Monetary Policy with Endogenous Money Supply

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Motivation:

- Asset prices, borrowing and debt stock are closely related in an economy.
- We observe that asset prices are correlated to money supply.
- Asset prices and general price level do not follow the same path which requires a dual price system, general price level and asset prices.
Existing Literature

- Eggertsson and Krugman (2012) blame the representative agent model for not accounting for the role of debt in New Keynesian models. They divide agents into borrowers and lenders. We think it is not necessary.

- Palivos, Wang, and Zhang (1993) use a modified cash in advance (CIA) model to show high-money-growth equilibrium imply lower welfare and low-money-growth equilibrium imply higher welfare.

- Salyer (1991) analyzes the timing of the markets and how it influences the characteristics of real balances.
Model

- A representative agent model
- A modified Cash-in Advance (CIA) model.
- In a CIA model, “money” is in the form of cash only, where in our model bank deposits are money as well.
- Consequently, more borrowing (loans) implies more money in the economy. $D_s^t$ is the debt stock of an agent at time $t$, $D_t$ is period borrowing (or saving if negative) at period $t$. $D_s^t = D_{t-1}^s + D_{t-1}$. In real terms:

$$d_t^s = \frac{1}{1 + \pi_t} (d_{t-1}^s + d_{t-1})$$
Assets

- Households have the opportunity to build up wealth in the form of assets which have no returns other than their nominal appreciation, or depreciation. $A_t^s$ is the nominal value of an agent’s asset stock at period $t$, $A_t$ is how much assets is purchased at period $t$.

- The nominal value of assets is determined from the quantity of assets purchased by multiplying the quantity of assets ($q_t$) by the general price level ($P_t$) and the asset price level ($\bar{P}_t$), $A_t = P_t \bar{P}_t q_t$ & $A_t^s = P_t \bar{P}_t q_t^s$.

- In quantity, $q_t^s = \phi q_{t-1}^s + q_{t-1}$ where $\phi$ is period asset depreciation and in real terms $a_t^s = (1 + \pi_t)(\phi a_{t-1}^s + a_{t-1})$ where $\bar{P}_t/\bar{P}_{t-1} = 1 + \pi_t$, $a_t = A_t/P_t$, $a_t^s = A_t^s/P_t$.

- Asset supply is constant at “s” every period. Therefore, asset market equilibrium condition is $a_t = s$, demand equals supply.
The Budget Constraint

- $P_t c_t + E_t\{P_{t+1}\}E_t\{\tilde{P}_{t+1}\}q_{t+1}^s + i_{t-1}D_t^s \leq P_t y_t + P_t \tau_t + P_t \tilde{P}_t \phi q_t^s + D_t$

- Households have a constant stream of real income, $y_t$. They can borrow funds, $D_t$ (saving if $D_t < 0$). They receive an extra income of $\tau_t$ from production of new assets every period.

- Households pay the interest cost on their debt stock but they carry the balance to the next period, unless they are saving (a negative $D_t$).

- Households decide how many assets to purchase that determines their asset stock. The choice variable, $a_t$, is in real terms.

- For solving our model we will assume naive expectations, $E_t\{P_{t+1}\} = P_t$ and $E_t\{\tilde{P}_{t+1}\} = \tilde{P}_t$. 
Monetary Policy, Assets

- The central bank determines the interest rate $i_t$ that affects the debt burden in households’ budget. Higher interest rate means less money to spend on consumption and assets, low interest rate means more money to spend on consumption and assets.

- Although assets do not have a return, their significance will be allowing agents to borrow more, which will be explained later.
Constraint on Purchases

- Similar to a CIA model, money is required to purchase consumption goods and assets. But the definition of money is extended to include money created by bank loans. We use the following “money-in-advance” constraint:

$$P_t(c_t + a_t) \leq D_t$$

- As people borrow more, $D_t$ increases which allows for more purchases. As people save more, $D_t$ decreases which restricts people’s purchases.

- For the sake of simplicity we ignored government printed money
Timing of the Markets

Unlike a CIA model, how much money to hold is not a choice variable in this model. It is accumulated debt accrued from prior borrowing decisions.
Optimization

- Maximize the Lagrangian w.r.t. $c_t$, $a_t$, $d_t$. Part of the function is as follows:

$$u(c_t) + \lambda_t \left[ y_t + \phi a_t^s + \tau_t + d_t - c_t - \frac{1}{1 + \tilde{\pi}_{t+1}} a_{t+1}^s - i_{t-1} d_t^s \right]$$

$$+ \lambda_{t+1} \beta \left[ y_{t+1} + \phi a_{t+1}^s + \tau_{t+1} + d_{t+1} - c_{t+1} - \frac{1}{1 + \tilde{\pi}_{t+2}} a_{t+2}^s - i_t d_{t+1}^s \right]$$

$$+ \mu_t (d_t^s - c_t - a_t)$$

$$+ \mu_{t+1} \beta (d_{t+1}^s - c_{t+1} - a_{t+1})$$
Lambda and Euler Equation

\[ \lambda_{t+1} = \frac{u'(c_t)}{\beta \phi (1 + \tilde{\pi}_{t+1})} \]

