Efficiency or resiliency?
Corporate choice between operational and financial hedging
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Abstract

We study the corporate choice between financial efficiency and operational resilience. Firms substitute between saving cash for financial hedging, which mitigates the risk of financial default, and spending on operational hedging which mitigates the risk of operational default and failure to deliver on their obligations to customers. This tradeoff is particularly strong for financially constrained firms and is reflected in a positive correlation between operational spread (markup) and financial leverage or credit risk. We present empirical evidence supporting this correlation, the effect being stronger for constrained firms.

KEYWORDS: FINANCIAL DEFAULT, OPERATIONAL DEFAULT, LIQUIDITY, FINANCIAL CONSTRAINTS, RISK MANAGEMENT

JEL: G31, G32, G33

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1. Introduction

The Covid-19 crisis has raised the issue of corporate resilience to shocks following disruptions in supply chains which adversely affect operations. Companies tackle such negative supply-chain shocks by operationally hedging against them. This includes diversifying the supply chains by allocating resources to increase the pool of suppliers and shifting some of them to nearby, more secure locations; maintaining backup capacity; and, holding excess inventory. In essence, companies endure a higher cost of production — through holding spare capacity and excess inventory, and rearranging their supply chains — in order to mitigate the risk of operational disruption.

A global survey by Institute for Supply Management finds that by the end of May 2020, 97% of organizations reported that they would be or had already been impacted by coronavirus-induced supply-chain disruptions.[1] Consequently, U.S. manufacturing is operating at 74% of normal capacity while in Europe capacity is at 64%. The survey also finds that while firms in North America report that operations have or are likely to have inventory to support current operations, confidence has declined to 64% in the U.S., 49% in Mexico and 55% in Canada. In Japan and Korea too, many firms are not confident that they will have sufficient inventory for Q4; and, almost one-half of the firms are holding inventory more than usual. In response, 29% of organizations are planning or have begun to re-shore or near-shore some or most operations.[2] However, such operational resiliency is not being favored by all firms as several corporate chief executive officers (CEOs) and investors contend that

[1] `https://www.prnewswire.com/news-releases/covid-19-survey-round-3-supply-chain-disruptions-continue-globally-301096403.html`. See also “Businesses are proving quite resilient to the pandemic”, The Economist, May 16th 2020, and “From ‘just in time’ to ‘just in case’”, Financial Times, May 4th 2020.

[2] “Reshoring” and “nearshoring” is the process of bringing the manufacturing of goods to the firm’s country or a country nearby, respectively.

operational hedging is costly and occurs at the cost of financial efficiency.[3]

Our paper studies one aspect of the tension between operation resiliency and financial efficiency, viz., the tradeoff between the firm's allocation of cash to operational hedging and to the prevention of financial distress. While operational hedging may be beneficial on its own, it may compete for resources with the firm's demand for financial hedging. The need to optimally balance these two hedging needs — operational hedging and financial hedging — can help explain the lack of operational resilience in some firms.

In our theoretical setting, a levered company faces two important risks. First, it faces a risk of financial default, because cash flows from assets in place are risky. Second, the firm faces operational risk due to an existing commitment to deliver goods to costumers. The two risks — financial default and operational default — are possibly related. For example, an aggregate shock may affect both the firm's cash flows, possibly enough to induce financial default, and the firm's suppliers, who may be unable to deliver to the firm, causing the firm to default on its contract to deliver goods to its customers. Both financial and operational defaults lead to a loss in the franchise value of the firm.

The firm can use its cash inflow to build up cash buffers and mitigate the risk of financial default. The firm can also use the cash inflow to increase the likelihood that it will deliver on its promise to customers by allocating resources to operational hedging that includes greater expenses on supply chains, maintaining backup capacity, and holding excess inventory. Naturally, such operational hedging raises the firm's cost of production or reduces its operational spread, viz., the "markup" or the price-to-cost margin per unit. Even an unlevered firm will in general optimally choose an interior level of operational hedging in order to protect its profitability while recognizing that an operational default leads to a loss of its franchise value.

[3] https://www.ft.com/content/4ee0817a-809f-11ea-b0fb-13524ae1056b

Because operational hedging reduces the risk of delivering to the firm's customers, it can potentially also reduce the risk of financial default by increasing the firm's capacity to raise financing against future cash flows. However, this is feasible only if the firm can pledge the benefits of operational hedging to outside investors. For firms that are financially constrained, such pledgeability may be low; in turn, financial and operating hedging become substitutes: the firm must decide between using cash to mitigate the risk of financial default, or spending cash on contracting with higher-cost suppliers, holding excess inventory, or maintaining spare capacity.

Our principal theoretical result is that for a financially constrained firm, the optimal amount of operational hedging decreases with the credit spread which is increasing in financial default risk. Operational hedging also reduces the operational spread as it increases firm's cost of production. In other words, the firm optimally sacrifices operational resiliency for financial efficiency. This creates a negative relation between the credit spread and the operational spread. More financial hedging that reduces the credit spread also reduces operational hedging and this is reflected in a wider operational spread. Similarly, our model also predicts that higher existing leverage is associated with a wider operational spread. This positive relation between leverage and operational spread is also pronounced for financially constrained firms since unconstrained firms can engage in operational hedging and simultaneously pledge superior operating cash flows to avoid default should there be a shortfall in cash.

We provide empirical tests of our model's prediction on the tradeoff between operational hedging and credit risk, or specifically, between the operational spread (or markup) and financial leverage or measures of credit risk, and relate that to firms' financially constraint.

We start by documenting that markup or operational spread is correlated with proxies for operational hedging in the expected way: higher inventory and greater supply chain diversification reduce the operational spread. This supports the use of operational spread as a summary measure of the extent of operational hedging that the firm engages in. We then examine whether leverage and credit risk are correlated with the operational spread in the way predicted by our model. We find that higher leverage and higher credit risk, measured using Altman's z-score, which necessitate allocation of cash to financial hedging, are positively related to the operational spread or markup, implying a reduction in operational hedging. To gauge the economic significance of the effect, one standard deviation increase in the firm's negative z-score raises the firm's markup by 0.04 standard deviation, or 13% of the sample median markup. We find that the positive relation between the operational spread and leverage is stronger for the short-term portion of the long-term debt which matures in the next two years. Higher short-term portion of the long-term debt raises the operational leverage about twice as strongly as does long-term debt. This is consistent with our model by which the near-term need to avert financial default diverts funds from longer-term operational hedging, and this is reflected in a wider operational spread when there is more short-term debt due. Notably, the near-term of debt is exogenous to the current state of the firm, having been determined in the past when the long-term debt was issued.

We conduct two tests to address concerns about the usefulness of measures of financial constraints and about the endogeneity of leverage. First, we exploit an exogenous shock to credit supply to firms following Chodorow-Reich (2014), who studies the negative impact of the subprime mortgage crisis and Lehman Brothers' collapse on lenders' abilities to extend credit to borrowers. A firms's exposure to this shock, in terms of its relationship banks being affected by the shock, reflects a tightening of its financing constraint. We find that exposed firms that were more highly levered prior to the crisis reduced operational hedging by more than less exposed firms. This test uses time-series variation in financing constraints to measure our predicted tension between operational hedging and financial hedging. This test also helps address concerns about the endogeneity of leverage in that existing literature on the impact of the financial crisis has shown that pre-crisis leverage is an important determinant of real effects post-crisis through a liquidity channel (e.g., Giroud and Mueller, 2016)

Second, we exploit an exogenous increase in the inducement to raise debt following Heider and Ljungqvist (2015) who find that increases in state taxes induced increases in leverage of the exposed firms. By our model, these firms should reduce operational hedging, reflected in widening of their operational spread or markup. This test can help identify the effect of leverage on operational hedging under the reasonable assumption that changes in state taxes do not directly affect operational hedging. We find that an increase in state tax was associated with an increase in the operational spread which indicates a reduction in expenses on operational hedging. We further test our model's prediction that leverage more strongly affects the operational spread in financially-constrained firms. We find that the positive effect of increased state taxes on operational leverage prevails for constrained firms, using various common measures of financial constraint.

Our paper is related to studies of the real effects of financing constraints (see Stein (2003) for a review) which show that financing frictions can affect investment decisions and employment (Lemmon and Roberts, 2010; Duchin et al., 2010; Almeida et al., 2012; Giroud and Mueller, 2016). The literature also studies the effect of financial constraints and financial distress on financial policies such as cash, credit lines, and risk management (e.g., Almeida et al., 2004; Sufi, 2009; Bolton et al., 2011; Acharya et al., 2012). Our paper also relates closely to those of Rampini and Viswanathan (2010) and Rampini et al. (2014), who show that more constrained or distressed firms may reduce their engagement in risk management in order to preserve debt capacity for investment and other current expenditures. These studies focus on financial hedging through derivatives while our focus is on operational hedging. Our paper also relates to Froot et al. (1993), who propose a theory for the rationale for corporate hedging. In Froot et al. (1993), hedging against cash shortfalls helps the firm mitigate the risk of not being able to finance valuable investment opportunities. In our model, however, operational hedging is not a means to avoid financing shortfall but it is rather the other way around: A shortfall of cash that presents a financial default risk reduces the resources allocated to operational hedging.

Studies of the relationship between a firm's liquidity position and its markup (e.g., Gilchrist et al., 2017; Dou and Ji, 2020) propose that financially constrained firms that need to increase short-term profits may raise their product price and thus their markup. Such an ability to extract higher profit by raising prices implicitly assumes market power. Our analysis controls for market power which is known to be positively associated with the firm's markup. We find that the effect of market power on the operational spread is larger for constrained firms. Yet we find that our model's predicted positive association between operational spread and leverage or credit risk persists after controlling for market power.

Broadly speaking, our contribution in this paper is to study both theoretically and empirically the determinants of operational hedging and its tradeoff with financial hedging, especially for financially-constrained firms. To our best knowledge, the positive relationship between operational spread (markup) and financial leverage or credit risk has not been documented in the literature.

2. The model

2.1 Model setup

This section develops a model of a levered firm's optimal operational hedging policy in the presence of costly financial default (default on debt service) and operational default (default on the supplier contract). Our main goal is to show that the firm faces a tension between operational hedging and financial hedging, where we model financial hedging as the firm saving cash in order to avoid default on its debt maturing before the settlement date of supply contracts with its customers.

Our model introduces operational hedging in the setting of Acharya et al. (2012) who study the impact of credit risk on the firm' s cash holding. The model features a single-levered firm with existing debt F in a three-period economy: $t = 0, 1, 2$. The firm has assets in place that generate a cash flow x_t at each period $t = 0, 1$. x_2 represents the franchise or the continuation value. Additionally, the firm has an outstanding supplier contract that stipulates a delivery of I units of goods at unit price p at $t = 2$. Our goal is to analyze the tension between the firm's cash holding and operational hedging decisions to avoid financial and operational defaults, respectively.

For our purpose, it is important to introduce a random shock u that affects both the firm's cash flow at $t = 1$ and its capacity to fulfill the supplier contract. Specifically, the firm's cash flow from assets in place at $t = 1$ is given by $x_1 = \bar{x}_1 + u$, and its production capacity is reduced from I to $(1 - \delta(u))I$, where $\delta(u)$ is decreasing and convex in u with continuous and finite first and second order derivatives. The probability distribution of u is given by the density function $g(u)$ with support $[0, \infty)$, the associated cumulative distribution function being $G(u)$ and the hazard function $h(u)$ being defined as

$$h(u) = \frac{g(u)}{1 - G(u)} . \tag{2.1}$$

To derive analytical results, we assume that u is exponentially distributed on $[0, \infty)$ with density function $g(u) = \alpha e^{-\alpha u}$. Then the cumulative distribution function $G(u) = 1 - e^{-\alpha u}$ and the hazard function $h(u)$ is a constant α. Figure 1 illustrates the timeline of the model.

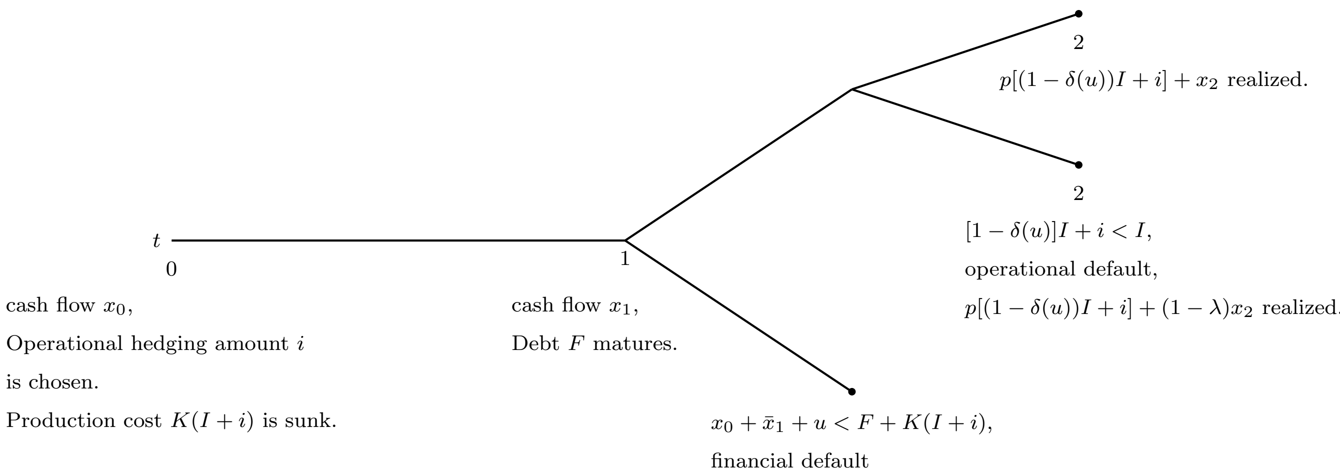

Figure 1: The timeline of the model

At date $t = 0$, the assets in place generate a positive cash flow $x_0 > 0$. At this time, the firm starts producing I units of goods scheduled for delivery at $t = 2$. Moreover, the firm can choose to hedge the operational risk by investing in excess inventory i, resulting in the total units of delivered goods being $(1 - \delta(u))I + i$. Note that i can also be interpreted as spare production capacity. The cost of the production and operational hedging is summarized by an increasing and convex cost function $K(I + i)$ with continuous and finite first and second order derivatives. We assume that the firm is a price-taker in its supplier contracts. We also make the following assumption regarding the unit product price p and the marginal cost of production commitment I:

Assumption 2.1.

$$p > K'(I) \ . \tag{2.2}$$

Assumption 2.1 says that the firm enjoys some positive markup over the marginal cost of the production commitment. Since the production cost function K is convex, this assumption says that the firm can potentially choose a positive hedging policy i and still enjoy a positive marginal profit from the supplier contract. Market frictions preclude the firm from accessing outside financing, so that the firm's disposable cash at date-0 comes entirely from its internal cash flow. Thus, the cash reserve is $c = x_0 - K(I + i)$.

At date $t = 1$, the firm must make a debt payment of F, which is assumed to be predetermined (a legacy of the past). We assume that debt cannot be renegotiated due to high bargaining costs; for example, it might be held by dispersed bondholders prone to coordination problems. Failure to repay the debt in full at $t = 1$ results in financial default and liquidation, in which case future cash flow from the contractual delivery investment, $p[(1 - \delta(u)) + i]$, and franchise value, x_2, are lost. Since the period-1 cash flow, x_1, is random, there is no assurance that the firm has enough liquidity to repay the debt in full. Moreover, failure to deliver I units of goods results in operational default, also leading to a loss of the franchise value, x_2, by a portion $\lambda \in [0, 1)$. This can be interpreted as, for example, a loss of reputation with some of its customers who can switch to alternate suppliers.

Due to market frictions, external financing is unavailable also at $t = 1$, and hence the debt payment must be made out of the firm's internal funds. The financing friction gives rise to a tension between financial hedging versus operational hedging decisions at $t = 0$: On the one hand, the firm has incentive to spend on excess inventory i, to hedge against the operational shock and reduce the probability of operational default; on the other hand, such operational hedging worsens its liquidity position between periods 0 and 1, thereby increasing the probability of a future cash shortfall and financial default.

2.2 Discussion

Before proceeding further, we want to stress that the exact specification of the model can vary widely without affecting the results qualitatively, as long as two assumptions are satisfied. First, default involves deadweight costs. Although we assume that all future cash flows are lost in default, an extension to a partial loss is straightforward. Second, external financing cannot be raised against the full income from the supplier contract settlement at date-2. If the firm can pledge a large enough fraction of its income from fulfilling the supplier contract as collateral, then current and future cash holdings can be viewed as time substitutes, and there is no role for precautionary savings of cash. As a result, the tension between financial hedging and operational hedging breaks down. In reality, the condition of partial pledgeability is likely to be universally met. While the base case model assumes that external financing is prohibited, Section 3.1 extends the model by allowing the firm to borrow up to a certain fraction τ of its cash flow from contract settlement at $t = 2$, and shows that our main results hold as long as financing constraints are sufficiently binding, i.e., τ is sufficiently small.

