac{P_i I_i}{I} \frac{s_I}{\alpha_K} \]
Relative prices

Relative price of Investment - Equipment

- Computers
- CommEquipt
- Photocopy
- Office
- MedicalEquipt
- Autos
- ElectricalNEC
- OtherNonRes
- Electrical
- Nonmedical
- ServiceInd
- Trucks
- Furniture
- Metalworking
- GeneralInd
- Engines
- FabrMetals
- SpecialInd
- Railroad
- Ships
- Agricultural
- Construction
- Aircrafts
Log relative prices

Log Investment Prices - Equipment Categories

- Computers
- CommEquipt
- Photocopy
- Office
- MedicalEquipt
- Autos
- ElectricalNEC
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- GeneralInd
- Engines
- FabrMetals
- SpecialInd
- Railroad
- Railroad
- Ships
- Agricultural
- Construction
- Aircrafts
I-w vs. K-w prices

Log Price of Investment vs. Capital

I-weighted
K-weighted
I-w vs. K-w prices

Average contrib. to change in rel. invt price: I vs. K weights

Capital Type

% per year
Depreciation and Price Trend

Relative price of Investment vs. Depreciation rate

- Avg depreciation rate (%)
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60
  - 70
  - 80

- Avg growth rate of investment price (%)
  - -16
  - -14
  - -12
  - -10
  - -8
  - -6
  - -4
  - -2
  - 0
  - 2
  - 4

- Relative price of Investment vs. Depreciation rate
  - Computers
  - CommEquipt
  - Photocopy Office
  - OilGasStruct
  - BrokerComm
  - OtherResidStruct
  - PrepackSoftware
  - CustomSoftware
  - OwnaccountSoftware
  - Music
  - OilGasStruct

- Avg depreciation rate (%)
  - 0
  - 10
  - 20
  - 30
  - 40
  - 50
  - 60
  - 70
  - 80

- Avg growth rate of investment price (%)
  - -16
  - -14
  - -12
  - -10
  - -8
  - -6
  - -4
  - -2
  - 0
  - 2
  - 4

- Computers
- CommEquipt
- Photocopy Office
- OilGasStruct
- BrokerComm
- OtherResidStruct
- PrepackSoftware
- CustomSoftware
- OwnaccountSoftware
- Music
Comparison of contribution of ISTC

Contributions of ISTC: I-w vs. R-w
Centered rolling window 15 years

Y/L
I-w
R-w
Comparison of contribution of ISTC: Percentages

Contributions (in %) of Prices: I-w vs. R-w
Centered rolling window 15 years

I-w
R-w
Macroeconomic Puzzles

- Decline of investment
- High profitability
- Decline of labor share
- Decline of $r^*$ (TBA)
Decline in net I/K

gross and net I/K for: Private fixed assets

Slope=-0.032;SE=0.01
Slope=-0.055;SE=0.009
Write BGP condition, adding an error term:

\[
\frac{I_{it}}{K_{it}} = \delta_i + g_Y - \frac{\dot{p}_{it}}{p_{it}} + \varepsilon_{it}.
\]

True at any level of aggregation (w. stock-weighted indices)
Evolution of net I/K: computers
Evolution of net I/K: non-res equipment
Stability of Profit/K

Profit-Capital Ratio

Slope=0.04; SE=0.009
## Data

<table>
<thead>
<tr>
<th></th>
<th>DlogY/H</th>
<th>Inv/Prof</th>
<th>Price IW</th>
<th>Price KW</th>
<th>Price RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1970-2017</strong></td>
<td>1.19</td>
<td>0.51</td>
<td>-1.02</td>
<td>0.23</td>
<td>-0.41</td>
</tr>
<tr>
<td><strong>1970-1984</strong></td>
<td>1.17</td>
<td>0.55</td>
<td>-0.23</td>
<td>0.65</td>
<td>0.16</td>
</tr>
<tr>
<td><strong>1985-2005</strong></td>
<td>1.49</td>
<td>0.52</td>
<td>-1.49</td>
<td>0.09</td>
<td>-0.73</td>
</tr>
<tr>
<td><strong>2006-2017</strong></td>
<td>0.68</td>
<td>0.45</td>
<td>-1.12</td>
<td>-0.01</td>
<td>-0.51</td>
</tr>
</tbody>
</table>

Avg. growth of Y/L, I/Profits, and I-w, K-w, R-w prices