Time Consistency, Temporal Resolution Indifference and the Separation of Time and Risk

Felix Kubler, Larry Selden and Xiao Wei

University of Zurich, Columbia University, Fudan University

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- Also under these conditions, quite surprisingly the KP and Selden and Stux DOCE (dynamic ordinal certainty equivalent) models generate identical demands. (Our goal is not to evaluate relative superiority of two models.)
- DOCE Preferences are constructed from separate time U and risk preferences V (for simplicity assume $T = 3$): $U\left(c_1, \widehat{c}_2, \widehat{c}_3\right)$, where $\widehat{c}_i = V^{-1}\left(EV\left(\widetilde{c}_i\right)\right)$ $(i = 2, 3)$.

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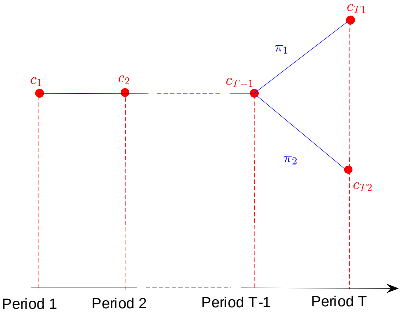
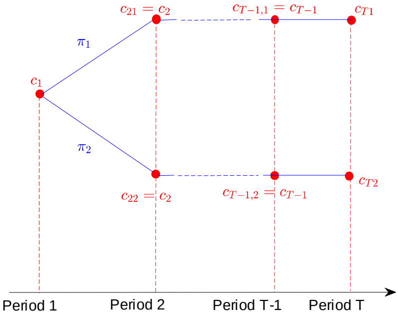


(a)



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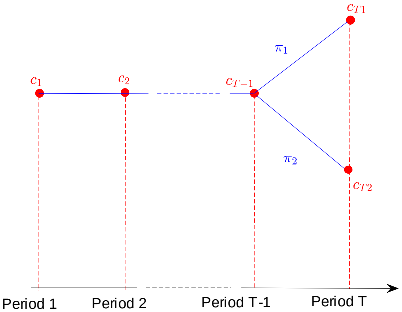

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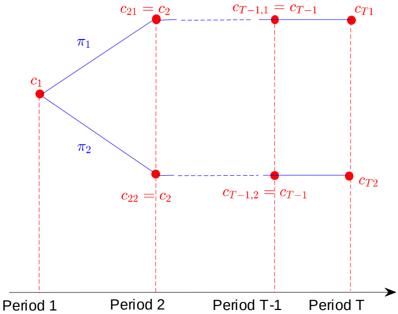

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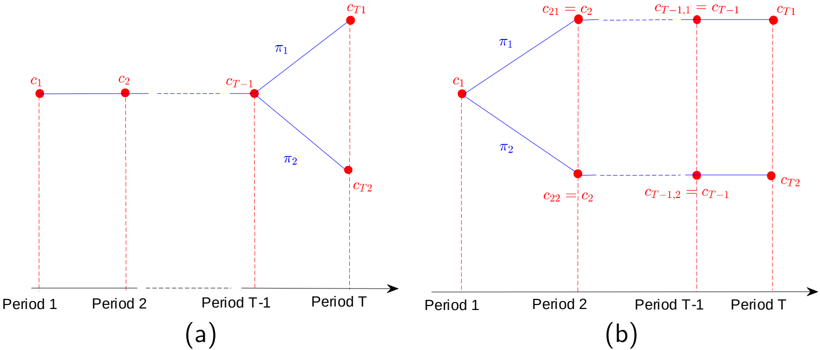
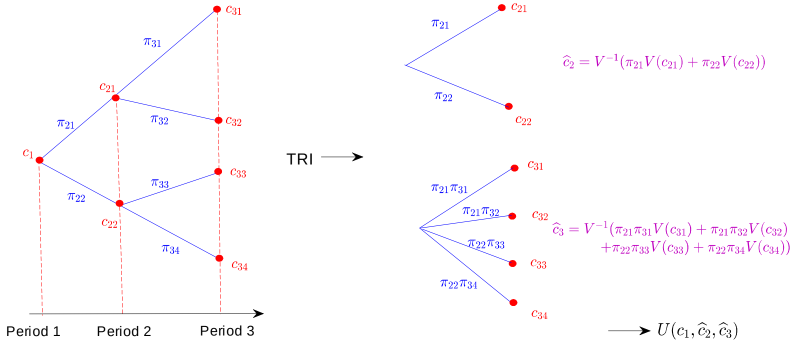


- DOCE explicitly assumes indifference between trees (a) and (b).
- KP preferences, based on typical specifications of (U, V), always prefer tree (b).

