Contracting and Search with Heterogeneous Principals and Agents*

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Abstract

This paper incorporates a risk-neutral principal-agent model with contractual constraints into a random search model to study the interaction between contracting and search in partial and general equilibrium. I introduce heterogeneity in principals' and agents' production technologies in terms of the distribution of output across states of nature and characterize equilibrium in this context. Principals tailor contracts to agents' production technologies to incentivize agents' effort, generating dispersion in optimal contracts. The resulting endogenous dispersion in agency rents introduces an option value of search, which can generate inefficient equilibria with lengthy search and a persistent fraction of unmatched principals and agents. I show that a reduction in search costs and contractual innovations can reduce welfare. The results have implications for labor and financial economics and imply a novel theory of specialization in decentralized markets.

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JEL Classifications: D83, D86, G32, J41, J63, J64.

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1 Introduction

In contrast to the standard paradigm of demand and supply, trading in many important markets, such as labor and financial markets, is often decentralized and subject to frictions. It typically takes time and other resources to locate a suitable trading partner. In addition, even once a suitable trading partner is located, the exchange process itself is regularly subject to frictions such as moral hazard. While both search frictions and moral hazard jointly affect many important markets, the interaction between these two trading frictions remains an open question.

Answering this question is important for several reasons. While it is well known that both search frictions and moral hazard can lead to inefficiencies independent of each other, it is important to understand whether and how the two frictions interact and whether the interaction has welfare implications. In particular, it is important to understand how changes to the contracting environment affect search behavior and vice versa in decentralized markets. For example, how do contractual innovations affect agents' search behavior and welfare in decentralized markets?

This paper develops a theory of contracting and search to address these questions. I incorporate a standard risk-neutral principal-agent model with contractual constraints into a standard random search model. The contractual constraints capture, for example, wealth constraints and a minimum wage in labor markets or statutory limited liability in financial markets. The key innovation is to introduce heterogeneity in principals' and agents' production technologies in terms of the distribution of output across states of nature. As such, production technologies in my framework capture the ability of economic agents to substitute output across states of nature (see, e.g., Cochrane, 2019). In a principal-agent context, the distribution of output clearly matters in addition to average output. The challenge is to incorporate this heterogeneity into a search problem with endogenous contracts. I develop a novel and tractable framework that allows me to study contracting and search with heterogeneous production technologies and endogenous contracts in partial and general equilibrium.

The heterogeneity in principals' and agents' production technologies captures the idea that economic transactions differ not only in how much surplus they generate, but also in the characteristics of the contracting problem associated with the production of the surplus. For example, the contracting problem between firms and workers may differ across workers, such that different types of workers require different incentive contracts to generate the same surplus. Similarly, the contracting problem between investors and entrepreneurs may differ across entrepreneurs, such that different types of entrepreneurs require different financial contracts to generate the same surplus. Consistent with this view, firms tailor incentive contracts to employees and investors tailor financial contracts to entrepreneurs.[1] Whereas the existing literature focuses on heterogeneity in surplus, this paper studies the notion of technological heterogeneity.

I show that the interaction between contracting and search frictions can lead to lengthy and inefficient search by principals. The contracting problem forces principals to tailor contracts to the characteristics of agents' production technologies to incentivize agents' effort, which generates dispersion in optimal contracts. The resulting endogenous dispersion in agency rents gives rise to an option value of search, which can generate inefficient equilibria in which principals pass up valuable contracting opportunities to search for agents who require a lower agency rent. A reduction in search costs and contractual innovations can magnify this effect and reduce welfare. The welfare loss that arises due to the interaction between contracting and search frictions crucially depends on the nature and degree of heterogeneity in principals and agents in the economy. For example, a higher degree of heterogeneity in human capital in labor markets and a higher degree of heterogeneity in investment projects in financial markets can reduce welfare in these markets.

Before expanding on these results, I first lay out the basic ingredients of the partial equilibrium model. I consider a risk-neutral principal with a production technology that generates a random and independent output in each period. Without an agent, all output states have a positive probability. The principal can contract with a single agent in each period. Agents have different production technologies in the sense that they generate different output distributions when they exert effort. An agent's effort is noncontractible. If the principal is not contracting with an agent at the end of a period, she can pay a search cost to draw an agent at random at the beginning of the next period. Given an agent draw, the principal can pay a fixed cost and offer a one-period contract to the agent, which is repeated in all future periods. An agent's contractual payoff cannot be lower than a lower bound in each state, capturing contractual constraints. At the end of each period, a principal-agent match breaks up exogenously with some probability. To isolate the paper's novel contribution, I

[1]For example, Murphy (1999, 2013), Frydman and Jenter (2010), and Edmans et al. (2017) survey evidence on the cross-sectional variation in executive incentive contracts, and Kaplan and Strömberg (2000, 2001, 2003) provide evidence on the cross-sectional variation in financial contracts between venture capitalists and entrepreneurs.

assume that all agents generate the same expected value of effort, incur the same disutility of effort, and have the same reservation utility, implying the same expected surplus of effort.

There are several interpretations of the heterogeneity in agents' production technologies. For example, a worker with experience in an established technology can increase the probability of a firm's intermediate output through effort whereas a worker with experience in cutting-edge technology can increase the probability of high output. Similarly, a moderately innovative entrepreneur can increase the probability of intermediate output through effort whereas a highly innovative entrepreneur can increase the probability of high output.

I first show that in a benchmark economy without contractual constraints, the equilibrium is efficient. In the absence of contractual constraints, the principal can offer an agent a contract with an expected payoff that matches his reservation utility. As a result, the principal obtains the full expected surplus from search, searches when it is efficient to do so, and contracts with the first agent she draws. Since all agents generate the same expected surplus, she has no incentive to continue searching once she meets an agent. In particular, the search frictions and moral hazard do not generate inefficiencies in the absence of contractual constraints.

Next, I consider the contracting problem in the presence of contractual constraints. As is standard, agents earn agency rents if their reservation utility is sufficiently low. To minimize the agency rent, the principal rewards an agent in a state with the maximum likelihood ratio, which is most informative about the agent's effort. Since agents have heterogeneous production technologies that imply different maximum likelihood ratio states and levels, different agents receive different optimal contracts and earn different agency rents. I derive agency rents as a function of agents' production technologies—the agency rent function—which provides a tractable characterization of equilibrium agency rents. Combined with random search, it implies an endogenous probability distribution of agency rents.

The agency rent function summarizes each contracting problem in a single statistic, which makes the principal's search problem tractable. Using this result, I show that the endogenous dispersion in agency rents can lead to lengthy and inefficient search by principals. Although the principal cannot increase the expected surplus through search, there is an option value of search. If the principal draws an agent with a high agency rent, she has an incentive to pass up this contracting opportunity to continue searching for an agent with a lower agency rent. In many search theories, continued search can increase the surplus and is therefore socially desirable. In my model, continued search arises endogenously and is driven by a rent reduction motive, which is the result of optimal contracting. The welfare implications therefore differ substantially.

An equilibrium in which the principal continues to search is constrained inefficient. A social planner ignores agency rents, since they are a transfer from the principal to the agent. Since all agents generate the same expected surplus through their effort, continued search does not generate any additional expected surplus, but results in search costs and forgone output. Importantly, the inefficiencies arise only due to the interaction between contracting and search frictions. In the absence of contractual constraints, agents earn zero agency rent, there is no continued search, and the equilibrium is constrained efficient. In the absence of search frictions, the principal would immediately identify and contract with the agent equipped with the minimum agency rent production technology. This finding highlights that the welfare implications of the two frictions cannot be assessed separately and require a theoretical framework that accounts for their interaction.

My framework allows me to study how changes to the contracting environment affect the principal's search behavior and welfare. An increase in the degree of contractual constraints makes the contracting problem more severe and leads to more search and reduces welfare. By contrast, contractual innovations improve contracting but can lead to more search and reduce welfare. These findings highlight that changes in policy and technological innovations, which affect contracting between economic agents, can have important implications for the functioning of decentralized markets. I further show that a lower search cost, despite its positive impact on expected surplus, can reduce welfare through an increase in inefficient search.

In the second part of the paper, I extend the model to a general equilibrium model with a continuum of risk-neutral principals and risk-neutral agents. There are two types of principals, who generate different output distributions without an agent, and two types of agents, who differ in how they shift probability mass across states of nature when they exert effort. An agent's effort is noncontractible. At the beginning of each period, each unmatched principal draws an agent from the set of unmatched agents at random and can offer a one-period contract to the agent, which is repeated in all future periods. An agent's contractual payoff cannot be lower than zero in each state, capturing contractual constraints. At the end of each period, an exogenous share of principal-agent matches breaks up. All agents receive the same utility when unmatched—their outside option. I consider the economy in steady-state equilibrium.

I first show that in a benchmark economy without contractual constraints, there exists a unique full matching equilibrium in which each principal type offers a contract to each agent type. As a result, all principals and agents are matched during production in each period and the equilibrium is efficient.

Next, I consider the contracting problem in the presence of contractual constraints. Optimal contracts depend on the principal-agent match and can be written as the sum of an incentive payoff, which ensures incentive compatibility, and a fixed payoff, which ensures the agent's participation. The contracting problem can generate endogenous dispersion in agents' incentive payoffs since some agents have production technologies that, when matched with a principal's production technology, generate a relatively higher informativeness of output, which implies a lower incentive payoff. This endogenous dispersion in agents' incentive payoffs can generate dispersion in agency rents. I consider symmetric equilibria in which the two types of principals prefer opposite types of agents in terms of agency rents.

Using this characterization of optimal contracts, I show that the dispersion in agency rents can lead to partial matching in steady-state equilibrium, in which principals pass up valuable contracting opportunities. If an unmatched principal draws an agent at the beginning of a period who requires a high agency rent under optimal contracts, the principal has an incentive to pass up this contracting opportunity to search for an agent who requires a lower agency rent. As in the partial equilibrium model, the heterogeneity in agency rents introduces an option value of search, which generates partial matching in general equilibrium. In a partial matching equilibrium, each principal type contracts only with a particular agent type. My framework therefore implies a novel theory of specialization, which is not driven by exogenous productive complementarities but arises endogenously due to the interaction between contracting and search frictions.

Partial matching is constrained inefficient. Each principal-agent match creates the same expected surplus. The dispersion in agency rents affects only how the expected surplus is shared between principals and agents. Principals should therefore never pass up contracting opportunities.

A constrained social planner can implement a full matching equilibrium by offering incentive-compatible contracts to agents with the same expected payoff for all agents. As in the partial equilibrium model, the inefficiencies arise only due to the interaction between contracting and search frictions.

Related Literature. This paper contributes to the small but growing literature that studies contracting under asymmetric information in the presence of search frictions. Most closely related is the subset of the literature that considers moral hazard. Inderst and Müller (2004) develop a model of contracting between venture capitalists and entrepreneurs in the presence of search frictions and study the effect of capital market characteristics on capital market competition and value creation by entrepreneurial firms. Shimer and Wright (2004) study a competitive search model in which firms cannot observe workers' effort and workers cannot observe the realization of the match-specific productivity shock. They characterize optimal contracts and show that asymmetric information unambiguously reduces the vacancy-unemployment ratio and reduces the probability that a meeting results in a match. Demougin and Helm (2011) show that higher unemployment benefits can strengthen the effort incentives and increase the productivity of low-skilled workers in a frictional labor market. Moen and Rosén (2011) study a competitive search model with workers' private information on match quality and effort and show that it is less costly to provide incentives for workers in a frictional labor market since workers' rents can speed up the hiring rate. Moen and Rosén (2013) study the trade-off between improving workers' incentives to exert effort and distorting their on-the-job search decisions and show that multiple equilibria arise in this context. Another subset of the literature focuses on adverse selection (see, e.g., Inderst, 2001, 2004, 2005; Guerrieri, 2008; Guerrieri et al., 2010; Lester et al., 2019). The existing literature either considers homogeneous agents or heterogeneity in agents' (expected) surplus. The contribution of this paper is to introduce heterogeneity in principals' and agents' production technologies in terms of the distribution of output across states of nature and to characterize equilibrium in this context. As discussed, this captures the idea that economic transactions differ not only in how much surplus they generate, but also in the characteristics of the contracting problem associated with the production of the surplus. As such, this paper studies a novel source of inefficiency in decentralized markets. In addition, I provide a novel theory of specialization in decentralized markets, which arises endogenously due to the interaction between contracting and search frictions.

There is extensive literature on moral hazard and, specifically, the role of contracts as incentive devices (see, e.g., Mirrlees, 1975, 1976; Holmström, 1979; Grossman and Hart, 1983). Several papers highlight the role of the stochastic structure of output or information (see, e.g., Holmström, 1979; Grossman and Hart, 1983; Kim, 1995; Oyer, 2000; Poblete and Spulber, 2012; Chaigneau et al., 2018; Starmans, 2020) on optimal contracting. The contribution of this paper is to incorporate the idea of heterogeneity in principals' and agents' production technologies in terms of the distribution of output across states of nature into a standard random search model (see, e.g., McCall, 1970; Mortensen, 1970; Gronau, 1971) to study contracting in a decentralized market in partial and general equilibrium.

My analysis is also related to the literature on wage dispersion, which rationalizes how workers with the same productivity can earn different wages (see, e.g., Mortensen, 2005; Rogerson et al., 2005). Shapiro (2006) demonstrates that search frictions can also lead to dispersion in incentive contracts. Relatedly, Stevens (2004) studies wage-tenure contracts, and Burdett and Coles (2003) show that these can differ between workers if they are risk-averse. This paper provides a novel theory of equilibrium contract dispersion.

The paper proceeds as follows: Section 2 contains the partial equilibrium framework: Section 2.1 introduces the model; Section 2.2 considers a benchmark model without contractual constraints; Section 2.3 studies the equilibrium with contractual constraints; Section 2.4 discusses welfare; Section 2.5 discusses implications. Section 3 contains the general equilibrium framework and follows the same structure as Section 2. Section 4 concludes. The Appendix contains all proofs and additional material.