A related feature in our model is that the outstanding debt matures before the supplier contract settlement date, giving rise to a maturity mismatch between debt contract and supplier contract. Effectively, the cash flow from the supplier contract can neither be fully pledged nor used as cash to cover debt obligations. In practice, supplier contracts often stipulate a considerable elapsed time from initiation to settlement, especially in the case of durable goods industry. Meanwhile, firms often have to borrow short-term debt or draw down their credit lines to finance their working capital needs during the production process. What is crucial here is that the shock at $t = 1$ can be severe enough to make the cash flow at date-1 fall short of the debt contract obligation and make the production capacity fall short of the supplier contract obligation. This creates the needs for both financial and operational hedging. The two needs compete with each other in our model. Although we assume a single uncertainty state affects both the cash flows from assets in place and the ability to fulfill the supplier contract, extending our model to different sources of uncertainties is possible.

In general, in addition to saving and investing, firms can also distribute some of the cash to their shareholders. Fixed, pre-committed dividends at $t = 0$ amount to a reduction in the net cash flow x_0, and can be easily incorporated in the model. Although modeling an optimal dividend policy at $t = 0$ would complicate the analysis considerably by introducing a second choice variable, the intuition on the effect of optimal dividends is as follows. Most firms in the model would choose not to pay any dividend at either date-0 or date-1. This is because the cash can be saved to avoid costly financial bankruptcy, or it can be invested in inventory or spare production capacity to weather operational disruptions. However, if the firm is very risky, meaning the shock at $t = 1$ is sufficiently detrimental to the firm's cash flow x_1 and production capacity, the precautionary motive for saving cash and investment in operational hedging can be dominated by the incentives to engage in asset substitution, i.e., to pay out a large immediate dividend at the expense of making the firm even riskier (Jensen and Meckling, 1976). In our base-case model, we assume that in order to prevent such behavior, discretionary dividends are prohibited by debt covenants.

Moreover, if the firm has access to external capital at $t = 0$, it can choose to raise additional capital at that time to invest in operational hedging and/or cash holdings. Selling equity can be viewed as making a negative dividend payment. By the same logic as above, in our model a firm may find it desirable to raise equity, as long as the marginal value of an additional dollar of operational hedging is greater than one. However, if the firm is very risky, instead of contributing the proceeds from additional equity issuance to the operational hedging, shareholders would have incentives to pay themselves a dividend and not engage in operational risk management.

Finally, shareholders may find it optimal to raise debt maturing at $t = 1$ that is more senior than the existing debt and invest in operational hedging. This occurs when sensitivity of function $\delta(u)$ with respect to u and the cost of operational default (captured by λ) are both sufficiently high.[4] In our base case model, we assume that financing constraints at $t = 0$

[4] By contrast, as in Acharya et al. (2012), raising debt maturing at $t = 1$ solely to increase the cash reserve is value-neutral in this setting, as the increase in cash is exactly offset by the increase in the required debt repayment (i.e., cash is negative debt in this setting of "short-term" debt).

preclude the firm from accessing any additional financing. Explicitly modeling endogenous capital structure polices at date-0 is an interesting extension of our model. In Section 4.2, we allow for tax shields of debt and numerically solve for the firm's optimal debt policy (F) at date-0 as a function of the pledgeability of its cash flow and the corporate tax rate.

2.3 Optimal hedging policies

In general, the firm has a positive amount of existing debt ($F > 0$). At date 0, the firm faces the following trade-off between investing its cash in the operational hedging and retaining it until the next period. The firm's optimal hedging policies depend on the relative likelihood of financial default and operational default, as will be shown below.

The amount of cash available for debt service at date 1 is $x_0 - K(I+i) + x_1$, where $x_0 - K(I+i)$ is the cash reserve and $x_1 = \bar{x}_1 + u$ is the interim-period cash flow from assets. The "financial default boundary", u_F, is the minimum cash flow shock that allows the firm to repay F in full and avoid default:

$$\begin{aligned} u_F &= F + K(I+i) - x_0 - \bar{x}_1 \\ &= \bar{F} + K(I+i) \ , \end{aligned} \tag{2.3}$$

where $\bar{F} = F - x_0 - \bar{x}_1$ is the net debt, i.e., debt minus date 0 and 1 predictable cash flows. The financial default boundary u_F increases with the level of net debt ($\bar{F}$) and operational hedging amount (i). For all realizations of u between 0 and u_F, the firm defaults on its debt contract and equity holders are left with nothing.

We also allow the firm to default on the supplier contract. The amount of goods that the firm can deliver at date-2 is $(1 - \delta(u))I + i$. If this amount is less than the production commitment I, the firm defaults on the supplier contract. Correspondingly, the "operational default boundary", u_O, is the minimum shock that allows the firm to deliver its contractual amount of goods in full and avoid operational default:

$$(1 - \delta(u_O))I + i = I, \text{ or}$$
$$u_O = \delta^{-1}\left(\frac{i}{I}\right) \ . \tag{2.4}$$

Since the loss function δ is decreasing in u, its inverse function δ^{-1} is decreasing in i. This means that the operational default boundary u_O is decreasing with i, the amount of operational hedging the firm chose at date-0. In this sense, operational hedging reduces the operational default risk. For all realizations of u between 0 and u_O, the firm defaults on its supplier contract and equity holders lose a portion λ of the franchise value x_2.

Define $D(i, \bar{F})$ as the difference between financial and operational default thresholds for given net debt level $\bar{F}$ and operational hedging policy i:

$$D(i, \bar{F}) \equiv u_F - u_O = \bar{F} + K(I + i) - \delta^{-1}\left(\frac{i}{I}\right) \ . \tag{2.5}$$

$D(i, \bar{F})$ is continuously differentiable in both i and $\bar{F}$ with partial derivatives:

$$\frac{\partial D}{\partial i} = K'(I + i) - \frac{1}{I\delta'(u_O)} \ , \tag{2.6a}$$
$$\frac{\partial D}{\partial \bar{F}} = 1 \ . \tag{2.6b}$$

Note that $\frac{\partial D}{\partial i} > 0$ because $K'(I + i) > 0$ and $\delta'(u) < 0$ by assumption.

2.3.1 Benchmark: Optimal hedging policy when $F = 0$

Consider first a benchmark case when the debt level $F = 0$. In this case, financial default is irrelevant: $u_F = 0$. In this case, the firm will choose the hedging policy $\bar{i}$ that maximizes the unlevered date-0 equity value:

$$\bar{E} = \int_0^{\infty} \Big[x_0 - K(I+i) + \bar{x}_1 + u + p\left[(1-\delta(u))I + i\right] + x_2\Big] g(u)du - \int_0^{u_O} \lambda x_2 g(u)du \ . \quad (2.7)$$

The last term of Equation (2.7) reflects the proportional loss of franchise value in case of operational default. The first-order condition is

$$\frac{\partial \bar{E}}{\partial i} = p - K'(I+i) - \lambda x_2 \frac{g(u_O)}{I\delta'(u_O)} = 0 \ , \quad (2.8)$$

where $u_O = \delta^{-1}\left(\frac{i}{I}\right)$. Define $\bar{i}$ being the solution for the first-order condition (2.8). In Appendix A.1, we show that $\bar{i}$ is also the unique optimal hedging level that maximizes the equity value (2.7), under mild technical conditions.

The following assumption ensures that the firm has enough cash flow at date-0 to choose the highest possible optimal operational hedging, which occurs when the firm is debt-free. It also ensures that u_F is continuous in $\bar{F}$ and $u_F = 0$ for sufficiently small $\bar{F}$:

Assumption 2.2.

$$K(I+\bar{i}) < x_0 + x_1 \ . \quad (2.9)$$

Since $D(i, \bar{F})$ is continuous in $\bar{F}$, u_F is always smaller than u_0 regardless of the value of i for sufficiently small $\bar{F}$.

As will be clear later, operational default boundary u_O only enters into equity value function if it is larger than the financial default boundary u_F. Thus, the main challenge in solving the model is that both u_F and u_O are endogenously determined by the firm's hedging policy. In what follows, we first solve for the firm's optimal hedging policy that maximizes the equity value; then we characterize the relationship between the hedging policy and the net debt level.[5] We do this in steps by considering the relative position of thresholds for financial and operational defaults, u_F and u_O, respectively, and then addressing its endogeneity to hedging policy and model primitives (such as leverage).

2.4 Optimal hedging policy when $u_F \geq u_O$

If the firm's inherited debt level is so high that the financial default boundary is greater than the operational default boundary, then the firm would have already declared financial default at date-1 for the shock values that would trigger the operational default. Thus, operational default boundary does not enter the equity value function in this case. The total payoff to equity holders is the sum of cash flows from assets in place and the payoff from the contractual fulfillment to customers, less the production cost, the operational hedging cost and the debt repayment, provided that the firm does not default on its debt in the interim. The market value of equity is therefore given as:

$$\begin{aligned} E &= \int_{u_F}^{\infty} \Big[x_0 - K(I+i) + \bar{x}_1 + u + p\left[(1-\delta(u))I + i\right] + x_2 - F\Big] g(u)du \\ &= \int_{u_F}^{\infty} \Big[u + p\left[(1-\delta(u))I + i\right] + x_2 - K(I+i) - \bar{F}\Big] g(u)du \\ &= \int_{u_F}^{\infty} \Big[u - u_F + p\left[(1-\delta(u))I + i\right] + x_2\Big] g(u)du \ , \end{aligned} \tag{2.10}$$

where the last equality is from the definition of financial default boundary u_F in (2.3). Here,$(u - u_F)$ is the amount of cash left in the firm after debt F is repaid, and $p\left[(1-\delta(u))I + i\right] +$

[5] It is straightforward to consider hedging being undertaken by a manager who maximizes equity value net of personal costs arising from firm"s bankruptcy (see, for example, Gilson (1989)). This extension is available upon request.

x_2 is period-2 cash flow, conditional on the firm not defaulting in the interim.

Equity holders choose the level of operational hedging i to maximize equity value E in (2.10) which yields the following first-order condition:

$$\begin{aligned}\frac{\partial E}{\partial i} &= \int_{u_F}^{\infty}\Big[p - K'(I+i)\Big]g(u)du - \Big[p\left[(1-\delta(u_F))I + i\right] + x_2\Big]g(u_F)\frac{\partial u_F}{\partial i} \\ &= \Big[1 - G(u_F)\Big]\Big[p - K'(I+i) - V(u_F, i)h(u_F)K'(I+i)\Big] = 0 \ . \end{aligned} \tag{2.11}$$

Define $V(u_F, i) \equiv p\left[(1-\delta(u_F))I + i\right] + x_2$, which is the firm's date-2 franchise value at the financial default boundary. Substituting that $\frac{\partial u_F}{\partial i}$ equals $K'(I+i)$, we can rewrite this first-order condition in terms of the "markup", $p - K'(I+i)$, as:

$$p - K'(I+i) = V(u_F, i)h(u_F)K'(I+i) \ . \tag{2.12}$$

The first order condition (2.12) is intuitive. On the one hand, a marginal increase in operational hedging will yield the firm a marginal profit $p - K'(I+i)$. On the other hand, a marginal increases in operational hedging also increases the expected cost of financial default, which is the product of three terms on the right-hand side of Equation (2.12): the first term is the loss of date-2 franchise value had a financial default occurred; the second term is the hazard rate of a financial default; and, the last term is the marginal effect of additional operational hedging on the financial default boundary u_F. The first order condition says that in equilibrium the firm chooses the optimal hedging policy i^* such that the marginal profit is equal to the marginal increase of the expected financial default cost.

Since u is exponentially distributed on $[0, \infty)$ with $g(u) = \alpha e^{-\alpha u}$ and $h(u) = \alpha$, the first order condition (2.12) simplifies to

$$p - K'(I+i) = V(u_F, i)\alpha K'(I+i) \ . \tag{2.13}$$

Define i^* is the firm'optimal hedging policy that satisfies (2.13). The following assumption guarantees that a positive interior solution i^* exists and $D(i^*, \bar{F}) > 0$ for sufficiently large $\bar{F}$:[6]

Assumption 2.3. $p - K'(I) > (pI + x_2)\alpha K'(I)$.

Appendix A.2 proves that

Lemma 2.1. *If Assumption 2.3 holds and $\bar{F}$ is sufficiently large, then the first order condition* (2.13) *admits a positive interior solution i^* is a uniquely solution that maximizes E subject to $D(i, \bar{F}) > 0$.*

Next, we study the correlation between the firm's optimal operational hedging policy and its inherited net debt level when u_F is always greater than u_O. Lemma 2.2, also proved in Appendix A.2, states that the optimal optimal operational hedging policy decreases in the firm's net debt level maturing in the interim:

Lemma 2.2. *When $\bar{F}$ is sufficiently high such that $D(i^*, \bar{F}) > 0$, the equilibrium operational hedging policy i^* decreases in the firm's net debt level $\bar{F}$.*

2.5 Optimal hedging policy when $u_F < u_O$

We now focus on the case in which the firm's inherited debt level is sufficiently low such that the financial default boundary is always below the operational default boundary. In this case,

[6] We assume that initial cash holdings are high enough to avoid the corner solution, i.e., the production cost of I units of goods and the chosen operational hedging level i, is less than $x_0 + x_1$.

the operational default boundary enters the equity value function. The total expected payoff to the equity holders is as in Section 2.4, less an expected cost proportional to unlevered firm value at date-2, λx_2, if the firm defaults on its supplier contract, provided that the firm does not default on its debt in the interim. The market value of equity is therefore,

$$\hat{E} = E - \int_{u_F}^{u_O} \lambda x_2 g(u) du \ , \tag{2.14}$$

where the E is the equity value when $u_F > u_O$, as specified in (2.10).

Equity holders choose the optimal level of operational hedging i to maximize $\hat{E}$, which yields the following first-order condition:

$$\frac{\partial \hat{E}}{\partial i} = [1 - G(u_F)] \left[p - K'(I+i) - [V(u_F, i) - \lambda x_2] h(u_F) K'(I+i) - \frac{\lambda x_2 g(u_O)}{1 - G(u_F)} \frac{\partial u_O}{\partial i} \right] = 0 \ . \tag{2.15}$$

Substituting that $\frac{\partial u_O}{\partial i} = \frac{1}{I\delta'(u_O)}$ and rearranging, we can rewrite this first-order condition as:

$$p - K'(I+i) = [V(u_F, i) - \lambda x_2] h(u_F) K'(I+i) + \frac{\lambda x_2 g(u_O)}{[1 - G(u_F)] I \delta'(u_O)} \ . \tag{2.16}$$

Define $\hat{i}^*$ as the firm's optimal hedging policy that satisfies (2.16). Similar to the case in which $u_F > u_O$, a marginal increase in investment on spare production will yield the firm a marginal profit $p - K'(I+i)$. However, the effect of a marginal increase in i on the firm's expected loss from operational default and financial default is opposite. On the one hand, a marginal increases in operational hedging increase the expected cost of financial default by increasing the financial default boundary u_F.[7] On the other hand, a marginal increase

[7]Notice that the loss conditional on a financial default is reduced by λx_2. This is because the firm would declare operational default if $u_F < u < u_O$.

in operational hedging decreases the expected cost of operational default since it reduces the operational default boundary u_O, which is captured by the last term of the first order condition (2.16). Therefore, the first-order condition (2.16) says that in equilibrium the firm chooses the optimal hedging policy $\hat{i}^*$ such that the marginal profit ("markup") is equal to the marginal increase of the expected financial default cost net of the marginal decrease of the expected operational default cost. We show in Appendix A.3 that

Lemma 2.3. *If the production commitment I is sufficiently high and $\frac{K'(I+\bar{i})}{I}$ is sufficiently low, then $\hat{i}^*$ that satisfies* (2.16) *uniquely maximizes $\hat{E}$.*[8]

Intuitively, the condition that I is sufficiently high means that the supply contract value is important economically. The condition that $\frac{K'(I+\bar{i})}{I}$ is sufficiently low means that the firm's marginal production cost does not increase too fast as the production quantity increases. This condition makes sure that the firm has enough operational flexibility to do the operational hedging even if the production quantity is high and it has zero debt.

In summary, when the firm's inherited net debt $\bar{F}$ is sufficiently low such that the operational default boundary u_O always dominates the financial default boundary u_F, operational default risk is main concern of equity holders. Thus, the firm will invest more on operational hedging, i.e., $\hat{i}^* > i^*$. The following lemma, proved in Appendix A.3 confirms this intuition:

Lemma 2.4. *If Lemma 2.3 holds, the operational hedging policy $\hat{i}^*$ that satisfies Equation* (2.16) *is higher than the operational hedging policy i^* that satisfies Equation* (2.12)*, i.e., $\hat{i}^* > i^*$.*

Similar to the $u_F > u_O$ case, Appendix A.3 derives that when u_F is always less than u_F, the firm's optimal operational hedging policy $\hat{i}^*$ decreases in its inherited net debt level:

[8]Appendix A.3 also provide the conditions that I and $\frac{K'(I+\bar{i})}{I}$ need to satisfy. We continue to assume that initial cash holdings are high enough for the first-order condition to avoid corner solution.

