Principal Trading Arrangements: When Are Common Contracts Optimal?

Markus Baldauf
University of British Columbia

Christoph Frei
University of Alberta

Joshua Mollner
Northwestern University

AFA

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Big Picture

A (risk-neutral) client

- seeks to buy a large position
- lacks expertise in "working" orders directly on the market
- outsources the complexities of the trade to a (risk-averse) dealer

The arrangement

- at time 0, the client contracts that at time $T + 1$, she will purchase the position from the dealer (in an off-market trade)
- in the interim, the dealer will acquire the position (via on-market trades)
- the client's payment will be a function of market prices and volumes $\tau(p_1, \ldots, p_T, v_1, \ldots, v_T)$

Question: what contract τ should the client use?

Hidden action: the client cannot observe the dealer's on-market trades, which influence $(p_1, \ldots, p_T)$ and $(v_1, \ldots, v_T)$

Results

Two contacts that are used in practice:

1. Guaranteed market-on-close ("guaranteed MOC")
   - *result:* generally not optimal
2. Guaranteed volume-weighted average price ("guaranteed VWAP")
   - *result:* uniquely optimal under certain conditions

Literature

Empirics

- *Broker-dealer/investor conflicts:* Battalio, Corwin and Jennings (2016); Battalio, Hatch and Sağlam (2019); Anand, Samadi, Sokobin and Venkataraman (2019); Barbon, Di Maggio, Franzoni and Landier (2019)
- *Trade-based benchmark manipulation:* Harris (1989); Felixson and Pelli (1999); Carhart, Kaniel, Musto and Reed (2002); Hillion and Suominen (2004); Ben-David, Franzoni, Landier and Moussawi (2013); Comerton-Forde and Putniņš (2011, 2014); Griffin and Shams (2017); Henderson, Pearson and Wang (2019)

Theory

- *Broker-dealer/investor conflicts:* Röell (1990); Fishman and Longstaff (1992); Bernhardt and Taub (2008); Saakvitne (2016)
- *Financial benchmarks:* Duffie and Dworczak (2018); Coulter, Shapiro and Zimmerman (2018); Ingersoll, Goetzmann, Spiegel and Welch (2007); Duffie, Dworczak and Zhu (2017)
- *Optimality of simple contracts:* Holmström and Milgrom (1987); Carroll (2015)
- *Volume participation strategies:* Kato (2015); Humphery-Jenner (2011); Frei and Westray (2015); Cartea and Jaimungal (2016)

General Model

Trading

- Trading periods $t \in \{1, 2, \ldots, T\}$
- Market conditions $\boldsymbol{\eta} = (\eta_t)_{t=1}^T$
  - realizations are learned by dealer
  - stochastic from client perspective
- Dealer chooses a trading schedule $\boldsymbol{x} = (x_t)_{t=1}^T$
  - $x_t \geq 0$
  - $\sum_{t=1}^T x_t = 1$
- Market outcomes linked to $\boldsymbol{x}$ and $\boldsymbol{\eta}$
  - prices $\boldsymbol{p} = (p_t)_{t=1}^T$
  - volumes $\boldsymbol{v} = (v_t)_{t=1}^T$

Contracts

Contracts

- characterized by how the client pays the dealer for the share
- any real-valued, measurable function $\tau(\boldsymbol{p}, \boldsymbol{v})$

Interpretation

- prices and volumes are publicly observable
- market conditions and dealer's trades are not

Examples

1. arrival price: $\tau \in \mathbb{R}$
2. guaranteed market-on-close: $\tau^{MOC} \equiv p_T$
3. guaranteed TWAP: $\tau^{TWAP} \equiv \dfrac{1}{T} \sum_{t=1}^{T} p_t$
4. guaranteed VWAP: $\tau^{VWAP} \equiv \dfrac{\sum_{t=1}^{T} p_t v_t}{\sum_{s=1}^{T} v_s}$

Timing

- Client offers contract τ
- Dealer accepts or rejects
  - *reject:* $\begin{cases} \text{dealer's payoff:} & u(0) \\ \text{client's payoff:} & -\infty \end{cases}$
  - *accept:* continue…
- Dealer learns $\boldsymbol{\eta}$ and chooses $\boldsymbol{x}$
- $\boldsymbol{p}$ and $\boldsymbol{v}$ realized
- Dealer delivers the share; client pays according to τ

$$\begin{cases} \text{dealer's payoff:} & u\big(\tau(\boldsymbol{p},\boldsymbol{v}) - \boldsymbol{p}\cdot\boldsymbol{x}\big) \\ \text{client's payoff:} & -\tau(\boldsymbol{p},\boldsymbol{v}) \end{cases}$$

Client's Problem

Second-Best Problem

- Minimize payment to dealer

$$\min_{\tau, \boldsymbol{x}(\cdot)} \mathbb{E}\left[\tau\left(\boldsymbol{p}, \boldsymbol{v}\right)\right]$$

- subject to

$$\mathbb{E}\left[u\left(\tau\left(\boldsymbol{p}, \boldsymbol{v}\right) - \boldsymbol{p} \cdot \boldsymbol{x}(\boldsymbol{\eta})\right)\right] \geq u(0) \qquad \text{(IR)}$$

$$\forall \hat{\boldsymbol{x}}(\cdot) : \mathbb{E}\left[u\left(\tau\left(\boldsymbol{p}, \boldsymbol{v}\right) - \boldsymbol{p} \cdot \boldsymbol{x}(\boldsymbol{\eta})\right)\right] \geq \mathbb{E}\left[u\left(\tau\left(\boldsymbol{p}, \boldsymbol{v}\right) - \boldsymbol{p} \cdot \hat{\boldsymbol{x}}(\boldsymbol{\eta})\right)\right] \qquad \text{(IC)}$$

MOC Contract Not Optimal

$$\tau^{MOC} \equiv p_T$$

Proposition

If Condition 1 (a weak technical condition) holds, then τ^{MOC} is not optimal.