2 Partial Equilibrium Framework

In this section, I consider a partial equilibrium model of contracting and search with a single principal and different types of agents. I consider an extension of the model to a general equilibrium model in Section 3.

2.1 Model

I incorporate a standard risk-neutral principal-agent model with contractual constraints (see, e.g., Bolton and Dewatripont, 2005) into a standard random search model (see, e.g., Rogerson et al., 2005) and discuss applications to labor and financial economics at the end of this section.

Time is discrete and infinite: $t \in \mathbb{N}_0 := \{0, 1, \ldots\}$. There is a single risk-neutral principal (referred to as *she*) and risk-neutral agents (each referred to as *he*) with the same time-discount factor $\beta \in (0,1)$. The principal has access to a production technology that generates a random output $\tilde{x}$ in each period $t \in \mathbb{N} := \{1, 2, \ldots\}$. Specifically, the production technology pays $x_i \in \mathbb{R}_+ := [0, \infty)$ in state $i \in \Omega := \{1, \ldots, n\}$, where $n > 2$ and $x_i \neq x_j$ for $i \neq j$. Output is independent across periods. Without an agent, the output $\tilde{x}$ is drawn according to the probability distribution $q \in \mathbb{R}^n$, where $q_i > 0$ denotes the probability of state $i \in \Omega$. I refer to q as the "baseline production technology."[2]

There is a set $\mathscr{K} := \{1, \ldots, K\}$ of agents, where $K > 1$. In each period $t \in \mathbb{N}$, the principal can contract with a single agent. The agent decides between exerting effort and shirking, which is noncontractible. Agents have heterogeneous production technologies, denoted by $\Delta^k \in \mathbb{R}^n$, $k \in \mathscr{K}$. If agent $k \in \mathscr{K}$ shirks, the output $\tilde{x}$ is drawn according to the baseline production technology q. If agent $k \in \mathscr{K}$ exerts effort, the output is drawn according to the production technology $p^k := q + \Delta^k \in \mathbb{R}^n$, where p^k_i denotes the probability of state $i \in \Omega$. In particular, an agent's type $\Delta^k = p^k - q$ describes the change in the output distribution from shirking to exerting effort. All agents have a disutility of effort $c > 0$ and a reservation utility of $u \geq 0$ in each period $t \in \mathbb{N}$.

Assumption 1. *All agents $k \in \mathscr{K}$ generate the same expected value of effort $\pi > 0$, that is, for all $k \in \mathscr{K}$, $\mathbb{E}_{p^k}[\tilde{x}] - \mathbb{E}_q[\tilde{x}] = \sum_{i=1}^n \Delta^k_i x_i = \pi$.*

Since all agents generate the same expected value of effort π, incur the same disutility of effort c, and have the same reservation utility u, all agents generate the same "expected surplus of effort" $\pi - c - u$. Equalizing the expected surplus of effort across agents eliminates increasing expected surplus as a motivation for search, which allows me to isolate the novel contribution of this paper.

[2]The assumption that the principal produces the expected output $\mathbb{E}_q[\tilde{x}]$ without an agent is not important. The distribution q matters to the extent that it determines the output distribution if an agent shirks. In particular, the model can be extended to feature zero output without an agent.

The model can be extended to feature agents with different expected values of effort.[3]

If the principal is not contracting with an agent at the end of a period $t \in \mathbb{N}_0$, the principal can pay the search cost $\kappa \geq 0$ to draw an agent $k \in \mathscr{K}$ at the beginning of the next period $t+1 \in \mathbb{N}$ (i.e., before production in period $t+1$) according to the probability distribution $\mu \in \mathbb{R}^K$, where $\mu_k > 0$ denotes the probability of drawing agent $k \in \mathscr{K}$. I denote the random variable describing the random draw of an agent $k \in \mathscr{K}$ according to the probability distribution μ by $\tilde{k}$.

Given an agent draw $k \in \mathscr{K}$ at the beginning of period $t \in \mathbb{N}$, the principal can pay the fixed cost $I \geq 0$ to offer a one-period contract $s = (s_i)_{i \in \Omega} \in \mathbb{R}^n$ to the agent, which is repeated in each future period until the match breaks up, where s_i is the agent's contractual payoff and $x_i - s_i$ is the principal's contractual payoff in state $i \in \Omega$ in each period. I denote the random variable by $\tilde{s}$. The contracting problem is kept deliberately simple to focus on the interaction between contracting and search frictions.[4] At the end of each period, a principal-agent match breaks up exogenously with probability $\delta \in (0,1)$.

Assumption 2. $\frac{\pi - c - u}{1 - \beta(1-\delta)} - I \geq 0.$

Assumption 2 implies that contracting with an agent is efficient independent of its type in the absence of search frictions and excludes the uninteresting cases in which contracting with an agent is inefficient even in the absence of search frictions.

Assumption 3. *For all $i \in \Omega$, the contract s satisfies $s_i \geq \theta \geq 0$.*

Assumption 3 requires that the contractual payoff to an agent cannot be lower than $\theta \geq 0$ in each state and captures contractual constraints. Since an increase in θ reduces the set of feasible contracts, I refer to θ as the "degree of contractual constraints." The contractual constraints capture, for example, wealth constraints and a minimum wage in labor markets or statutory limited liability in financial markets.

An equilibrium is a set of contract offers and search decisions by the principal, accept and reject decisions by each agent, and effort decisions by agents such that the principal and each agent maximizes his/her expected discounted payoff.

[3] See Appendix B for an extension of the model with heterogeneity in the expected value of effort.

[4] In Appendix C, I study long-term contracts of length T with the same contractual constraints from Assumption 3 in each period. I show that the restriction to one-period contracts is without loss of generality in this class of long-term contracts in the sense that optimal long-term contracts can be implemented by a series of optimal one-period contracts.

Application 1 (Labor economics)**.** *There are K workers (agents) with heterogeneous production technologies. A firm (principal) can hire a worker. Without a worker's effort, the firm's output distribution is given by q. If the firm hires worker $k \in \mathscr{K}$, incurring the fixed cost $I \geq 0$, and the worker exerts effort, the firm's output distribution is given by $q + \Delta^k$. Intuitively, the baseline production technology q captures job-specific characteristics. A worker's production technology Δ^k captures worker-specific characteristics, such as human capital. For example, a worker with experience in cutting-edge technology increases the probability of developing more novel products through effort, and thus improves a higher region of the output distribution, compared with a worker with experience in an established technology. To find a worker, the firm has to search. A worker's effort is noncontractible and there is a wealth constraint or a minimum wage, that is, $\theta \geq 0$.*

Application 2 (Financial economics)**.** *There are K penniless entrepreneurs (agents) with heterogeneous production technologies. Entrepreneur $k \in \mathscr{K}$ has an investment project that requires an upfront investment of $I > 0$ from the investor (principal) and generates an output distribution $q + \Delta^k$ if the entrepreneur exerts effort. All entrepreneurs have the same output distribution q when they shirk. Intuitively, the baseline production technology q captures uncertainty that entrepreneurs cannot control (e.g., demand fluctuations). An entrepreneur's production technology Δ^k captures the uncertainty the entrepreneur can (partially) control (e.g., idiosyncratic production risk), capturing entrepreneur-specific characteristics, such as human capital, or project-specific characteristics, such as assets. Differences in entrepreneurs' production technologies can be interpreted as different degrees of innovation. Intuitively, a more innovative entrepreneur increases the probability of developing more innovative products through effort, and thus improves a higher region of the output distribution.[5] To find an entrepreneur, the investor has to search. An entrepreneur's effort is noncontractible and entrepreneurs are protected by limited liability, that is, $\theta = 0$.*

[5] This interpretation is consistent with evidence in Krieger et al. (2019), who show that more innovative pharmaceutical drugs are more valuable conditional on approval, but are less likely to be approved.

2.2 Benchmark Model without Contractual Constraints

In this section, I consider a benchmark model without contractual constraints from Assumption 3 such that the principal can choose a contract s without any constraints. Consider an arbitrary agent $k \in \mathscr{K}$. An optimal one-period contract, denoted by $s^*(k)$, satisfies

$$s^*(k) \in \arg\max_s \mathbb{E}_{p^k}[\tilde{x} - \tilde{s}] \text{ s.t. } \mathbb{E}_{p^k}[\tilde{s}] - c \geq \mathbb{E}_q[\tilde{s}], \ \mathbb{E}_{p^k}[\tilde{s}] - c \geq u. \tag{1}$$

The first constraint is the agent's incentive constraint. The second constraint is the agent's participation constraint.

Lemma 1. *Consider an agent $k \in \mathscr{K}$. Each optimal contract $s^*(k)$ satisfies the agent's participation constraint with equality, that is, $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = u$.*

The principal can choose a contract that gives the agent an expected payoff equal to his reservation utility u such that the agent is indifferent between accepting the contract and not accepting the contract. Using Lemma 1 and Assumption 1, the principal's expected payoff under an optimal contract is given by $\mathbb{E}_{p^k}[\tilde{x} - \tilde{s}^*(k)] = \mathbb{E}_q[\tilde{x}] + \pi - c - u$. Importantly, the principal's expected payoff is independent of the agent's type $k \in \mathscr{K}$. In particular, the principal's decision whether to contract with an agent is independent of the agent's type.

Proposition 1. *If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right) \geq \kappa$, the principal searches at the end of each period in which she is not matched with an agent and contracts with the agent drawn in the next period independent of the agent's type. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right) < \kappa$, the principal does not search in any period $t \in \mathbb{N}_0$.*

The result is intuitive. Assume that the principal searches at the end of each period in which she is not matched with an agent and contracts with the agent drawn at the beginning of the next period independent of the agent's type. Then, the principal's expected discounted payoff in period $t = 0$ is given by

$$-\frac{1-\beta(1-\delta)}{1-\beta}\kappa + \beta\left(\frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u}{1-\beta} - \frac{1-\beta(1-\delta)}{1-\beta}I\right).$$

If the principal does not search in any period $t \in \mathbb{N}_0$, her expected discounted payoff is given by

$\beta \frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$. Indeed, the former is larger or equal to the latter if and only if

$$\beta \left(\frac{\pi - c - u}{1 - \beta(1-\delta)} - I \right) \geq \kappa. \tag{2}$$

The principal incurs the search cost κ and the fixed cost I not only when she searches initially but also every time she searches after separation. This recurring cost can be translated into a lower effective time-discount factor $\beta(1-\delta) < \beta$ in (2) (see, e.g., Rogerson et al., 2005). In words, the principal searches if the effective benefit from search $\beta \left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I \right)$, which is positive by Assumption 2, exceeds the search cost κ.

Notice that since the principal captures the whole expected surplus in this benchmark model, the equilibrium maximizes the sum of the principal's and agents' expected discounted payoffs and is first-best efficient. This implies that, in the absence of contractual constraints, the search frictions and moral hazard alone do not generate inefficiencies in my framework.

2.3 Equilibrium with Contractual Constraints

In this section, I study the equilibrium with contractual constraints from Assumption 3. In Section 2.3.1, I solve the one-period contracting problem for an arbitrary agent $k \in \mathscr{K}$. In Section 2.3.2, I determine the dynamic equilibrium.

2.3.1 Optimal Contracts and Agency Rents

Consider an arbitrary agent $k \in \mathscr{K}$. An optimal one-period contract, denoted by $s^*(k)$, satisfies

$$s^*(k) \in \arg\max_{s} \mathbb{E}_{p^k}[\tilde{x} - \tilde{s}] \text{ s.t. } \mathbb{E}_{p^k}[\tilde{s}] - c \geq \mathbb{E}_q[\tilde{s}],\ \mathbb{E}_{p^k}[\tilde{s}] - c \geq u,\ \forall i \in \Omega : s_i \geq \theta. \tag{3}$$

Definition 1. *For each agent $k \in \mathscr{K}$, the likelihood ratio $\phi(k) = (\phi_i(k))_{i \in \Omega} \in \mathbb{R}^n$ is defined as $\phi_i(k) := \frac{\Delta_i^k}{q_i}$, $i \in \Omega$. I denote the maximum likelihood ratio by $\phi^*(k) := \max_{i \in \Omega} \phi_i(k)$.*

Intuitively, the likelihood ratio $\phi_i(k)$ measures how informative the state $i \in \Omega$ is about agent k's effort.

Lemma 2. *Consider an agent $k \in \mathscr{K}$. Let $j \in \arg\max_{i \in \Omega} \phi_i(k)$.*

(i) If $u < \theta + \frac{c}{\phi^*(k)}$, then the agent's incentive constraint binds and an optimal contract is given by $s_i^*(k) = \theta + \mathbb{1}_{\{i=j\}} \frac{c}{\Delta_j^k}$, $i \in \Omega$, which satisfies $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = \theta + \frac{c}{\phi^*(k)} > u$.

(ii) If $u \geq \theta + \frac{c}{\phi^*(k)}$, then the agent's participation constraint binds and an optimal contract is given by $s_i^*(k) = u - \frac{c}{\phi^*(k)} + \mathbb{1}_{\{i=j\}} \frac{c}{\Delta_j^k}$, $i \in \Omega$, which satisfies $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = u$.

In Case (i) of Lemma 2, the agent's reservation utility u is low such that the agent's incentive constraint binds. The principal has to give the agent at least $\theta \geq 0$ in all states. As is standard, it is optimal to give the agent more than θ only in a state with the maximum likelihood ratio. Intuitively, this state is most informative about the agent's effort. In particular, the agent earns an agency rent, given by

$$\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c - u = \theta + \frac{c}{\phi^*(k)} - u > 0.$$

A higher maximum likelihood ratio $\phi^*(k)$ implies that output is more informative about the agent's effort, which implies a lower agency rent.

