Collateral, Haircuts and Rates: a Theory of Repo

[ Dmitry Chebotarev ]  INSEAD

What is Repo?

- Repo is a form of lending collateralized by a portfolio of securities.
- Repo market is systemically important (Gorton and Metrick, 2012), with a daily turnover of $3 trillion globally (ICMA, 2019).
- A repo deal has not only a price condition (interest rate, \( r \)), but also a degree of collateralization (haircut, \( h \)).

Two Questions

Q1: How does collateral quality affect repo parameters?

Finding 1: Value-at-Risk (VaR) and Expected Shortfall (ES) arise endogenously as sufficient statistics of the quality of collateral, i.e. its return distribution.

Finding 2: \( ES \uparrow \Rightarrow h^\dagger, r^\dagger \)

Finding 3: VaR \( \uparrow \Rightarrow h^\dagger, r^\dagger \)

Q2: How do borrower’s properties affect repo parameters?

Finding 4: While riskier borrowers face higher haircuts, they do not necessarily pay higher rates.

Finding 5: Borrowers that possess more profitable investment opportunities borrow with a smaller haircut at a cost of paying a higher rate.

In Brief, This Paper...

1. Endogenizes the effect of collateral quality on haircuts and rates (Adrian and Shin (2013), Dang et al. (2013)).
2. Suggests a solution to the VaR vs ES debate (Artzner (1999), Acerbi and Tasche (2002), BIS (2016)).
3. Suggests a framework to resolve some puzzling empirical patterns (Benmelech and Bergman (2009), Auh and Landoni (2016)).

Model

A two-period model with two risk-neutral agents, borrower (\( b \)) and lender (\( l \)).

Borrower: penniless, has a private investment opportunity, possesses one unit of pledgeable financial asset worth $1.

Lender: competitive, deep-pocketed, can invest in a riskless asset with return \( (1 + r_f) \) or lend the borrower some amount (\( M \)).

Investment opportunity: binomial, scalable (CRS).

Pledgeable financial asset: return \( R \) distributed with a cdf \( F(R) \in C^1 \), independent of the borrower’s investment opportunity.

Assumption 1: difference in beliefs. Agent \( i \) believes \( P = P_i, i \in \{ b, l \} \), so that
\[ NPV_b \triangleq (1 + \rho) \times (1 - P_b) - (1 + r_f) > 0, \]
\[ NPV_l \triangleq (1 + \rho) \times (1 - P_l) - (1 + r_f) < 0. \]

Assumption 2: borrower prefers to keep the financial asset rather than selling it (i.e., due to immediate selling costs).

Borrower’s expected utility:
\[ W(r, M) = \frac{M \times (1 - P_b)}{\max\{R, (1 + r_f)M, 0\}} \times P_b. \]

Lender’s expected utility:
\[ U(r, M) = \frac{(1 + r_f)M}{\max\{R, (1 + r_f)M, 0\}} \times P_l. \]

Equilibrium

Definition (haircut): \((1 + h) \triangleq \frac{1}{\alpha}\).

Definition (equilibrium): The repo market equilibrium is a contract \((r_q, h_q)\) such that the borrower’s utility \( W \) is maximized subject to the lender’s break-even condition \( U(r, M) = 0 \).

\[ 1 + r_q = (1 + r_f) \times \left( \frac{PD}{1 - \frac{ES(\alpha) - VaR(\alpha)}{1 - VaR(\alpha)}} \right)^{-1}, \]

where \( \alpha \text{-const.}, \alpha \in [0,1] \).

Comparative Statistics -1 (Collateral)

VaR and ES are tightly related, but represent different aspects of market risk. One needs to first orthogonalize them.

\[ \frac{dh}{dES(\alpha)} = \frac{VaR(\alpha)}{ES(\alpha) - VaR(\alpha)} > 0, \]
\[ \frac{dr}{dES(\alpha)} = \frac{VaR(\alpha)}{ES(\alpha) - VaR(\alpha)} > 0, \]
\[ \frac{dh}{dVaR(\alpha)} = \frac{ES(\alpha) - VaR(\alpha)}{ES(\alpha) - VaR(\alpha)} < 0, \]
\[ \frac{dr}{dVaR(\alpha)} = \frac{ES(\alpha) - VaR(\alpha)}{ES(\alpha) - VaR(\alpha)} < 0, \]

where \( dES(\alpha) \) and \( dVaR(\alpha) \) are defined in terms of an \( \alpha \)-QPS and \( \alpha \)-OTQS respectively.

Comparative Statistics -2 (Borrower)

The main parameters of the borrower are
- the probability of failure \( P_b \)
- the return on the borrower’s project \( \rho \).

\[ \frac{dh_q}{dp} < 0, \quad \frac{dr_q}{dp} > 0, \quad \frac{dh_q}{dP_l} > 0, \]

where \( \kappa \) is the elasticity of the cdf \( F(\cdot) \) at \( K_q = \frac{1}{1 + h_q} \), and \( \kappa > 0 \text{- const.} \).