Euler eqn:

\[ \frac{u'(c_{t-1})}{\beta \phi (1 + \tilde{\pi}_t)} - \frac{u'(c_t)}{\phi (1 + \tilde{\pi}_{t+1})} \frac{1 + i_t}{1 + \pi_{t+1}} + \beta \frac{u'(c_{t+1})}{1 + \pi_{t+1}} = 0 \]
Market Clearing

- To close the money market, we have a borrowing limit: \( d_t = a_t^s - d_t^s \)

- The borrowing limit is the net worth of households, the difference between their asset stock and debt stock. We assume that banks lend, as long as net worth is positive. More debt than assets imply a negative \( d_t \), savings!

- Asset market eq-m condition: \( a_t = s_t \) where \( a_t \) is asset demand, \( s_t \) is asset supply. \( s_t \) is exogenous, \( a_t \) is determined in the model. Asset prices make sure that demand equals supply
Dynamics

- At the beginning of the period, households learn about their endowments, \( y_t \), asset supply, \( s_t \), interest rate, \( i_t \).

- Then they decide how much to borrow, \( d_t \), how much assets to demand, \( a_t \), and how much to consume, \( c_t \).

- Debt and asset stock flow equations determine the stock variables. The money market and asset market clearing equations, the Euler equation and the two constraints close the model.
Steady States

- \( a = \left( \frac{1}{1 + \tilde{\pi}} - \phi \right) a^s \)
- \( d = \pi d^s \)
- \( a = y + \tau + d - c - id^s \)
- \( d = a^s - d^s \)
- \( c + a = d^s \)
- \( a = s \)

These equations produce the following two equations that determine the general price inflation, \( \pi_t \), and asset price inflation, \( \tilde{\pi}_t \):

- \( 1 + \pi = \beta(1 + i) - \beta^2 \phi(1 + \tilde{\pi}) \)
- \( 1 + \tilde{\pi} = \frac{(1 + \pi)(y + s)}{s(1 + i - \pi) + \phi(1 + \pi)(y + s)} \)
Results

- \[ a = \left( \frac{1}{1 + \tilde{\pi}} - \phi \right) a^s > 0 \] as long as real value of asset stock is decreasing, \( \phi(1 + \tilde{\pi}) < 1 \)

- We assumed asset stock, \( a^s \), has a steady state too. This requires buying assets to keep asset stock steady when assets are losing value.
Results, Monetary Policy

• \( \frac{\partial \tilde{\pi}}{\partial i} > 0 \) when \( \phi(1 + \tilde{\pi}) < 1 \), as long as there is a demand for assets, \( a > 0 \), an expansionary monetary policy lowers steady state asset price inflation.

• \( \frac{\partial \pi}{\partial i} > 0 \), regardless of asset demand, an expansionary monetary policy lowers steady state general price inflation too.
Results, Debt

- In the special case of $s = 0$,

- $\frac{\partial d^s}{\partial i} = \frac{-y(1 - \frac{\partial \pi}{\partial i})}{(1 + i - \pi)^2} < 0$ which is an expected result, an expansionary monetary policy will increase money stock in the economy.
Balance Sheet Recessions

- For the special case \( s = 0 \), and for the general case \( \pi < 0 \). Through \( d = \pi d^s \) this implies a negative borrowing, or saving for households.

- Borrowers save as much as the difference between the value of their assets and debt, \( d = a^s - d^s \). This is similar to what happened in Japan and US after the collapse of their housing markets.

- After a collapse in asset prices, debt stock exceeded asset stock which forced households to save.
Conclusion

- Adding a second price system to a model where money is needed to purchase not only goods, but assets too is necessary to analyze the effects of monetary policy on inflation, and asset price inflation.

- A sudden drop in asset prices induces households to save rather than borrow.

- A low interest rate policy is intended to increase asset prices. But as long as the real value of asset stock is steady, this will lead to negative inflation rates and household saving instead of borrowing, as it is the case for Japan.
Further Research

- This model assumes debt stock and asset stock are steady, $d_i^s = d^s \forall t$ and $a_i^s = a^s \forall t$, which is a significant restriction on the model.

- Analyzing the dynamics of the model rather than the steady state of the model is necessary.
References