In Case (ii) of Lemma 2, the agent's reservation utility u is high such that the agent's participation constraint binds. In this case, the optimal contract from Case (i) satisfies the agent's incentive constraint but violates the agent's participation constraint. The principal can then simply add a fixed component to the contract from Case (i) to make the participation constraint bind. In particular, the agent does not earn an agency rent, that is, $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c - u = 0$.

Define the agency rent function $A : \mathscr{K} \to \mathbb{R}_+$ by

$$A(k) := \max\left\{\theta + \frac{c}{\phi^*(k)} - u, 0\right\}, \; k \in \mathscr{K}. \tag{4}$$

Using the definition of the agency rent function, an agent's expected payoff under an optimal contract is given by $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = u + A(k)$. The principal's expected payoff under an optimal contract is given by $\mathbb{E}_{p^k}[\tilde{x} - \tilde{s}^*(k)] = \mathbb{E}_q[\tilde{x}] + \pi - c - u - A(k)$. As in the benchmark model without contractual constraints in Section 2.2, the principal has to compensate the agent for his disutility of effort c and his reservation utility u. However, in the presence of contractual constraints, the agent earns an agency rent if $A(k) > 0$, which depends on the agent's type $k \in \mathscr{K}$.

The agency rent function provides a complete characterization of the principal's and agents' expected payoffs under optimal contracts.[6] I can therefore consider a random draw of an agency rent $A(k)$, rather than a random draw of an agent $k \in \mathscr{K}$. Put differently, the agency rent function captures all information about the contracting problem, which is relevant for the principal's and agents' decisions. Thus, the agency rent function summarizes each contracting problem in a single statistic, which makes the principal's search problem tractable. In particular, the principal draws the agency rent $A(k)$ with probability μ_k, $k \in \mathscr{K}$. Importantly, the probability distribution of agency rents is endogenous since it is the result of optimal contracting. To simplify the exposition, I reorder the index such that $A(1) \leq \cdots \leq A(K)$.

Assumption 4. $\frac{\pi - c - u - A(K)}{1-\beta(1-\delta)} - I \geq 0$.

Assumption 4 states that the principal is generally willing to contract with any agent in the presence of contractual constraints and ensures that the contracting problem alone does not generate inefficiencies in my framework. This implies that in the absence of search frictions, the principal would immediately identify and contract with agent $k = 1$, who requires the lowest agency rent.

2.3.2 Dynamic Equilibrium

Consider a principal at the end of period $t \in \mathbb{N}$ and denote the principal's optimal expected discounted payoff if she is unmatched by V and if she is matched with a type k agent by $V_m(k)$. Recall that there are two reasons that the principal can be unmatched at the end of a period. First, if the principal did not contract with an agent at the beginning of the period, she is unmatched at the end of the period. Second, if the principal is matched with an agent at the beginning of the period but the match breaks up exogenously at the end of the period, she is unmatched at the end of the period. Then

$$V_m(k) = \beta \left(\mathbb{E}_q[\tilde{x}] + \pi - c - u - A(k) + (1-\delta)V_m(k) + \delta V \right).$$

Thus, $V_m(k)$ satisfies

$$V_m(k) = \beta \frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u - A(k) + \delta V}{1-\beta(1-\delta)}. \tag{5}$$

[6] As discussed, the contracting problem is kept deliberately simple to focus on the interaction between contracting and search frictions. Note that the agency rent function can also be derived for more complex contracting problems. For example, Starmans (2020) derives the agency rent function for a contracting problem with standard limited liability and monotonicity constraints and shows that heterogeneity in agents' production technologies generates endogenous dispersion in agency rents. In particular, the main insights regarding the interaction between contracting and search frictions, which relies on the dispersion in agency rents, holds for more complex contracting problems.

If the principal is unmatched and does not search, her expected discounted payoff is given by $\beta(\mathbb{E}_q[\tilde{x}]+V)$. If the principal is unmatched and searches and does not contract with an agent at the beginning of the next period, her expected discounted payoff is given by $-\kappa+\beta(\mathbb{E}_q[\tilde{x}]+V)$. If the principal is unmatched, searches and contracts with a type k agent, her expected discounted payoff is given

$$-\kappa+\beta(\mathbb{E}_q[\tilde{x}]+\pi-c-u-A(k)-I+(1-\delta)V_m(k)+\delta V). \tag{6}$$

Substituting (5) into (6) yields

$$-\kappa+\beta\left(\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u-A(k)+\delta V}{1-\beta(1-\delta)}-I\right).$$

The expected discounted payoff from search is therefore given by

$$-\kappa+\beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u-A\left(\tilde{k}\right)+\delta V}{1-\beta(1-\delta)}-I,\mathbb{E}_q[\tilde{x}]+V\right\}\right].$$

As a result, V satisfies

$$V=\max\left\{\beta(\mathbb{E}_q[\tilde{x}]+V),-\kappa+\beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u-A\left(\tilde{k}\right)+\delta V}{1-\beta(1-\delta)}-I,\mathbb{E}_q[\tilde{x}]+V\right\}\right]\right\}. \tag{7}$$

Consider the case in which it is optimal for the principal to search, such that

$$V=-\kappa+\beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u-A\left(\tilde{k}\right)+\delta V}{1-\beta(1-\delta)}-I,\mathbb{E}_q[\tilde{x}]+V\right\}\right]. \tag{8}$$

In particular, the principal contracts with agent $k\in\mathscr{K}$ if

$$\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u-A(k)+\delta V}{1-\beta(1-\delta)}-I\geq\mathbb{E}_q[\tilde{x}]+V. \tag{9}$$

Notice that the right-hand side of (9) is independent of $A(k)$. Thus, there exists a threshold R such that (9) holds if and only if $A(k) \leq R$. In particular, R satisfies

$$\frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u - R + \delta V}{1 - \beta(1-\delta)} - I = \mathbb{E}_q[\tilde{x}] + V. \tag{10}$$

Substituting (10) into (8) and solving for V implies

$$V = -\frac{1-\beta(1-\delta)}{1-\beta}\kappa + \frac{\beta}{1-\beta}\left(\mathbb{E}_q[\tilde{x}] + \pi - c - u - (1-\beta(1-\delta))I\right) - \frac{\beta}{1-\beta}\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), R\right\}\right]. \tag{11}$$

Substituting (11) into (10) and solving for R implies

$$R = T(R) := (1-\beta(1-\delta))((1-\delta)\kappa + \pi - c - u) - (1-\beta(1-\delta))^2 I + \beta(1-\delta)\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), R\right\}\right]. \tag{12}$$

Lemma 3. *The function $T : \mathbb{R} \to \mathbb{R}$ defined in (12) is a contraction mapping on $\mathbb{R}_+$.*

The Banach fixed-point theorem then implies that the equilibrium condition (12) has a unique solution, namely that there exists a unique $R^* \geq 0$ such that $T(R^*) = R^*$.

To understand the intuition for the equilibrium condition (12), we can rewrite it as

$$(1-\delta)\kappa + \pi - c - u - R - I = \beta(1-\delta)\left(\frac{\mathbb{E}_\mu\left[\max\left\{R - A\left(\tilde{k}\right), 0\right\}\right]}{1-\beta(1-\delta)} - I\right). \tag{13}$$

The benefit of contracting with the marginal agent R rather than continuing to search is captured by the left-hand side of equation (13). Intuitively, the principal avoids the search cost κ with probability $1-\delta$ and generates the additional expected payoff $\pi - c - u - R - I$ today. The cost of contracting with the marginal agent R is that the principal loses the opportunity to find an agent k with a lower agency rent $A(k) < R$ tomorrow. This "option value of search" is captured by the right-hand side of equation (13).

Proposition 2. *Let $R^* \geq 0$ be the solution to the equilibrium condition (12).*

(i) If $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} - I \geq 0$, the principal searches if she is not matched with an agent at the end of a period and contracts with the agent drawn at the beginning of the next period if $A(k) \leq R^*$.

(ii) If $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} - I < 0$, the principal does not search in any period $t \in \mathbb{N}_0$.

In Case (i) of Proposition 2, it is profitable for the principal to search, and the decision whether to contract with the agent drawn in the next period follows a standard threshold strategy. If the principal draws an agent k with an agency rent (weakly) below the equilibrium threshold R^*, then the principal contracts with the agent. If the principal draws an agent k with an agency rent above the equilibrium threshold R^*, then the principal does not contract with the agent and continues to search. The equilibrium threshold R^* determines the set of agents the principal is willing to contract with, which is given by $\{k \in \mathscr{K} : A(k) \leq R^*\}$. In particular, the agency rent function makes the principal's search problem tractable by allowing for a simple threshold strategy. This allows me to solve the search problem in a setting in which agent types are probability distributions and contracts are endogenous. In Case (ii), it is not profitable for the principal to search. As a result, the principal never searches in any period.

Importantly, the option value of search arises endogenously from optimal contracting and requires two assumptions. First, it requires contractual constraints. As shown in Section 2.2, the option value of search does not arise in the benchmark model without contractual constraints. Second, it requires dispersion in agency rents across agents, which in turn requires heterogeneity in agents' production technologies. In particular, the equilibrium with continued search in Case (i) of Proposition 2 crucially depends on the nature and degree of heterogeneity in agents' production technologies in the economy.

2.4 Constrained Efficiency and Discussion

In this section, I consider a social planner who is subject to the same frictions and constraints as the principal and maximizes the sum of the principal's and agents' expected discounted payoffs. As discussed in Section 2.2, the first-best allocation is given in Proposition 1. Note that the social planner can implement this allocation in the presence of contractual constraints since there exist incentive-compatible contracts that agents accept. Put differently, the social planner can contract with any agent. In particular, the constrained efficient search and contracting policy is also first-best efficient.

In the following, I discuss the efficiency of the equilibrium with contractual constraints from Proposition 2. Recall that $A(1) \leq \cdots \leq A(K)$.

Proposition 3. *Consider the equilibrium from Proposition 2 with the equilibrium threshold R^*.*

(i) Let $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)}-I \geq 0$. If $A(K) \leq R^*$, the equilibrium is constrained efficient. If $A(K) > R^*$, the equilibrium is constrained inefficient.

(ii) Let $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)}-I < 0$. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)}-I\right) \leq \kappa$, the equilibrium is constrained efficient. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)}-I\right) > \kappa$, the equilibrium is constrained inefficient.

Proposition 3 shows that there are two cases in which the interaction between contracting and search frictions leads to inefficiencies. In Case (i), the principal searches and an inefficiency arises if the principal does not contract with all agents and continues to search if she draws an agent with a high agency rent. Intuitively, the principal searches too much in this case. The social planner would search and contract with agents independent of their type. In contrast, if $A(K) > R^*$, there are agents with high agency rents the principal does not contract with. If she draws one of these agents, she continues to search. The inefficiency arises due to the option value of search. If there is dispersion in agency rents, the principal has an incentive to not contract with agents with high agency rents and to continue to search for agents with lower agency rents.

In Case (ii), the principal does not search at all and an inefficiency arises if search is constrained efficient. Intuitively, the principal searches too little in this case. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)}-I\right) \geq \kappa$, the social planner would search and contract with agents independent of their type. In contrast, if $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} < I$, the principal does not search at all. The inefficiency arises due to the presence of agency rents. The principal does not obtain the full expected discounted surplus from search and therefore has a lower incentive to search compared to the social planner. Notice that this second case can arise even if there is no dispersion in agency rents. In addition, if there is dispersion in agency rents and $\beta\left(\frac{\pi-c-u-A(1)}{1-\beta(1-\delta)}-I\right) \geq \kappa$, then the principal would search if she could locate agent $k=1$ in the next period. In this case, it is the fact that search is random and the principal cannot locate agent $k=1$ with a probability of one that generates the inefficiency.

Importantly, the inefficiencies arise due to the interaction between contracting and search frictions. To see this, first recall that in the absence of contractual constraints in Section 2.2, the equilibrium is constrained efficient. Second, in the absence of search frictions, the principal would immediately identify and contract with agent $k=1$ equipped with the minimum agency rent production technology since Assumption 4 implies $\beta\left(\frac{\pi-c-u-A(1)}{1-\beta(1-\delta)}-I\right)\geq 0$, which would also implement the constrained efficient allocation. Recall that the option value of search, which leads to the overly intense search in Case (i), requires both contractual constraints and heterogeneity in agents' production technologies, which jointly generate endogenous dispersion in agency rents. In particular, the welfare loss that arises due to the interaction between contracting and search frictions crucially depends on the nature and degree of heterogeneity in agents in the economy.

2.5 Discussion and Implications

Next, I study how changes to the search and contracting environment affect the search equilibrium in Case (i) of Proposition 3 and its efficiency. Note that the search equilibrium is inefficient if $A(K) > R^*$, that is, if the probability of continued search $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*\right)$ is positive. In particular, all else being equal, a reduction in the probability of continued search increases welfare.

2.5.1 Search Cost

In this section, I study the search cost $\kappa \geq 0$ and write $R^*(\kappa)$ for the equilibrium threshold.

Proposition 4. *$R^*(\kappa)$ is increasing in κ. The probability of continued search $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*(\kappa)\right)$ is nonincreasing in κ. If $R^*(0) < A(K)$, then the probability of continued search $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*(\kappa)\right)$ is decreasing for some κ.*

An increase in the search cost κ increases the equilibrium threshold $R^*(\kappa)$. Intuitively, when search is more costly, the principal searches less and is willing to contract with agents with higher agency rents. As a result, the effect of a reduction in the search cost on welfare is ambiguous. While a reduction in the search cost can increase the probability of continued search, which reduces welfare, each unit of search becomes less costly, which increases welfare.

Proposition 5. *There exist cases in which a reduction in the search cost κ reduces welfare.*

Importantly, if the welfare loss from the increase in the probability of continued search dominates the reduction in the equilibrium expected discounted search cost, then a reduction in the search cost reduces welfare. Proposition 5 shows that such cases exist. In particular, less severe search frictions can reduce welfare in an environment in which contracting and search frictions interact.

