Identification at the Zero Lower Bound

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Motivation

- The ZLB is a challenge for conduct and analysis of macroeconomic policy:
    - Liquidity trap
    - Solving models more complicated
    - Econometric issues due to censoring
- So, ZLB generally viewed as a *nuisance/problem*
- I propose to view this as an *opportunity* to learn about the causal effects of policy!

Main insight

- When policy hits bound, agents' decision rules (may) change
- This change generates additional variation that can be used to identify causal effect of policy
- Intuition: behavior of economy across ZLB and non-ZLB regimes differs only due to impact of policy
  - If policy shuts down at ZLB (no unconventional policy), we identify the impact of (conventional) policy
  - If unconv pol is partially effective, we partially identify causal effects of policy and relative efficacy of conv v unconv policy
  - If unconv pol is fully effective (ZLB irrelevance), we get no identification

What the paper does

1. Shows that occasionally binding constraints identify causal effects

   - If constraint fully (partially) binding, we get point (set) identification
   - Similar to identification via heteroskedasticity (Rigobon, 2003) or via structural change (Magnusson and Mavroeidis, 2014)
   - But change is endogenous – so different methods needed

2. Develops methodology for identification and estimation of SVARs with occasionally binding constraint

   - Method provides formal tests of "empirical (ir)relevance of the ZLB" (Debortoli et al 2019)
   - If empirical irrelevance is assumed, method can estimate SVARs with conventional identification schemes

Related literature

Long literature... Most closely related papers:

- Macro: Wu and Xia (2016), Hayashi and Koeda (2019), Wu and Zhang (2019), Aruoba et al (2020)
- Micro: Amemiya (1974), Nelson and Olson (1978), Gourieroux Laffont and Monfort (1980), Blundell and Smith (1994)
- None of the above points out implications for identification

Outline

1 Identification

2 Estimation

3 Application

Simultaneous equations model

- Toy monetary policy model

$$\pi_t = c + \beta\left(r_t - r^n\right) + \varepsilon_{1t}$$
$$r_t^* = r^n + \gamma\pi_t + \varepsilon_{2t}$$
$$r_t = \max\left(r_t^*, 0\right)$$

- π: inflation, r: policy rate, ε_1 inflation shock, ε_2 policy shock
- Without ZLB constraint $(r_t = r_t^*)$, model is *under-identified*
- 'Shadow rate' r_t^* represents desired policy stance

Add unconventional policy channel

- Model of bond mkt segmentation (Chen et al 2012, Ikeda et al 2020)

$$\pi_t = c + \beta \left(r_t - r^n + \varphi b_{L,t} \right) + \varepsilon_{1t}$$

- QE reduces $b_{L,t}$, amount of long-term bonds held by private sector
- φ depends on fraction of restricted households and elasticity of term premium wrt long-term bond holdings
- If $b_{L,t} = \min \left(\alpha r_t^*, 0 \right)$, $\alpha \geq 0$, $r_t^* = r^n + \gamma \pi_t$ (Taylor rule), then

$$\pi_t = c + \beta \left(r_t - r^n \right) + \underbrace{\beta^*}_{\lambda\beta} \min \left(r_t^*, 0 \right) + \varepsilon_{1t}$$

Solution of the model/decision rules

- If $(1-\gamma\beta)\,/\,(1-\lambda\gamma\beta) > 0$ (coherency), the solution for π_t is

$$\pi_t = \begin{cases} \frac{c}{1-\gamma\beta} + \frac{\varepsilon_{1t}+\beta\varepsilon_{2t}}{1-\gamma\beta}, & \text{if } r_t > 0 \\ \frac{c-r^n\beta(1-\lambda)}{1-\lambda\gamma\beta} + \frac{\varepsilon_t+\lambda\beta\varepsilon_{2t}}{1-\lambda\gamma\beta} & \text{if } r_t = 0 \end{cases}$$

- This is piecewise linear and continuous at the kink
- Behaviour changes across regimes if $\lambda \neq 1$