Lemma 2.5. *If Lemma 2.3 holds, the equilibrium operational hedging policy $\hat{i}^*$ decreases in the firm's net debt level $\bar{F}$.*

2.6 Optimal hedging policy and net debt $\bar{F}$

We can now formally characterize the correlation between the firm's optimal operational hedging policy and its inherited net debt level $\bar{F}$. We will show that the firm's optimal operational hedging policy is $\hat{i}^*$ when the net debt level $\bar{F}$ is sufficiently low and is i^* when the net debt level is sufficiently high. The operational hedging policy is at a level $\tilde{i}$ in the "sliding region" such that $D = 0$, when the net debt level is intermediate. Recall that $D = u_F - u_O$ is defined in Equation (2.5).

Let $\bar{F}_{fb}$ is such that $\bar{F}_{fb} + K(I + \bar{i}) = 0$, i.e., $\bar{F}_{fb}$ is the maximal net debt level such that the firm is able to pay back the debt at date-1 when it chooses the maximal optimal hedging policy $\bar{i}$ that maximizes the unlevered firm value, as derived in Section 2.3.1. When $\bar{F} \leq \bar{F}_{fb}$, short-term debt is riskless and the firm chooses the optimal hedging policy as if the short-term debt level is zero. Moreover, we introduce $D^*(\bar{F}) \equiv D(i^*(\bar{F}), \bar{F})$ and $\hat{D}^*(\bar{F}) \equiv D(\hat{i}^*(\bar{F}), \bar{F})$, i.e., D^* and $\hat{D}^*$ are the differences between financial default boundary u_F and operational default boundary u_O when the firm chooses the operational hedging policy i^* and $\hat{i}^*$ respectively. Define $\bar{F}_0$ to be such that $\hat{D}^*(\bar{F}_0) = 0$ and $\bar{F}_1$ such that $D^*(\bar{F}_1) = 0$. Appendix A.4 shows that $\bar{F}_0$ and $\bar{F}_1$ exist and are unique with $\bar{F}_0 < \bar{F}_1$; $D^* < 0$ if $\bar{F} < \bar{F}_1$; and, $D^* > 0$ if $\bar{F} > \bar{F}_1$. Similarly, $\hat{D}^* < 0$ if $\bar{F} < \bar{F}_0$ and $\hat{D}^* > 0$ if $\bar{F} > \bar{F}_0$. The following proposition formalizes this relationship between the firm's optimal operational hedging policy and its net debt level maturing at date-1:

Proposition 2.1. *If Lemma 2.3 holds, then*

I. If $0 \leq \bar{F} \leq \bar{F}_{fb}$, the firm's optimal operational hedging policy is $\bar{i}$.

II. If $\bar{F}_{fb} < \bar{F} \leq \bar{F}_0$, the firm's optimal operational hedging policy is $\hat{i}^*$.

III. If $\bar{F}_0 < \bar{F} < \bar{F}_1$, the firm's optimal operational hedging policy is $\tilde{i}$.

IV. If $\bar{F} \geq \bar{F}_1$, the firm's optimal operational hedging policy is i^*.

The next proposition states the main results of in our paper: saving cash to hedge against the financial default risk and invest it in spare production capacity/inventory to hedge the operational default risk compete with each other. When the firm is more financially leveraged in the interim, i.e., having higher net debt levels $\bar{F}$ maturing at date-1, financial hedging motive dominates the operational hedging motive; such a firm cuts investment in operational hedging to conserve more cash, in order to better weather the financial default. As a result, the intensity of equilibrium operational hedging, denoted by i^{**}, is lower.

Proposition 2.2. *When $\bar{F} > \bar{F}_{fb}$, the firm's optimal operational hedging policy i^{**} decreases in net debt $\bar{F}$.*

3. Model extensions

3.1 The effect of partial pledgeability

In our base-case model of Section 2, the firm has no access to external financing. In this subsection, we extend the model to consider the effect of partial pledgeability of cash flows from supplier contract settlement. We use subscript FC to denote respective quantities for this extension.

Suppose that at $t = 1$ the firm can use a fraction τ of its proceeds from date-2 supplier contract settlement (which is $\tau p[(1 - \delta(u))I + i]$) as collateral for new financing, where $0 \leq \tau \leq 1$. Here, $\tau = 0$ corresponds to our base case of extreme financing frictions, when the firm cannot raise any external financing against its future cash flow, whereas $\tau = 1$ implies frictionless access to external capital with payment backed by future cash flow. In practice, τ can also represent the ease of access to cash flow financing.

Conditional on survival, raising new financing at $t = 1$ in this setting is value-neutral. Therefore, we can assume without loss of generality that the firm always raises the amount equal to the cash shortfall when the cash flow shock hits the financial default boundary $u_{F,FC}$. Thus, cash available for debt service at date 1 is $x_0 - K(I+i) + x_1 + \tau p[(1-\delta(u_{F,FC}))I + i]$, which is the sum of the cash reserve $x_0 - K(I+i)$, the random cash flow $x_1 = \bar{x}_1 + u$, and the newly borrowed amount $\tau p[(1-\delta(u_{F,FC}))I + i]$. The financial default boundary is now given as:[9]

$$u_{F,FC} = \bar{F} + K(I+i) - \tau p[(1-\delta(u_{F,FC}))I + i] \ . \tag{3.1}$$

In turn, the value of equity when $u_{F,FC} > u_O$ can be written as

$$E_{FC} = \int_{u_{F,FC}}^{\infty} \Big[(u - u_{F,FC}) - \tau p[(1-\delta(u_{F,FC}))I+i] + p[(1-\delta(u_{F,FC}))I+i] + x_2\Big] g(u) du \ . \tag{3.2}$$

The partial pledgeability case can be solved in an analogues manner as the zero pledgeability case. We define $\hat{i}^*_{FC}$ as the optimal hedging policy that maximizes the equity value when $u_{F,FC} < u_O$; $\tilde{i}_{FC}$ as the optimal hedging policy that equalizes the operational and financial default boundaries $u_O(\tilde{i}_{FC}) = u_{F,FC}(\tilde{i}_{FC}, \bar{F})$; and, i^*_{FC} as the optimal hedging policy that maximizes the equity value when $u_{F,FC} > u_O$. Specifically, i^*_{FC} and $\hat{i}^*_{FC}$ are given respectively by the following first-order conditions:

$$p - K'(I + i^*_{FC}) = V(u_{F,FC}, i^*_{FC}) \ h(u_{F,FC}) \ \frac{[K'(I + i^*_{FC}) - \tau p]}{[1 - \tau p \delta'(u_{F,FC}) I]} \ , \tag{3.3}$$

[9] The operational default boundary u_O is the same as the base case.

$$p - K'(I + \hat{i}^*_{FC}) = \Big[V(u_{F,FC}, \hat{i}^*_{FC}) - \lambda x_2\Big] \; h(u_{F,FC}) \, \frac{[K'(I + \hat{i}^*_{FC}) - \tau p]}{[1 - \tau p \delta'(u_{F,FC}) I]} + \frac{\lambda x_2 g(u_O)}{[1 - G(u_{F,FC})] I \delta'(u_O)} \; . \tag{3.4}$$

As long as the the firm is sufficiently financially constrained, i.e., τ is sufficiently low, the optimal hedging policy is of the same form as that in the baseline case. Consequently, the intensity of equilibrium operational hedging, denoted by i^{**}, is lower when the inherited net debt level $\bar{F}$ is higher.

Define $\bar{F}_{fb,FC}$ to be such that

$$\bar{F}_{fb,FC} + K(I + \bar{i}_{FC}) = \tau * p * \bar{i}_{FC} \; . \tag{3.5}$$

In other words, $\bar{F}_{fb,FC}$ is the maximal net debt level such that the firm is able to pay back the debt at date-1 even if the production shock u is severe enough to obliterate the entire production capacity I. $\bar{F}_{0,FC}$ and $\bar{F}_{1,FC}$ are defined analogously to the respective thresholds in Proposition 2.1: $\bar{F}_{0,FC}$ is such that $u_{F,FC}(\hat{i}^*_{FC}, \bar{F}_{0,FC}) = u_O(\hat{i}^*_{FC})$; $\bar{F}_{1,FC}$ is such that $u_{F,FC}(i^*_{FC}, \bar{F}_{1,FC}) = u_O(i^*_{FC})$. The following proposition characterizes the firm's optimal hedging policy as a function of $\bar{F}$ when the pledgeability is imperfect, i.e., $\tau < \bar{\tau} < 1$:[10]

Proposition 3.1. *There exists $\bar{\tau} < 1$ such that if $\tau < \bar{\tau}$, then*

I. If $0 \leq \bar{F} \leq \bar{F}_{fb,FC}$, the firm's optimal operational hedging policy is $\bar{i}$.

II. If $\bar{F}_{fb,FC} < \bar{F} \leq \bar{F}_{0,FC}$, the firm's optimal operational hedging policy is $\hat{i}^*_{FC}$.

III. If $\bar{F}_{0,FC} < \bar{F} < \bar{F}_{1.FC}$, the firm's optimal operational hedging policy is $\tilde{i}_{FC}$.

[10]The proofs of Proposition 3.1 and Proposition 3.2 are very similar to the base case although the algebra is much more involved. The proofs are available upon request.

IV. If $\bar{F} \geq \bar{F}_{1,FC}$, the firm's optimal operational hedging policy is i^*_{FC}.

Proposition 3.2. *If $\tau < \bar{\tau}$ and $\bar{F} > \bar{F}_{fb,FC}$, the firm's optimal operational hedging policy i^{**} decreases in $\bar{F}$.*

3.2 Operational spread and credit spread

Consistent with Acharya et al. (2012), the credit spread is defined by the ratio between the face value F and the market value of debt D minus 1. The market value of debt is given as:

$$\begin{aligned} D &= F - \int_0^{u_F} \left[F - (x_0 - K(I+i)) - \bar{x}_1 - u - \tau p\left((1-\delta(u))I + i\right)\right] g(u)du \\ &= F - \int_0^{u_F} \left[u_F - u - \tau p\left(\delta(u_F) - \delta(u)\right) I\right] g(u)du \ . \end{aligned} \tag{3.6}$$

The second term of Equation (3.6) is the expected bankruptcy cost. Then, the credit spread s is

$$s = \frac{F}{D} - 1 \ . \tag{3.7}$$

The operational spread is the markup, $p - K'(I+i)$. Our model predicts that the operational spread and credit spread are positively correlated. The operational spread is the markup, $p - K'(I+i)$. To see this, note that $\frac{di^{**}}{dcs} = \frac{\partial i^{**}}{\partial F}\frac{dF}{ds} = \frac{\partial i^{**}}{\partial F}D$. By Proposition 2.2, the above quantity is smaller than zero, meaning that in equilibrium, the operational hedging level i^{**} decreases in the credit spread s. The operational spread also decreases in the operational hedging level i. Thus, we have the following proposition:

Proposition 3.3. *In equilibrium, operational spread and credit spread are positively correlated.*

3.3 Debt maturity

So far, we focused on the case in which the firm's existing debt matures at date-1, before the supply contract delivery. What happens if the debt matures at date-2, at the same date as the contract delivery? If the debt maturity date is aligned with the delivery date of the supplier contract, then the firm can use its entire cash flow from the supply contract payment to pay off its debt. Thus, the optimal operational hedging policy in the "long-term" debt case is the same as the case of perfect pledgeability ($\tau = 1$). In fact, although we interpret τ as the pledgeability of the cash flow from the supplier contract, we can also treat $(1-\tau)$ as the proportion of the firm's debt that matures before the contract delivery, i.e., the mismatch between the firm's debt maturity structure and the duration of its operational cash flows.

3.4 Supply chain diversification

We can modify our model slightly to accommodate the case in which the firm hedges against the operational default risk by choosing multiple suppliers instead of choosing spare production capacity or excess inventory. Suppose that the production function becomes $K = K(I, n)$, in which $n \geq \underline{n}$ denotes the measure of suppliers that the firm chooses to enlist in the production process, and $\underline{n}$ denotes the minimal measure of suppliers that the firm needs to keep the production running.[11] We assume that it is more costly if the firm chooses a more diversified supply chain, i.e., n being large. Mathematically, it means that the first- and second-order partial derivatives of K with respect to n are both positive: $K_n(I, n) > 0$ and $K_{nn}(I, n) > 0$. We assume that the production loss function $\delta(u, n)$ depends on both the production shock u and the measure of suppliers n. Consistent with the baseline model, $\delta(u, n)$ is decreasing and convex in both u and n with continuous and finite first- and second-

[11] We assume that n represents the measure, instead of number of suppliers, in order to use the first-order conditions, consistent with our baseline model.

order derivatives, $\delta_u(u,n) < 0$, $\delta_n(u,n) < 0$, $\delta_{uu}(u,n) > 0$ and $\delta_{nn}(u,n) > 0$. In addition, we assume that the cross-partial derivative of $\delta(u,n)$. $\delta_{un}(u,n) < 0$.

In this setting, the operational default threshold u_O is such that $\delta(u_O, n) = 0$. Then $\frac{\partial u_O}{\partial n} = -\frac{\delta_n(u_O,n)}{\delta_u(u_O,n)} < 0$. It can be verified that the second-order derivative of u_O with respect to n is greater than zero, which is the same as the baseline case. In this setting, our previous lemmas and propositions still go through.[12] In particular, operational hedging measured as supply chain diversification (n) is decreasing in firm's financial leverage and credit risk.

4. Numerical analysis

We show comparative statics from the model using numerical analysis. First, we treat the debt level (F) as predetermined and try to show the evolution of the optimal hedging policy i^{**} for different levels of debt F maturing at date-1. Then we numerically solve for the optimal debt, taking into consideration that the firm may have tax-shield benefits of debt that have to be traded off against the cost of lower operational hedging at higher leverage. These results will relate to some of our empirical results.

Throughout this section, we focus on the generalized version of the model in Section 3.1, which features a pledgeability $\tau \in [0,1]$. As mentioned in Section 2.1, the cash flow shock u follows an exponential distribution with rate parameter $\alpha = 0.05$, i.e., the probability density function of u, $g(u) = 0.05e^{-0.05u}$. The production loss function is assumed to be $\delta(u) = e^{-u}$. Consistent with neoclassic investment literature (Bolton et al., 2011), we assume that a quadratic production cost function $K(I+i) = \kappa(I+i)^2$, in which $\kappa = 0.1$. All parameter values are in Table 1. Given the stylized nature of our model, we do not attempt to match

[12] There are some technical modifications. The term p ceases to enter the left hand-sides of first-order conditions (2.12) and (2.16). Consequently, the firm will not add additional suppliers when $u_F > u_O$, i.e., $u^* = \underline{u}$. The second-order conditions and the optimal hedging policies across different levels of debt F are qualitatively identical.

any model-implied quantities with respective empirical moments. Therefore, the choice of the parameter values is just for illustrative purpose. That being said, the numerical results in exogenous debt case are based on analytical results in Proposition 3.1 and Proposition 3.2; thus, they are robust to specific parameter choices.

[Table 1 HERE]

4.1 Exogenous F

Figure 1 presents the firm's optimal operational hedging policies i^{**} given different short-term debt levels F. The blue, red and yellow lines represent the cases of low ($\tau = 0$), intermediate ($\tau = 0.4$) and high pledgeability ($\tau = 0.8$) cases, respectively. In all three cases, the optimal hedging policy i^{**} is flat when the debt level F is low. This corresponds to the scenario I of Proposition 3.1: debt does not affect the firm's optimal hedging policy when the debt level is sufficiently low, i.e., the debt is guaranteed to be paid off at date-1 even if the worst production shock occurs at date-1 that "wipes out" the firm's entire production capacity. As the debt level F increases, the optimal hedging policy i^{**} exhibits a negative correlation with the debt level maturing at date-1. Moreover, the negative slope is steeper and holds for a wider range of debt levels F the lower is the pledgeability τ. Overall, the optimal operational hedging policy intensity decreases in the amount of debt maturing in the interim, especially if the firm is financially constrained, i.e., has a low pledgeability τ.[13]

When τ is high, the tension between choosing operational hedging and financial hedging should be relaxed. Nevertheless, as the yellow line of Figure 1 shows, the negative relationship between operational hedging policy and inherited debt level F is still present in our numerical examples. This demonstrates a "debt overhang" effect when firm chooses how much to

[13] From Equation (3.5), $\bar{F}_{fb}$ increases in the pledgeability τ. Thus the F-region in which debt level does not affect the optimal hedging policy increases with τ.

hedge against the operational default risk. Intuitively, the benefit of operational hedging is "truncated" at the cash flow threshold F below which the firm declares bankruptcy, in which case the equity holders lose the franchise value anyways. Thus as the debt level increases, this cash flow threshold increases and equity holders' benefit of operational hedging diminishes. In response, equity holders lower the operational hedging activity.

