Volatility Uncertainty and Jumps

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The 1987 stock market crash showed that option pricing models fail to price options with short TTM and deep-OTM puts

→ Solution: state-dependent jump intensity that is linked to volatility (Bates, 2000)

\[ \lambda_t = \alpha_0 + \lambda^V V_t + \ldots \]

→ Implications:

- **Strong linear** link between jump intensity and volatility
- Only **source of time-variation** in jump risks is volatility
Motivation

- Linking jump risks to volatility seems reasonable
  - Negative jumps in stock market occur when volatility is high
- **Andersen, Fusari, Todorov (2015, 2019):** After turbulent times, left tail stays elevated long after volatility mean-reverts
  - Disconnect between time-series dynamics?
This Paper

In an almost non-parametric setting, we ask:

- Are expected jump risks and volatility linearly tied?
  - Very weak relationship at best
  - Significance completely gone once higher moments are included

- Which moment is related to jump risks? Volatility Uncertainty
  - Main driver of evolution of jump risks
  - Higher volatility uncertainty increases downside risk and decreases upside potential
  - Predicts realized price jumps

- How can option pricing models account for our findings?
  - Decoupling jump risk evolution from volatility is crucial
  - Separately modeling left and right tail necessary

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Event Study - Large VIX and VVIX Shocks

→ Changes in volatility uncertainty have an isolated effect on tails

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Higher Moments and Tail Measure

- Main analysis based on option-implied information (under risk-neutral measure)
- We extract higher moments in standard-fashion with portfolios of weighted option prices
  - Vol$^2$ and SKEW using S&P500 options
  - VolVol$^2$ using VIX options
- For tail measure, we follow Bollerslev, Todorov, and Xu (2015)
  - Use (deep) out-of-the-money options
  - Fit them to jump intensity
    \[
    \nu_t(dy) = \left( \phi_t^+ \times e^{-\alpha t} y 1_{\{y>0\}} + \phi_t^- \times e^{\alpha t} y 1_{\{y<0\}} \right)
    \]
  - independent left (LJV) and right (RJV) tail
  - time-variations in shape of tail possible
Data

- Option Metrics: monthly and weekly S&P500 options, monthly VIX options
- Basic filters; Time-to-maturity of options: $1 < \text{TTM} < 45$
- Calculate our measures on a **weekly** basis, then
  1. orthogonalize them due to correlations
  2. take first differences due to autocorrelation
  3. standardize measures
Evolution of Left Tail

\[ \Delta \text{LJV}_t = \alpha + \beta \Delta X_t + \epsilon_t \]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \text{Vol}^2 )</td>
<td>0.2578</td>
<td>0.1954</td>
<td>0.2241</td>
<td>0.2241</td>
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<tr>
<td></td>
<td>(1.67)</td>
<td>(1.41)</td>
<td>(1.63)</td>
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<tr>
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<tr>
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<td>0.2303</td>
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<tr>
<td></td>
<td>(3.01)</td>
<td>(4.22)</td>
<td>(3.30)</td>
<td></td>
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<tr>
<td>( \Delta \text{SKEW} )</td>
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<td>-0.2652</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(-4.66)</td>
<td>(-4.75)</td>
<td></td>
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</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.0644</td>
<td>0.0845</td>
<td>0.0996</td>
<td>0.1153</td>
<td>0.1345</td>
</tr>
</tbody>
</table>

The table shows results for the regressions with left-jump variation as dependent variable. All input variables are weekly averages and normalized by their full-sample standard-deviation. Newey-West robust t-statistics are given in the parentheses below. \( \perp \) represents the orthogonal part.
Evolution of Right Tail

\[ \Delta \text{RJV}_t = \alpha + \beta \Delta X_t + \epsilon_t \]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>(\Delta \text{Vol}^2)</td>
<td>-0.0515</td>
<td>-0.0097</td>
<td>-0.0090</td>
<td>-0.0090</td>
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<tr>
<td></td>
<td>(-1.74)</td>
<td>(-0.46)</td>
<td>(-0.41)</td>
<td>(-0.41)</td>
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<tr>
<td>(\Delta \text{VolVol}^2)</td>
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<td>-0.1220</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(-3.17)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\Delta \text{VolVol}^2,\perp)</td>
<td></td>
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<td>-0.1356</td>
<td>-0.1297</td>
<td>-0.1331</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-3.09)</td>
<td>(-3.28)</td>
<td>(-3.05)</td>
</tr>
<tr>
<td>(\Delta \text{SKEW})</td>
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<td></td>
<td></td>
<td>-0.1006</td>
<td>-0.0696</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-1.88)</td>
<td>(-1.38)</td>
</tr>
<tr>
<td>adj. (R^2)</td>
<td>0.0006</td>
<td>0.0129</td>
<td>0.0153</td>
<td>0.0276</td>
<td>0.0248</td>
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</tbody>
</table>