2.5.2 Contractual Constraints

In this section, I study the degree of contractual constraints θ and write $R^*(\theta)$. Note that the parameter θ only affects contracting, but neither affects the search frictions nor agents' expected value of effort $\pi - c - u$. In contrast to the search cost κ, the parameter θ affects both the equilibrium threshold $R^*(\theta)$ and the agency rent distribution $A(\tilde{k})$, which jointly determine the equilibrium probability of continued search.

Proposition 6. *$R^*(\theta)$ is nondecreasing in θ. The probability of continued search $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*(\theta)\right)$ is nondecreasing in θ. If $R^*(0) > A(1)$, then the probability of continued search $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*(\theta)\right)$ is increasing for some θ.*

As is evident from the agency rent function (4), an increase in the degree of contractual constraints θ leads to an upward shift in the agency rent distribution. Specifically, if $A(k) > 0$, then $A(k)$ increases one-to-one with θ. Intuitively, an increase in θ makes it more costly to contract with agents today. In addition, an increase in the agency rent distribution makes it more costly to contract with agents in the future, which increases the equilibrium threshold $R^*(\theta)$. Since the principal discounts the future, the increase in $R^*(\theta)$ is less than one-to-one with θ. As a result, agency rents today increase more than the equilibrium threshold $R^*(\theta)$, which implies that the probability of continued search is nondecreasing in θ and increasing for some θ.

Proposition 6 shows how contracting affects search in my framework. Specifically, a higher degree of contractual constraints makes incentive provision more costly for the principal, which can increase her equilibrium search intensity and reduce the set of agents the principal contracts with in equilibrium. In particular, a higher degree of contractual constraints reduces welfare. Note that contracts are endogenous and the welfare loss arises despite the fact that the principal optimally adjusts the contracts as the degree of contractual constraints increases. Notice also that an increase in the degree of contractual constraints does not reduce welfare in the absence of search frictions as long as Assumption 4 is satisfied. The welfare loss arises only due to the interaction of contracting and search frictions.

2.5.3 Contractual Innovations

In this section, I study how fundamental changes in the contracting environment affect equilibrium. Specifically, consider a partition of the state space Ω, denoted by $\hat{\Omega}$, where $\hat{\Omega} \neq \{\Omega\}$ and $\hat{\Omega} \neq \{\{1\}, \ldots, \{n\}\}$. I consider contracts that can only depend on events in $\hat{\Omega}$, that is, for all $\hat{\omega} \in \hat{\Omega}$ and for all $i, j \in \hat{\omega}$, $s_i = s_j$. A change of the state space from $\hat{\Omega}$ to Ω has several interpretations such as an improvement in the legal system or an improvement in the monitoring technology, which I jointly refer to as a "contractual innovation." Alternatively, a change of the state space from Ω to $\hat{\Omega}$ can be interpreted as legal changes that reduce the set of permissible contracts.

Lemma 4. *Let $\hat{\Omega}$ be a partition of Ω, where $\hat{\Omega} \neq \{\Omega\}$ and $\hat{\Omega} \neq \{\{1\}, \ldots, \{n\}\}$. Denote the agency rent function induced by the contracting environment with state space $\hat{\Omega}$ by $\hat{A}(k)$ and the agency rent function induced by the contracting environment with state space Ω by $A(k)$. Then for all $D \geq 0$, we have $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) \leq D\right) \geq \mathbb{P}_\mu\left(\hat{A}\left(\tilde{k}\right) \leq D\right)$.*

If the inequality in Lemma 4 is strict for some D, then the agency rent distribution induced by the contracting environment with state space $\hat{\Omega}$ first-order stochastically dominates the agency rent distribution induced by the contracting environment with state space Ω.[7] Intuitively, a contractual innovation increases the maximum likelihood ratio, which increases informativeness and reduces agency rents. Put differently, a contractual innovation allows the principal to write more complex contracts, which makes it less costly to incentivize agents.

Lemma 5. *An increase in the agency rent distribution in the sense of first-order stochastic dominance weakly increases the equilibrium threshold R^*.*

Lemmas 4 and 5 imply that a contractual innovation leads to a downward shift in the agency rent distribution and reduces the equilibrium threshold R^*. As a result, the effect of a contractual innovation on the probability of continued search is ambiguous.

Proposition 7. *There exist cases in which a contractual innovation increases the probability of continued search and reduces welfare.*

[7] The proof of Lemma 4 shows that for all $k \in \mathscr{K}$, $\phi^*(k) \geq \hat{\phi}^*(k)$, where $\phi^*(k)$ is the maximum likelihood ratio in the case of a state space Ω and $\hat{\phi}^*(k)$ in the case of $\hat{\Omega}$. Thus, if there exists a $k \in \mathscr{K}$ such that $\phi^*(k) > \hat{\phi}^*(k)$ and $A(k) > 0$, then the agency rent distribution induced by the contracting environment with state space $\hat{\Omega}$ first-order stochastically dominates the agency rent distribution induced by the contracting environment with state space Ω.

As shown in Proposition 7, there are cases in which a contractual innovation can lead to an increase in the probability of continued search and therefore reduce welfare. The intuition is that if a contractual innovation significantly reduces the agency rents of only some agents, the dispersion in agency rents increases, which in turn increases the option value of search. Notice also that a contractual innovation does not reduce welfare in the absence of search frictions. The welfare loss arises only due to the interaction of contracting and search frictions.

2.5.4 Set of Agents

The framework also allows me to study how a change in the set of agents affects equilibrium. In other words, this addresses the question of how a change in agent heterogeneity affects equilibrium. Note that a change in the set of agents affects the search equilibrium through its effect on the agency rent distribution. For example, if we interpret the probability μ_k as the proportion of type k agents in the economy, then a change in the proportions of agent types in the economy changes the agency rent distribution. For example, the set of agents changes due to a change in the size of the market, changes in agents' production technologies, or changes in the search technology.

As discussed in Section 2.5.3, changes in the probability distribution of agency rents lead to a change in the equilibrium threshold R^*. In particular, the effect of a change in the set of agents on the probability of continued search is ambiguous. However, it follows from the analysis in Section 2.5.3 that there are cases in which a change in the set of agents can lead to an increase in the probability of continued search and therefore reduce welfare. The intuition is that a change in the set of agents can increase the dispersion in agency rents, which can in turn increase the option value of search. Notice also that a change in the set of agents does not reduce welfare in the absence of search frictions. In particular, the welfare loss that arises due to the interaction between contracting and search frictions crucially depends on the nature and degree of heterogeneity in agents in the economy. The results show that due to the interaction between contracting and search frictions, the functioning of decentralized markets depends crucially on the nature and degree of heterogeneity in market participants. In particular, my results suggest that markets with a higher degree of heterogeneity, for example, a higher degree of heterogeneity in human capital in labor markets or a higher degree of heterogeneity in investment projects in financial markets, may be less efficient compared with markets with a lower degree of heterogeneity.

3 General Equilibrium Framework

In this section, I extend the partial equilibrium model from Section 2 to a general equilibrium model.

3.1 Model

Time is discrete and infinite: $t \in \mathbb{Z} := \{\ldots,-1,0,1,\ldots\}$. There is a continuum of risk-neutral principals with measure 1 (each referred to as *she*) and a continuum of risk-neutral agents with measure 1 (each referred to as *he*) with the same time-discount factor $\beta \in (0,1)$. There are two types of principals, indexed by $j \in \{1,2\}$, each with measure $\frac{1}{2}$. Each principal has access to a production technology, which generates a random output $\tilde{x}$ in each period $t \in \mathbb{Z}$. Specifically, the production technology pays $x_i \in \mathbb{R}_+ := [0,\infty)$ in state $i \in \Omega := \{1,\ldots,n\}$, where $n > 2$. Output is independent across periods and across principals. Without an agent, the output $\tilde{x}$ of a type $j \in \{1,2\}$ principal is drawn according to the probability distribution $q^j \in \mathbb{R}^n$, where $q^j_i > 0$ denotes the probability of state $i \in \Omega$. I refer to q^j as the "baseline production technology" of a type j principal.[8]

There are two types of agents, indexed by $k \in \{1,2\}$, each with measure $\frac{1}{2}$. In each period $t \in \mathbb{Z}$, each principal can contract with a single agent. The agent decides between exerting effort and shirking, which is noncontractible. Agents have heterogeneous production technologies, denoted by $\Delta^k \in \mathbb{R}^n$, $k \in \{1,2\}$. If an agent shirks, then output $\tilde{x}$ of a type $j \in \{1,2\}$ principal is drawn according to the principal's baseline production technology q^j. If a type $k \in \{1,2\}$ agent exerts effort, then output $\tilde{x}$ of a type $j \in \{1,2\}$ principal is drawn according to the production technology $p^{j,k} := q^j + \Delta^k$, where $p^{j,k}_i$ denotes the probability of state $i \in \Omega$. All agents incur a disutility of effort $c > 0$ and receive utility $b > 0$ in a period $t \in \mathbb{Z}$ when not contracting with a principal. I refer to b as an agent's "outside option."

[8]The assumption that a type j principal produces the expected output $\mathbb{E}_{q^j}[\tilde{x}]$ without an agent is not important. The distribution q^j matters to the extent that it determines the output distribution if an agent shirks. In particular, the model can be extended to feature zero output without an agent.

Assumption 5. *All agents generate the same expected value of effort* $\pi > 0$*, that is, for all* $j \in \{1,2\}$ *and for all* $k \in \{1,2\}$, $\mathbb{E}_{p^{j,k}}[\tilde{x}] - \mathbb{E}_{q^j}[\tilde{x}] = \sum_{i=1}^n \Delta_i^k x_i = \pi$.

Since all agents generate the same expected value of effort π, incur the same disutility of effort c, and receive the same outside option b, all agents generate the same expected surplus of effort $\pi - c - b$. Equalizing the expected surplus of effort across agents eliminates increasing expected surplus as a motivation for search, which allows me to isolate the novel contribution of the paper. The model can be extended to feature agents with different expected values of effort.[9]

At the beginning of each period $t \in \mathbb{Z}$, each unmatched principal (who is not contracting with an agent at the beginning of the period) randomly draws an agent from the set of unmatched agents (who are not contracting with a principal at the beginning of the period). Given an agent draw at the beginning of period $t \in \mathbb{Z}$, a principal can offer a one-period contract $s = (s_i)_{i \in \Omega} \in \mathbb{R}^n$ to the agent, which is repeated in each future period, where s_i is the agent's contractual payoff and $x_i - s_i$ is the principal's contractual payoff in state $i \in \Omega$ in each period.[10] I denote the random variable by $\tilde{s}$. At the end of each period, a fraction $\delta \in (0,1)$ of existing principal-agent matches breaks up at random.

Assumption 6. $\pi - c - b > 0$.

Assumption 6 implies that contracting with an agent is efficient independent of its type and corresponds to Assumption 2 in the partial equilibrium model.

Assumption 7. *For all* $i \in \Omega$*, the contract* s *satisfies* $s_i \geq 0$.

Assumption 7 requires that an agent's contractual payoff cannot be lower than 0 in each state and corresponds to Assumption 3 in the partial equilibrium model with $\theta = 0$.

An equilibrium is a set of contract offers and search decisions by each principal, accept and reject decisions by each agent, and effort decisions by each agent such that each principal and each agent maximizes his/her expected discounted payoff. I consider the economy in steady-state equilibrium, in which the measures of unmatched principals and unmatched agents of each type

[9]See Appendix B for an extension of the partial equilibrium model with heterogeneity in the expected value of effort.

[10]See Footnote 4 and Appendix C for a discussion of long-term contracts, which applies equally to the general equilibrium framework.

and the measures of principal-agent matches $(j,k) \in \{1,2\} \times \{1,2\}$ are constant over time. Note that uniqueness in my framework is defined as uniqueness in terms of expected payoffs; in other words, an equilibrium is unique up to variations in incentive-compatible contract offers that give principals and agents the same expected payoffs.

Compared to the partial equilibrium model from Section 2, I assume that the search cost κ is equal to zero, the fixed cost I is equal to zero, and the degree of contractual constraints θ is set to zero, which simplifies the exposition. The model can be extended to include the additional parameters κ, I, and θ.

Applications 1 and 2 discussed in Section 2.1 apply to the general equilibrium model. The additional dimension of the general equilibrium model is that there are different types of principals. In particular, the output distribution depends on both the principal's and agent's type. In labor markets, differences between principals' baseline production technologies can capture differences between different types of jobs a worker can apply for. In financial markets, differences between principals' baseline production technologies can capture differences in expertise between different types of investors an entrepreneur can obtain capital from.

3.2 Benchmark Model without Contractual Constraints

In this section, I consider a benchmark model without contractual constraints from Assumption 7 such that principals can choose a contract s without any constraints. Consider an exogenous reservation utility $u \geq 0$, which I endogenize below, and an arbitrary principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$. An optimal one-period contract, denoted by $s^*(j,k)$, satisfies

$$s^*(j,k) \in \arg\max_{s} \mathbb{E}_{p^{j,k}}[\tilde{x} - \tilde{s}] \text{ s.t. } \mathbb{E}_{p^{j,k}}[\tilde{s}] - c \geq \mathbb{E}_{q^j}[\tilde{s}],\ \mathbb{E}_{p^{j,k}}[\tilde{s}] - c \geq u. \tag{14}$$

Note that this contracting problem is identical to contracting problem (1) and the following corollary follows directly from Lemma 1.

Corollary 1. *Consider a principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$. Each optimal contract $s^*(j,k)$ satisfies the agent's participation constraint with equality, that is, $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] - c = u$.*

Using Corollary 1 and Assumption 5, the principal's expected payoff under an optimal contract is given by $\mathbb{E}_{p^{j,k}}[\tilde{x}-\tilde{s}^*(j,k)] = \mathbb{E}_{q^j}[\tilde{x}] + \pi - c - u$. Importantly, the principal's expected payoff is independent of the agent's type $k \in \{1,2\}$.