Reduced form

- The reduced-form can be written as

$$\pi_t = \mu_1 + u_{1t} - \widetilde{\beta} D_t \left(\mu_2 + u_{2t}\right), \;\; D_t := 1_{\{r_t = 0\}} \tag{1}$$

$$r_t = \max\left(\mu_2 + u_{2t}, 0\right), \quad \text{where} \tag{2}$$

$$\widetilde{\beta} = \frac{(1-\lambda)\,\beta}{1-\lambda\beta\gamma}, \quad u_{1t} = \frac{\varepsilon_{1t} + \beta\varepsilon_{2t}}{1-\gamma\beta}, \quad u_{2t} = \frac{\gamma\varepsilon_{1t} + \varepsilon_{2t}}{1-\gamma\beta}$$

- Reduced form parameter $\widetilde{\beta}$ is identified
  - (1) is 'incidentally kinked' regression, estimable by variant of Heckit
- This gives extra bit of information about structural parameters

Point identification when constraint is fully binding

- With Gaussian errors, β estimable via control function approach

$$E(\pi_t | r_t) = c + \beta(r_t - r^n) + \rho h_t(\mu_2, \omega_{22}),$$

$$h_t(\mu_2, \omega_{22}) = (1 - D_t)(r_t - \mu_2) - D_t \frac{\sqrt{\omega_{22}}\phi(a)}{\Phi(a)}, \quad a = -\frac{\mu_2}{\sqrt{\omega_{22}}}$$

- γ can then be obtained from orthogonality of errors $E(\varepsilon_{1t}\varepsilon_{2t}) = 0$

$$\gamma = \frac{\omega_{12} - \omega_{22}\beta}{\omega_{11} - \omega_{12}\beta}, \quad \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{pmatrix} = var(u_t)$$

Partial identification when constraint is partially binding

- We've established that $\widetilde{\beta} = \frac{(1-\lambda)\beta}{1-\lambda\beta\gamma}$ is identified
- Without further restrictions, β is not point-identified
- BUT we get bounds on β (and λ) by solving

$$\widetilde{\beta} = \frac{(1-\lambda)\,\beta}{1-\lambda\beta\frac{\omega_{12}-\omega_{22}\beta}{\omega_{11}-\omega_{12}\beta}}$$

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SVAR with occasionally binding constraint

- $Y_t = (Y'_{1t}, Y_{2t})'$ vector of $k = k_1 + k_2$ endogenous variables
- Y_{1t} are k_1 unconstrained variables (e.g., inflation, unemployment)
- Y_{2t} are k_2 variables subject to occ bin constraint (e.g., policy rate)
- *Censored and Kinked* SVAR (CKSVAR): includes contemporaneous and lagged values of both observed Y_{2t} and latent (shadow rate) Y^*_{2t}
  - Important to capture forward-guidance, e.g., a la Reifschneider & Williams (2000) or Debortoli et al (2019)
- Special cases:
  1. *Kinked* SVAR (KSVAR): no lags of latent variable Y^*_{2t}
  2. *Censored* SVAR (CSVAR): linear SVAR in (Y_{1t}, Y^*_{2t})

CKSVAR(p)

Ignoring deterministic terms and external variables:

$$A_{11}Y_{1t} + A_{12}Y_{2t} + A_{12}^{*}Y_{2t}^{*} = B_{11}X_{1t} + \sum_{j=1}^{p} B_{12,j}Y_{2,t-j} + \sum_{j=1}^{p} B_{1,j}^{*}Y_{2,t-j}^{*} + \varepsilon_{1t},$$

$$A_{22}^{*}Y_{2t}^{*} + A_{22}Y_{2t} + A_{21}Y_{1t} = B_{21}X_{1t} + \sum_{j=1}^{p} B_{22,j}Y_{2,t-j} + \sum_{j=1}^{p} B_{2,j}^{*}Y_{2,t-j}^{*} + \varepsilon_{2t},$$

$$Y_{2t} = \max\left(Y_{2t}^{*}, b_t\right),$$

X_{1t} includes lags of Y_{1t}, and the lower bound b_t is observed

Reparametrization

$$Y_{1t} = \overbrace{-A_{11}^{-1}\left(A_{12} + A_{12}^{*}\right)}^{\beta} Y_{2t} + \overbrace{\left(-A_{11}^{-1}A_{12}^{*}\right)}^{\beta^{*}=\lambda\beta} D_t Y_{2t}^{*} + \overbrace{A_{11}^{-1}\varepsilon_{1t}}^{\bar{\varepsilon}_{1t}} + lags,$$