The table shows results for the regressions with right-jump variation as dependent variable. All input variables are weekly averages and normalized by their full-sample standard-deviation. Newey-West robust t-statistics are given in the parentheses below. \(\perp\) represents the orthogonal part.
Predicting Realized Risks

Analysis so far under risk-neutral measure. Can volatility uncertainty also explain realized risks?

- Determine realized variance and tri-power variation
- Difference isolates realized price jumps

Run predictive regressions of form

\[
\text{Realized Risk}_{[t+h-1,t+h]} = \gamma + \beta_{Vol} \ Vol_t^2 + \beta_{VolVol} \ VolVol_t^2 + \epsilon_t, \\
h = 2, \ldots, 25.
\]

and compare the $R^2$ of multiple regression to $R^2$ of simple regression.

**Note:** Non-overlapping regressions, we predict the weekly avg. in $t + h$. Standard errors are HAC-estimators that correct for autocorrelation.
Almost no predictive power of volatility uncertainty on total risk
Realized Price Jumps

- Price jumps can be well predicted by volatility uncertainty.
- Vol uncertainty not only explains expected jump risks ($\mathbb{Q}$) but also realized jump risks ($\mathbb{P}$)

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Testing Option Pricing Models

- What happens if jump intensity is only tied to volatility?
- Test model of Eraker (2004)

\[
\frac{dS_t}{S_t} = (r - \mu)dt + \sqrt{V_t}dW_t^{S,Q} + dJ_t^{S,Q}
\]

\[
dV_t = \kappa^Q(\theta^Q - V_t)dt + \sigma_V \sqrt{V_t}dW_t^{V,Q} + dJ_t^{V,Q}
\]

\[
\lambda_t = \lambda_0 + \lambda_1 V_t
\]

- How do we test? For each week
  - Extract state variables by minimizing distance between model’s variance expectations and model-free IV
  - Simulate model 50,000 times
  - Determine model-implied option prices and risk measures
## Eraker Model - Results

<table>
<thead>
<tr>
<th></th>
<th>ΔLJV</th>
<th></th>
<th>ΔRJV</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>ΔVol^2</td>
<td>0.8102</td>
<td>0.8115</td>
<td>0.1248</td>
<td>0.1254</td>
</tr>
<tr>
<td></td>
<td>(3.49)</td>
<td>(3.61)</td>
<td>(1.92)</td>
<td>(1.99)</td>
</tr>
<tr>
<td>ΔVolVol^2</td>
<td>-0.3104</td>
<td>-0.0835</td>
<td>-0.0835</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-4.77)</td>
<td>(-1.99)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔVolVol^2,⊥</td>
<td>-0.1015</td>
<td></td>
<td>-0.0525</td>
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</tr>
<tr>
<td></td>
<td>(-2.45)</td>
<td></td>
<td>(-1.40)</td>
<td></td>
</tr>
<tr>
<td>adj. R^2</td>
<td>0.6557</td>
<td>0.0945</td>
<td>0.6654</td>
<td>0.0136</td>
</tr>
</tbody>
</table>

- Volatility is clearly the main driver
- Counterfactual negative link between left tail and volatility uncertainty
- VolVol^2 irrelevant for right tail

→ Overall, OTM option price dynamics are not in line with data
Summary

- Paper analyzes the interdependencies between expected tail risks and higher moments of return distribution.
- We show that volatility uncertainty has a distinct impact on both tails of the risk-neutral distribution.
- Expected volatility uncertainty predicts realized price jumps but not realized volatility.
- Findings present a challenge for many modern option pricing models.
  - Model tests suggest that decoupling the intensity from volatility is necessary.
  - Separately model left and right jump intensity.
## Backup – Liquidity of SPX Options

The table shows descriptives for daily liquidity measures of SPX options across different moneyness buckets. The quantities for volume (Vol) and open interest (OI) are quoted in millions. # refers to the absolute number of contracts, $ refers to the corresponding dollar value and % refers to the relative number of traded or open contracts. The Bid-Ask Spread is calculated as $2(\text{Ask} - \text{Bid}) / (\text{Ask} + \text{Bid})$.  

