The U.S. Public Debt Valuation Puzzle

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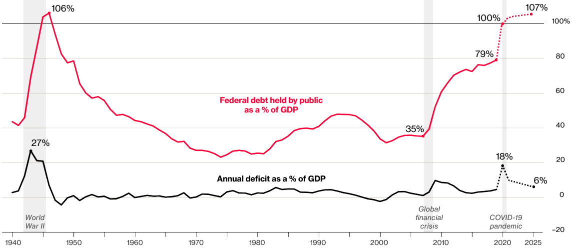
What is U.S. Government's Debt-Bearing Capacity?

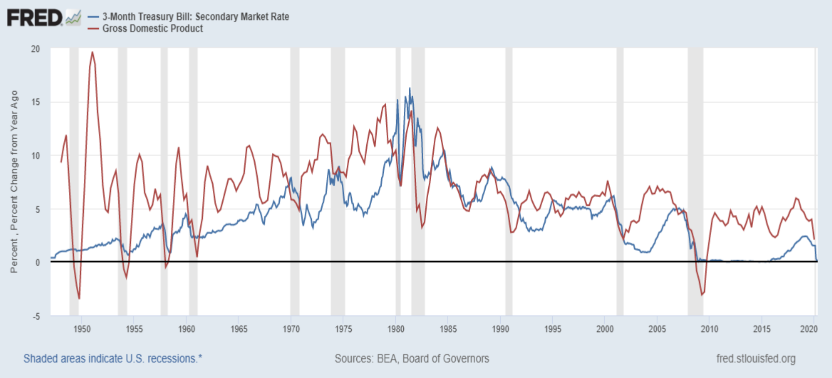
- U.S. federal government is the largest borrower in the world. The outstanding debt held by the public was $17.67 trillion at end of 2019.
- Doubled from 35% of GDP before the Great Recession to 79% of GDP in 2019. Further deteriorated during the Covid crisis.



- Can the U.S. government continue to borrow trillions more? Or should it reduce the deficit to avoid a debt market crash?

"... public debt may have no fiscal cost."

$$r^f < g$$



- Olivier Blanchard's AEA presidential address (2019)

Government Bond Portfolio

- We adopt an asset pricing approach.
- One-period government budget:

$$\underbrace{G_t}_{\text{Govt Spending}} + \underbrace{Q^1_{t-1}}_{\text{Expiring Debt}} = \underbrace{T_t}_{\text{Tax Revenue}} + \underbrace{\sum_{h=1}^{H} \left(Q^h_t - Q^{h+1}_{t-1} \right) P^h_t}_{\text{Net New Issuance}} .$$

- Iterate forward and impose no-arbitrage: $P^h_t = \mathbb{E}_t \left[M_{t,t+1} P^{h-1}_{t+1} \right]$

$$D_t = \sum_{h=0}^{H} Q^{h+1}_{t-1} P^h_t = \mathbb{E}_t \left[\sum_{j=0}^{T} M_{t,t+j} (T_{t+j} - G_{t+j}) \right] + E_t \left[M_{t,t+T} D_{t+T} \right]$$

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- Impose a TVC: $E_t \left[M_{t,t+T} D_{t+T}\right] \to 0$ as $T \to \infty$

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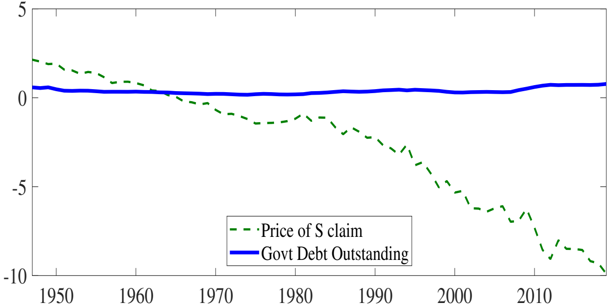
$$\underbrace{D_t = \sum_{h=0}^{H} Q^{h+1}_{t-1} P^h_t}_{\substack{\text{the market value of} \\ \text{government debt}}} = \underbrace{\mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]}_{\substack{\text{the expected risk-adjusted PDV} \\ \text{of future primary surpluses}}}$$