[Figure 1 HERE]

In what follows, we plot the firm's credit spread against its operational spread, i.e., the markup $p - K'(I + i)$. Proposition 3.3 is confirmed in Figure 2: When the firm chooses optimal hedging policy given the debt level F, the credit spread and operational spread are positively correlated. This positive relationship is stronger when the firm is more financially constrained, i.e., its pledgeability τ is lower. This is consistent with the novel implication of our model: when the firm's credit spread is higher, the financially constrained firm cuts the operational hedging activity by a larger extent to save more cash at date-0, hoping to avoid financial default given the more likely cash shortfall at date-1.

[Figure 2 HERE]

4.2 Endogenous F

Next, we examine the firm's optimal debt policy, i.e., how the firm chooses the optimal debt level F that matures in the interim, knowing that it will always choose an operational hedging policy that maximizes the equity value given the existing debt level. To introduce the benefit of debt to firm value, we assume that the firm enjoys a tax deduction for debt payments. Assuming t is the corporate tax rate, we can write firm value as:

$$\begin{aligned}
&E + D = \\
&\underbrace{(1-t)\left[\int_0^{\infty}\left[x_0 - K(I+i) + \bar{x}_1 + u + p\left((1-\delta(u))I + i\right) + x_2\right] g(u)du - \lambda x_2 G(u_O)\right]}_{\text{unlevered firm value}} \\
&- \underbrace{\left[(1-t)\int_0^{u_F}(1-\tau)p\left((1-\delta(u))I + i\right) g(u)du + x_2 G(u_F) - \lambda x_2 G\left(\min\{u_O, u_F\}\right)\right]}_{\text{expected net bankruptcy cost}} \\
&+ \underbrace{tF[1 - G(u_F)]}_{\text{tax benefit}} \qquad (4.1)
\end{aligned}$$

As shown in the first term of Equation (4.1), debt level F creates a real inefficiency in the sense that the optimal operational hedging policy in the presence of F deviates from the first-best policy that maximizes the unlevered firm value. The second term is the expected bankruptcy cost: If the firm declares bankruptcy, with probability $G(u_F)$, equity holders lose the the expected unpledgeable income from supply contract delivery, as well as franchise value x_2. Unlike the standard risky debt models (e.g., Acharya et al., 2012), by declaring bankruptcy, equity holders avoid the cost of operational default, which is $\lambda x_2 G\left(\min\{u_O, u_F\}\right)$. The last term captures the tax-shield benefit of debt.

Figure 3 presents the equilibrium debt policy and the equilibrium operational hedging policy against $\tau \in [0, 0.8]$. First, as shown in Figure 3a and Figure 3b, both the optimal F and the optimal operational hedging policy i increase in τ. A fraction τ of operational hedging amount i can be pledged to the creditors. As the pledgeability increases, operational hedging policy resembles the cash savings policy in terms of lowering the probability of financial default. Thus, the firm is willing to invest more in operational hedging activity. Since the bankruptcy probability, and in turn the expected bankruptcy cost, is lower due to higher pledgeability, the firm will also borrow more at date-0. Correspondingly, Figure 3c illustrates that the optimal debt policy and operational hedging policy mutually reinforce each other as the pledgeability τ increases for most of τ range. Finally, when τ is higher, the increase in equilibrium debt level and operational hedging level outweigh the debt risk reduction from higher pledgeability. Consequently, the credit spread increases in τ, leading to a negative relationship between credit spread and operational spread, as shown in Figure 3d.

[Figure 3 HERE]

Our next exercise characterizes the firm's equilibrium debt and operational hedging polices for various levels of corporate tax rate t. We vary t from 0.1 to 0.7. A higher tax rate increases the marginal benefit of debt, thus the firm chooses a higher debt level F, as illustrated in Figure 4a. Figure 4b shows that the optimal operational hedging policy decreases in the tax rate t. Taken together, Figure 4a and Figure 4b lead to a negative relationship between the optimal debt policy F and operational hedging policy i, as shown in Figure 4c. Moreover, in Figure 4c, the slope of the blue line ($\tau = 0$ case) is steeper than that of the red line ($\tau = 0.4$ case), which is in turn steeper than that of the yellow line ($\tau = 0.8$ case), meaning that the negative relationship between debt policy and operational hedging policy is stronger when the firm is more financially constrained, i.e., τ is lower. The pattern is explained by the main message of the paper: when higher tax benefit induces the firm to take on more debt, it reduces the operational hedging activity as it saves more cash in order to reduce the financial default risk, especially when the firm lacks the ability to pledge the income from supplier contract fulfillment to creditors. Lastly, the firm chooses the optimal debt level that equalizes the marginal benefit (tax savings) and the marginal cost (from financial and operational defaults). As the marginal benefit increases as a result of a higher tax rate, the firm chooses a higher optimal F that also leads to a higher marginal cost, which manifests itself in a higher credit spread and lower operational hedging. As a result, the markup increases in the tax rate, as shown in Figure 4b. Thus, the credit spread and the operational spread are positively correlated, especially for a firm with lower τ, i.e., a financially constrained firm, as can be seen in Figure 4d.[14]

[Figure 4 HERE]

5. Empirical analysis

5.1 Empirical predictions

Our model shows that operational hedging declines with the amount of the firm's credit risk, captured by its credit spread. A higher credit risk induces the firm to allocate more resources to avert financial default and spend less on operational hedging, which result in lower costs and higher price-unit cost margin or operational spread. Our model also predicts that higher existing leverage, especially shorter-term leverage that imposes a liquidity requirement, induces the firm to allocate resources to avoid financial default while spending less on operational hedging, which again raises the operational. We further show that the positive relation between operational spread and both leverage and credit risk should be stronger when the firm is financially constrained.

We test this implication as follows. First, we document that the Markup, our measure of operational spread, is negatively correlated with indicators associated with operational hedging, as we predict. Specifically, we find that Markup declines in the level of inventory whose hoarding indicates the propensity of the firm to engage in operational hedging, and it also declines in measures of supply chain diversification. The estimation controls for firm

[14] In untabulated results, we also numerically solve the firm's optimal debt policies and operational hedging policies for various levels of managerial risk aversion θ. It turns out that neither policy is sensitive to the managerial risk aversion for a wide range of θ values.

characteristics and for market power that affect Markup. This initial test suggests that the Markup can be taken as a summary measure of the extent of operational hedging that the firm engages in. We then examine whether measures of leverage and credit risk affect the operational spread in the way predicted by our model. In particular, we test whether Markup in an increasing function of credit risk and leverage, especially the portion of debt that matures within two years. This breakdown is particularly important because our model focuses on the consequences for operational hedging of the immediate liquidity needs to avoid financial default, which is especially intense when debt is due for redemption. Notably, the maturity time of the short-term portion of the long-term debt has been determined in the past when the long-term debt was issued and thus it is exogenous to the current state of the firm and its current production plans. We propose that the need to avoid financial default induces the form to reduce operational hedging, leading to a wider Markup. At the same time, there should be a muted effect of the long-term debt on operational hedging and on Markup.

Next, we exploit an exogenous shock to credit supply to firms to test our prediction that the positive effects of credit risk and leverage on Markup are greater when firms are financially constrained. We analyze the negative impact of the subprime mortgage crisis of 2008 on lenders' abilities to extend credit to borrowers, following Chodorow-Reich (2014). Specifically, we test whether exposed firms whose credit risk and leverage were higher prior to the crisis reduced operational hedging by more than less exposed firms, leading to a higher Markup. This test uses time series variation in financing constraints to measure the key tension between operational hedging and liquidity emphasized in our paper, and it helps address concerns about the endogeneity of leverage; studies of the impact of the financial crisis show that pre-crisis leverage is an important determinant of the post-crisis real effects through a liquidity channel (e.g., Giroud and Mueller, 2016).

Then we address the concern of leverage being endogenously determined thus being correlated with variables that also affect the operational spread through channels that are different from the one proposed in our model. We exploit an exogenous increase in the marginal tax benefit of debt, which in turn affected firms' propensity to raise leverage, and study its effect on Markup. Heider and Ljungqvist (2015) find that increases in state taxes lead to a significant subsequent increase in leverage of exposed firms. Our model then suggests that these same firms should reduce operational hedging, leading to a rise in Markup.

5.2 Data and empirical definition

We employ quarterly data from 1973 to April 2020, a span of 189 quarters, from Compustat. We exclude firms in the financial industries (SIC codes 6000-6999) and utility industries (SIC codes 4900-4949), and firms involved in major mergers (Compustat footnote code AB). We include firm-quarter observations with market capitalization greater than \$10 million and quarterly sales more than \$1 million at the beginning of the quarter. Our sample includes 17,363 firms with an average asset value of \$2.3 billion dollars (inflation adjusted to 2004). Altogether we have $553,056$ firm-quarters.

5.2.1 Variable definitions

Our dependent variable is the operational spreads of Markup, which we define empirically as sales ($SALEQ$) minus cost of goods sold ($COGSQ$) divided by sales. This measure of the price-unit cost spread proxies for our model's marginal cost of production of the contracted output quantity, which we use the measure the effect of operational hedging cost. Our independent variables of interest are proxies for the firm's ability to pay off its debt liabilities. Our independent variables of interest are proxies for the firm's ability to pay off its debt liabilities. We use two measures: z-score (e.g. Altman, 2013)[15] and financial leverage, the financial debt ($DLTTQ + DLCQ$) divided by total assets (ATQ). We use the negative value of z-score so that a higher value means that the firm has greater financial risk. We include variables to control for the firm's investment needs and its debt capacity. We control for firm size by including total assets in logarithms. To control for the firm's investment opportunities we include Tobin's Q, the sum of common shares outstanding ($CHOQ$) multiplied by the stock price at the close of the fiscal quarter ($PRCCQ$), preferred stock value ($PSTKQ$) plus dividends on preferred stock ($DVPQ$), and liabilities (LTQ), scaled by total assets, to control for the firm's potential investment.[16] To control for the firm's debt capacity, we include cash holdings ($CHEQ$), cash flow ($IBQ + DPQ$) and tangible assets ($PPENTQ$), all scaled by total assets. We use three variables to control for market power, given that Markup is associated with monopoly power (Lerner, 1934) and with inventory behavior (e.g. Amihud and Medenelson, 1989). One variable is "top 3 industry seller", which equal one if the firm's sales ranks among the top three sellers in the industry in a given quarter, using Fama and French's 38 industries, and zero otherwise. The second variable is the firm's Sales/Industry sales, and the third is Herfindahl's index for the industry.

We use variables that are associated with operational hedging. The disruptions of supply chains during the 2020 Covid-19 pandemic highlighted the importance of a new form of operational hedging, supply chain diversification. Indeed our model accommodates supply chain diversification as a measure of operational hedging (see Section 3.4). We thus create operational hedging measures using information on firms' supply chains using information

[15]z-score is computed using the following formula: z-score = 1.2×(current assets ($ACTQ$)−current liabilities ($LCTQ$))/assets+1.4× retained earnings (REQ)/assets+3.3×EBIT ($OIBDPQ$)/assets+0.6×market value of equity ($PRCCQ \times CSHOQ + PSTKQ + DVPQ$)/total liabilities ($LTQ$)+1.0×sales/assets. We use $OIBDP$ instead of $EBIT$ because the latter is not available in Compustat quarterly data.

[16]The definition follows, Covas and Den Haan (e.g. 2011).

from the Factset Revere relationship database.[17] It contains a comprehensive relationship-level data between firms, starting from April 2003. An observation in this database is the relationship between two firms with information about the identities of the related parties, the start and end date of the relationship, the type of the relationship (e.g., competitor, supplier, customer, partner, etc.), and importantly, the firms' geographic origins.

We aggregate the relationship-level data to firm-quarter level and calculate three measures of supply chain diversification for each firm in each quarter: (i.) ln(1+Number of suppliers); (ii.) ln(1+Number of supplier regions), where supplier regions are country and state/province combination; (iii.) ln(1+Number of out-of-region suppliers), that is, suppliers that are not from the firm's region. We merge the supply-chain data to our main sample, yielding a total of $148,230$ firm-quarter observations covering $6,066$ firms, from mid-2003 to the first quarter of 2020. The median firm has 4 suppliers from 3 regions in a given quarter, out of which 2 suppliers are not from the same region as the firm. We create three composite measures of supply chain diversification using the three aforementioned individual measures.

(1) Supply chain diversification index, the first principal component score from a principal component analysis using three individual measures: supply chain diversification index = $0.5809 \times$ ln(1+ Number of suppliers) + $0.6077 \times$ ln(1+Number of supplier regions) + $0.5414 \times$ ln(1+Number of out-of-region suppliers).[18] A higher supply chain diversification index indicates a more diversified supply chain network.

(2) Supply chain diversification ranking, the average across the three supply chain variables of the ranking of the firm-quarter ranking for each of the individual measure. The ranking for each of the three series is scaled by the number of non-missing variables. A

[17] Factset Revere has much better coverage of supply chain information than the COMPUSTAT segment data and used by some studies about supply chain (e.g. Ding et al., 2020).

[18] The first principal component explains 86% of the sample variance.

smaller value of supply chain diversification ranking indicates a more diversified supply chain network.

(3) Standardized supply chain diversification, the average of the value of the three supply chain variables where each is subtracted by the respective sample mean then scaled by the standard deviation of the variable. A higher standardized supply chain diversification indicates a more diversified supply chain network.

Finally, our analysis includes inventory ($INVTQ$) divided by sales as an indicator of operational hedging.

Table 2 presents summary statistics of the variables in our study. All continuous variables in our analysis are winsorized at the 1% and 99% tails. We find that Markup has a mean of 0.329 and its median is 0.341, which is close. The median firm has a z-score of 2.116 which, by Altman's analysis, indicates a state which is close to financial distress, and the mean is 3.567. The median leverage is 0.207 and the mean is 0.238. Measures of market power indicate that most firms operate in a competitive environment: For 75% of the sample, the Herfindahl index is below 0.068 and the firm's sales is 0.003 of the industry sales. Thus, normally it can be expected that Markup reflects a magnitude that is close to the average competitive magnitude and deviations around it arise, among other things, by the considerations that are analyzed by our model.

5.3 The relationship between markup and operational hedging

Our model implies that higher operational hedging activities translates into lower markup through increased marginal production cost. We test whether this implication is supported by the evidence. We estimate the following model using data for firm j in quarter t,

$$Y_{j,t} = \sum_k \beta_k X_{k,j,t-1} + \sum_m Control\ variables_{m,j,t-1} + firm\ FE + year\ FE \tag{5.1}$$

The dependent variable $Y_{j,t}$ is $\ln(Markup_{j,t})$ and $X_{k,j,t-1}$ are the explanatory variables that we focus on which include either of the three supply chain diversification measures and $\ln(inventory/sales)$. Inventory serves here as an indicator of the firm's propensity to expend resources for the purpose of operational hedging, consistent with our model in which the firm produces a higher output than contracted for as a means to avert the cost of a shortfall on its contract with customers in case of a negative shock to output. The control variables are Tobin's Q, Cash holdings, Cash flow, Asset tangibility, and the three variables that measure market power, which is known to affect Markup. The model includes firm and year fixed effects with standard errors clustered by firm and by year.

[INSERT Table 3]

By the results in Table 3, markup is negatively affected by indicators of operational hedging. It is significantly lower when the firm spends more on supply chain diversification and when it engages in increasing inventory. All three measures of supply chain diversification indicate that. Markup is declining in the PCA index of the three supply chain diversification variables; it is declining in the index of standardized average of these variables; and it is increasing in the ordinal index of average ranking by which a lower number means a higher ranking. To illustrate the economic significance of the estimated effect, by the estimation in Column (2), a rise of one place in the ranking of supply chain diversification, which means a decline by one unit, increases log(Markup) by 0.052 which is 5% of its mean. By the estimation in column (1), one standard deviation increase in Supply chain diversification index will lower markup by 1%, and 10% increase in inventory-sales ratio lowers Markup by

0.5%. Overall, the results suggest that markup is a reasonable summary of firms' operational hedging activities, as our model implies.[19] Thus, in the following sections, we examine the relationship between markup and firms' liquidity positions.

5.4 Baseline results

We estimate the main prediction of our model of the tradeoff between allocating funds to avert financial default and spending on operational hedging. We propose that firms in financial distress and with high leverage will reduce spending on operational hedging, resulting in a higher operation spread which we proxy by Markup. We estimate Model (5.1) where $Y_{j,t} = Markup_{j,t}$ and the explanatory variables $X_{k,j,t-1}$ include the variables that indicate he firm's liquidity needs: -(z-score), since the credit spread increases in this variable, and leverage. As before, the control variables are Tobin's Q, Cash holdings, Cash flow, Asset tangibility, and the three measures of market power, as well as firm and year fixed effects.