Consider a full matching equilibrium in which each principal type offers a contract to each agent type, and agents accept the contract offers.

Lemma 6. *In a full matching equilibrium, the measures of principal-agent matches* $(1,1)$*,* $(1,2)$*,* $(2,1)$*, and* $(2,2)$ *after contracting and before separation are equal to* $\frac{1}{4}$*. The probability of drawing a type* 1 *agent at the beginning of a period before contracting is given by* $\gamma = \frac{1}{2}$*.*

Random search and the equal proportions of principal and agent types imply a uniform steady-state distribution of principal-agent matches in a full matching equilibrium. A share δ of these matches breaks up after production. At the beginning of the next period, all principals meet an agent at random and vice versa. Since each principal type offers a contract to each agent type, all unmatched principals and unmatched agents are matched again at the beginning of the next period. Intuitively, while there are unmatched principals and unmatched agents after production due to exogenous separation, all principals and agents are matched again before production in the next period.

Proposition 8. *There exists a unique full matching equilibrium in which contract offers give agents the expected contractual payoff* $c+b$ *in all matches.*

The benchmark model without contractual constraints has a simple equilibrium. As shown in Corollary 1, agents receive their reservation utility under optimal contracts. In the absence of contractual constraints, a principal can therefore offer a contract that gives the agent an expected payoff of b per period. I will refer to the portion of the expected surplus of effort $\pi - c - b > 0$, which an agent obtains in a principal-agent match, as the agent's agency rent. In particular, a principal can offer a contract that gives an agent an agency rent of zero. A particular principal type therefore does not prefer a particular agent type. As a result, each principal type offers a contract to each agent type. In other words, there is never a situation in which a principal meets an agent and does not offer a contract to the agent.

It is also clear that the equilibrium maximizes the sum of the principals' and agents' expected discounted payoffs and is first-best efficient. In each period $t \in \mathbb{Z}$, a principal-agent match (j,k)

generates expected surplus $\mathbb{E}_{p^{j,k}}[\tilde{x}] - c = \mathbb{E}_{q^j}[\tilde{x}] + \pi - c$. If a principal does not contract with an agent, the expected surplus generated by the unmatched principal and the unmatched agent is given by $\mathbb{E}_{q^j}[\tilde{x}] + b$. We have $\mathbb{E}_{q^j}[\tilde{x}] + \pi - c > \mathbb{E}_{q^j}[\tilde{x}] + b \Leftrightarrow \pi - c - b > 0$, which holds by Assumption 6. This implies that, in the absence of contractual constraints, the search frictions and moral hazard alone do not generate inefficiencies in my framework.

3.3 Equilibrium with Contractual Constraints

In this section, I study the equilibrium with contractual constraints from Assumption 7. In Section 3.3.1, I discuss the one-period contracting problem for an arbitrary principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$. In Sections 3.3.2 and 3.3.3, I determine the dynamic equilibrium.

3.3.1 Optimal Contracts and Agency Rents

Consider an arbitrary principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$ and an exogenous reservation utility $u \geq 0$, which I endogenize below. An optimal one-period contract, denoted by $s^*(j,k)$, satisfies

$$s^*(j,k) \in \arg\max_{s} \mathbb{E}_{p^{j,k}}[\tilde{x} - \tilde{s}] \text{ s.t. } \mathbb{E}_{p^{j,k}}[\tilde{s}] - c \geq \mathbb{E}_{q^j}[\tilde{s}],\ \mathbb{E}_{p^{j,k}}[\tilde{s}] - c \geq u,\ \forall i \in \Omega : s_i \geq 0. \quad (15)$$

Definition 2. *For each principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$, the likelihood ratio $\phi(j,k) = (\phi_i(j,k))_{i \in \Omega} \in \mathbb{R}^n$ is defined as $\phi_i(j,k) := \frac{\Delta_i^k}{q_i^j}$, $i \in \Omega$. I denote the maximum likelihood ratio by $\phi^*(j,k) := \max_{i \in \Omega} \phi_i(j,k)$.*

Definition 2 extends Definition 1 from the partial equilibrium model. Intuitively, the likelihood ratio $\phi_i(j,k)$ measures how informative the state $i \in \Omega$ is about the effort of a type k agent when matched with a type j principal. The contracting problem (15) is a special case of the contracting problem (3) and the following corollary follows directly from Lemma 2.

Corollary 2. *Consider a principal-agent pair $(j,k) \in \{1,2\} \times \{1,2\}$. Let $l \in \arg\max_{i \in \Omega} \phi_i(j,k)$.*

(i) If $u < \frac{c}{\phi^*(j,k)}$, then the agent's incentive constraint binds and an optimal contract is given by $s_i^*(j,k) = \mathbb{1}_{\{i=l\}} \frac{c}{\Delta_l^k}$, $i \in \Omega$, which satisfies $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] - c = \frac{c}{\phi^*(j,k)} > u$.

(ii) If $u \geq \frac{c}{\phi^*(j,k)}$, then the agent's participation constraint binds and an optimal contract is given by $s_i^*(j,k) = u - \frac{c}{\phi^*(j,k)} + \mathbb{1}_{\{i=l\}} \frac{c}{\Delta_l^k}$, $i \in \Omega$, which satisfies $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] - c = u$.

In Case (i) of Corollary 2, the agent's reservation utility u is low such that the agent's incentive constraint binds. The principal has to give the agent at least 0 in all states. As is standard, it is optimal to give the agent more than 0 only in a state with the maximum likelihood ratio. Intuitively, this state is most informative about the agent's effort. In particular, the agent's expected contractual payoff is given by $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] = c + \frac{c}{\phi^*(j,k)}$. Let $A(j,k) := \frac{c}{\phi^*(j,k)}$. I refer to $c + A(j,k)$ as an agent's "incentive payoff." A higher maximum likelihood ratio $\phi^*(j,k)$ implies that output is more informative about the agent's effort, which implies a lower incentive payoff.[11]

In Case (ii) of Corollary 2, the agent's reservation utility u is high such that the agent's participation constraint binds. In this case, the optimal contract from Case (i) satisfies the agent's incentive constraint but violates the agent's participation constraint. The principal can then simply add a fixed component to the contract from Case (i) to make the participation constraint bind. In particular, the agent's expected contractual payoff is given by $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] = c + u$.

Without loss of generality, I can therefore write the expected contractual payoff of an optimal contract from Corollary 2 as the sum of the incentive payoff $c + A(j,k)$ and a "fixed payoff" $r(j,k) \geq 0$, that is, $\mathbb{E}_{p^{j,k}}[\tilde{s}^*(j,k)] = c + A(j,k) + r(j,k)$. Using Assumption 5, the principal's expected payoff is then given by $\mathbb{E}_{p^{j,k}}[\tilde{x} - \tilde{s}^*(j,k)] = \mathbb{E}_{q^j}[\tilde{x}] + \pi - c - A(j,k) - r(j,k)$. Importantly, in the presence of contractual constraints, a principal's expected payoff under optimal contracts depends on the type of agent and vice versa. I will refer to the portion of the expected surplus of effort $\pi - c - b > 0$, which an agent obtains in a principal-agent match, as the agent's agency rent. Specifically, an agent's agency rent is given by $A(j,k) + r(j,k) - b$.

To simplify the exposition, I assume the following.

Assumption 8. *We have $A(1,1) = A(2,2) =: \underline{A}$ and $A(1,2) = A(2,1) =: \bar{A}$.*

Assumption 9. *We have $\underline{A} < \bar{A}$.*

Assumption 8 allows me to focus on symmetric equilibria, which simplifies the exposition.[12]

[11] See Footnote 6 for a discussion of the contracting problem.

[12] As will become clear during the analysis, the symmetric general equilibrium model has desirable properties. First,

Assumption 9 implies dispersion in incentive pay, which allows me to focus on the interesting cases of the model.

Assumption 10. *We have* $c+\bar{A}<\pi$.

Assumption 10 states that it is profitable for each principal type to contract with each agent type in a static setting. If Assumption 10 is violated, some principal types do not contract with some agent types even in a static setting. Note that while $A(1,1)$, $A(2,2)$, $A(1,2)$, and $A(2,1)$ are endogenous quantities, there exist production technologies q^j, $j \in \{1,2\}$, and Δ^k, $k \in \{1,2\}$, such that Assumptions 8, 9, and 10 are satisfied.

3.3.2 Full Matching Equilibrium

As in Section 3.2, in this section, I consider a full matching equilibrium in which each principal type offers a contract to each agent type, and agents accept the contract offers. Thus, the steady-state measures of principal-agent matches are given in Lemma 6.

As discussed in Section 3.3.1, a type j principal needs to offer a type k agent a contractual payoff of at least $c+\underline{A}$ if $k=j$, and of at least $c+\bar{A}$ if $k \neq j$, since the agent does not exert effort otherwise. It therefore remains to determine the fixed payoffs $r(j,k) \geq 0$, $(j,k) \in \{1,2\} \times \{1,2\}$. The equilibrium contracts are then given by, for all $(j,k) \in \{1,2\} \times \{1,2\}$, $s_i^*(j,k)+r(j,k)$, $i \in \Omega$, where $s^*(j,k)$ is an optimal one-period contract from Case (i) of Corollary 2.

Proposition 9. *If* $b \geq \bar{A}$, *then there exists a unique full matching equilibrium in which contract offers give agents the expected contractual payoff* $c+b$ *in all matches, that is,* $r(1,1)=r(2,2)=b-\underline{A}$ *and* $r(2,1)=r(1,2)=b-\bar{A}$.

If $b \geq \bar{A}$, the agents' outside option determines the lower bound for agents' expected contractual payoffs as in the benchmark model without contractual constraints discussed in Section 3.2. In particular, a principal can therefore offer a contract that gives an agent an agency rent of zero. As a result, principals offer agents contracts with expected payoffs equal to the outside option that imply an agency rent of zero.

in all symmetric equilibria, both agent types receive contract offers in equilibrium. Second, both agent types have the same expected discounted payoff if unmatched at the end of a period. This implies that an agent would be indifferent between both production technologies before entering the market. Third, the assumption in the partial equilibrium framework that all agents have the same reservation utility is satisfied in all symmetric equilibria.

Proposition 10. *Let $b < \bar{A}$ and $\theta := \frac{2(1-\beta(1-\delta))}{2-\beta(1-\delta)}$. We have $\theta \in (0,1)$.*

(i) If $(1-\theta)\bar{A} + \theta b \leq \underline{A}$ and $\bar{A} \leq \theta(\pi - c) + (1-\theta)\underline{A}$, then there exists a unique full matching equilibrium in which contract offers give agents the expected contractual payoff $c + \underline{A}$ in matches $(1,1)$ and $(2,2)$, and $c + \bar{A}$ in matches $(1,2)$ and $(2,1)$, that is, $r(1,1) = r(2,2) = r(1,2) = r(2,1) = 0$.

(ii) If $(1-\theta)\bar{A} + \theta b > \underline{A}$ and $\bar{A} \leq \frac{1}{2-\theta}(\pi - c) + \frac{1-\theta}{2-\theta}b$, then there exists a unique full matching equilibrium in which contract offers give agents the expected contractual payoff $c + (1-\theta)\bar{A} + \theta b$ in matches $(1,1)$ and $(2,2)$, and $c + \bar{A}$ in matches $(1,2)$ and $(2,1)$, that is, $r(1,1) = r(2,2) = (1-\theta)\bar{A} - \underline{A} + \theta b > 0$, and $r(1,2) = r(2,1) = 0$.

Proposition 10 shows that if $b < \bar{A}$, there is dispersion in agents' expected contractual payoffs in a full matching equilibrium. In Case (i), a type 1 agent receives a low expected contractual payoff $c + \underline{A}$ when matched with a type 1 principal, and a higher expected contractual payoff $c + \bar{A}$ when matched with a type 2 principal. In particular, a type 1 agent receives an agency rent of $\underline{A} - b \geq 0$ when matched with a type 1 principal and a higher agency rent of $\bar{A} - b > 0$ when matched with a type 2 principal. The first condition $(1-\theta)\bar{A} + \theta b \leq \underline{A}$ ensures that the agents' outside option is sufficiently low since agents would not accept the low contract offers and would continue to search for a high contract offer otherwise. The second condition $\bar{A} \leq \theta(\pi - c) + (1-\theta)\underline{A}$ ensures that the dispersion in the incentive payoffs is not too large since principals would not make high contract offers and would continue to search for an agent who accepts a low contract offer otherwise.

While the equilibrium in Proposition 10 generates full matching in each period, the equilibrium is different from the full matching equilibrium in the benchmark model without contractual constraints in Proposition 8. In the benchmark model, each agent receives the expected contractual payoff $c + b$ in equilibrium, which implies an agency rent of zero, independent of the match. In the full matching equilibrium with contractual constraints, there is dispersion in expected contractual payoffs and agency rents both across and within agents.

In Case (ii) of Proposition 10, the agents' outside option is sufficiently high such that agents would not accept contract offers with the low expected contractual payoff $c + \underline{A}$ in matches $(1,1)$ and $(2,2)$. As a result, the principals have to offer a positive fixed payoff in these matches. Interestingly, the expected contractual payoff for a particular principal-agent match then depends on other agency problems in the economy. This is the case because an agent's reservation utility depends on the other matching opportunities in the economy and therefore on the other contracting problems the agent faces with other principals.

3.3.3 Partial Matching Equilibrium

In this section, I consider a partial matching equilibrium in which type j principals offer contracts only to type $k = j$ agents, and agents accept the contract offers but do not offer contracts to type $k \neq j$ agents. Let $\lambda_k \in \left[0, \frac{1}{2}\right]$ denote the steady-state measure of type k agents who are unmatched during period $t \in \mathbb{Z}$ (i.e., after contracting and before separation). Due to the symmetry of the model, λ_k is equal to the measure of unmatched type $j = k$ principals during period $t \in \mathbb{Z}$. Thus, $\frac{1}{2} - \lambda_k$ is the measure of matched type k agents and $\frac{1}{2} - \lambda_k$ is the measure of matched type $j = k$ principals.