  - Holds ex ante both in real and nominal terms
  - Holds when we allow for sovereign default (extension)

Government Bond Valuation Puzzle

$$D_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} (T_{t+j} - G_{t+j}) \right]$$



- The wedge between the MV of outstanding debt and the risk-adjusted PV of future surpluses is 2.6x GDP; keeps widening
- We estimate the PV using realistic SDF M and cash flow dynamics $T - G$

State Variables

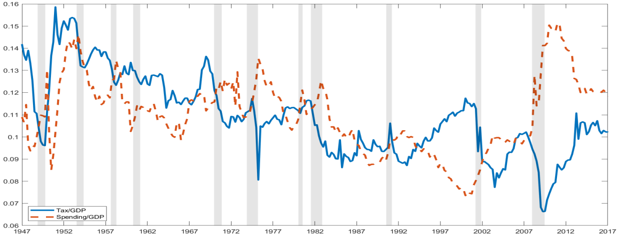
We assume that the vector of state variables $\boldsymbol{z}$ follows a Gaussian first-order VAR:

$$\boldsymbol{z}_t = \Psi \boldsymbol{z}_{t-1} + \boldsymbol{\Sigma}^{\frac{1}{2}} \boldsymbol{\varepsilon}_t$$

Position	Variable	Mean	Description
1	π_t	π_0	Log Inflation
2	x_t	x_0	Log Real GDP Growth
3	$y_t^{\$}(1)$	$y_0^{\$}(1)$	Log 1-Year Nominal Yield
4	$yspr_t^{\$}$	$yspr_0^{\$}$	Log 5-Year Minus 1-Year Nominal Yield Spread
5	pd_t	$\overline{pd}$	Log Stock Price-to-Dividend Ratio
6	Δd_t	μ_d	Log Stock Dividend Growth
7	$\Delta \log \tau_t$	μ_τ	Log Tax Revenue-to-GDP Growth
8	$\Delta \log g_t$	μ_g	Log Spending-to-GDP Growth
9	$\log \tau_t$	$\log \tau_0$	Log Tax Revenue-to-GDP Level
10	$\log g_t$	$\log g_0$	Log Spending-to-GDP Level

Key Ingredient I: Cash Flow Risk in $\{T, G\}$

1. Business cycle-frequency risk
   - Tax revenues are pro-cyclical. Government spendings are strongly counter-cyclical.



   - $S = T - G$ is strongly pro-cyclical: In recessions, investors who hold all Treasury has negative cash flows
   - If we correctly price the business cycle risks, we can infer the risk premium associated with these risks on the fiscal cash flows.

Key Ingredient I: Cash Flow Risk in $\{T, G\}$

1. Business cycle-frequency risk
2. Long-run risk
   - The levels of τ and g in the VAR imply tax and spending are cointegrated with GDP
     - Cointegration indicates (long-run) automatic stabilizers (Bohn, 1998)
   - They share the same long-run risk: The expected return on a long-dated tax or spending strip = expected return on long-dated GDP strip
   - Investor who is net long govt debt portfolio faces substantial long-run GDP risk

Some Details on Cash Flow Dynamics

- In the VAR, tax and spending can depend on lagged macro variables.
- Model delivers reasonable impulse-responses of fiscal variables
- Results robust to
  - Zeroing out insignificant elements in VAR companion matrix
  - Using quarterly instead of annual VAR
  - Starting sample in 1970
  - Adding debt/gdp as a predictor in the VAR (see appendix G)

Key Ingredient II: SDF M

- Affine log SDF with market prices of risk Λ_t (Ang and Piazzesi, 2003)

$$
\begin{aligned}
m^{\$}_{t+1} &= -y^{\$}_t(1) - \frac{1}{2}\boldsymbol{\Lambda}_t'\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t'\boldsymbol{\varepsilon}_{t+1} \\
\boldsymbol{\Lambda}_t &= \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 z_t
\end{aligned}
$$

- Fits individual nominal and real bond yields of various maturities
- Fits stock price-dividend ratio and equity risk premium
- Has a sufficiently large permanent component (Alvarez and Jermann, Borovicka, Hansen, Scheinkman)
    - A claim to government surplus has substantial long-run risk premium