[INSERT Table 4]

Table 4 presents our baseline results. As predicted in Proposition 3.2 of our model, the operational spread, measured by Markup, is positively affected by the firm's liquidity needs measured by either the credit spread or the leverage ratio. To gauge the economic significance of the effect, one standard deviation increase in the firm's negative z-score raises the firm's markup by 0.04 standard deviation, or 13% of the sample median markup. And, one standard deviation increase in leverage results in 0.02 standard deviation increase in markup. In Column (3) we include both -(z-score) and Leverage and find that the coefficient

[19] Using individual supply chain diversification measures instead of composite measures yields qualitatively similar results.

of -(z-score) remains unchanged in both magnitude and statistical significance while the coefficient of Leverage declines in magnitude and it is no longer significant.

In our theoretical model, it is the liquidity need to avoid financial default that presses the firm to divert resources from operation hedging. This is because the firm's existing debt matures before the contracted delivery date of its output. This maturity mismatch between debt obligations and operational cash flow contributes to the tension between financial hedging and operational hedging. It follows that short-term leverage should have a larger impact on operational hedging or Markup than the impact of long-term leverage.

We test this hypothesis by studying the effect of the portion of debt which matures in the coming two years. Importantly, the short-term part of the long-term debt had its maturity determined in the past when the debt was issued. Thus it is not determined simultaneously with operational hedging policies in response to the current state of the firm and its environment.

The results Column (4) of Table 4 show that the effect on markup of the short-term debt — the part of long-term that matures in the next two years — is more than twice as large as that of the remaining long-term debt maturing in more than two years, with the difference between the coefficients being statistically significant at the 0.05 level.[20] The result supports our theoretical prediction that it is the pressing liquidity need that induces firms to shift funds from operational hedging to the accommodation of the need to avoid financial default. In Column (5), we include -(z-score) together with the two leverage variables, the short term and long-term part of leverage. We find that the coefficients of both -(z-score) and the short-term part of the long-term debt are positive and significant, as predicted by our model, while the coefficient of the remaining long-term debt is positive but smaller in

[20] In untabulated result, we also test for the significance of the difference between coefficients on long-term debt maturing in the next two years and remaining long-term leverage, the difference is 0.039 with t-statistics equal to 2.60.

magnitude and statistically insignificant.

Figure 5 presents binned scatter plots of the relationship between operational spread and either -(z-score) or Leverage. Following the methodology of Rampini et al. (2014),[21] we first residualize markup, -(z-score) and Leverage with respect to the baseline control variables (including the firm and year fixed effects), as in Table 4. We then add back the unconditional mean of the respective variables in the estimation sample to facilitate interpretation of the scale. Finally we divide the x-axis variable into twenty equal-sized groups (vingtiles) and plot the means of the y-variable within each bin against the mean value of x-variable within each bin. Figure 5a and Figure 5b correspond to the estimations in Columns (1) and (2) of Table 4, respectively. We see that the markup monotonically increases with one-period lagged values of both -(z-score) and Leverage. Notably, the monotonic relationships between the firm's liquidity need and Markup across all the bins shows that that our results are driven by extreme observations. These results support our prediction on the negative relationship between a firm's liquidity needs and the intensity of its operational hedging, reflected in a positive relation between the credit spread and leverage and its operational spread or Markup.

5.5 Effect of financial constraint: the consequences of a shock to credit supply

We exploit the financial shocks during the crisis of 2008 to test our prediction that when a firm becomes financially constrained, there is a stronger effect of its credit risk and leverage on operational hedging. In our model, greater financial constraint is captured by a lower pledgeability parameter τ. During the financial crisis of 2008, a number of banks could no longer extend credit to firms with which they had lending relationship beforehand. We test

[21] We thank Raj Chetty for making the relevant STATA program available.

whether for firms that were adversely affected by this shock to credit, the effect of -(z-score) and Leverage on Markup became stronger.

We first find the relationship between our sample firms and bank lenders using data from the LPC-Dealscan database. We then follow Chodorow-Reich (2014) who use three variables to measure the negative impact of the subprime mortgage crisis on lenders' abilities to extend credit to the borrowers.[22] The first variable is a direct measure of changes in loan supply for a firm's lenders. For each lender, it calculates the percentage changes in the (weighted) number of loans that the lender extended to all the firms other than the firm in question, between the 9-month period from October 2008 to June 2009, and the average of 18-month period containing October 2005 to June 2006 and October 2006 to June 2007. The weight is the lender's share of each loan package commitment. The second measure is Lehman exposure, the exposure to Lehman Brothers through the fraction of a bank's syndication portfolio where Lehman Brothers had a lead role. The third measure captures banks' exposure to toxic mortgage-backed securities (ABX exposure), which is calculated using the correlation between banks' daily stock return and the return on the ABX AAA 2006-H1 index. Then, for each firm and each of the three variables, it calculates a weighted average of the measure over all members of the last pre-crisis loan syndicate of the firm, in which the weight is each lender's share in the firm's last pre-crisis loan syndicate.[23] We construct the three variables in a way so that a larger value implies a larger exposure to the financial crisis on the lenders' side.

[22]We thank Chodorow-Reich for sharing his data with us.

[23]Please refer to Chodorow-Reich (2014) for detailed constructions of the three variables.

We use the following regression specification.

$$\begin{aligned} Markup_{j,t} = \alpha + \beta_1 \times X_{j,2007} \times Lender\ exposure_{j,t} + \beta_2 \times Lender\ exposure_{j,t} \\ + \sum_k \beta_{3,k} \times Control\ variable_{j,t-1} \\ + \sum_k \beta_{4,k} \times Controls\ variables_{j,t-1} \times Lender\ exposure_{j,t} + \theta_j + \eta_t + \epsilon_{j,t} \end{aligned} \tag{5.2}$$

We estimate the differential effect on $Markup_{i,t}$ for firms that entered the post-crisis period with different levels $X_{i,2007}$ being either -(z-score) or Leverage, given different levels of the firm's exposure to the crisis. Notably, $X_{i,2007}$ is fixed before the crisis as of the end of 2007. The comparison is between the two-year period before the crisis (July 2006 to June 2008) and the two-year period after the crisis (January 2009 to December 2010). The lenders' exposure to the financial crisis equals zero for the two-year period before the crisis, and equals its actual respective values for the two-year period after the crisis. The control variables are the same as in the baseline regression (Table 4) and they are fixed at the end of year 2007 for the post-crisis years, to be consistent with $X_{i,2007}$. Our test focuses on β_1, the coefficient of the interaction between the crisis exposure and the liquidity variables. The model includes firm and year fixed effects and standard errors are at firm levels.28 Naturally, in these regressions which are confined to a short time period, the number of observations is much smaller.

INSERT Table 5.

Table 5 presents the results. We find that the coefficient β_1 is positive and significant for all interactive terms except for the $-$(z-score) $\times$ Lehman exposure. Our results mean that the effect of credit risk, proxied by -(z-score), of Leverage was greater for firms whose lenders were adversely affected by the financial crisis. These firms because financially constrained and thus their liquidity needs forced them to reduce spending on operational hedging which we capture by the widening of Markup. To gauge the economic significance of the joint impacts of the firm's liquidity position and its exposure to financial crisis on the borrower's operational spread, taking Column (1) as an example, one unit increase in the firm's negative z-score yields additional 1.6% markup when the firm's lenders reduce number of loans to other borrowers by 10% more during the financial crisis. According to Column (2), a firm that enters the crisis period with 0.01 higher leverage ratio witnesses an additional 0.001 markup when the firm's lenders reduce number of loans to other borrowers by 10% more during the financial crisis. Using two alternative exposures to financial crisis yields qualitatively similar results.

Overall, the results show that the tension between operational hedging spending and the liquidity needs to avoid financial default is stronger when the firm is hit by a negative shock to its ability to raise capital. Then, it foregoes spending on operational hedging activities and diverts cash to service its financial needs. As shown in Table 5 and consistent with our model's predictions, the positive relationship between markup and the illiquidity position is stronger when the firm becomes more financially constrained.

5.6 State tax change

Our base-case tests in Table 4 find that higher leverage induces firms to reduce spending on operational hedging, resulting in a wider Markup. As discussed in Section 4.2, firms may design their capital structures taking into account the impact of their financial policies on their operational hedging policies and operational default risks. Our empirical results so far are consistent with the case in which the firms choose their operational hedging polices taking their debt levels as given. This was highlighted by the fact that the long-term debt that happens to come to maturity in the near future exerts the greatest effect on the operational spread. Nevertheless, to further alleviate the concern of simultaneous determination of leverage and operational hedging we use changes in firms' leverage that are induced by an external shock: the change in the value of tax subsidy on leverage.

We follow Heider and Ljungqvist (2015) who show that changes in the state tax law affect firms' capital structure. The tax effect on capital structure is asymmetric: Firms raise leverage following tax increases but do not reduce leverage after tax cuts. In our study, we use all the state tax law changes from 1989 to 2012.[24] We test whether the tax-induced increase in leverage caused firms to reduce spending on operational hedging, which would result in wider operational spread or Markup.

Following Heider and Ljungqvist (2015), our dependent variable is the quarterly change of Markup. The independent variables are Tax increase indicator and Tax decrease indicator, a dummy variable that equals one if the firm's headquarter state had a tax law change that increased or decreased the state tax, respectively. The quarter that is indicated by the dummy variable is the first quarter in the year that follows the year of change in the state tax law and the preceding three quarters.[25] All other control variables are the same as those in baseline regressions, used in their first-order difference form. As in Heider and Ljungqvist (2015), we include $industry \times year$ fixed effects to remove any time-varying industry shocks. Robust standard errors are clustered at state level.[26]

Table 6 presents the results. In Column (1) we present the results for our entire sample, The coefficient of Tax increase indicator is positive and significant suggesting that the inducement to raise leverage following the increase in the tax advantage of leverage causes the firm to reduce spending on operational leverage which in turn makes Markup larger. Table 6

[24]These changes are listed in appendix A and appendix B of Heider and Ljungqvist (2015). They state that they include all U.S. state corporate income tax rises in 1989 — 2012 affecting firms in fiscal years 1989 — 2011. Accordingly, our sample period is from 1990 to 2012.

[25]We assume that the tax change in a year may have happened in any quarter during that year.

[26]The empirical specification here is the same as Heider and Ljungqvist (2015).

shows that the firm on average increases its markup by 0.004 unit if its headquarter state has a tax increase in the previous quarter. However, a tax rate decreases in a firm's headquarter state does not lead to any changes in markup, which is consistent with Heider and Ljungqvist (2015) finding that tax decrease does not affect leverage. That is, the asymmetric impact of state tax laws on leverage is mirrored in an asymmetric effect on Markup.

The rest of the estimations in Table 6 test our theoretical prediction that the Markup-Leverage relationship is stronger for firms that are financially constrained. A higher debt level will push firm into lowering its spending on operational hedging, which raises Markup, if they face greater cost of raising capital to avert financial default. In terms of our model in Section 4.2, if the pledgeability of future operational cash flow to creditor is low, indicated by lower τ, the effect of the firm's liquidity needs on the operational spread is greater.

We test this prediction by using three alternative measures of financial constraints that are standard in the literature: (i) The White-Wu index (Whited and Wu, 2006); (ii) The size-age index (Hadlock and Pierce, 2010); and (iii) A dummy variable equal to one if the firm does not pay any dividend in a given quarter, and zero otherwise, following Fazzari et al. (1988). A firm is financially constrained if it does not pay any dividend in the previous quarter.[27] For the first two continuous measures of financial constraint we classify a firm financially constrained in a given quarter if the value of that financial constraint measure for the firm is higher than the bottom tercile in the previous quarter. For measure (iii), the firms are classified by whether the value of the dummy variable is 0 or 1, meaning that they are either financially constrained or not financially constrained, respectively.

The estimation results are consistent with our predictions. As shown in Column (3), (5) and (7), the coefficients on tax increase are positive and significant for financially constrained

[27]The formula of Whited-Wu index is on page 543 of (Whited and Wu, 2006). The formula of size-age index is on page 1929 of (Hadlock and Pierce, 2010).

firms that experience a tax increase, which induces increased leverage. For firms that are not financially constrained, the tax increase has no significant effect on their Markup. Such firms are able to raise capital in order to avert financial default should there be a cash shortfall and therefore, even if they increase leverage they do not need to divert finding from operational hedging. Consequently, in such forms, Markup is unaffected by their propensity to increase leverage following the tax increase.

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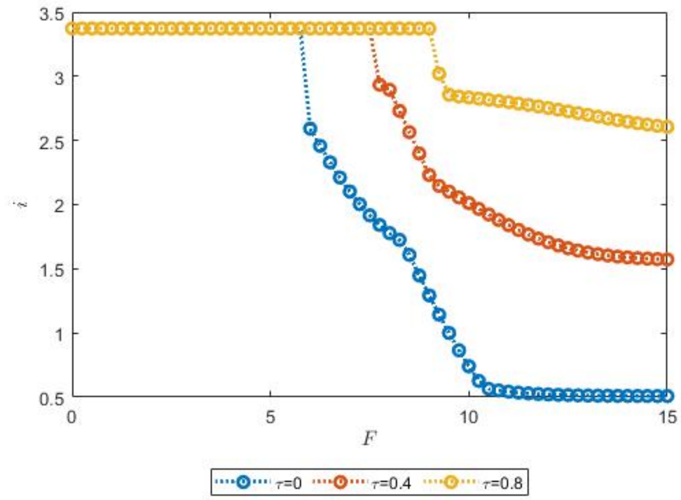

Figure 1: **Firm's optimal hedging policy i^{**} and debt level F**

Optimal i^{**} given F for $\tau = 0$, $\tau = 0.4$ and $\tau = 0.8$. All other parameters are presented in Table 1.

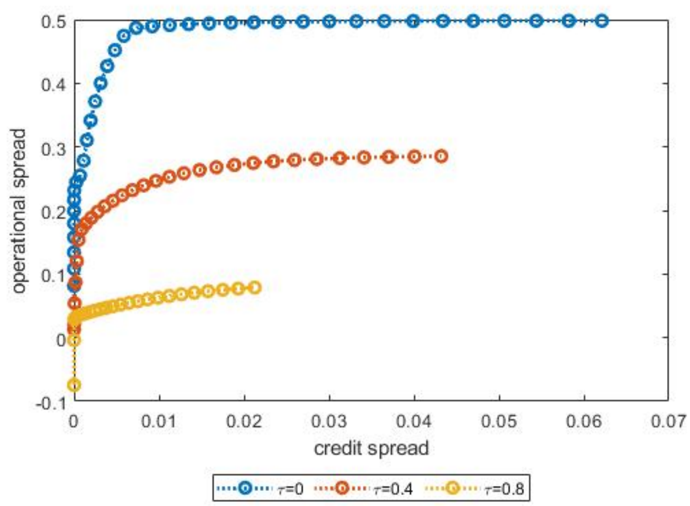

Figure 2: **Credit spread and operational spread**

The credit spread and operational spread under the optimal hedging policy i^{**} given F for $\tau = 0$, $\tau = 0.4$ and $\tau = 0.8$. All other parameters are presented in Table 1.