Computing DOCE Utility

- Assume given (U, V) – Property SEP satisfied



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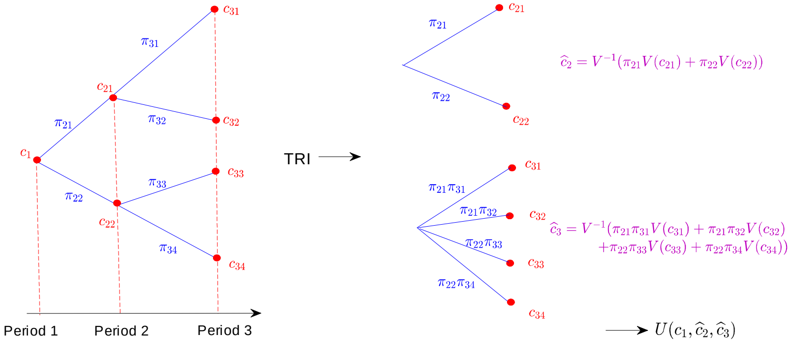

- Property TRI $\Longrightarrow$ three period tree decomposed into (i) period 2 tree and (ii) single stage compound "lottery" paying off $c_{31}, \ldots, c_{34}$.

Special Utility Building Blocks

- Extensively use Modified Bergson/HARA utilities[1]

$$u(c_t) = -\frac{1}{\delta_1}(c_t - b)^{-\delta_1} \quad \text{and} \quad V(c_t) = -\frac{1}{\delta_2}(c_t - b)^{-\delta_2} \;\; (\delta_1, \delta_2 \geq -1),$$

$$u(c_t) = -\frac{\exp(-\kappa_1 c_t)}{\kappa_1} \quad \text{and} \quad V(c_t) = -\frac{\exp(-\kappa_2 c_t)}{\kappa_2} \;\; (\kappa_1, \kappa_2 > 0),$$

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- CES and CRRA time and risk preference utilities used in EZ special case of KP preferences

$$u(c_t) = -\frac{1}{\delta_1}{c_t}^{-\delta_1} \quad \text{and} \quad V(c_t) = -\frac{1}{\delta_2}c_t^{-\delta_2} \;\; (\delta_1, \delta_2 > -1).$$

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- For this case, EIS and Pratt-Arrow relative risk aversion measures given by

$$EIS = \frac{1}{1+\delta_1} \qquad \text{and} \qquad -c_t\frac{V''(c_t)}{V'(c_t)} = 1 + \delta_2.$$

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Optimization Problem

- Since DOCE preference can be time inconsistent, need to consider standard Strotz-Pollak resolute, naive and sophisticated solution techniques.

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- Period 1 consumption-portfolio optimization problem

$$P_1 : \max_{c_1, c_{21}, c_{22}, c_{31}, c_{32}, c_{33}, c_{34}} U^{(1)}\left(c_1, c_{21}, c_{22}, c_{31}, c_{32}, c_{33}, c_{34}\right)$$

$$S.T.\ c_1 + n_1 + n_{f1} = I.$$

Period 2 optimization problem if upper state realized[2]

$$P_{21} : \max_{c_{21}, c_{31}, c_{32}} U^{(21)}\left(c_{21}, c_{31}, c_{32}\right)$$

$$S.T.\ c_{21} + n_2 + n_{f2} = R_{21} n_1 + R_{f2} n_{f1}$$

$$c_{31} = R_{31} n_2 + R_{f3} n_{f2} \text{ and } c_{32} = R_{32} n_2 + R_{f3} n_{f2}.$$

Period 2 optimization problem if lower state realized

$$P_{22} : \max_{c_{22}, c_{33}, c_{34}} U^{(22)}\left(c_{22}, c_{33}, c_{34}\right)$$

$$S.T.\ c_{22} + n_2 + n_{f2} = R_{22} n_1 + R_{f2} n_{f1}$$

$$c_{33} = R_{33} n_2 + R_{f3} n_{f2} \text{ and } c_{34} = R_{34} n_2 + R_{f3} n_{f2}.$$

[2] All assets have one period maturity.