Pricing Claims to Revenue T and Spending G

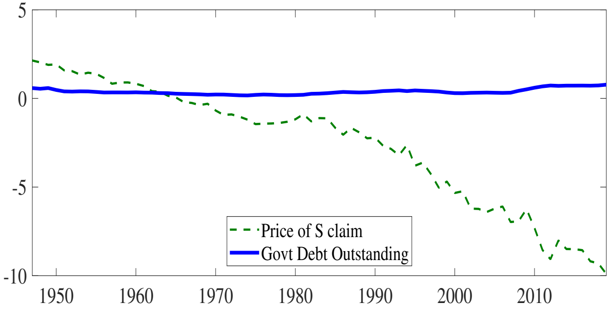
- With VAR dynamics and the SDF in hand, we can value T and G claims

$$
\begin{aligned}
P_t^T &= \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j} T_{t+j}\right] \\
P_t^G &= \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j} G_{t+j}\right]
\end{aligned}
$$

- When the TVC holds, the debt valuation satisfies $P_t^T - P_t^G = D_t$

And we get the Government Bond Valuation Puzzle

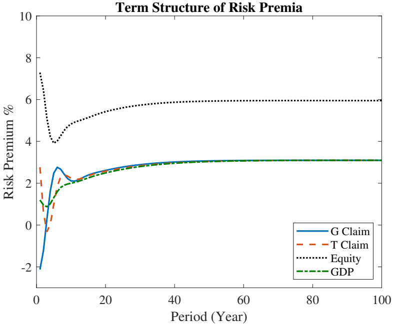
$$D_t = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j}(T_{t+j} - G_{t+j}) \right]$$



- We generate this puzzle while generating a good fit for individual bond prices (and stock prices).

Term Structure of Risk Premia

$$D_t = \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j}(T_{t+j} - G_{t+j})\right]$$



- Short-run: G claim risk premium $<$ T claim risk premium
- Long-run: G and T have the same risk premium as GDP strip
- Overall: G strip has a higher valuation than T strip, whereas the average T and G cash flows are similar in sample.

Potential Resolution 1: Convenience Yield

- Convenience yield $\lambda_t \Leftrightarrow$ Treasury bonds paying lower yields than the discount rate implied from SDF:

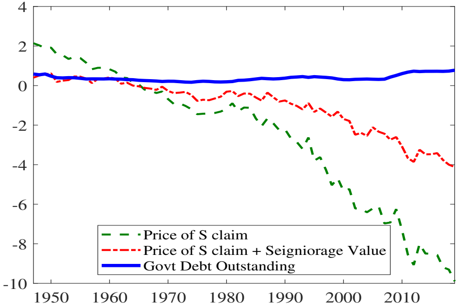
$$\begin{aligned} E_t[M_{t+1}] &= P_t^1 e^{-\lambda_t}, \\ E_t[M_{t+1}P_{t+1}^1] &= P_t^2 e^{-\lambda_t}, \\ E_t[M_{t+1}P_{t+1}^K] &= P_t^{K+1} e^{-\lambda_t}. \end{aligned}$$

- Debt now also backed by a convenience service/seigniorage term:

$$D_t = E_t\left[\sum_{j=0}^{\infty} M_{t,t+j}\left(T_{t+j} - G_{t+j} + (1 - e^{-\lambda_{t+j}})D_{t+j}\right)\right]$$

Can Convenience Yields Close the Gap?

- Measure λ_t as the weighted average of CP–T-bill spread and AAA–T-bond spread (Krishnamurthy and Vissing-Jorgensen, 2012).
- Reduces puzzle but does not resolve it
  - PDV of convenience services averages 12% of GDP



- Leaves open possibility that convenience yields are much larger and counter-cyclical than conventionally thought

