Estimating Macroeconomic Models of Financial Crises: An Endogenous Regime-Switching Approach

Gianluca Benigno\textsuperscript{1}  Andrew Foerster\textsuperscript{2}  Christopher Otrok\textsuperscript{3}  Alessandro Rebucci\textsuperscript{4}

\textsuperscript{1}Federal Reserve Bank of New York, LSE, and CEPR
\textsuperscript{2}Federal Reserve Bank of San Francisco
\textsuperscript{3}University of Missouri and Federal Reserve Bank of St Louis
\textsuperscript{4}Johns Hopkins Carey Business School, CEPR and NBER

SNDE-ASSA–Jan 3, 2021

The views expressed herein are solely those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of New York, San Francisco, or St. Louis, or the Federal Reserve System.
Motivation

- The global financial crisis (GFC) reignited a strong interest in the causes, consequences, and remedies of financial crises

- DSGE models with occasionally binding frictions proved successful for positive and normative purposes

- Structural estimation is important for inference, counterfactual analysis, and forecasting but in this environment is difficult
This Paper

- New approach to specifying, solving, estimating models of occasionally binding constraints
  - Proposes a new formulation of the occasionally binding friction as an *endogenous* regime-switching process
  - Develops a general perturbation-based solution method for such a framework that is fast, scalable, and accurate
  - Estimate using full-information Bayesian methods

- Applies the framework to study the anatomy of Mexico’s business cycle and financial crisis history since 1981
Empirical Results

- Estimate coverage intervals on critical parameters governing the occasionally binding constraint
- Estimated model fits cycles and crises well without relying on large shocks
- Identifies financial crises of *varying* duration and *intensity* in line with Mexico’s history
- Different shocks matter for different *variables* over the business cycles and drive different crisis *episodes* and *phases*
Related Literature

- Literature on estimating DSGE models and a few papers attempting to estimate models with occasionally binding constraints
- Larger literature on solving and estimating Regime-Switching DSGE models
- Large Literature on business cycles and financial crises in emerging markets
Model Overview

▶ A workhorse medium-scale DSGE model
  • Two endogenous state variables and six shocks
  • Structure same as in Mendoza (2010) except for the borrowing constraint formulation
  • A broad set of shocks as in Garcia-Cicco, et. al. (2010)

▶ Distinctive feature: economy endogenously switches between two regimes
  • Binding regime: the borrowing constraint holds with equality
  • Non-binding regime: borrowing is unconstrained
  • Switch is a *stochastic* rather then *deterministic* function of the endogenous level of leverage
Preferences and Technology

- Representative household-firm with preferences

\[ U \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{1}{1-\rho} \left( C_t - \frac{H_t^\omega}{\omega} \right)^{1-\rho} \right\} \]

- GDP is gross output less intermediate expenditures

\[ Y_t = A_t K_{t-1}^\eta H_t^\alpha V_t^{1-\alpha-\eta} - P_t V_t \]

- Investment with adjustment costs

\[ I_t = \delta K_{t-1} + (K_t - K_{t-1}) \left( 1 + \frac{\iota}{2} \left( \frac{K_t - K_{t-1}}{K_{t-1}} \right)^2 \right) \]

- Budget constraint: working capital \( \phi \), debt \( B_t < 0 \)

\[ C_t + I_t + E_t = Y_t - \phi r_t (W_t H_t + P_t V_t) - \frac{1}{1 + r_t} B_t + B_{t-1} \]
Exogenous Processes

- **Technology**
  \[ \log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t} \]

- **Terms of Trade**
  \[ \log P_t = (1 - \rho_P) \log P^* + \rho_P \log P_{t-1} + \sigma_P \varepsilon_{P,t} \]

- **Preference**
  \[ \log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t} \]

- **Expenditure**
  \[ \log E_t = (1 - \rho_E) \log E^* + \rho_E \log E_{t-1} + \sigma_E \varepsilon_{E,t} \]

- **Country interest rate**
  \[ r_t = r_t^* + \sigma_r \varepsilon_{r,t} + \psi_r \left( e^{\tilde{B} - B_t} - 1 \right) \]
  \[ r_t^* = (1 - \rho_r^*) \tilde{r}^* + \rho_r^* r_{t-1}^* + \sigma_r^* \varepsilon_{r^*,t} \]
Collateral Constraint