Time Consistency for DOCE Preferences

- Key Assumptions:
  [ICER]: Assume that for all s^t, $t < T$,

$$\sum_{s^{t+1} \succ s^t} \pi(s^{t+1}|s^t) \left(R(s^{t+1}) \cdot \widetilde{\mathbf{n}}(s^t) \right)^{-\delta_2} = \left(\hat{R}_{pt} \sum_j \widetilde{\mathbf{n}}_j(s^t) \right)^{-\delta_2},$$

  where $\hat{R}_{pt}$ is non-stochastic.[3]
  [RF]: there exists a risk free asset.

[3] Stronger assumption that asset returns i.i.d. (independently and identically distributed) made in important papers Levhari and Srinivasan (1969), Samuelson (1969), Weil (1993), Campbell and Cochrane (1999) and Barro (2009).

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  where $\hat{R}_{pt}$ is non-stochastic.[3]
  [RF]: there exists a risk free asset.
- The consumer's preferences satisfy TC (time consistency) if and only if at any time t and all $\mathbf{c}$, $\mathbf{c}'$ with some payoff history s^t,

$$\mathcal{U}\left(\mathbf{c}|s^{t+1}\right) \geq \mathcal{U}\left(\mathbf{c}'|s^{t+1}\right) \quad \left(\forall s^{t+1} \succ s^t\right) \Rightarrow \mathcal{U}\left(\mathbf{c}|s^t\right) \geq \mathcal{U}\left(\mathbf{c}'|s^t\right),$$

  where $c\left(s^t\right) = c'\left(s^t\right)$.

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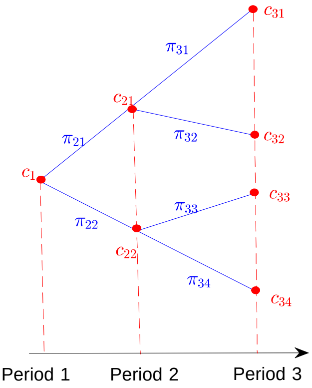

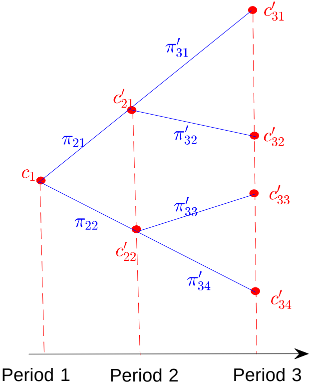

$$(c_{21}, c_{31}, c_{32}) \succeq (c'_{21}, c'_{31}, c'_{32})$$
$$(c_{22}, c_{33}, c_{34}) \succeq (c'_{22}, c'_{33}, c'_{34})$$

$$\longrightarrow \quad (c_1, c_{21}, c_{22}, c_{31}, c_{32}, c_{33}.c_{34}) \succeq (c_1, c'_{21}, c'_{22}, c'_{31}, c'_{32}, c'_{33}, c'_{34})$$

- For homothetic DOCE preferences and [ICER], optimal intertemporal choice in set

$$\mathcal{I}_0 = \left\{ \begin{array}{c} \mathbf{c} \in \mathbb{R}_+^7 : \mathbf{c} = (c_1, c_{21}, c_{22}, \alpha_1 c_{21}, \alpha_2 c_{21}, \alpha_3 c_{22}, \alpha_4 c_{22}), \\ (\alpha_1, ..., \alpha_4) \in \mathbb{R}_+, \pi_{31}\alpha_1^{-\delta_2} + \pi_{32}\alpha_2^{-\delta_2} = \pi_{33}\alpha_3^{-\delta_2} + \pi_{34}\alpha_4^{-\delta_2} \end{array} \right\}.$$

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Proposition

Assume set of 4 branch consumption trees satisfying [ICER]. Homothetic DOCE preferences corresponding to

$$u(c) = -\frac{c^{-\delta_1}}{\delta_1} \quad \textit{and} \quad V(c) = -\frac{c^{-\delta_2}}{\delta_2} \quad (\delta_1 > -1, \delta_2 \geq -1)$$

satisfy TC over domain $\mathcal{I}_0$.