Figure 3A: Equilibrium debt policy and τ

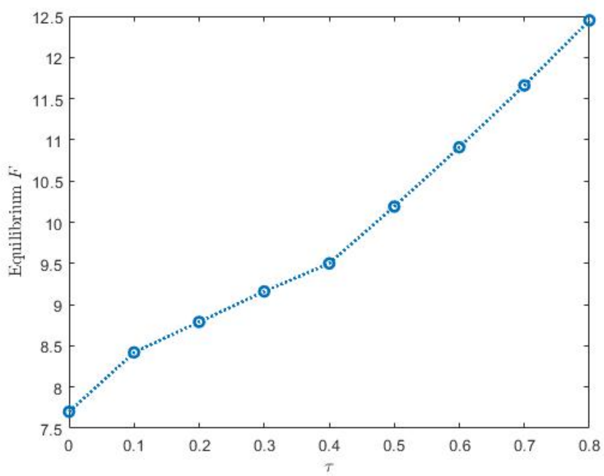

Figure 3B: Equilibrium hedging policy and τ

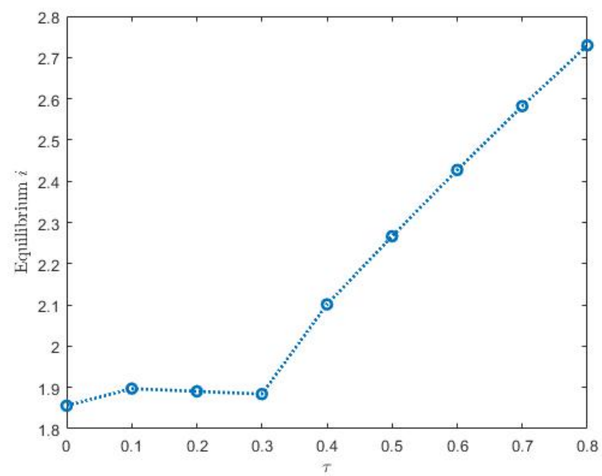

Figure 3C: Equilibrium debt policy and hedging policy

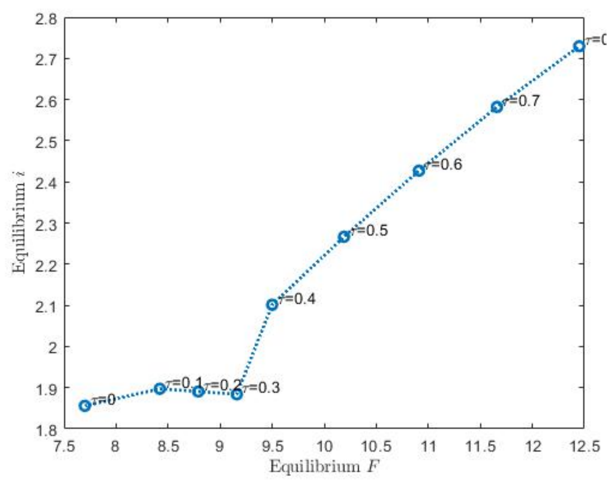

Figure 3D: Equilibrium credit spread and operational spread

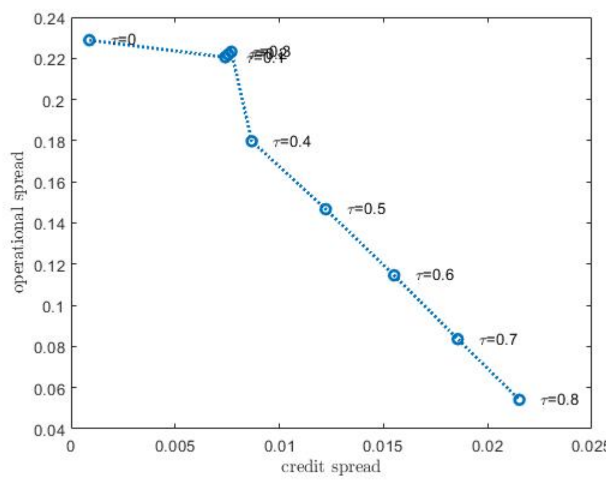

Figure 3: **Equilibrium debt policy, operational hedging policy and τ**

All parameters are presented in Table 1.

Figure 4A: Equilibrium debt policy and t

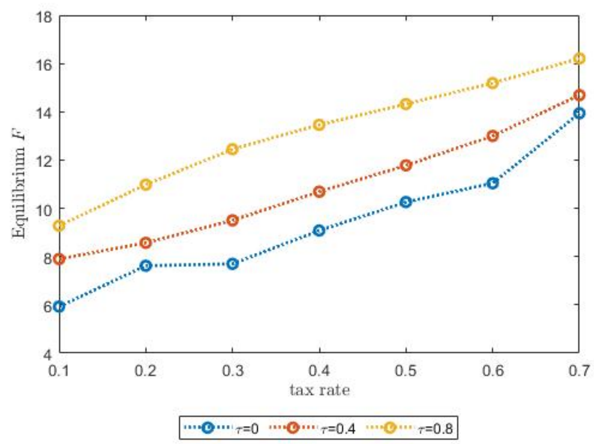

Figure 4B: Equilibrium hedging policy and t

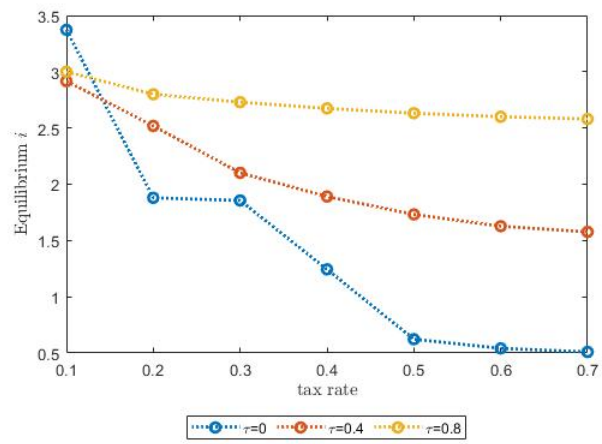

Figure 4C: Equilibrium debt policy and hedging policy

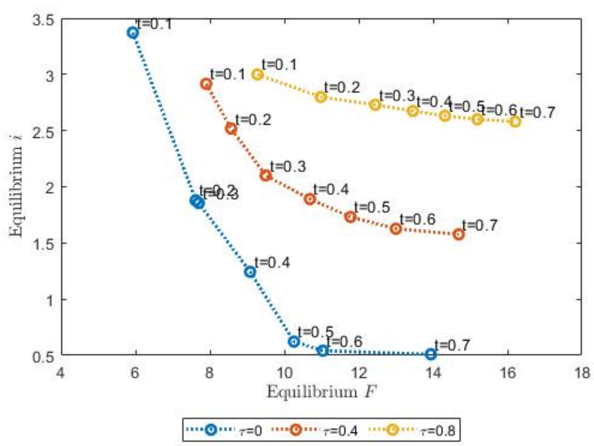

Figure 4D: Equilibrium credit spread and operational spread

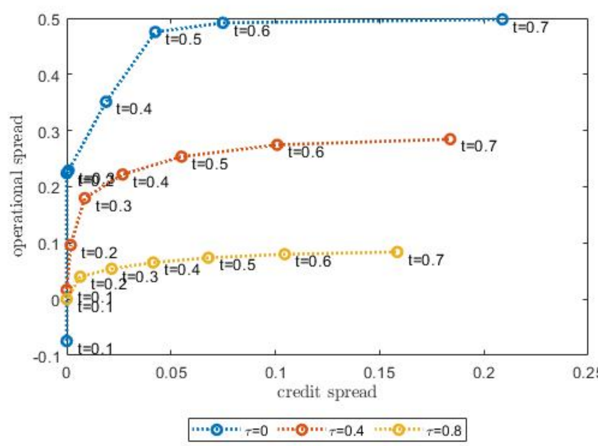

Figure 4: **Equilibrium debt policy, operational hedging policy and t**

All parameters are presented in Table 1.

Figure 5A: Negative z-score and markup

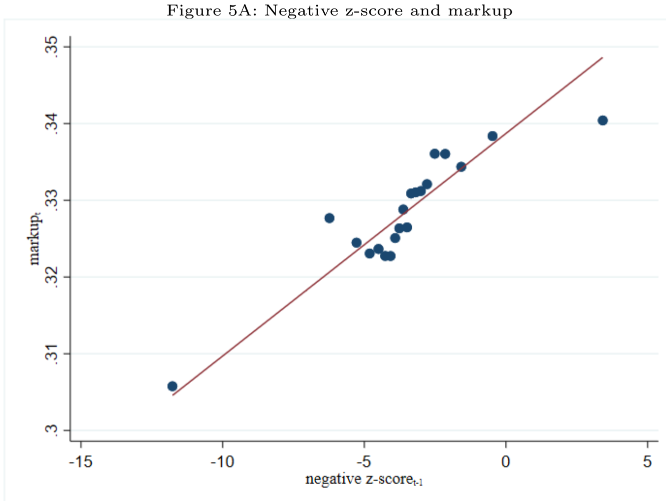

Figure 5B: Financial leverage and markup

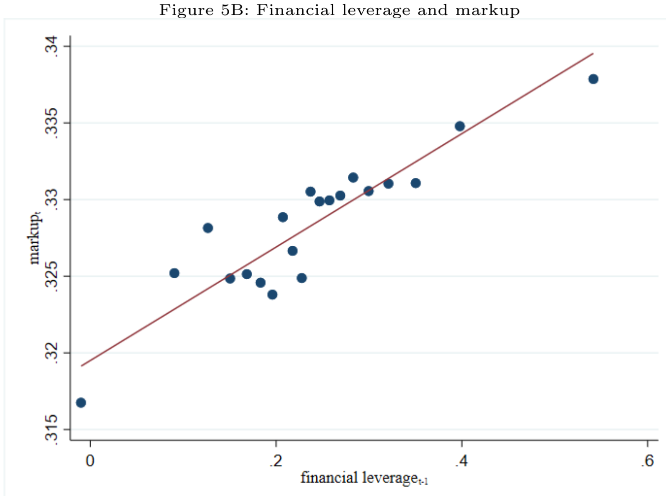

Figure 5: **Liquidity position and markup**

We first residualize the y-axis variable and x-axis variable with respect to the baseline control vector (including the fixed effects) in Table 4. We then add back the unconditional mean of the y and x variables in the estimation sample to facilitate interpretation of the scale. Finally we divide the (residualized) x-axis variable into twenty equal-sized groups (vingtiles) and plot the means of the y-variable within each bin against the mean value of x-variable within each bin.

Table 1: **Parameter values for numerical analysis**

Parameter	Interpretation	Value
α	Rate of the exponential distribution of u	0.05
I	Contractual delivery amount	3
κ	Production cost parameter	0.1
λ	Proportional cost of operational default	0.5
p	Unit price	1.2
t	Tax rate	0.3
x_0	Cash flow at date-0	5
$\bar{x}_1$	Certain cash flow at date-1	5
x_2	Franchise value at date-2	10

Table 2: **Summary statistics** — COMPUSTAT 1973-2020

This table presents the summary statistics of the variables in our sample from 1973 to April 2020. The data are quarterly from COMPUSTAT; The variable names are in parentheses. Markup = (sales(SALEQ) − cost of goods sold(COGSQ))/$Sales$. z-score is Altman (2013)'s measure calculated from quarterly data. Leverage = (long-term debt(DLTTQ) + short-term debt(DLCQ))/total assets(ATQ). Tobin's Q = (common shares outstanding(CHOQ) × stock price at the close of the fiscal quarter(PRCCQ)+preferred stock value(PSTKQ)+dividends on preferred stock(DVPQ)+ liabilities(LTQ))/total assets. Cash holdings ($CHEQ$), Cash flow ($= IBQ + DPQ$) and Tangible assets ($PPENTQ$) are divided by Total assets. Market power is measured by the following three variables, all employing Fama and French's 38 industries: Top 3 industry seller = 1 if the firm's sales are among the top three sellers in the industry (0 otherwise); Sales/Industry sales; and Herfindahl index. The operational hedging variables include Inventory ($INVQ$)/Sales and Supply chain diversification index, Supply chain diversification ranking and Standardizes supply chain diversification. They are composed from three raw measures: (i) log(1+number of suppliers), (ii) log(1+number of supplier regions), (iii) log(1+number of suppliers not from the firm's region). Data are quarterly (source: Factset), covering $6,066$ firms from mid-2003 to the first quarter of 2020. Supply chain diversification index is the first principal component score from a principal component analysis that equals $0.5809 \times$ (i) $+ 0.6077 \times$ (ii) $+ 0.5414 \times$ (iii) where (i)-(iii) indicate the above three measures. Supply chain diversification ranking is the average ranking of the firm-quarter ranking in terms of each of the individual measures. A smaller value of supply chain diversification ranking indicates a more diversified supply chain network. Standardized supply chain diversification is the average of the individual measures in each quarter standardized by their cross-section standard deviation for the quarter. A higher standardized supply chain diversification indicates a more diversified supply chain network.

The sample requires that the lagged firm capitalization is at least $10 million and quarterly sales are at least $1 million. All continuous variables are winsorized at both the 1st and 99th percentiles.

VARIABLES	N	mean	sd	p25	p50	p75
Markup: (sales-cogs)/sales	553,056	0.329	0.395	0.211	0.341	0.513
-(Z-score)	526,852	-3.567	5.687	-4.051	-2.116	-1.113
Financial leverage	538,459	0.238	0.213	0.048	0.207	0.363
Tobin's Q	553,056	1.962	1.525	1.087	1.454	2.198
Cash holdings	553,056	0.170	0.211	0.024	0.082	0.234
Cash flow	553,056	0.014	0.052	0.007	0.022	0.036
Asset tangibility	553,056	0.318	0.255	0.108	0.246	0.472
Top 3 industry seller	553,056	0.021	0.144	0.000	0.000	0.000
Sales/industry sales	553,056	0.006	0.017	0.000	0.001	0.003
Herfindahl index	553,056	0.058	0.053	0.027	0.043	0.068
Total assets	553,056	2,314.591	6,834.820	77.159	279.206	1,203.518
Inventory/sales	542,264	0.478	0.512	0.060	0.373	0.699
Supply chain diversification index	107,496	0.098	1.650	-1.165	-0.268	1.042
Supply chain diversification ranking	107,496	0.436	0.230	0.249	0.448	0.624
Standardized supply chain diversification	107,496	0.082	0.952	-0.641	-0.125	0.627

Table 3: **Markup and operational hedging**

Estimation of the relationship between Markup and measures of operational hedging. The variables are defined in Table 2. The control variables include Tobin's Q, ln(Total assets), Cash holdings, Cash flow, Tangible assets, Top 3 industry seller, Sales/total sales, and Herfindahl index. All explanatory variables are lagged. The regressions include firm and year fixed effects. Standard errors are clustered at firm and year levels. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	Log(markup)		
	(1)	(2)	(3)
Supply chain diversification index	-0.007**		
	(0.003)		
Supply chain diversification ranking		0.052***	
		(0.018)	
Standardized supply chain diversification			-0.012**
			(0.005)
Log(inventory/sales)	-0.045***	-0.045***	-0.045***
	(0.009)	(0.009)	(0.009)
Control variables		Yes	
Firm fixed effects		Yes	
Year fixed effects		Yes	
Observations	104,580	104,580	104,580
R-squared	0.868	0.868	0.868

Table 4: **Markup and liquidity position**

Estimation of the relationship between Markup, -(z-score) and Leverage. Leverage is also divided into the short-term debt maturing in 2 years and the remainder, both scaled by total assets. The control variables include Tobin's Q, ln(Total assets), Cash holdings, Cash flow, Tangible assets, Top 3 industry seller, Sales/total sales, and Herfindahl index. All explanatory variables are lagged. The regressions include year and firm fixed effects. Standard errors are clustered at firm and year levels. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively. The complete table is presented in Table A.1.

VARIABLES	Markup				
	(1)	(2)	(3)	(4)	(5)
-(Z-score)	0.003***		0.003***		0.003***
	(0.001)		(0.001)		(0.001)
Financial leverage		0.037***	0.010		
		(0.011)	(0.011)		
Long-term debt maturing in the next 2 years/total assets				0.073***	0.049***
				(0.017)	(0.016)
Remaining long-term leverage				0.034***	0.012
				(0.012)	(0.012)
Control variables			Yes		
Firm fixed effects			Yes		
Year fixed effects			Yes		
Observations	526,027	537,595	511,400	467,825	442,175
R-squared	0.630	0.623	0.630	0.641	0.646

Table 5: **Markup and liquidity position: Exposure to the financial crisis**

Regressions of Markup on firms' -(z-score) and Leverage that interact with the extent of exposures to the 2008 financial crisis. The two-year periods before and after the crisis are July 2006 to June 2008, and January 2009 to December 2010, respectively. The three measures for crisis exposure are % # Loans reduction, Lehman exposure and ABX exposure, using Chodorow-Reich (2014)'s definitions and data. The values of -(z-score) and Leverage are as of the end of 2007. The firm-level control variables, as in Table 4, are fixed at the end of year 2007, for the post-crisis years. The variable definitions are in Table 2. The regressions include year and firm fixed effects. Standard errors are clustered by firm. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

	Panel A: % # Loans reduction		Panel B: Lehman exposure		Panel C: ABX exposure	
VARIABLES	Markup		Markup		Markup	
	(1)	(2)	(3)	(4)	(5)	(6)
-(Z-score) × avg. lender exposure	0.160***		0.132		0.136***	
	(0.050)		(0.097)		(0.041)	
Leverage × avg. lender exposure		1.410***		2.413***		1.236***
		(0.376)		(0.702)		(0.338)
Avg. lender exposure	0.033	-0.320	0.207	-0.524	-0.073	-0.334
	(0.490)	(0.492)	(0.865)	(0.855)	(0.402)	(0.402)
Control variables	Yes					
Control variables × exposure	Yes					
Firm fixed effects	Yes					
Year fixed effects	Yes					
Observations	59,259	60,866	59,259	60,866	59,259	60,866
R-squared	0.743	0.742	0.743	0.742	0.743	0.742

Table 6: **Markup and leverage: State tax changes**

Regression of ΔMarkup, the quarterly change in Markup, on changes in state-level tax from 1990 to 2012. Tax increase (decrease) indicator is a dummy variable that equals one if the firm's headquarter state had a tax law change that increased (decreased, respectively) the state tax in the year that has just ended and in the preceding three quarters in that year. with the regression includes the first differences in the control variables in Table 4.