- Agent faces a regime-specific constraint

- In the binding regime \((s_t = 1)\), borrowing is a fraction of the collateral value

\[
\frac{1}{(1 + r_t)} B_t - \phi (1 + r_t) (W_t H_t + P_t V_t) = -\kappa q_t K_t, \quad \text{with multiplier } \lambda_t
\]

- In the non-binding regime \((s_t = 0)\), borrowing is unconstrained with “borrowing cushion” defined as

\[
B_t^* = \frac{1}{(1 + r_t)} B_t - \phi (1 + r_t) (W_t H_t + P_t V_t) + \kappa q_t K_t,
\]
Endogenous Switching

- Transition between regimes is logistic
- In the non-binding regime, the probability that constraint binds next period is
  \[ \Pr(s_{t+1} = 1 | s_t = 0) = \frac{\exp(-\gamma_0 B^*_t)}{1 + \exp(-\gamma_0 B^*_t)} \]
- In the binding regime, probability that constraint doesn’t bind next period is
  \[ \Pr(s_{t+1} = 0 | s_t = 1) = \frac{\exp(-\gamma_1 \lambda_t)}{1 + \exp(-\gamma_1 \lambda_t)} \]
- Regime in \( t \) determined before shocks at \( t \)
Remarks on Endogenous Switching Specification

- **One:** Agents in the model have rational expectations about endogenous switches
Remarks on Endogenous Switching Specification (Cont.)

**Two:** Both $B_t^*$ and $\lambda_t$ can be positive or negative, switches triggered stochastically

- Allows for both preemptive and delayed switches
- Consistent with growing body of evidence that switches occur stochastically
  - Borrowers and lenders renegotiate covenants as credit limits are approached, rather than triggering them as financial stress arises (Chodorow-Reich and Falato, 2017; Greenwald, 2019)
  - The likelihood of a financial crisis increases with leverage, but high leverage does not necessarily lead to a financial crisis (Jorda et al., 2013)

**Three:** Nests a specification where the logistic becomes a step-like function or flat function (exogenous switching)

**Four:** As is common in the extant literature, specification is not derives as outcome of an optimal contract, but could be derived from variation of costly state verification (e.g., Martin, 2008) or from heterogeneous agent environment (e.g., Fernandez-Villaverde et. al. 2020)
Solution Method

- Full set of structural equations (23 equilibrium conditions)
  - First-order conditions, resource constraints, prices
  - Exogenous processes

- Use *regime-switching slackness condition* to map regimes into traditional parameter switching

- Compute an approximate solution suitable for likelihood-based estimation building on the perturbation method of Foerster et. al. (2016)

- Single approximation point, which is the ergodic mean of the regimes
  - Between steady states of “binding” and “non-binding” regimes
  - Depends on relative frequency of regimes, which is itself endogenous
  - A fixed-point problem that can be solved with an iterative procedure
Regime Switching Slackness Condition

- Introduce the regime-switching parameters $\varphi(s_t) = \nu(s_t) = s_t$, where
  - $\varphi(s_t)$ is a parameter that affects the level of the economy
  - $\nu(s_t)$ is a parameter that affects the dynamics of the economy

\[
\varphi(s_t) B^*_ss + \nu(s_t) (B^*_t - B^*_ss) = (1 - \varphi(s_t)) \lambda_{ss} + (1 - \nu(s_t)) (\lambda_t - \lambda_{ss})
\]

- This formulation is continuously differentiable and implies that

\[
s_t = 0 = \varphi(0) = \nu(0) \Rightarrow \lambda_t = 0
\]

\[
s_t = 1 = \varphi(1) = \nu(1) \Rightarrow B^*_t = 0
\]
Properties of the Solution Method

- **Proposition** (Irrelevance of Endogenous Switching in a First-Order Approximation): The first-order solution of the model is identical to the first-order solution of an exogenous regime-switching model.

- This means that impact of *endogenous* switching on decision rules (precautionary behavior) requires at least second-order approximation.