Time Consistent DOCE Demands: HARA Class

Theorem

Assumptions [ICER] and [RF] hold and consumer solves consumption-portfolio problem. Then DOCE demands are time consistent iff one of following holds

$$(i)\ u(c) = -\frac{(c-b)^{-\delta_1}}{\delta_1} \quad \text{and} \quad V(c) = -\frac{(c-b)^{-\delta_2}}{\delta_2}\ (\delta_1, \delta_2 > -1),$$

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$$\text{(iii) } u(c) = \frac{(b-c)^{-\delta_1}}{\delta_1} \quad \text{and} \quad V(c) = \frac{(b-c)^{-\delta_2}}{\delta_2} \ (\delta_1, \delta_2 < -1).$$

- Intuition: For special tree structure assumed, quasihomothetic preferences and consumption-portfolio problem, assumed existence of risk free asset effectively translates quasihomothetic case into homothetic one.

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Then demands can be also rationalized by the KP preferences, where[a]

$$U(c_t, x) = -\frac{\left(c_t^{-\delta_1} + \beta\left(-\delta_2 x\right)^{\frac{\delta_1}{\delta_2}}\right)^{\frac{\delta_2}{\delta_1}}}{\delta_2} \quad \text{and} \quad V_T(x) = -\frac{x^{-\delta_2}}{\delta_2}.$$

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- Similar result for quasihomothetic DOCE and KP (u, V) building blocks if [ICER] and [RF] hold.

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Examples

Although time preferences are characterized by $(\delta_1, \beta) = (0, 1)$ and risk preferences are characterized by an arbitrary δ_2, TRI holds:

$$\begin{aligned}
\mathcal{U}(\mathbf{c}) &= \exp\left(\ln c_1 - \frac{1}{\delta_2}\ln\left(\begin{array}{c} \pi_{21}\exp\left(-\delta_2\left(\ln c_2 + \ln c_{31}\right)\right) \\ +\pi_{22}\exp\left(-\delta_2\left(\ln c_2 + \ln c_{32}\right)\right)\end{array}\right)\right) \\
&= c_1 c_2\left(\pi_{21}c_{31}^{-\delta_2} + \pi_{22}c_{32}^{-\delta_2}\right)^{-\frac{1}{\delta_2}} \\
&= \exp\left(\ln c_1 + \ln c_2 + \ln\left(\pi_{21}c_{31}^{-\delta_2} + \pi_{22}c_{32}^{-\delta_2}\right)^{-\frac{1}{\delta_2}}\right).
\end{aligned}$$

Time preference parameter value cannot vary and hence fails the SEP Definition.

Extension of Two Period KPS Demand Properties

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Then in each period t, optimal asset ratios $n_f\left(s^t\right) / n_j\left(s^t\right) = \eta_j\left(s^t\right)$ are same for KP and DOCE preferences and are independent of δ_1 and β.

Concluding Comments

- One key insight: in intertemporal demand problems TC behavior does not depend just on preferences, but prices (or asset returns) can also play a crucial role. This differs from typical decision theoretic analyses (KP and Johnsen and Donaldson 1985) where the conditions for TC are implicitly assumed to hold for all prices. ([ICER] and [IR] have been shown to essentially be restrictions on return distributions or, for complete markets, on contingent claim prices.)

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- It would be interesting to consider more generally when joint restrictions on preferences and prices such that dynamic choice behavior is time consistent.
- For instance, assuming a simplified tree structure and $\beta R_{f3} = 1$ (and no restrictions on risky returns), a consumer with CES time and CARA risk preference DOCE utility satisfies TC (and SEP and TRI).

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