The transition of the economy from period $t \in \mathbb{Z}$ to period $t+1$ is summarized in Figure 1. After production in period t, a share δ of principal-agent matches $(1,1)$ and $(2,2)$ breaks up at random. Type 1 principals only offer contracts to type 1 agents and type 2 principals only offer contracts to type 2 agents. I denote the probability that an unmatched principal draws a type 1 agent at the beginning of a period before contracting by γ. Before production in period $t+1$, a share γ of unmatched type 1 agents and a share $1-\gamma$ of unmatched type 2 agents meet a type 1 principal and a type 2 principal, respectively, and receive a contract offer. Since there is a measure $\lambda_k + \delta\left(\frac{1}{2} - \lambda_k\right)$ of unmatched type k agents after separation, we have

$$\gamma = \frac{\lambda_1 + \delta\left(\frac{1}{2} - \lambda_1\right)}{\lambda_1 + \delta\left(\frac{1}{2} - \lambda_1\right) + \lambda_2 + \delta\left(\frac{1}{2} - \lambda_2\right)} = \frac{(1-\delta)\lambda_1 + \frac{\delta}{2}}{(1-\delta)(\lambda_1 + \lambda_2) + \delta}.$$

Lemma 7. *In a partial matching equilibrium, the measures of unmatched type* 1 *agents and unmatched type* 2 *agents after contracting and before separation are given by* $\lambda_1 = \lambda_2 = \frac{\delta}{2(1+\delta)}$. *The measures of* $(1,1)$ *principal-agent matches and* $(2,2)$ *principal-agent matches are equal to* $\frac{1}{2(1+\delta)}$. *The probability of drawing a type* 1 *agent at the beginning of a period is given by* $\gamma = \frac{1}{2}$.

As discussed in Section 3.3.1, a type j principal needs to offer a type $k = j$ agent an expected contractual payoff of at least $c + \underline{A}$, since the agent does not exert effort otherwise. It therefore

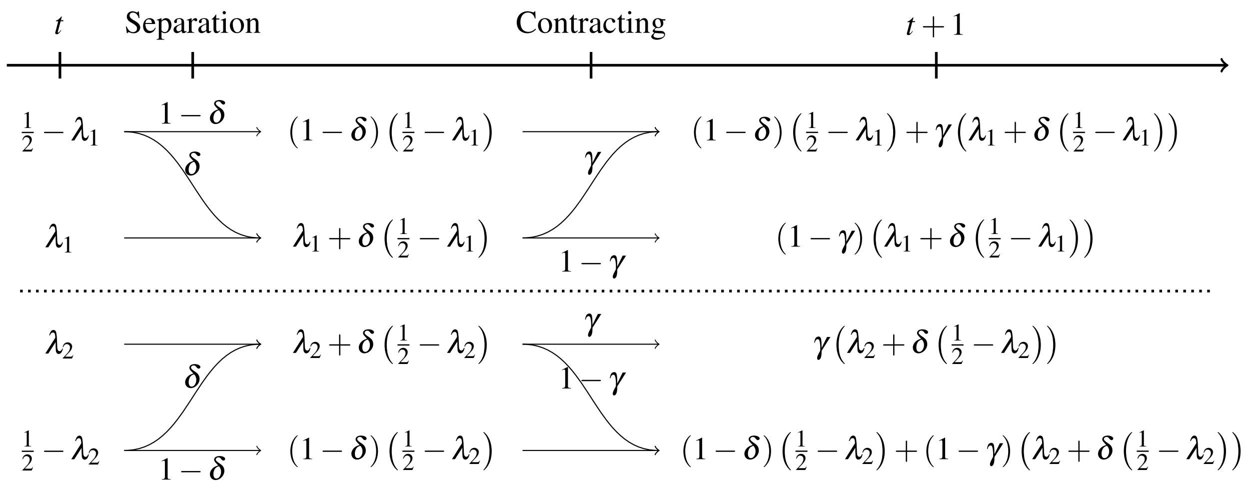

Figure 1: Separation and contracting in a partial matching equilibrium. This figure summarizes the matching dynamics in a partial matching equilibrium in which type j principals offer contracts only to type $k = j$ agents but not to type $k \neq j$ agents. δ is the exogenous separation rate at the end of each period, and γ is the endogenous probability of drawing a type 1 agent at the beginning of each period.

remains to determine the fixed payoffs $r(j,j) \geq 0$, $j \in \{1,2\}$. The equilibrium contracts are then given by, for all $(j,k) \in \{1,2\} \times \{1,2\}$, $s_i^*(j,k) + r(j,k)$, $i \in \Omega$, where $s^*(j,k)$ is an optimal one-period contract from Case (i) of Corollary 2.

Proposition 11. *If $b \geq \bar{A}$, then there does not exist a partial matching equilibrium.*

If $b \geq \bar{A}$, the agents' outside option determines the lower bound for agents' expected contractual payoffs as in the benchmark model without contractual constraints in Section 3.2. As a result, principals offer agents contracts with expected payoffs equal to the outside option that imply an agency rent of zero. Since agents have the same outside option, the principal has no incentive not to contract with some agent types.

Proposition 12. *Let $b < \bar{A}$ and $\theta := \frac{2(1-\beta(1-\delta))}{2-\beta(1-\delta)}$. We have $\theta \in (0,1)$. If $\bar{A} \geq \theta(\pi - c) + (1-\theta)\max\{\underline{A}, b\}$, then there exists a unique partial matching equilibrium in which contract offers give agents the expected contractual payoff $c + \max\{\underline{A}, b\}$ in matches $(1,1)$ and $(2,2)$, that is, $r(1,1) = r(2,2) = \max\{b - \underline{A}, 0\}$.*

In the partial matching equilibrium in Proposition 12, a type 1 principal makes contract offers only to type 1 agents but not to type 2 agents, and a type 2 principal makes contract offers only to type 2 agents but not to type 1 agents. In particular, when a type 1 principal meets a type 2 agent, both the principal and the agent remain unmatched. The condition $\bar{A} \geq \theta(\pi - c) + (1-\theta)\max\{\underline{A}, b\}$ ensures that the dispersion in the incentive payoffs is sufficiently large since principals would make contract offers to agents that require high incentive payoffs otherwise.

Importantly, the partial matching equilibrium is not the result of the contracting problem per se. A type j principal could offer a contract to a type $k \neq j$ agent with the expected contractual payoff $c + \bar{A}$, implying an agency rent of $\bar{A} - b > 0$. This would generate the additional expected payoff of $\pi - c - \bar{A}$ per period for the principal, which is positive by Assumption 10. The reason that the principal does contract with the agent is the option value of search. While the principal loses the additional expected payoff $\pi - c - \bar{A} > 0$ per period if she does not contract with the agent, she can find an agent with a lower agency rent in the next period.

3.3.4 Uniqueness of Equilibrium

In this section, I discuss uniqueness of equilibrium. Recall that I consider uniqueness in terms of expected payoffs. The previous analysis identifies conditions under which a unique full matching equilibrium and a unique partial matching equilibrium exist. However, there are two other classes of potential symmetric equilibria. First, there is a class of potential symmetric equilibria in which both principal types make no contract offers to any agent type. Clearly, such an equilibrium cannot exist because a type 1 principal has an incentive to deviate and offer a contract to a type 1 agent. Second, there is a class of potential symmetric equilibria in which type 1 principals offer contracts only to type 2 agents and type 2 principals offer contracts only to type 1 agents. Such an equilibrium cannot exist since a type 1 principal has an incentive to deviate and offer a contract when she meets a type 1 agent, since she can offer a contract with a lower expected contractual payoff to that agent. As a result, the unique full matching equilibrium and the unique partial matching equilibria are the only possible symmetric equilibria.

Note that there may be a region in which both the partial and the full hiring equilibrium exist. Specifically, assume that $\bar{A} \geq \theta(\pi - c) + (1-\theta)\underline{A}$. Then Propositions 9 and 10 imply that a full matching equilibrium exists if

$$b > \bar{b}_1 := \frac{2-\theta}{1-\theta}\bar{A} - \frac{1}{1-\theta}(\pi - c).$$

Proposition 12 implies that a partial matching equilibrium exists if

$$b \leq \bar{b}_2 := \frac{1}{1-\theta}\bar{A} - \frac{\theta}{1-\theta}(\pi - c).$$

Note that $\bar{b}_2 > \bar{b}_1 \Leftrightarrow \pi - c - \bar{A} > 0$. Hence, there are parameters such that both the full matching and the partial matching equilibrium exist. The reason for the existence of multiple equilibria is the following. In the full matching equilibrium, the expected contractual payoff in matches $(1,1)$ and $(2,2)$ may be determined by agents' endogenous reservation utility, which is a function of the expected contractual payoff in matches $(2,1)$ and $(1,2)$, respectively. This can be interpreted as a strategic complementarity (see, e.g., Bulow et al., 1985; Cooper and John, 1988). If a share of type 1 principals starts offering contracts to type 2 agents, this increases the reservation utility of type 2 agents when matched with type 2 principals, which increases their expected contractual payoff in these matches. This in turn makes type 2 principals more likely to offer contracts to type 1 agents. This strategic complementarity gives rise to multiple equilibria.

3.4 Constrained Efficiency

In this section, I consider a social planner who is subject to the same frictions and constraints as the principals and maximizes the sum of the principals' and agents' expected discounted payoffs.

In each period $t \in \mathbb{Z}$, each principal and agent can be matched or unmatched. Note that the search frictions do not prevent each principal and each agent from being matched in each period. At the beginning of each period, each unmatched agent meets an unmatched principal and vice versa. If all principals would contract with the agents they meet, independent of their type, there would be no unmatched principals and agents during production.

If a type j principal is matched with an agent in period $t \in \mathbb{Z}$, then the expected surplus of this match in period t is given by $\mathbb{E}_{q^j}[\tilde{x}] + \pi - c$. If a type j principal is unmatched in period $t \in \mathbb{Z}$, then the expected surplus of this non-match in period t is given by $\mathbb{E}_{q^j}[\tilde{x}] + b$. The social planner prefers a match if $\mathbb{E}_{q^j}[\tilde{x}] + \pi - c > \mathbb{E}_{q^j}[\tilde{x}] + b \Leftrightarrow \pi - c - b > 0$, which holds by Assumption 6. The social planner can implement a full matching equilibrium by offering incentive-compatible contracts with an expected contractual payoff $c+\max\{b,\bar{A}\}$ to all agents, such that all agents accept the contract offers. In particular, the partial matching equilibrium is constrained inefficient, whereas the full matching equilibrium is constrained efficient.

Importantly, the inefficiencies arise due to the interaction between contracting and search frictions. To see this, first recall that in the absence of contractual constraints in Section 3.2, the equilibrium is constrained efficient. Second, in the absence of search frictions, each principal would immediately identify and contract with the low incentive payoff agent, which would also implement the constrained efficient allocation. Recall that the option value of search, which leads to the overly intense search in the partial matching equilibrium, requires both contractual constraints and heterogeneity in principals' and agents' production technologies, which jointly generate endogenous dispersion in agents' incentive payoffs and agency rents.

3.5 Discussion and Implications

First note the role of the agents' outside option b. Proposition 9 shows that a unique full matching equilibrium exists if b is sufficiently high. Proposition 11 shows that there does not exist a partial matching equilibrium if b is sufficiently high. In particular, given the discussion in Section 3.3.4 regarding uniqueness, this implies that the full matching equilibrium is the unique symmetric equilibrium if b is sufficiently high. This implies that, while an inefficient partial matching equilibrium may exist if b is low, an increase in b can always restore efficiency by making the full matching equilibrium the unique symmetric equilibrium. The intuition is that the dispersion in agency rents is lower if b is higher, which reduces the option value of search and therefore reduces principals' incentives to forgo valuable contracting opportunities.

In the context of a labor market, b captures unemployment insurance. In particular, the results imply that an increase in unemployment insurance can reduce unemployment and increase welfare. In the context of a financial market, b captures the outside option of entrepreneurs such as alternative employment opportunities. The results imply that better alternative employment opportunities for entrepreneurs can make it easier for entrepreneurs to obtain capital and increase welfare.

In addition to the differences in the measures of matched and unmatched principals and agents in the full matching compared with the partial matching equilibrium, the two equilibria differ in their matching patterns. Specifically, in the full matching equilibrium, each principal type contracts with each agent type. In the context of a labor market, this means that each type of firm employs each type of worker. In the context of a financial market, this means that each type of investor provides capital to each type of entrepreneur. In contrast, in the partial matching equilibrium, there is specialization in the sense that a type 1 principal contracts only with a type 1 agent and a type 2 principal contracts only with a type 2 agent. For example, each firm specializes in one type of worker or each investor specializes in one type of entrepreneur.

As such, my framework provides a novel theory of specialization that is not driven by exogenous productive complementarities and is consistent with the specialization of both firms (see, e.g., Krugman, 1981) and investors (see, e.g., Carey et al., 1998). In my model, specialization arises endogenously due to the interaction between contracting and search frictions. Note that this "contractual specialization" is inefficient as it is driven by a rent reduction motive of principals.

4 Concluding Remarks

This paper develops a theory of contracting and search by incorporating a standard risk-neutral principal-agent model with contractual constraints into a standard random search model. The principal-agent model has been applied extensively to labor and financial economics. The contractual constraints capture, for example, wealth constraints and a minimum wage in labor markets or statutory limited liability in financial markets.

The key innovation in this paper is to introduce heterogeneity in principals' and agents' production technologies in terms of the distribution of output across states of nature. This notion of production technologies captures the intuitive idea that economic agents differ not only in their average output but also in their ability to substitute output across states of nature (see, e.g., Cochrane, 2019). In a principal-agent context, the distribution of output clearly matters in addition to average output. The challenge I address in this paper is to incorporate this heterogeneity into a search problem with endogenous contracts. My model is tractable and allows for the analysis of contracting and search in partial and general equilibrium in this context.