- Solution is fast, scalable, applicable to a general class of models, and accurate
  - We compare a stripped down version of the model with Mendoza and Villalvazo (2020)
  - Similar moments and dynamics in 1 second rather than 810
  - Euler equation errors in line with other perturbation approaches
Accuracy and Computing Time

<table>
<thead>
<tr>
<th>Euler Equation Errors (log_{10} units)</th>
<th>FiPlt</th>
<th>End. Switch.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond - Mean</td>
<td>-6.27</td>
<td>-2.92</td>
</tr>
<tr>
<td>Bond - Max</td>
<td>-1.56</td>
<td>-1.61</td>
</tr>
<tr>
<td>Capital - Mean</td>
<td>-7.04</td>
<td>-3.61</td>
</tr>
<tr>
<td>Capital - Max</td>
<td>-6.68</td>
<td>-2.41</td>
</tr>
<tr>
<td>Computing Time (seconds)</td>
<td>810</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**FiPlt:** Mendoza and Villalvazo (2020) solution of Mendoza (2010). **End. Switch.:** BFOR endogenous switching model. Both solutions yield moments and ergodic distributions very close to Mendoza (2010), but BFOR is closer to the data.
Estimating the Nonlinear Model

- Three challenges: multiple regimes, second-order, endogenous transition.
  - Bianchi (2013) solves the first challenge.

- We use unscented Kalman Filter (UKF) with sigma points to evaluate the likelihood function (Binning and Maih, 2015) to solve the second and third challenges.

- Bayesian estimation with standard MCMC methods.

- We calibrate parameters that can be pin down from the first moments of the data and estimate critical ones.
Data for Estimation

- Data for Mexico from 1981:Q1 to 2016:Q4

- Observables
  - GDP growth
  - Consumption growth
  - Investment growth
  - Country interest rate constructed as in *Uribe and Yue (2006)*
  - Current account to GDP ratio
  - Import prices

- Measurement errors restricted to 5% of the variance of each observable
## Estimation Results: Parameters of Interest

<table>
<thead>
<tr>
<th>Par.</th>
<th>Description</th>
<th>Prior</th>
<th>Prior</th>
<th>Mode</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\iota$</td>
<td>Capital Adj.</td>
<td>N(10,5)</td>
<td>12.70</td>
<td>12.65</td>
<td>12.70</td>
<td>12.72</td>
<td></td>
</tr>
<tr>
<td>$\phi$</td>
<td>Working Cap.</td>
<td>U(0,1)</td>
<td>0.71</td>
<td>0.710</td>
<td>0.720</td>
<td>0.721</td>
<td></td>
</tr>
<tr>
<td>$r^*$</td>
<td>Mean Int. Rate</td>
<td>N(0.0177,0.01)</td>
<td>0.017</td>
<td>0.012</td>
<td>0.017</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Leverage</td>
<td>U(0,1)</td>
<td>0.17</td>
<td>0.16</td>
<td>0.17</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Autocor. TFP</td>
<td>B(0.6,0.2)</td>
<td>0.9796</td>
<td>0.9653</td>
<td>0.9793</td>
<td>0.9881</td>
<td></td>
</tr>
<tr>
<td>$\rho_e$</td>
<td>Autocor. Exp</td>
<td>B(0.6,0.2)</td>
<td>0.9111</td>
<td>0.9066</td>
<td>0.9132</td>
<td>0.9237</td>
<td></td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>Autocor. Imp Price</td>
<td>B(0.6,0.2)</td>
<td>0.9711</td>
<td>0.9609</td>
<td>0.9754</td>
<td>0.9549</td>
<td></td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Autocor. Pref.</td>
<td>B(0.6,0.2)</td>
<td>0.9810</td>
<td>0.9753</td>
<td>0.9810</td>
<td>0.9843</td>
<td></td>
</tr>
<tr>
<td>$\rho_r^*$</td>
<td>Autocor. Persist. Int. Rate</td>
<td>B(0.6,0.2)</td>
<td>0.8929</td>
<td>0.8782</td>
<td>0.8896</td>
<td>0.8995</td>
<td></td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>SD TFP</td>
<td>IG(0.01,0.01)</td>
<td>0.008</td>
<td>0.007</td>
<td>0.008</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>SD Exp.</td>
<td>IG(0.1,0.1)</td>
<td>0.18</td>
<td>0.17</td>
<td>0.18</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>SD Imp. Price</td>
<td>IG(0.1,0.1)</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>SD Pref.</td>
<td>IG(0.1,0.1)</td>
<td>0.11</td>
<td>0.10</td>
<td>0.11</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>SD Trans. Int. Rate</td>
<td>IG(0.01,0.01)</td>
<td>0.003</td>
<td>0.001</td>
<td>0.003</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>$\sigma_r^*$</td>
<td>SD, Persist Int. Rate</td>
<td>IG(0.01,0.01)</td>
<td>0.005</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Logistic, Enter Binding</td>
<td>U(0,150)</td>
<td>13.55</td>
<td>10.90</td>
<td>13.71</td>
<td>18.01</td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>Logistic, Exit Binding</td>
<td>U(0,150)</td>
<td>17.80</td>
<td>15.78</td>
<td>17.80</td>
<td>19.81</td>
<td></td>
</tr>
</tbody>
</table>
Estimated Logistic Functions and Their Endogenous Drivers
Model Fits Mexican Cycles and Crises Well without Large Shocks