The regressions are estimated for the whole sample and for the sample split by three measures of financial constraint: Whited-Wu's measure, Size-age and Dividend payment. Firms are classified into the financially-constrained group, denoted FC, are the non-FC group every quarter by the values of these variables in the previous quarter, with 2/3 of the firms in the FC group.

The regression include fixed effects that are the product of the year and industry, using Fama and French's 38 industries. Robust standard errors are clustered at state level. $*$, $**$, $***$ denote significance below 10%, 5%, and 1% levels, respectively.

VARIABLES	ΔMarkup						
	Full sample	Whited-Wu		Size-age		Dividend payment	
		Non-FC	FC	Non-FC	FC	Non-FC	FC
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Tax increase indicator	0.004***	0.002	0.005***	-0.000	0.005***	-0.000	0.004***
	(0.001)	(0.003)	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)
Tax decrease indicator	-0.001	-0.002	-0.001	-0.001	-0.002	-0.001	-0.002
	(0.001)	(0.002)	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)
ΔControl variables				Yes			
Industry × year fixed effects				Yes			
Observations	328,709	105,808	219,546	110,264	214,993	88,529	239,915
R-squared	0.025	0.067	0.025	0.038	0.030	0.047	0.028

A. Appendix

Table A.1: **Markup and liquidity position — Complete table**

This table reports the complete table of Table 4.

VARIABLES	Markup				
	(1)	(2)	(3)	(4)	(5)
-(Z-score)	0.003***		0.003***		0.003***
	(0.001)		(0.001)		(0.001)
Financial leverage		0.037***	0.010		
		(0.011)	(0.011)		
Long-term debt maturing in the next 2 years/total assets				0.073***	0.049***
				(0.017)	(0.016)
Remaining long-term leverage				0.034***	0.012
				(0.012)	(0.012)
Tobin's Q	0.020***	0.014***	0.019***	0.013***	0.020***
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Log assets	0.010***	0.006*	0.010***	0.005	0.009***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Cash holdings	-0.025**	-0.042***	-0.024**	-0.041***	-0.020*
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Cash flow	0.881***	0.870***	0.883***	0.837***	0.855***
	(0.051)	(0.051)	(0.050)	(0.052)	(0.051)
Asset tangibility	0.020*	0.025**	0.020	0.027**	0.018
	(0.012)	(0.012)	(0.012)	(0.012)	(0.013)
Top 3 industry seller	0.011**	0.011**	0.010*	0.009*	0.009
	(0.005)	(0.005)	(0.006)	(0.005)	(0.006)
Sales/industry sales	-0.888***	-0.798***	-0.864***	-0.704***	-0.776***
	(0.136)	(0.136)	(0.137)	(0.132)	(0.134)
Herfindahl index	0.115***	0.111***	0.116***	0.106***	0.111***
	(0.035)	(0.034)	(0.035)	(0.031)	(0.032)
Firm fixed effects			Yes		
Year fixed effects			Yes		
Observations	526,027	537,595	511,400	467,825	442,175
R-squared	0.630	0.623	0.630	0.641	0.646

A.1 Second-order condition in benchmark case ($F = 0$)

The second-order derivative of $\bar{E}$ with respect to i is:

$$\frac{\partial^2 \bar{E}}{\partial i^2} = -K''(I+i) - \frac{\lambda x_2}{I^2} \frac{g'(u_O) - g(u_O)\frac{\delta''(u_O)}{\delta'(u_O)}}{[\delta'(u_O)]^2} < 0 \tag{A.1}$$

Since $\delta(u)$ is decreasing and convex in u, $\frac{\partial^2 \bar{E}}{\partial i^2}$ is always negative if the production commitment I is sufficiently high. In other words, the objective function $\bar{E}$ is concave in i. Thus, $\bar{i}$ is the unique optimal solution that maximizes the equity value (2.7).

A.2 Optimal hedging policy when $u_F \geq u_O$

We begin this subsection by proving Lemma 2.1. First, we show that i^* that satisfies the first-order condition (2.12) is the unique optimal solution for the maximization problem when $u_F > u_O$. Define $S = p - K'(I+i) - V(u_F, i)h(u_F)K'(I+i)$.[28] Taking the derivative of S with respect to i:

$$\frac{\partial S}{\partial i} = -\left[\begin{array}{c} K''(I+i) + \frac{\partial V(u_F,i)}{\partial i} h(u_F) K'(I+i) \\ +V(u_F,i)\frac{\partial h(u_F)}{\partial u_F}\frac{\partial u_F}{\partial i} K'(I+i) + V(u_F,i)h(u_F)\frac{\partial^2 u_F}{\partial i^2} \end{array} \right] \tag{A.2}$$

$$\frac{\partial V(u_F, i)}{\partial i} = p[1 - \delta'(u_F) I K'(I+i)] > 0 \tag{A.3}$$

and

$$\frac{\partial^2 u_F}{\partial i^2} = K''(I+i) > 0 \tag{A.4}$$

[28] S the term inside the bracket after $[1 - G(u_F)]$ of the left hand-side of the first-order condition (2.11).

Using these quantities,

$$\frac{\partial S}{\partial i} = -\left[\begin{array}{c} K''(I+i) + p[1-\delta'(u_F)IK'(I+i)]h(u_F)K'(I+i) \\ +V(u_F,i)\frac{\partial h(u_F)}{\partial u_F}[K'(I+i)]^2 + V(u_F,i)h(u_F)K''(I+i) \end{array}\right] \tag{A.5}$$

$\frac{\partial S}{\partial i}$ is smaller than zero. Thus, the second-order condition for maximization $[1-G(u_F)]\frac{\partial S}{\partial i}$ at $i = i^*$ is smaller than zero. By the first-order condition (2.11), $S = 0$ if $i = i^*$. Since $\frac{\partial S}{\partial i} < 0$, we have $S > 0$ if $i < i^*$ and $S < 0$ if $i > i^*$. Since $\frac{\partial}{\partial i}E = [1-G(u_F)]S$, E increases in i for $i < i^*$ and decreases in i for $i > i^*$. Therefore i^* is the unique optimal solution to the maximization problem.

Now we prove that Assumption 2.3 is sufficient condition that guarantees a positive interior solution i^* and $D(i^*, \bar{F}) > 0$ when $\bar{F}$ is sufficiently large. Denote $\underline{i}$ such that $p - K'(I+\underline{i}) = (p(I+\underline{i}) + x_2)\alpha K'(I+\underline{i})$. Notice that $\underline{i}$ must be strictly greater than zero. This is because the left hand-side of the above equation decreases with i, the right hand-side increases with i, and left hand-side is strictly greater than the right hand-side when $i = 0$ by Assumption 2.3.[29] For any $\bar{F} > 0$, the right hand-side of the first-order condition (2.13) when $i = \underline{i}$ is $V(u_F, \underline{i})\alpha K'(I+\underline{i})$, which is smaller than $(p(I+\underline{i})+x_2)\alpha K'(I+\underline{i}) = p - K'(I+\underline{i})$. The left hand-side of the first-order condition (2.13) decreases with i. The right hand-side of the first-order condition (2.13) increases with i.[30] So the optimal i^* that satisfies the first-order condition (2.13) must be strict greater than $\underline{i}$. Denote $\bar{F}_M$ such that $D(\underline{i}, \bar{F}_M) = 0$. Then for any $\bar{F} \geq \bar{F}_M$, we must have $D(i^*(\bar{F}), \bar{F}) > D(\underline{i}, \bar{F}) > 0$. This is because $D(\bar{F}, i)$ increases in $\bar{F}$ and i, and $i^*(\bar{F}) > \underline{i}$. Thus, we have proved that for $\bar{F} > \bar{F}_M$, the first-order condition (2.13) admits a positive interior solution i^* and the financial default boundary u_F is greater than the operational default boundary u_O when the firm chooses the optimal hedging policy

[29] The monotonicities of the left and right hand-side are due to the fact that $K(I+i)$ is convex in i.

[30] This is because u_F increases with i and $\delta(u)$ decreases with u. Consequently, $(1-\delta(u_F))$ increases with i. $K'(I+i)$ increases with i because the convexity of K in i.

i^*. Since we have proved that the first-order condition (2.13) is also the sufficient condition for the solution of the constrained maximization problem subject to $D(i, \bar{F}) > 0$, we have proved Lemma 2.1.

In what follows, we proof Lemma 2.2: The firm's optimal operational hedging policy i^* decreases in $\bar{F}$. Define $M(i^*(\bar{F}), \bar{F}) \equiv E(i^*(\bar{F}), \bar{F})$ the value function under optimal hedging policy i^*.[31] By the first-order condition, $\frac{\partial M}{\partial i^*} = 0$. Differentiating both sides with respect to $\bar{F}$:

$$\frac{\partial^2 M}{\partial i^{*2}} \frac{\partial i^*}{\partial \bar{F}} + \frac{\partial M}{\partial i \partial \bar{F}} = 0 \tag{A.6}$$

From equation (A.6) we get $\frac{\partial i^*}{\partial \bar{F}} = -\frac{\partial^2 M}{\partial i^* \partial \bar{F}} / \frac{\partial^2 M}{\partial i^{*2}}$. Since $\frac{\partial^2 M}{\partial i^{*2}} < 0$ by the second-order condition, so the sign of $\frac{\partial i^*}{\partial \bar{F}}$ is the same as the sign of $\frac{\partial M}{\partial i^* \partial \bar{F}}$. Taking the partial derivative of the first-order condition (2.11) with respect to $\bar{F}$

$$\begin{aligned}
\frac{\partial^2 M}{\partial i^* \partial \bar{F}} &= [1 - G(u_F)] \left[pI\delta'(u_F) \frac{\partial u_F}{\partial \bar{F}} h(u_F) K'(I + i^*) - V(u_F, i^*) \frac{\partial h(u_F)}{\partial u_F} \frac{\partial u_F}{\partial \bar{F}} K'(I + i) \right] \\
&= [1 - G(u_F)] \left[pI\delta'(u_F) h(u_F) K'(I + i^*) - V(u_F, i^*) \frac{\partial h(u_F)}{\partial u_F} K'(I + i) \right]
\end{aligned} \tag{A.7}$$

Since u follows a exponential distribution, $\frac{\partial h(u_F)}{\partial u_F} = 0$. Thus, Equation (A.7) is smaller than zero. Therefore, $\frac{\partial i^*}{\partial \bar{F}} < 0$.

A.3 Optimal hedging policy when $u_F < u_O$

We begin this subsection by proving Lemma 2.3. First, we show that $\hat{i}^*$ that satisfies the first-order condition (2.16) is the unique optimal solution for the maximization problem. Define $\hat{S} = p - K'(I + i) - [V(u_F, i) - \lambda x_2] h(u_F) K'(I + i) - \frac{\lambda x_2 g(u_O)}{1 - G(u_F)} \frac{\partial u_O}{\partial i}$.[32] Taking the

[31] The optimal hedging policy i^* and the associated financial default boundary u_F are all functions of $\bar{F}$.
[32] $\hat{S}$ the term inside the bracket after $[1 - G(u_F)]$ of the left hand-side of the first-order condition (2.15).

derivative of $\hat{S}$ with respect to i:

$$\frac{\partial \hat{S}}{\partial i} = -\left[\begin{array}{c} K''(I+i) + \frac{\partial V(u_F,i)}{\partial i} h(u_F)K'(I+i) + [V(u_F,i) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}\frac{\partial u_F}{\partial i}K'(I+i) \\ +[V(u_F,i) - \lambda x_2]h(u_F)\frac{\partial^2 u_F}{\partial i^2} + \lambda x_2 \frac{\partial}{\partial i}\left[\frac{g(u_O)}{[1-G(u_F)]I\delta'(u_O)}\right] \end{array}\right] \tag{A.8}$$

$$\frac{\partial}{\partial i}\left[\frac{g(u_O)}{[1-G(u_F)]I\delta'(u_O)}\right] = \left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I+i)g(u_O)}{[1-G(u_F)]^2}\right]\frac{1}{I\delta'(u_O)} \tag{A.9}$$

The absolute value of (A.9) is small if the production commitment I is sufficiently high and $\frac{K'(I+\bar{i})}{I}$ is sufficiently low. In the numerical analysis, we assume that $K(I+i)$ is of quadratic form, $K(I+i) = \kappa(I+i)^2$, where $\kappa > 0$, which is standard in the investment literature. Then $\frac{K'(I+\bar{i})}{I}$ is sufficiently low if κ is sufficiently small. Using quantities (A.3), (A.4) and (A.9), $\frac{\partial \hat{S}}{\partial i}$ is

$$\frac{\partial \hat{S}}{\partial i} = -\left[\begin{array}{c} K''(I+i) + p[1-\delta'(u_F)IK'(I+i)]h(u_F)K'(I+i) \\ +[V(u_F,i) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}[K'(I+i)]^2 + [V(u_F,i) - \lambda x_2]h(u_F)K''(I+i) \\ +\lambda x_2\left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I+i)g(u_O)}{[1-G(u_F)]^2}\right]\frac{1}{I\delta'(u_O)} \end{array}\right] \tag{A.10}$$

$\frac{\partial S}{\partial i}$ is always smaller than zero, thus, the second-order condition for maximization $[1 - G(u_F)]\frac{\partial \hat{S}}{\partial i}$ at $i = \hat{i}^*$ is smaller than zero. By the first-order condition (2.15), $S = 0$ if $i = \hat{i}^*$. Since $\frac{\partial \hat{S}}{\partial i} < 0$, we have $\hat{S} > 0$ if $i < \hat{i}^*$ and $\hat{S} < 0$ if $i > \hat{i}^*$. Since $\frac{\partial}{\partial i}\hat{E} = [1 - G(u_F)]\hat{S}$, $\hat{E}$ increases in i for $i < \hat{i}^*$ and decreases in i for $i > \hat{i}^*$. Therefore $\hat{i}^*$ is the unique optimal solution to the maximization problem.

Now we prove Lemma 2.4: $\hat{i}^* > i^*$. i^* satisfies the first-order condition (2.12):

$$\begin{aligned} p - K'(I+i^*) &= V(u_F, i^*)h(u_F)K'(I+i^*) \\ &> V(u_F, i^*)h(u_F)K'(I+i^*) - \lambda x_2 h(u_F)K'(I+i^*) + \frac{\lambda x_2 g(u_O)}{[1-G(u_F)]I\delta'(u_O)} \end{aligned} \tag{A.11}$$

The inequality holds because $\lambda x_2 h(u_F)K'(I+i^*) > 0$ and $\frac{\lambda x_2 g(u_O)}{[1-G(u_F)]I\delta'(u_O)} < 0$. Now taking derivative of both sides of the first-order condition in $u_O > u_F$ case, (2.16), with respect to i. The derivative of the left-hand side is $-K''(I+i) < 0$. The derivative of the right-hand side is

$$\begin{aligned} & p[1-\delta'(u_F)IK'(I+i)]h(u_F)K'(I+i) + [V(u_F,i) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}[K'(I+i)]^2 \\ & + [V(u_F,i) - \lambda x_2]h(u_F)K''(I+i) \\ & + \lambda x_2 \left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I+i)g(u_O)}{[1-G(u_F)]^2}\right]\frac{1}{I\delta'(u_O)} \end{aligned} \tag{A.12}$$

The quantity (A.12) is always greater than zero if the production commitment I is sufficiently high and $\frac{K'(I+\bar{i})}{I}$ is sufficiently low. Thus the left-hand side of Equation (2.16) decreases in i and the right-hand side of Equation (2.16) increases in i. Since $\hat{i}^*$ satisfies the first-order condition in $u_O > u_F$ case, (2.16). We must have $\hat{i}^* > i^*$.

In what follows, we prove Lemma 2.5: the firm's optimal operational hedging policy $\hat{i}^*$ decreases in $\bar{F}$. Define $\hat{M}(\hat{i}^*(\bar{F}), \bar{F}) \equiv E(\hat{i}^*(\bar{F}), \bar{F})$ the value function under optimal hedging policy $\hat{i}^*$.[33] Similar to the $u_F > u_O$ case, $\frac{\partial \hat{i}^*}{\partial \bar{F}} = -\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}} / \frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$. Since $\frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}} < 0$ by the second-order condition, so the sign of $\frac{\partial \hat{i}^*}{\partial \bar{F}}$ is the same as the sign of $\frac{\partial \hat{M}}{\partial \hat{i}^* \partial \bar{F}}$. Taking the partial

[33] The optimal hedging policy $\hat{i}^*$ and the associated financial default boundary u_F are all functions of $\bar{F}$.

derivative of the first-order condition (2.11) with respect to $\bar{F}$

$$\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}} = [1 - G(u_F)] \left[\begin{array}{c} pI\delta'(u_F)\frac{\partial u_F}{\partial \bar{F}} h(u_F)K'(I + \hat{i}^*) - [V(u_F, \hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}\frac{\partial u_F}{\partial \bar{F}}K'(I + i) \\ -\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)}\frac{\partial u_F}{\partial \bar{F}} \end{array} \right]$$

$$= [1 - G(u_F)] \left[\begin{array}{c} pI\delta'(u_F)h(u_F)K'(I + i^*) - [V(u_F, i^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}K'(I + i) \\ -\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} \end{array} \right] \tag{A.13}$$

Since u follows an exponential distribution, $\frac{\partial h(u_F)}{\partial u_F} = 0$. Thus, Equation (A.13) is always smaller than zero if the production commitment I is sufficiently high. Therefore, $\frac{\partial i^*}{\partial \bar{F}} < 0$ if the production commitment I is sufficiently high.