$$Y_{2t}^{*} = \begin{cases} \overbrace{-\left(A_{22}^{*} + A_{22}\right)^{-1} A_{21}}^{\gamma} Y_{1t} + \overbrace{\left(A_{22}^{*} + A_{22}\right)^{-1} \varepsilon_{2t}}^{\bar{\varepsilon}_{2t}} + lags, & \text{if } D_t = 0 \\ \underbrace{-\left(A_{22}^{*}\right)^{-1} A_{21}}_{\gamma^{*}=\zeta\gamma} Y_{1t} + \underbrace{\left(A_{22}^{*}\right)^{-1} \varepsilon_{2t}}_{\zeta\bar{\varepsilon}_{2t}} + lags, & \text{if } D_t = 1 \end{cases}$$

- $\zeta = \left(A_{22}^{*} + A_{22}\right) / A_{22}^{*}$ allows policy response to vary across regimes
- Coherency condition: $\zeta\left(1 - \gamma\beta\right)\left(1 - \gamma^{*}\beta^{*}\right) > 0$

Reduced form

Proposition 2: If CC holds, the reduced-form of CKSVAR for $t \geq 1$ is

$$Y_{1t} = \overline{C}_1 X_t + \overline{C}_1^* \overline{X}_t^* + u_{1t} - \widetilde{\beta} D_t \left(\overline{C}_2 X_t + \overline{C}_2^* \overline{X}_t^* + u_{2t} - b_t \right) \quad (1)$$

$$Y_{2t} = \max\left(\overline{Y}_{2t}^*, b_t \right), \qquad D := 1_{\{Y_{2t} = b_t\}}$$

$$\overline{Y}_{2t}^* = \overline{C}_2 X_t + \overline{C}_2^* \overline{X}_t^* + u_{2t} \quad (2)$$

$$Y_{2t}^* = (1 - D_t) \overline{Y}_{2t}^* + D_t \left(\kappa \overline{Y}_{2t}^* + (1 - \kappa) b_t \right) \quad (3)$$

where $u_t \sim N(0, \Omega)$, $\overline{Y}_{2t}^*$: reduced-form shadow rate, $\overline{X}_t^*$: lags of $D_t \overline{Y}_{2t}^*$, $\kappa = \zeta (1 - \gamma\beta) / (1 - \zeta\gamma\beta^*)$ (unidentified) and

$$\widetilde{\beta} = \left(A_{11} - A_{12}^* A_{22}^{*-1} A_{21} \right)^{-1} \left(A_{12}^* A_{22}^{*-1} A_{22} - A_{12} \right) \quad (4)$$

Identification of structural parameters

- Reduced-form parameters (including k_1 coefficients on kink $\widetilde{\beta}$) are identified iff $0 < \Pr(D_t = 1) < 1$
- If policy is completely ineffective at the ZLB, monetary policy shock ε_{2t} and its IRF are point-identified!
- Otherwise, we get bounds on β, γ and $\xi = \lambda\zeta$, by collecting all solutions to

$$\widetilde{\beta} = (1 - \xi)(I - \xi\beta\gamma)^{-1}\beta, \text{ and}$$
$$\gamma = (\Omega_{12}' - \Omega_{22}\beta')(\Omega_{11} - \Omega_{12}\beta')^{-1}$$

- Note that $\xi = \lambda\zeta$, so λ and ζ are not separately identified

Likelihood

- Gaussian likelihood computed by simulation (except for KSVAR) using two alternative algorithms developed in the paper:
  - SIS: Sequential Importance Sampling (Lee 1999)
  - FAPF: Fully Adapted Particle Filter (Pitt and Shephard 1999)
- Ox and Python code for frequentist estimation available from my website
- Bayesian implementation in follow-up paper

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Three-equation SVAR of Stock and Watson (2001)

- Intended purely as an empirical illustration of the method
- SVAR(4) in inflation (GDP deflator), unemployment rate and Federal funds rate
- Sample: 1960q1-2018q2,
- For a serious application, see Ikeda Li Mavroeidis and Zanetti (2020) "Testing the effectiveness of unconventional monetary policy in Japan and the USA"