The heterogeneity in principals' and agents' production technologies captures the idea that economic transactions differ not only in how much surplus they generate, but also in the characteristics of the contracting problem associated with the production of the surplus, which is a potentially important aspect of many markets.

The results show that due to the interaction between contracting and search frictions, the functioning of decentralized markets depends crucially on the nature and degree of heterogeneity in market participants. In particular, my results suggest that markets with a higher degree of heterogeneity, for example, a higher degree of heterogeneity in human capital in labor markets or a higher degree of heterogeneity in investment projects in financial markets, may be less efficient compared with markets with a lower degree of heterogeneity, with potentially important implications for market integration and specialization.

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A Proofs

A.1 Proof of Lemma 1

The problem is equivalent to minimizing $\mathbb{E}_{p^k}[\tilde{s}]$ subject to the constraints. A contract $s \in \mathbb{R}^n$ is feasible if $\mathbb{E}_{p^k}[\tilde{s}] - c \geq \mathbb{E}_q[\tilde{s}]$ and $\mathbb{E}_{p^k}[\tilde{s}] \geq c + u$. For any feasible contract s with $\mathbb{E}_{p^k}[\tilde{s}] > c + u$, consider a contract s', where $s'_i = s_i - \gamma$, $i \in \Omega$, and $\gamma = \mathbb{E}_{p^k}[\tilde{s}] - c - u$. The contract s' is still feasible, since $\mathbb{E}_{p^k}[\tilde{s}'] - \mathbb{E}_q[\tilde{s}'] = \mathbb{E}_{p^k}[\tilde{s}] - \mathbb{E}_q[\tilde{s}] \geq c$ and $\mathbb{E}_{p^k}[\tilde{s}'] = \mathbb{E}_{p^k}[\tilde{s}] - \gamma = c + u$. Further, $\mathbb{E}_{p^k}[\tilde{s}'] = c + u < \mathbb{E}_{p^k}[\tilde{s}]$ and s is not optimal. ■

A.2 Proof of Proposition 1

Consider a principal at the end of period $t \in \mathbb{N}$ and denote the principal's optimal expected discounted payoff if she is unmatched by V and if she is matched with an agent by V_m. Then

$$V_m = \beta\left(\mathbb{E}_q[\tilde{x}] + \pi - c - u + (1-\delta)V_m + \delta V\right).$$

Thus, V_m satisfies

$$V_m = \beta\frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u + \delta V}{1 - \beta(1-\delta)}. \tag{16}$$

If the principal is unmatched and does not search, her expected discounted payoff is given by $\beta(\mathbb{E}_q[\tilde{x}] + V)$. If the principal is unmatched and searches and does not contract with an agent at the beginning of the next period, her expected discounted payoff is given by $-\kappa + \beta(\mathbb{E}_q[\tilde{x}] + V)$. If the principal is unmatched, searches and contracts with an agent, her expected discounted payoff is given by

$$-\kappa + \beta(\mathbb{E}_q[\tilde{x}] + \pi - c - u - I + (1-\delta)V_m + \delta V). \tag{17}$$

Substituting (16) into (17) yields

$$-\kappa + \beta\left(\frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u + \delta V}{1 - \beta(1-\delta)} - I\right).$$

As a result, V satisfies

$$V = \max\left\{\beta(\mathbb{E}_q[\tilde{x}]+V), -\kappa+\beta\left(\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u+\delta V}{1-\beta(1-\delta)}-I\right)\right\}.$$

The solution to the fixed-point problem is $V = \beta(\mathbb{E}_q[\tilde{x}]+V) \Leftrightarrow V = \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$, or

$$V = -\kappa+\beta\left(\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u+\delta V}{1-\beta(1-\delta)}-I\right),$$

which is equivalent to

$$V = -\frac{1-\beta(1-\delta)}{1-\beta}\kappa+\beta\left(\frac{\mathbb{E}_q[\tilde{x}]+\pi-c-u}{1-\beta}-\frac{1-\beta(1-\delta)}{1-\beta}I\right).$$

Equivalently,

$$V = \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}+\max\left\{-\frac{1-\beta(1-\delta)}{1-\beta}\kappa+\beta\left(\frac{\pi-c-u}{1-\beta}-\frac{1-\beta(1-\delta)}{1-\beta}I\right), 0\right\}.$$

Thus, if $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)}-I\right) \geq \kappa$, it is optimal for the principal to search at the end of each period in which she is not matched with an agent and to contract with the agent drawn in the next period independent of the agent's type. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)}-I\right) < \kappa$, it is optimal for the principal to not search in any period $t \in \mathbb{N}_0$. ■

A.3 Proof of Lemma 2

First, I show that for any feasible contract s, we have

$$\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}] \leq \mathbb{E}_q[\tilde{x}]+\pi-c-u-\max\left\{\theta+\frac{c}{\phi^*(k)}-u, 0\right\}. \quad (18)$$

Equation (18) is equivalent to two inequalities. I first show that the first inequality $\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}] \leq \mathbb{E}_q[\tilde{x}]+\pi-c-\theta-\frac{c}{\phi^*(k)}$ holds. Let s be a feasible contract. Using Assumption 1 and the agent's incentive constraint, we get

$$\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}] = \mathbb{E}_q[\tilde{x}]+\pi-\mathbb{E}_{p^k}[\tilde{s}] \leq \mathbb{E}_q[\tilde{x}]+\pi-c-\mathbb{E}_q[\tilde{s}] = \mathbb{E}_q[\tilde{x}]+\pi-c-\theta-\mathbb{E}_q[\tilde{s}-\theta]. \quad (19)$$

It remains to show that $\mathbb{E}_q[\tilde{s}-\theta] \geq \frac{c}{\phi^*(k)}$. Using the agent's incentive constraint, it follows that

$$\phi^*(k)\mathbb{E}_q[\tilde{s}-\theta] = \phi^*(k)\sum_{i=1}^{n} q_i(s_i-\theta) \geq \sum_{i=1}^{n} \phi_i(k)q_i(s_i-\theta) = \mathbb{E}_{p^k}[\tilde{s}] - \mathbb{E}_q[\tilde{s}] \geq c, \tag{20}$$

and hence $\mathbb{E}_q[\tilde{s}-\theta] \geq \frac{c}{\phi^*(k)}$. I next show that the second inequality $\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}] \leq \mathbb{E}_q[\tilde{x}] + \pi - c - u$ holds. Using Assumption 1 and the agent's participation constraint, we get

$$\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}] = \mathbb{E}_q[\tilde{x}] + \pi - \mathbb{E}_{p^k}[\tilde{s}] \leq \mathbb{E}_q[\tilde{x}] + \pi - c - u.$$

Second, I prove statement (i). Assume that $\theta + \frac{c}{\phi^*(k)} > u$. Let $j \in \arg\max_{i\in\Omega} \phi_i(k)$, and consider the contract $s_i^*(k) = \theta + \mathbb{1}_{\{i=j\}}\frac{c}{\Delta_j^k}$, $i \in \Omega$. It is straightforward to verify that $s^*(k)$ is feasible. Further, it is straightforward to verify that $\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}^*(k)] = \mathbb{E}_q[\tilde{x}] + \pi - c - \theta - \frac{c}{\phi^*(k)}$. From (18), it follows that $s^*(k)$ is optimal. Thus, for an optimal contract $s^*(k)$, inequalities in (19) and (20) turn to equalities. In particular, the agent's incentive constraint binds and $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = \theta + \frac{c}{\phi^*(k)}$.

Third, I prove statement (ii). Assume that $\theta + \frac{c}{\phi^*(k)} \leq u$. Let $j \in \arg\max_{i\in\Omega} \phi_i(k)$, and consider the contract $s_i^*(k) = u - \frac{c}{\phi^*(k)} + \mathbb{1}_{\{i=j\}}\frac{c}{\Delta_j^k}$, $i \in \Omega$. It is straightforward to verify that $s^*(k)$ is feasible and the agent's participation constraint binds. Further, it is straightforward to verify that $\mathbb{E}_{p^k}[\tilde{x}-\tilde{s}^*(k)] = \mathbb{E}_q[\tilde{x}] + \pi - c - u$. From (18), it follows that $s^*(k)$ is optimal. Thus, for an optimal contract $s^*(k)$, the agent's participation constraint binds and $\mathbb{E}_{p^k}[\tilde{s}^*(k)] - c = u$. ■

A.4 Proof of Lemma 3

Assumption 2 and $\kappa \geq 0$ imply that for all $R \geq 0$, $T(R) \geq 0$. Let $R \geq 0$ and $\hat{R} \geq 0$. We then have

$$\left|T(R) - T\left(\hat{R}\right)\right| = \beta(1-\delta)\left|\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), R\right\} - \min\left\{A\left(\tilde{k}\right), \hat{R}\right\}\right]\right| \leq \beta(1-\delta)\left|R-\hat{R}\right|,$$

and hence T is a contraction mapping on $\mathbb{R}_+$ since $\beta(1-\delta) < 1$. ■

A.5 Lemma A.1

Lemma A.1. *Define the function* $h : \mathbb{R}_+ \to \mathbb{R}$ *by* $h(R) := T(R) - R$*, where* T *is defined in (12). Then* h *is decreasing in* R*.*

Proof. Let $R_1, R_2 \in \mathbb{R}_+$. From Lemma 3, we know that $|T(R_1) - T(R_2)| \leq \beta(1-\delta)|R_1 - R_2|$. Assume that $R_1 > R_2$. Then

$$\begin{aligned} h(R_1) = T(R_1) - R_1 &= T(R_1) - T(R_2) + T(R_2) - R_1 = T(R_1) - T(R_2) + h(R_2) - R_1 + R_2 \\ &\leq \beta(1-\delta)(R_1 - R_2) + h(R_2) - (R_1 - R_2) = h(R_2) - (1 - \beta(1-\delta))(R_1 - R_2) < h(R_2), \end{aligned}$$

as $\beta(1-\delta) \in (0,1)$ and $R_1 > R_2$. ■

A.6 Proof of Proposition 2

It remains to characterize the solution to equation (7) in terms of R^*, defined by $T(R^*) = R^*$, where T is defined in (12). From Lemma A.1 in Appendix A.5 we know that the function h defined as $h(R) = T(R) - R$ is decreasing and $h(R^*) = 0$. We further have $h(R) > 0 \Leftrightarrow R < R^*$.

First, assume that

$$\beta(\mathbb{E}_q[\tilde{x}] + V) > -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\mathbb{E}_q[\tilde{x}] + V, \frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u - A(\tilde{k}) + \delta V}{1 - \beta(1-\delta)} - I\right\}\right]. \quad (21)$$

In particular, we have $V = \beta(\mathbb{E}_q[\tilde{x}] + V) \Leftrightarrow V = \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$. Substituting $V = \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$ into equation (21) and rearranging implies

$$\begin{aligned} &(1 - \beta(1-\delta))\kappa - \beta(\pi - c - u - (1 - \beta(1-\delta))I) \\ &\qquad + \beta\mathbb{E}_\mu\left[\min\left\{A(\tilde{k}), \pi - c - u - (1 - \beta(1-\delta))I\right\}\right] > 0. \quad (22) \end{aligned}$$

Denoting $\hat{R} = \pi - c - u - (1 - \beta(1-\delta))I$, we can rewrite (22) as

$$(1 - \beta(1-\delta))\kappa - \beta\hat{R} + \beta\mathbb{E}_\mu\left[\min\left\{A(\tilde{k}), \hat{R}\right\}\right] > 0.$$

Further, simple algebra implies

$$h\left(\hat{R}\right) = (1-\beta(1-\delta))(1-\delta)\kappa - \beta(1-\delta)\hat{R} + \beta(1-\delta)\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), \hat{R}\right\}\right].$$

Thus, (21) implies $h\left(\hat{R}\right) > 0$, which further implies that $\hat{R} = \pi - c - u - (1-\beta(1-\delta))I < R^* \Leftrightarrow \frac{\pi-c-u-R^*}{1-\beta(1-\delta)} < I$.

Next, assume that

$$\beta(\mathbb{E}_q[\tilde{x}] + V) \leq -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\mathbb{E}_q[\tilde{x}] + V, \frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u - A\left(\tilde{k}\right) + \delta V}{1-\beta(1-\delta)} - I\right\}\right]. \quad (23)$$

In particular, $V \geq \beta(\mathbb{E}_q[\tilde{x}] + V)$, which is equivalent to $V \geq \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$ and $\beta\left(\mathbb{E}_q[\tilde{x}] + V\right) \geq \beta\frac{\mathbb{E}_q[\tilde{x}]}{1-\beta}$. Subtracting $\beta(\mathbb{E}_q[\tilde{x}] + V)$ from both sides, (23) can be rewritten as

$$\begin{aligned} 0 \leq & -(1-\beta(1-\delta))\kappa \\ & + \beta\mathbb{E}_\mu\left[\max\left\{0, \pi - c - u - (1-\beta(1-\delta))I - A\left(\tilde{k}\right) + \beta(1-\delta)\mathbb{E}_q[\tilde{x}] - (1-\beta)(1-\delta)V\right\}\right]. \quad (24) \end{aligned}$$

Using $\mathbb{E}_q[\tilde{x}] \leq \frac{1-\beta}{\beta}$, (24) implies

$$0 \leq -(1-\beta(1-\delta))\kappa + \beta\mathbb{E}_\mu\left[\max\left\{0, \pi - c - u - (1-\beta(1-\delta))I - A\left(\tilde{k}\right)\right\}\right]. \quad (25)$$

Defining $\hat{R} = \pi - c - u - (1-\beta(1-\delta))I$ and rearranging (25) implies

$$(1-\beta(1-\delta))\kappa - \beta\hat{R} + \beta\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), \hat{R}\right\}\right] \leq 0.$$

As discussed above, (23) therefore implies $h\left(\hat{R}\right) \leq 0$, which further implies that $\hat{R} = \pi - c - u - (1-\beta(1-\delta))I \geq R^* \Leftrightarrow \frac{\pi-c-u-R^*}{1-\beta(1-\delta)} \geq I$.