Figure: Fitted Output Growth

Figure: Estimated Technology Shock
Second Moments of Model In Line with Data

<table>
<thead>
<tr>
<th>Data Series</th>
<th>Relative Std. Dev.</th>
<th>Correlations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Output Growth</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption Growth</td>
<td>1.25</td>
<td>1.92</td>
</tr>
<tr>
<td>Investment Growth</td>
<td>5.37</td>
<td>5.75</td>
</tr>
<tr>
<td>Trade Balance to Output Ratio</td>
<td>1.24</td>
<td>0.80</td>
</tr>
<tr>
<td>Country Interest Rate</td>
<td>1.36</td>
<td>0.15</td>
</tr>
</tbody>
</table>
## Variance Decomposition: Different Shocks Drive Real/Financial Variables

<table>
<thead>
<tr>
<th>Variables / Shocks</th>
<th>TFP</th>
<th>Expend.</th>
<th>Import Prices</th>
<th>Pref.</th>
<th>Temp. Int. Rate</th>
<th>Pers. Int. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>33.2</td>
<td>17.2</td>
<td>15.7</td>
<td>25.4</td>
<td>2.5</td>
<td>6.0</td>
</tr>
<tr>
<td>Consumption</td>
<td>30.3</td>
<td>23.4</td>
<td>14.3</td>
<td>20.6</td>
<td>3.8</td>
<td>7.6</td>
</tr>
<tr>
<td>Investment</td>
<td>19.2</td>
<td>29.8</td>
<td>10.3</td>
<td>25.6</td>
<td>4.6</td>
<td>10.5</td>
</tr>
<tr>
<td>Trade Bal/Output</td>
<td>9.5</td>
<td>35.2</td>
<td>8.8</td>
<td>17.2</td>
<td>9.2</td>
<td>20.1</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>21.1</td>
<td>78.9</td>
</tr>
<tr>
<td>Borrowing Cush.</td>
<td>10.6</td>
<td>32.3</td>
<td>9.9</td>
<td>21.3</td>
<td>9.9</td>
<td>16.0</td>
</tr>
<tr>
<td>Debt/Output</td>
<td>15.2</td>
<td>25.5</td>
<td>7.6</td>
<td>40.9</td>
<td>1.4</td>
<td>9.5</td>
</tr>
<tr>
<td>Multiplier</td>
<td>9.5</td>
<td>40.5</td>
<td>9.5</td>
<td>18.1</td>
<td>9.6</td>
<td>12.8</td>
</tr>
</tbody>
</table>
Model Identifies Sudden Stops in Line with Mexico’s History of Crises

- Crisis episodes defined as consecutive periods in which the smoothed probability of binding regime (solid black line) is larger than 90%
- Crisis episodes (dashed vertical lines): Debt crisis 8 quarters; Tequila crisis 9 quarters; GFC 4 quarters
- Narrative Crisis Tally Index of Reinhart and Rogoff (2009) (grey bars): historical crisis episodes much more persistent than traditional model-based episodes (red bars)
Model Does Not Mistake Recessions for Crises

- OECD recession dates in light grey
- Recessions are not necessarily accompanied by binding borrowing constraint
### Every Crisis Is Different