<table>
<thead>
<tr>
<th>$m$</th>
<th>$(-\infty, -4]$</th>
<th>$(-4, -2.5]$</th>
<th>$(-2.5, -1]$</th>
<th>$(-1, 1]$</th>
<th>$(1, 2.5]$</th>
<th>$(2.5, 4]$</th>
<th>$(4, \infty)$</th>
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<tr>
<td>Vol[#]</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Vol[$$]</td>
<td>0.96</td>
<td>0.21</td>
<td>0.40</td>
<td>1.18</td>
<td>0.32</td>
<td>0.13</td>
<td>0.32</td>
</tr>
<tr>
<td>Vol[%]</td>
<td>0.36</td>
<td>0.07</td>
<td>0.09</td>
<td>0.20</td>
<td>0.09</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td>OI[#]</td>
<td>1.25</td>
<td>0.15</td>
<td>0.18</td>
<td>0.25</td>
<td>0.16</td>
<td>0.13</td>
<td>0.41</td>
</tr>
<tr>
<td>OI[$$]</td>
<td>43.55</td>
<td>3.80</td>
<td>4.69</td>
<td>6.97</td>
<td>4.34</td>
<td>3.32</td>
<td>21.58</td>
</tr>
<tr>
<td>OI[%]</td>
<td>0.47</td>
<td>0.06</td>
<td>0.08</td>
<td>0.11</td>
<td>0.07</td>
<td>0.06</td>
<td>0.16</td>
</tr>
<tr>
<td>OI[%]</td>
<td>0.52</td>
<td>0.06</td>
<td>0.07</td>
<td>0.10</td>
<td>0.06</td>
<td>0.05</td>
<td>0.14</td>
</tr>
<tr>
<td>Bid-Ask Spread</td>
<td>0.21</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
<td>0.08</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Bid-Ask Spread</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Backup – Self-Exciting Jump Model

- **Kaeck (2018)** uses a rich specification:

\[
\frac{dS_t}{S_t} = (r - q - \mu)dt + \sqrt{V_t}dW_t^{S,Q} + dJ_t^{\lambda,Q}
\]

\[
dV_t = \kappa_V^Q (m_t - V_t)dt + \sigma_V \sqrt{V_t} (\rho dW_t^{S,Q} + \sqrt{1 - \rho^2} dW_t^{V,Q}) + dJ_t^{\lambda,Q}
\]

\[
dm_t = \kappa_m^Q (\theta_m^Q - m_t)dt + \sigma_m \sqrt{m_t} dW_t^{m,Q}
\]

\[
dl_t = \kappa_l^Q (\theta_l^Q - \lambda_t)dt + \sigma_l \sqrt{\lambda_t} dW_t^{l,Q} + dJ_t^{\lambda,Q}
\]

- \(\lambda_t\) is the jump intensity for all jumps
  - follows independent process
  - can jump itself (self-exciting)
Backup – Kaeck Model Results

<table>
<thead>
<tr>
<th></th>
<th>$\Delta LJV$</th>
<th></th>
<th>$\Delta RJV$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$\Delta Vol^2$</td>
<td>-0.0094</td>
<td>-0.0449</td>
<td>0.0635</td>
<td>0.0268</td>
</tr>
<tr>
<td></td>
<td>(-1.38)</td>
<td>(-2.50)</td>
<td>(1.22)</td>
<td>(0.51)</td>
</tr>
<tr>
<td>$\Delta VolVol^2$</td>
<td>0.1634</td>
<td>0.1818</td>
<td>0.1818</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.10)</td>
<td>(2.06)</td>
<td></td>
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</tr>
<tr>
<td>$\Delta VolVol^{2,\perp}$</td>
<td>0.1670</td>
<td>0.1906</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(2.08)</td>
<td>(2.10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>-0.0017</td>
<td>0.1231</td>
<td>0.1213</td>
<td>0.0045</td>
</tr>
</tbody>
</table>

- Results for left tail close to empirics
- Counterfactual positive link between right tail and volatility uncertainty
  → Need to model left and right tail separately