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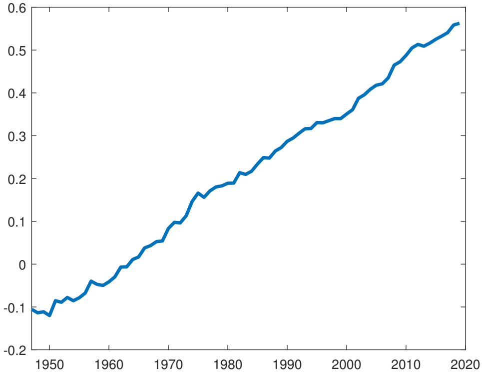
- Reduces puzzle but does not resolve it
- Leaves open possibility that convenience yields are much larger and counter-cyclical than conventionally thought
  - Krishnamurthy, Jiang, and Lustig (2019) find convenience yields for foreigners between 2 and 3%; Koijen and Yogo (2020) find 2.15% for U.S. long-term bonds
  - U.S. is world's designated supplier of dollar-denominated safe assets, *but that could change*; see Farhi and Maggiori (18)

Potential Resolution 2: Peso Problem

- Hypothesize that probability ϕ_t of a significant, **permanent** spending cut is priced in the surplus claim
- Such a rare spending cut never happened in post-war U.S. era, a peso event
- When realized, spending cut of 8% of U.S. GDP = 2×stdev of spending shock. Average spending is 11.5% of GDP in sample.
- How large should this spending cut probability ϕ_t be, so that the market value of the government debt = the present value of surpluses, period-by-period?

Potential Resolution 2: Peso Problem

- Large!



- Implied probability ϕ_t at odds with notion of a "rare" disaster
- A restatement of the puzzle

Potential Resolution 3: Bubble in Treasuries

- Bond markets are not enforcing TVC
  - Bubble = value of outstanding debt − value of surplus claim
  - We quantify the size of the bubble at 260% of GDP unconditionally
- TVC violations are hard to sustain in the presence of long-lived investors (Santos and Woodford, 97)
- Rise in sovereign CDS spread after GFC (Chernov et al. 16) seems inconsistent with rational bubble in Treasuries

Potential Resolutions 4: Pure Fiscal Risk is Priced

- Model assumes that fiscal shocks that are orthogonal to macro-economic and financial sources of risk are not priced
- Mechanically, one can close the wedge by changing this assumption. Allow for non-zero mpr on tax shock and let it depend on the state variables.
- Would need orthogonal tax revenue shocks to have a very large negative risk price to close the wedge
  - That would make the tax claim safer and increase its value, and hence the value of the surplus claim
  - Violates Cochrane and Saa-Requejo (2000) good-deal bound: adds 6.3 to the model's maximum Sharpe ratio.
  - Implausible that positive (orthogonal) tax revenues/GDP shocks occur in bad times
- Similarly, would need very large positive risk price to orthogonalized govt spending/gdp shock

Potential Resolutions 5: Government Assets

- Assets lower **net** government debt held by the public from 77.8% to 69.1% of the GDP; makes little difference for the puzzle
  - Outstanding student loans and other credit transactions, cash balances, and various financial instruments
  - Based on CBO data, total value of these government assets is 8.8% of GDP as of 2018.
- Other assets (national park land, defense assets, critical infrastructure, etc.) arguably off limits for political and military-strategic reasons
- If anything, massive off-balance sheet **liabilities** (Medicare, Social Security) will further **deepen** the puzzle in the future

Conclusion

- We evaluate the aggregate government debt portfolio by pricing the primary government surpluses: $D = PV(T - G)$.

- A *government debt valuation puzzle* emerges.

- The puzzle also manifests itself in the risk premium space.
  - *government debt risk premium puzzle*: The implied government debt yield should have been much higher.
  - Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2020b): Risk-free debt imposes tight restrictions on government surplus dynamics.

- Take-aways:
  1. Bond market investors fail to enforce the TVC.
  2. Convenience yields may be much larger than we think.
  3. Investors hold optimistic beliefs about future fiscal rectitude.

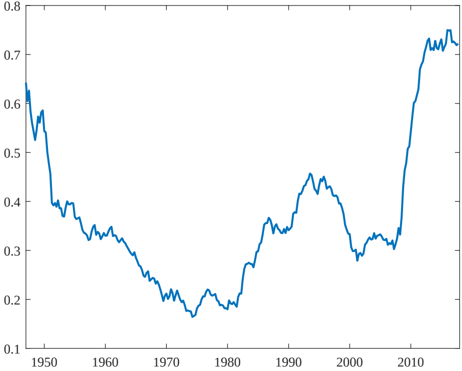
End of Presentation

Back up slides start from this point.