Tests of efficacy of unconventional policy

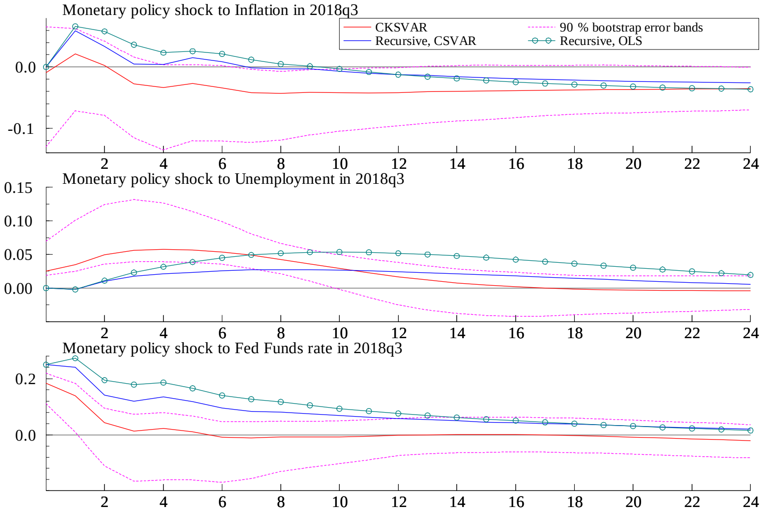
Model	log lik (SIS)	log lik (FAPF)	# of restr.	LR stat.	Asym. p-val.	Boot. p-val.
CKSVAR(4)	-81.64	-81.94				
KSVAR(4)	-97.05	-	12	30.82	0.002	0.011
CSVAR(4)	-94.86	-94.87	14	26.43	0.023	0.117

Sample: 1960q1-2018q2 (234 obs, 11% at ZLB)

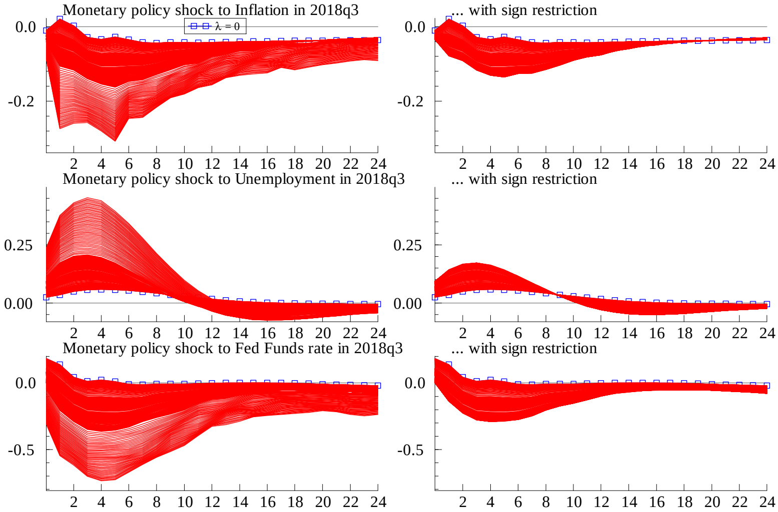
Estimated using 1000 particles, 999 bootstrap replications

- Reject hypothesis that UMP is completely ineffective (KSVAR) more strongly than the (opposite) hypothesis that ZLB is empirically irrelevant (CSVAR)
- BUT: simulations show LR test of latter is less powerful

IRFs


Monetary policy shock to Inflation in 2018q3
CKSVAR
Recursive, CSVAR
90 % bootstrap error bands
Recursive, OLS
0.0
-0.1
2
4
6
8
10
12
14
16
18
20
22
24
Monetary policy shock to Unemployment in 2018q3
0.15
0.10
0.05
0.00
Monetary policy shock to Fed Funds rate in 2018q3
0.2
0.0

Partially identified IRFs



Conclusions

- I developed a method for estimating SVARs with occasionally binding constraints
- This applies to models with a zero-lower bound constraint on policy rates
- I showed that the ZLB (partially) identifies the causal effects of policy
- The method can also be used to formally test whether ZLB is empirically irrelevant