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1983 Debt Crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two Quarters Prior (81Q1:Q2)</td>
<td>0.4</td>
<td>0.4</td>
<td>0.7</td>
<td>-3.2</td>
<td>0.9</td>
<td>0.8</td>
</tr>
<tr>
<td>During Crisis (81:Q3-83:Q2)</td>
<td>0.4</td>
<td>5.3</td>
<td>-2.0</td>
<td>-2.8</td>
<td>0.0</td>
<td>-0.8</td>
</tr>
<tr>
<td>Two-years After (83:Q3-85:Q2)</td>
<td>0.8</td>
<td>1.0</td>
<td>-0.6</td>
<td>0.2</td>
<td>-0.7</td>
<td>-0.7</td>
</tr>
<tr>
<td><strong>1995 Tequila Crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-years Prior (92:Q1-93:Q4)</td>
<td>-0.1</td>
<td>-1.0</td>
<td>0.4</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>During Crisis (94:Q1-96:Q1)</td>
<td>-2.2</td>
<td>-0.7</td>
<td>0.5</td>
<td>1.3</td>
<td>0.2</td>
<td>0.9</td>
</tr>
<tr>
<td>Two-years After (96:Q2-98Q1)</td>
<td>-0.1</td>
<td>-0.2</td>
<td>0.2</td>
<td>1.1</td>
<td>-0.6</td>
<td>-0.4</td>
</tr>
<tr>
<td><strong>2009 Global Fin. Crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two-years Prior (06:Q4-08:Q3)</td>
<td>-0.7</td>
<td>2.1</td>
<td>-0.7</td>
<td>-0.2</td>
<td>-0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>During Crisis (08:Q4-09:Q3)</td>
<td>0.2</td>
<td>-1.2</td>
<td>0.3</td>
<td>0.5</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Two-years After (09:Q4-11:Q3)</td>
<td>-0.4</td>
<td>-1.1</td>
<td>0.4</td>
<td>0.8</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Model Generates Long-lasting Crises as Rare Events

(a) Crisis Episodes of at least Four Consecutive Quarters

(b) Frequency of Crisis Episodes of Any Duration per Sample
Cocktails of Shocks Driving Crisis Dynamics

(a) Technology

(b) Import Prices

(c) Expenditure

(d) Preference

(e) Persist. Int. Rate

(f) Temp. Int. Rate
Crisis have Slow Buildups, Large Crashes, and Persistent Effects

- Output
- Consumption
- Investment
- CA/Y
- TB/Y
- EFPD
Conclusions

- We propose a new approach to specifying and solving models with occasionally binding frictions suitable for estimation.
- We use the framework to study Mexican history of cycles and crises.
- We find that the model fits the data well with its mechanisms rather than large shocks.
- Model identifies crisis episodes of variable duration and intensity driven by different shocks at different times.
# Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta = 0.9798$</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>$\rho = 2$</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>$\omega = 1.846$</td>
</tr>
<tr>
<td>Capital Share</td>
<td>$\eta = 0.3053$</td>
</tr>
<tr>
<td>Labor Share</td>
<td>$\alpha = 0.5927$</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta = 0.02277$</td>
</tr>
<tr>
<td>Import Price Mean</td>
<td>$P^* = 1.028$</td>
</tr>
<tr>
<td>Expenditure Mean</td>
<td>$E^* = 0.2002$</td>
</tr>
<tr>
<td>Interest Rate Debt Elasticity</td>
<td>$\psi_r = 0.001$</td>
</tr>
<tr>
<td>Neutral Debt Level</td>
<td>$\bar{B} = -6.117$</td>
</tr>
</tbody>
</table>
Model Fits Mexican Cycles and Crises Well

(a) Output Growth

(b) Consumption Growth

(c) Investment Growth

(d) Interest Rate

(e) Current Account to Output Ratio

(f) Import Price Growth
Large Shocks Not Required to Fit the Data

(a) TFP Shock

(b) Expenditure Shock

(c) Import Price Shock

(d) Preference Shock

(e) Transitory Interest Rate Shock

(f) Persistent Interest Rate Shock
Transition Probabilities show Exogenous Switching would be Misspecified