A.4 Optimal operational hedging policy and net debt $\bar{F}$

First of all, $\bar{i}$ in Appendix A.1 is the optimal equity-maximizing hedging policy given the inherited net short-term debt level $\bar{F}$ is sufficiently low, i.e., $\bar{F} \leq \bar{F}_{fb}$. $\bar{F}_{fb}$ is such that $\bar{F}_{fb} + K(I + \bar{i}) = 0$, i.e., $\bar{F}_{fb}$ is the maximal net debt level such that the firm is able to pay back the debt at date-1 it chooses the maximal optimal hedging policy $\bar{i}$ that maximizes the unlevered firm value. When $\bar{F} > \bar{F}_{fb}$, the firm has to choose the optimal hedging policy i that balances the concerns over financial and operational default, which we elaborate on below.

The following lemma is for technical purpose. It facilitates our proof that both $D^*(\bar{F}) = 0$ and $\hat{D}^*(\bar{F}) = 0$ has unique solutions, which we denote as $\bar{F}_0$ and $\bar{F}_1$, respectively.

Lemma A.1.

$$\frac{dD^*}{d\bar{F}} > 0 \ \textit{if} \ u_F(i^*) \geq u_O(i^*) \tag{A.14a}$$

$$\frac{d\hat{D}^*}{d\bar{F}} > 0 \ \textit{if} \ u_F(\hat{i}^*) \geq u_O(\hat{i}^*) \tag{A.14b}$$

Proof. First we prove the following inequality:

$$\frac{dD^*}{d\bar{F}} = \frac{\partial D^*}{\partial \bar{F}} + \frac{\partial D^*}{\partial i^*}\frac{\partial i^*}{\partial \bar{F}} > 0 \tag{A.15}$$

Using Equations (2.6a) and (2.6b) Inequality (A.15) is equivalent to

$$\left[K'(I+i^*) - \frac{1}{I\delta'(u_O)}\right]\left(-\frac{\partial i^*}{\partial \bar{F}}\right) < 1 \tag{A.16}$$

From Appendix A.2, $\frac{\partial i^*}{\partial \bar{F}} = -\frac{\partial^2 M}{\partial i^* \partial \bar{F}} / \frac{\partial^2 M}{\partial i^{*2}}$. $\frac{\partial^2 M}{\partial i^* \partial \bar{F}}$ is given by Equation (A.7). $\frac{\partial^2 M}{\partial i^{*2}}$ is given by $[1 - G(u_F)]\frac{\partial S}{\partial i^*}$ where $\frac{\partial S}{\partial i^*}$ is given by Equation (A.5) at $i = i^*$. Thus, Inequality (A.16) is equivalent to

$$\frac{V(u_F,i^*)\frac{\partial h(u_F)}{\partial u_F}K'(I+i^*) - pI\delta'(u_F)h(u_F)K'(I+i^*)}{\left[\begin{array}{c} K''(I+i^*) + p[1-\delta'(u_F)IK'(I+i^*)]h(u_F)K'(I+i^*) \\ +V(u_F,i^*)\frac{\partial h(u_F)}{\partial u_F}[K'(I+i^*)]^2 + V(u_F,i^*)h(u_F)K''(I+i^*) \end{array}\right]} \frac{1 - I\delta'(u_O)K'(I+i^*)}{-I\delta'(u_O)} < 1 \tag{A.17}$$

Algebraic simplification shows that the above inequality is equivalent to

$$\begin{aligned} &V(u_F,i^*)\frac{\partial h(u_F)}{\partial u_F}K'(I+i^*) + pI\left[\delta'(u_O) - \delta'(u_F)\right]h(u_F)K'(I+i^*) \\ &< \left[1 + V(u_F,i^*)h(u_F)\right]K''(I+i^*)\left[-I\delta'(u_O)\right] \end{aligned} \tag{A.18}$$

Since u follows a exponential distribution, $\frac{\partial h(u)}{\partial u} = 0$ and the first term of the left-hand side of Inequality (A.18) is equal to zero. the second term on the left-hand side is (weakly) smaller than zero if $u_F \geq u_O$ because $\delta(u)$ is convex in u. Therefore the left-hand side of Inequality (A.18) is (weakly) smaller than zero. The right-hand side of Inequality (A.18) is strictly greater than zero. Therefore, Inequality (A.18) holds and $\frac{dD^*}{d\bar{F}} > 0$.

Now we prove the following inequality:

$$\frac{d\hat{D}^*}{d\bar{F}} = \frac{\partial \hat{D}^*}{\partial \bar{F}} + \frac{\partial \hat{D}^*}{\partial \hat{i}^*}\frac{\partial \hat{i}^*}{\partial \bar{F}} > 0 \tag{A.19}$$

Inequality (A.15) is equivalent to

$$\left[K'(I+\hat{i}^*) - \frac{1}{I\delta'(u_O)}\right]\left(-\frac{\partial \hat{i}^*}{\partial \bar{F}}\right) < 1 \tag{A.20}$$

From Appendix A.3, $\frac{\partial \hat{i}^*}{\partial \bar{F}} = -\frac{\partial^2 \hat{M}}{\partial \hat{i}^* \partial \bar{F}} / \frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$. $\frac{\partial^2 \hat{M}}{\partial i^* \partial \bar{F}}$ is given by Equation (A.13). $\frac{\partial^2 \hat{M}}{\partial \hat{i}^{*2}}$ is given by $[1-G(u_F)]\frac{\partial \hat{S}}{\partial \hat{i}^*}$ where $\frac{\partial \hat{S}}{\partial \hat{i}^*}$ is given by Equation (A.10) at $i = \hat{i}^*$. Thus, Inequality (A.20) is equivalent to

$$\frac{\begin{bmatrix} [V(u_F,\hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}K'(I+\hat{i}^*) \\ -pI\delta'(u_F)h(u_F)K'(I+\hat{i}^*) \\ +\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)} \end{bmatrix}}{\begin{bmatrix} K''(I+\hat{i}^*) + p[1-\delta'(u_F)IK'(I+\hat{i}^*)]h(u_F)K'(I+\hat{i}^*) \\ +[V(u_F,\hat{i}^*) - \lambda x_2]\frac{\partial h(u_F)}{\partial u_F}[K'(I+\hat{i}^*)]^2 \\ +[V(u_F,\hat{i}^*) - \lambda x_2]h(u_F)K''(I+\hat{i}^*) \\ +\lambda x_2\left[\frac{g'(u_O)\delta'(u_O) - g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} + \frac{g(u_F)K'(I+\hat{i}^*)g(u_O)}{[1-G(u_F)]^2}\right]\frac{1}{I\delta'(u_O)} \end{bmatrix}} \frac{1 - I\delta'(u_O)K'(I+\hat{i}^*)}{-I\delta'(u_O)} < 1 \tag{A.21}$$

Algebraic simplification shows that the above inequality is equivalent to

$$[V(u_F,\hat{i}^*)-\lambda x_2]\frac{\partial h(u_F)}{\partial u_F}K'(I+\hat{i}^*)+pI\left[\delta'(u_O)-\delta'(u_F)\right]h(u_F)K'(I+\hat{i}^*)+\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)}$$
$$<\Big[1+[V(u_F,\hat{i}^*)-\lambda x_2]h(u_F)\Big]K''(I+\hat{i}^*)\left[-I\delta'(u_O)\right]-\lambda x_2\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} \tag{A.22}$$

Since u follows a exponential distribution, $\frac{\partial h(u)}{\partial u}=0$ and the first term of the left-hand side of Inequality (A.22) is equal to zero. the second term on the left-hand side is (weakly) smaller than zero if $u_F \geq u_O$ because $\delta(u)$ is convex in u. The first term of the right-hand side of Inequality (A.22) is strictly greater than zero. Therefore, to show that Inequality (A.22) holds, we need to show that:

$$\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)}<-\lambda x_2\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I} \tag{A.23}$$

Or, equivalently,

$$\frac{\lambda x_2}{I}\frac{g(u_O)g(u_F)}{[1-G(u_F)]^2\delta'(u_O)}+\lambda x_2\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{[1-G(u_F)][\delta'(u_O)]^2 I}<0$$
$$\Leftrightarrow \frac{\lambda x_2}{I[1-G(u_F)]\delta'(u_O)}\left[\frac{g(u_O)g(u_F)}{[1-G(u_F)]}+\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{\delta'(u_O)}\right]<0$$
$$\Leftrightarrow \frac{g(u_O)g(u_F)}{[1-G(u_F)]}+\frac{g'(u_O)\delta'(u_O)-g(u_O)\delta''(u_O)}{\delta'(u_O)}>0 \tag{A.24}$$

Since $g(u)=\alpha\exp(-\alpha u)$, $\alpha g(u)=-g'(u)$, and $\frac{g(u_F)}{[1-G(u_F)]}=\alpha$, the inequality (A.24) is equivalent to

$$\frac{\delta''(u_O)}{\delta'(u_O)}<0 \tag{A.25}$$

which always holds since $\delta(u)$ decreases and convex in u by assumption. Therefore, $\frac{d\hat{D}^*}{dF}>0$. Q.E.D.

Now we prove Proposition 2.1. First, i^* and $\hat{i}^*$ are continuously differentiable in $\bar{F}$ and $D(i, \bar{F})$ is continuously differentiable in both i and f. It follows that $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ are continuously differentiable, thus continuous in $\bar{F}$.

Secondly, from Section 2.4 and Section 2.5, we know that u_F is greater than u_O, i.e., $D^*, \hat{D}^* > 0$ when $\bar{F}$ is sufficiently high, i.e., $\bar{F} \geq \bar{F}_M$.[34] On the other hand, if $F = 0$, $u_F = 0$, which is always lower than u_O. Since $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ are continuous in $\bar{F}$, $D^*, \hat{D}^* < 0$ for all $\bar{F}$ that is sufficiently low. Again by the continuity of $D^*(\bar{F})$ and $\hat{D}^*(\bar{F})$ in $\bar{F}$, there must exist $\bar{F}_0$ and $\bar{F}_1$ such that $\hat{D}^*(\bar{F}_0) = 0$ and $D^*(\bar{F}_1) = 0$. By Lemma A.1, $\frac{d\hat{D}^*}{d\bar{F}} > 0$ whenever $\hat{D}^* \geq 0$ and $\frac{dD^*}{d\bar{F}} > 0$ whenever $D^* \geq 0$. It follows that $\bar{F}_0$ and $\bar{F}_1$ are unique. Moreover, $\hat{D}^* < 0$ for all $\bar{F} < \bar{F}_0$ and $\hat{D}^* > 0$ for all $\bar{F} > \bar{F}_0$. Similarly, $D^* < 0$ for all $\bar{F} < \bar{F}_1$ and $D^* > 0$ for all $\bar{F} > \bar{F}_1$.

From Lemma 2.4, $\hat{i}^* > i^*$ for any given $\bar{F}$. At $\bar{F} = \bar{F}_1$, $D^*(\bar{F}_1) = 0$. Since $\frac{\partial D}{\partial i} > 0$, we must have $\hat{D}^*(\bar{F}_1) = D(\hat{i}^*(\bar{F}_1), \bar{F}_1) > 0$. Thus, $\bar{F}_1 > \bar{F}_0$.

If $\bar{F} \leq \bar{F}_0$, then $D^* < 0$ and $\hat{D}^* \leq 0$. Thus, maximizing the firm's equity value minus the manager's aversion to financial default subject to $u_F \leq u_O$ will yield the optimal operational hedging policy $\hat{i}^*$. Meanwhile, maximizing the firm's equity value minus the manager's aversion to financial default subject to $u_F \geq u_O$ will yield a corner solution $\tilde{i} > i^*$ otherwise, in which $\tilde{i}$ is such that $D(\tilde{i}, \bar{F}) = 0$.[35] Since $\tilde{i}$ is also feasible for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \leq u_O$ and $\hat{E} = E$ when $i = \tilde{i}$, $\tilde{i}$ must yield a lower expected payoff for the shareholders and the manager combined compared with $\hat{i}^*$. Thus, the firm's optimal operational hedging policy is $\hat{i}^*$.

[34] From Lemma 2.1, $D^* > 0$ if $\bar{F} \geq \bar{F}_M$. From Lemma 2.4, for a given $\bar{F}$, $\hat{i}^* > i^*$. Since $D(i, \bar{F})$ increases in i, $\hat{D}^* > 0$ when $\bar{F} \geq \bar{F}_M$.

[35] Since $\frac{\partial D}{\partial i} > 0$, the feasible set of i for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \geq u_O$, if not empty, contains i values all higher than i^*. From Appendix A.2, E decreases in i for $i > i^*$.

If $\bar{F}_0 < \bar{F} < \bar{F}_1$, then $D^* < 0$ and $\hat{D}^* > 0$. Thus, maximizing the firm's equity value minus the manager's aversion to financial default subject to $u_F \leq u_O$ or subject to $u_F \geq u_O$ will yield the same corner solution $\tilde{i}$, in which $\tilde{i}$ is such that $D(\tilde{i}, \bar{F}) = 0$.[36] Thus, the firm's optimal operational hedging policy is $\tilde{i}$.

If $\bar{F} \geq \bar{F}_1$, then $D^* \geq 0$ and $\hat{D}^* > 0$. Thus, maximizing the firm's equity value minus the manager's aversion to financial default subject to $u_F \geq u_O$ will yield the optimal operational hedging policy i^*. Meanwhile, maximizing the firm's equity value minus the manager's aversion to financial default subject to $u_F < u_O$ will yield a corner solution $\tilde{i} < \hat{i}^*$.[37] Since $\tilde{i}$ is also feasible for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \geq u_O$ and $\hat{E} = E$ when $i = \tilde{i}$, $\tilde{i}$ must yield a lower expected payoff for the shareholders and the manager combined compared with i^*. Thus, the firm's optimal operational hedging policy is i^*.

Now we prove Proposition 2.2. From Proposition 2.1 and Lemma 2.2, when $\bar{F} > \bar{F}_1$, $i^{**} = i^*$ and thus decreases in $\bar{F}$. Similarly, from Proposition 2.1 and Lemma 2.5, when $\bar{F} < \bar{F}_0$, $i^{**} = \hat{i}^*$ and thus decreases in $\bar{F}$. Moreover, $\frac{\partial \tilde{i}}{\partial \bar{F}} = -\frac{\partial D}{\partial \bar{F}} / \frac{\partial D}{\partial \tilde{i}}$. Since both partial derivatives on the right-hand side are positive from Inequalities (2.6a) and (2.6b), $\frac{\partial \tilde{i}}{\partial \bar{F}} < 0$. When $\bar{F}_0 < \bar{F} < \bar{F}_1$, $i^{**} = \tilde{i}$ and thus decreases in $\bar{F}$. Lastly, at $\bar{F} = \bar{F}_1$, since $D^* = 0$, $i^* = \tilde{i}$, so $i^{**} = i^* = \tilde{i}$ at $\bar{F} = \bar{F}_1$ and thus is continuous in $\bar{F}$ at $\bar{F} = \bar{F}_1$. Similarly, at $\bar{F} = \bar{F}_0$, since $\hat{D}^* = 0$, $\hat{i}^* = \tilde{i}$, so $i^{**} = \hat{i}^* = \tilde{i}$ at $\bar{F} = \bar{F}_0$ and thus is continuous in $\bar{F}$ at $\bar{F} = \bar{F}_0$. Therefore, i^{**} decreases in $\bar{F}$.

[36] This is because, the feasible set i for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \geq u_O$ contains i values higher than i^* and from Appendix A.2, E decreases in i for $i > i^*$. Meanwhile, the feasible set i for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \leq u_O$ contains i values lower than $\hat{i}^*$ and from Appendix A.3, $\hat{E}$ increases in i for $i < i^*$.

[37] Since $\frac{\partial D}{\partial i} > 0$, the feasible set of i for the maximization problem of the firm's equity value minus the manager's aversion to financial default subject to $u_F \leq u_O$, if not empty, contains i values lower than $\hat{i}^*$ and from Appendix A.3, $\hat{E}$ increases in i for $i < \hat{i}^*$.