Related Literature

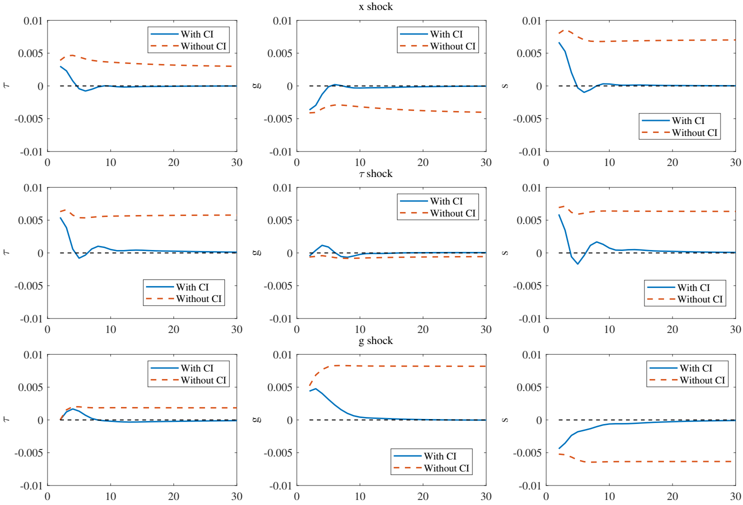
- **Affine no-arbitrage asset pricing models**: Campbell (91, 93, 96); Duffie and Kan (96); Dai and Singleton (00); Ang and Piazzesi (03); Lustig, Van Nieuwerburgh, and Verdelhan (13)
- **Fiscal policy** literature in macro: Hansen and Sargent (80); Lucas and Stokey (83); Hansen, Sargent, and Roberds (91); Angeletos (02); Buera and Nicolini (04); Hall and Sargent (11); Sargent (12); Karantounias (18); Bandhari, Golosov, Evans, and Sargent (17, 19); Blanchard (19), Cochrane (19, 20)
- **Specialness of U.S. bonds**: Longstaff (04); Krishnamurthy and Vissing-Jorgensen (12, 15); Greenwood, Hanson, and Stein (15); Nagel (16); Farhi and Maggiori (18) Du, Im, and Schreger (18); Binsbergen, Diamon, Grotteria (19); Jiang, Krishnamurthy, and Lustig (19)
- **Fiscal policy risk**: Croce, Nguyen, Schmid (12), Croce, Kung, Nguyen, and Schmid (19), Chernov, Schmid, and Schneider (19), Liu, Schmid, and Yaron (20)

The Market Value of Outstanding Debt to GDP

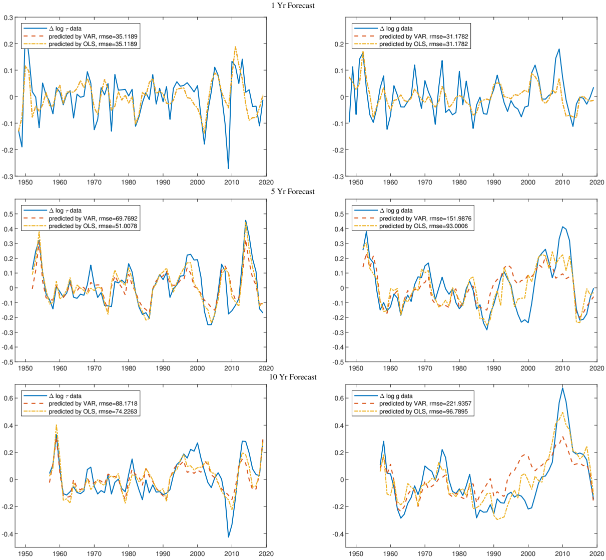


- Build up market value of government debt, cusip by cusip, stripped across horizons
- Follows Hall and Sargent (2011), extended to end of 2019
- Portfolio has low excess return over the T-bill rate: 1.11% per year