▸ Condition 1

Proof Sketch.

- The trading schedule $\boldsymbol{x}^{MOC} = (0, \ldots, 0, 1)$ would guarantee the dealer

$$\underbrace{\tau^{MOC}}_{\text{revenue}} - \underbrace{\boldsymbol{p} \cdot \boldsymbol{x}^{MOC}}_{\text{cost}} = p_T - p_T = 0$$

- Shifting δ volume to an earlier period:
  - an $O(\delta)$ increase in expected profits
  - an $O(\delta^2)$ increase in the variance of profits
- τ^{MOC} does not cause (IR) to bind □

Specialized Model

Prices and Volumes

$$p_t = h(x_t/\eta_t) + \varepsilon_t$$
$$v_t = v(x_t, \eta_t)$$

Strong assumptions

- no permanent price impact
- dealer can perfectly forecast the volume profile $\big(v(x_t, \eta_t)\big)_{t=1}^{T}$
  - only need for relative volume profile ▸ liquidity smile

Weak assumptions

- $\boldsymbol{\eta}$ distributed on $\mathbb{R}^T_{++}$
- $\mathbb{E}[\varepsilon_t|\boldsymbol{\eta}] = \mu$ for all t
- $yh(y)$ strictly convex
- $v(x_t, \eta_t)$ homogeneous of degree one and strictly increasing in η
- weakly concave dealer utility function $u(\cdot)$

▸ micro-foundation of price and volume

Optimality of VWAP

$$\tau^{VWAP} \equiv \frac{\sum_{t=1}^{T} p_t v_t}{\sum_{s=1}^{T} v_s}$$

Proposition (optimality)

The contract τ^{VWAP} is optimal.

Proposition (uniqueness)

If

1. *u is strictly concave and*
2. *$\boldsymbol{\varepsilon}$ and $\boldsymbol{v}(\boldsymbol{x}^{FB}(\boldsymbol{\eta}), \boldsymbol{\eta})$ have full support over $\mathbb{R}^T$ and $\mathbb{R}^T_{++}$,*

then a contract τ is optimal only if $\tau = \tau^{VWAP}$ almost everywhere on its domain.

Proof Sketch of VWAP Optimality Result (1/2)

Definition

A trading policy $\boldsymbol{x}(\cdot)$ is *first best* if, for all $\boldsymbol{\eta}$,

$$\boldsymbol{x}(\boldsymbol{\eta}) \in \arg\min_{\boldsymbol{x}} \mathbb{E}\left[\boldsymbol{p} \cdot \boldsymbol{x} \middle| \boldsymbol{\eta}\right]$$

Lemma

The first-best trading policy is a volume participation strategy

$$\boldsymbol{x}^{FB}(\boldsymbol{\eta}) = \left(\frac{v(x_t^{FB}(\boldsymbol{\eta}), \eta_t)}{\sum_{s=1}^{T} v(x_s^{FB}(\boldsymbol{\eta}), \eta_s)} \right)_{t=1}^{T}$$

Proof Sketch of VWAP Optimality Result (2/2)

Lemma

τ^{VWAP} incentivizes the dealer to use the first-best trading policy:

$$\forall \hat{\boldsymbol{x}}(\cdot) : \mathbb{E}\left[u\left(\tau^{VWAP}(\boldsymbol{p},\boldsymbol{v}) - \boldsymbol{p}\cdot\boldsymbol{x}^{FB}(\boldsymbol{\eta})\right)\right] \geq \mathbb{E}\left[u\left(\tau^{VWAP}(\boldsymbol{p},\boldsymbol{v}) - \boldsymbol{p}\cdot\hat{\boldsymbol{x}}(\boldsymbol{\eta})\right)\right]$$

Lemma

If the dealer uses the first-best trading policy, then τ^{VWAP} fully insures him and leaves him with zero surplus.

Proof Sketch.

Because $\boldsymbol{x}^{FB}(\cdot)$ is a volume participation strategy,

$$\underbrace{\tau^{VWAP}(\boldsymbol{p},\boldsymbol{v})}_{\text{revenue}} = \underbrace{\boldsymbol{p}\cdot\boldsymbol{x}^{FB}(\boldsymbol{\eta})}_{\text{cost}} \qquad \square$$

Intuition for Uniqueness Result

- Arrival price contracts: $\tau \in \mathbb{R}$
  - dealer must bear some price risk
- Guaranteed market-on-close: $\tau^{MOC} \equiv p_T$
  - dealer incentivized to tilt trades toward last period ('banging the close')
- Guaranteed TWAP: $\tau^{TWAP} \equiv \frac{1}{T} \sum_{t=1}^{T} p_t$
  - dealer incentivized to smooth trading

Contributions

Problem

- monitoring dealers is difficult
- legal obligations often opaque ▸ details

Applications

1. bilateral contracting in markets with public data (e.g., equities)
2. benchmark design in markets with incomplete public data (e.g., FX)
3. settlement prices of futures contracts
4. valuation of mutual funds

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