Responses of Tax and Spending

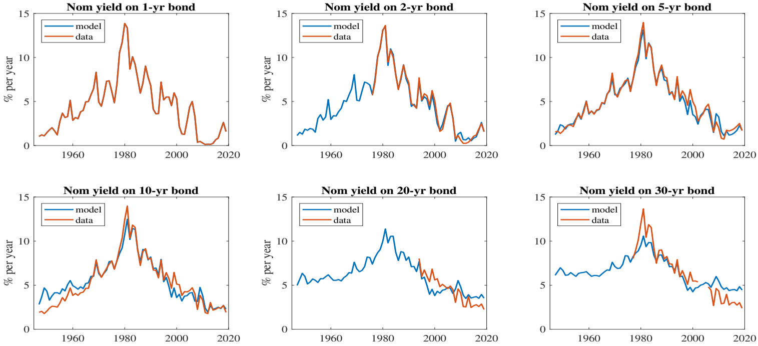


Forecasts of Revenue and Spending Growth



Estimation

- Estimate $\widehat{\Lambda}_0, \widehat{\Lambda}_1$ to match observed interest rates for bonds at various horizons, expected excess return on 5-year nominal bond (BRP), and observed stock valuation ratio and expected excess stock returns.



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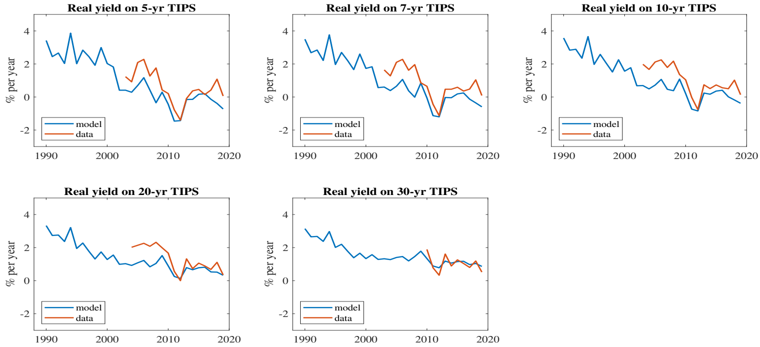

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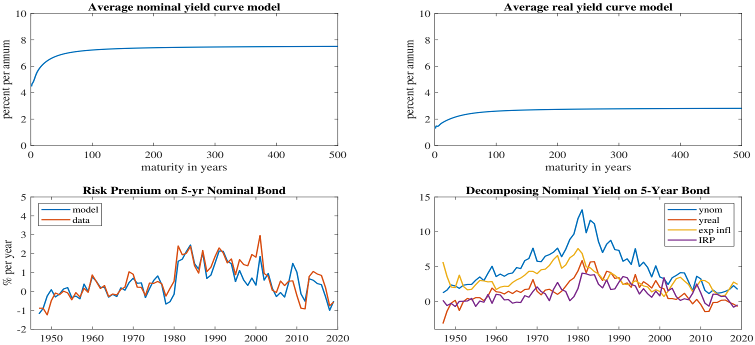

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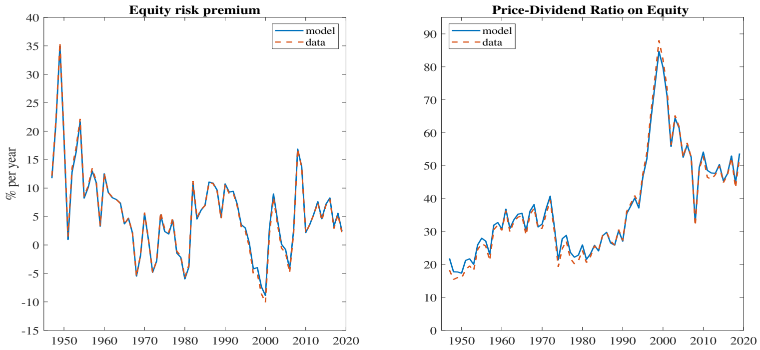

How Large a Convenience Yield to Close the Gap?

- Convenience services would need to be 20.57% of tax revenue
- They are only 1.9% in the data.
- Would need to be 41.35% of tax revenue in the last 20 years of sample