In summary, (21) implies $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} < I$ and (23) implies $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} \geq I$. In particular, if $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} < I$, then $V = \beta(\mathbb{E}_q[\tilde{x}] + V)$ and it is optimal to not search in any period $t \in \mathbb{N}_0$. In contrast, if $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} \geq I$, then

$$V = -\kappa + \beta \mathbb{E}_\mu \left[\max \left\{ \mathbb{E}_q[\tilde{x}] + V, \frac{\mathbb{E}_q[\tilde{x}] + \pi - c - u - A\left(\tilde{k}\right) + \delta V}{1-\beta(1-\delta)} - I \right\} \right],$$

and the principal searches if she is not matched with an agent at the end of a period and contracts with the agent drawn at the beginning of the next period if $A(k) \leq R^*$. ■

A.7 Proof of Proposition 3

Assume first that $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} \geq I$. Using equation (12), this implies

$$\beta \left(\frac{\pi - c - u - \mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), R^*\right\}\right]}{1-\beta(1-\delta)} - I \right) \geq \kappa,$$

which implies $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right) \geq \kappa$. Hence, if the principal searches, it is always efficient to search. However, the principal contracts only with agents $k \in \mathscr{K}$ such that $A(k) \leq R^*$. Hence, the principal contracts with the agent drawn at the beginning of a period independent of its type if $A(K) \leq R^*$, which coincides with the constrained efficient allocation. However, if $A(K) > R^*$, the principal does not contract with some agents at the beginning of a period. In this case, the equilibrium is constrained inefficient.

Assume next that $\frac{\pi-c-u-R^*}{1-\beta(1-\delta)} < I$, such that the principal does not search in any period. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right) < \kappa$, this is constrained efficient. If $\beta\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right) \geq \kappa$, search is constrained efficient and the equilibrium is constrained inefficient. ■

A.8 Proof of Proposition 4

To highlight its dependence on κ, denote the function h defined in Appendix A.5 by h_κ. Lemma A.1 in Appendix A.5 shows that $h_\kappa(R)$ is decreasing in R. Let $0 \leq \kappa_1 < \kappa_2$. We have

$$h_{\kappa_2}(R) = h_{\kappa_1}(R) + (1-\beta(1-\delta))(1-\delta)(\kappa_2 - \kappa_1).$$

Thus,

$$0 = h_{\kappa_2}\left(R^*(\kappa_2)\right) = h_{\kappa_1}\left(R^*(\kappa_2)\right) + (1-\beta(1-\delta))(1-\delta)(\kappa_2-\kappa_1),$$

which implies

$$h_{\kappa_1}\left(R^*(\kappa_2)\right) = -(1-\beta(1-\delta))(1-\delta)(\kappa_2-\kappa_1) < 0,$$

which further implies that $R^*(\kappa_2) > R^*(\kappa_1)$. Since μ does not depend on κ, it follows that $\mathbb{P}_\mu\left(A\left(\tilde{k}\right) > R^*(\kappa)\right)$ is nonincreasing in κ.

Note that Assumption 2 implies that $h_\kappa(R) \geq (1-\beta(1-\delta))(1-\delta)\kappa - R$. If $R < (1-\beta(1-\delta))(1-\delta)\kappa$, then $h_\kappa(R) > 0$ and $R < R^*(\kappa)$. The latter inequality is true for all $R < (1-\beta(1-\delta))(1-\delta)\kappa$. This implies that $R^*(\kappa) \geq (1-\beta(1-\delta))(1-\delta)\kappa$. In particular, $\lim_{\kappa\to\infty} R^*(\kappa) = \infty$. The probability of continued search is zero once $R^*(\kappa) \geq A(K)$. Thus, the probability of continued search is decreasing for some κ if $R^*(0) < A(K)$. ■

A.9 Proof of Proposition 5

First, I determine the expected discounted surplus in the search equilibrium. Denote the expected discounted surplus at the end of a period in which the principal is not matched by S. Let the expected discounted surplus at the beginning of a period in which the principal is matched be M. Further, define $P(\kappa) := \mathbb{P}_\mu\left(A\left(\tilde{k}\right) \leq R^*(\kappa)\right)$. Then S and M satisfy

$$S = -\kappa + \beta\left(P(\kappa)(M-I) + (1-P(\kappa))(\mathbb{E}_q[\tilde{x}] + u + S)\right), \tag{26}$$

and

$$M = \mathbb{E}_q[\tilde{x}] + \pi - c + \delta S + (1-\delta)\beta M. \tag{27}$$

Solving (26) and (27) implies

$$\begin{aligned}S(\kappa) = \frac{\beta}{1-\beta}(\mathbb{E}_q[\tilde{x}] + u) - \frac{1-\beta(1-\delta)}{(1-\beta)(1-\beta(1-\delta)(1-P(\kappa)))}\kappa \\ + \frac{\beta P(\kappa)(1-\beta(1-\delta))}{(1-\beta)(1-\beta(1-\delta)(1-P(\kappa)))}\left(\frac{\pi-c-u}{1-\beta(1-\delta)} - I\right).\end{aligned}$$

Let $a(\kappa)$ and $b(\kappa)$ be defined such that

$$S(\kappa) = \frac{\beta}{1-\beta}(\mathbb{E}_q[\tilde{x}] + u) - a(\kappa)\kappa + b(\kappa)\left(\frac{\pi - c - u}{1-\beta(1-\delta)} - I\right).$$

Let $\hat{\kappa} > 0$ such that $R^*(\hat{\kappa}) = A(k)$ for some $k \in \mathscr{K}$. Then for all $\tilde{\kappa} < \hat{\kappa}$ $a(\tilde{\kappa}) > a(\hat{\kappa})$ and $b(\tilde{\kappa}) < b(\hat{\kappa})$. In particular, we have

$$S(\hat{\kappa}) - S(\tilde{\kappa}) = \left(\frac{\pi - c - u}{1-\beta(1-\delta)} - I\right)(b(\hat{\kappa}) - b(\tilde{\kappa})) - a(\hat{\kappa})(\hat{\kappa} - \tilde{\kappa}) + \tilde{\kappa}(a(\tilde{\kappa}) - a(\hat{\kappa})).$$

Since the first and third terms are positive, there exists a $\tilde{\kappa} < \hat{\kappa}$ such that $S(\hat{\kappa}) > S(\tilde{\kappa})$. ■

A.10 Proof of Proposition 6

To highlight its dependence on θ, denote the function h defined in Appendix A.5 by h_θ. Lemma A.1 in Appendix A.5 shows that $h_\theta(R)$ is decreasing in R. To highlight its dependence on θ, denote the agency rent function $A(k)$ by $A_\theta(k)$. Let $0 \leq \theta_1 < \theta_2$. Then $A_{\theta_1}(k) \leq A_{\theta_2}(k)$ and for all R, we have $h_{\theta_1}(R) \leq h_{\theta_2}(R)$. Thus

$$h_{\theta_1}(R^*(\theta_2)) \leq h_{\theta_2}(R^*(\theta_2)) = 0 = h_{\theta_1}(R^*(\theta_1)),$$

which further implies that $R^*(\theta_2) \geq R^*(\theta_1)$.

Let $\hat{\theta} \geq 0$ be such that $A_{\hat{\theta}}(l) < R^*\left(\hat{\theta}\right) < A_{\hat{\theta}}(l+1)$ for some $l \in \{1, \ldots, K-1\}$, and for all $k \in \mathscr{K}$, $\hat{\theta} + \frac{c}{\phi^*(k)} - u \neq 0$. As will become clear, I can choose such an $\hat{\theta}$ and l if $R^*(0) > A(1)$. Hence,

$$\mathbb{E}_\mu\left[\min\left\{A_{\hat{\theta}}\left(\tilde{k}\right), R^*\left(\hat{\theta}\right)\right\}\right] = \sum_{k=1}^{l} \mu_k A_{\hat{\theta}}(k) + R^*\left(\hat{\theta}\right)\sum_{k=l+1}^{K} \mu_k.$$

Notice that for all $k \in \{1, \ldots, K\}$ such that $A_{\hat{\theta}}(k) > 0$, we have $A_{\hat{\theta}}(k) = \hat{\theta} + \frac{c}{\phi^*(k)} - u$. Let $m \in \{1, \ldots, l\}$, such that $A_{\hat{\theta}}(k) > 0 \Leftrightarrow j \in \{m, \ldots, K\}$. Hence

$$\mathbb{E}_\mu\left[\min\left\{A_{\hat{\theta}}\left(\tilde{k}\right), R^*\left(\hat{\theta}\right)\right\}\right] = \sum_{k=m}^{l} \mu_k\left(\hat{\theta} + \frac{c}{\phi^*(k)} - u\right) + R^*\left(\hat{\theta}\right)\sum_{k=l+1}^{K} \mu_k.$$

Using the equilibrium condition in equation (12), $R^*\left(\hat{\theta}\right)$ is equal to

$$v\left((1-\beta(1-\delta))((1-\delta)\kappa+\pi-c-u)-(1-\beta(1-\delta))^2 I+\beta(1-\delta)\sum_{k=m}^{l}\mu_k\left(\hat{\theta}+\frac{c}{\phi^*(k)}-u\right)\right), \quad (28)$$

where

$$v=\frac{1}{1-\beta(1-\delta)\sum_{k=l+1}^{K}\mu_k}.$$

Since $R^*(\theta)$ and $\theta+\frac{c}{\phi^*(k)}-u$ are continuous in θ, there exists a neighborhood around $\hat{\theta}$, such that $A_\theta(l)<R^*(\theta)<A_\theta(l+1)$ and for all $k\in\mathscr{K}$, $\theta+\frac{c}{\phi^*(k)}-u\neq 0$, for all θ in this neighborhood. In particular, for all θ in this neighborhood, (28) holds. In particular, $R^*(\theta)$ is linear in this neighborhood and

$$\frac{\mathrm{d}R^*(\theta)}{\mathrm{d}\theta}=\frac{1}{1-\beta(1-\delta)\sum_{k=l+1}^{K}\mu_k}\beta(1-\delta)\sum_{k=m}^{l}\mu_k>0.$$

I next show that

$$\frac{1}{1-\beta(1-\delta)\sum_{k=l+1}^{K}\mu_k}\beta(1-\delta)\sum_{k=m}^{l}\mu_k=\frac{\mathrm{d}R^*(\theta)}{\mathrm{d}\theta}<\frac{\mathrm{d}A_\theta(l)}{\mathrm{d}\theta}=\frac{\mathrm{d}A_\theta(l+1)}{\mathrm{d}\theta}=1.$$

Since the left-hand side of the inequality is increasing in $\beta(1-\delta)$, it is sufficient to show that the weak inequality holds for $\beta(1-\delta)=1$:

$$\frac{1}{1-\sum_{k=l+1}^{K}\mu_k}\sum_{k=m}^{l}\mu_k\leq 1\Leftrightarrow\sum_{k=m}^{l}\mu_k\leq\sum_{k=1}^{l}\mu_k,$$

which clearly holds.

In particular, we can partition the interval $[0,\infty)$ such that $A_\theta(l)<R^*(\theta)<A_\theta(l+1)\Leftrightarrow\theta\in(\theta_{l+1},\theta_l)$ for some $\theta_{l+1},\theta_l\in[0,\infty)$. Since there is a finite (or empty) set of θ, such that there exists a $k\in\mathscr{K}$, where $\theta+\frac{c}{\phi^*(k)}-u=0$, $R^*(\theta)$ is piecewise linear. The slope of $R^*(\theta)$ on each interval is lower than the slope of the bounds $A_\theta(l+1)$ and $A_\theta(l)$ if $\theta+\frac{c}{\phi^*(l)}-u>0$. Hence the probability of continued search is nondecreasing in θ, and increasing for some θ if $R^*(0)>A(1)$. ■

A.11 Proof of Lemma 4

For an agent $k \in \mathscr{K}$, denote the maximum likelihood ratio in the case of state space Ω by $\phi^*(k)$ and in the case of state space $\hat{\Omega}$ by $\hat{\phi}^*(k)$, where $\hat{\phi}_\omega(k) = \frac{\sum_{i\in\omega}\Delta_i^k}{\sum_{i\in\omega} q_i}$, $\omega \in \hat{\Omega}$. Optimal contracts for the new state space are natural adaptions of optimal contracts for the original state space in Lemma 2. For example, if $u < \theta + \frac{c}{\hat{\phi}^*(k)}$, an optimal contract is given by $s_i^*(k) = \theta + \mathbb{1}_{\{i\in\omega^*\}} \frac{c}{\sum_{i\in\omega^*}\Delta_j^k}$, $i \in \Omega$, where $\omega^* \in \arg\max_{\omega\in\hat{\Omega}} \hat{\phi}_\omega(k)$. It is clear from the proof of Lemma 2 that the change in the set of contractible events changes the agency rent function. First, I show that for all $k \in \mathscr{K}$, $\phi^*(k) \geq \hat{\phi}^*(k)$. We have for all $i \in \Omega$, $\phi_i(k) \leq \phi^*(k) \Leftrightarrow p_i^k \leq (\phi^*(k)+1)q_i$. Thus, for any $\omega \in \hat{\Omega}$,

$$\hat{\phi}_\omega(k) = \frac{\sum_{i\in\omega} p_i^k}{\sum_{i\in\omega} q_i} - 1 \leq (\phi^*(k)+1)\frac{\sum_{i\in\omega} q_i}{\sum_{i\in\omega} q_i} - 1 = \phi^*(k).$$

The claim then follows from taking the maximum over $\omega \in \hat{\Omega}$.

Denote the agency rents by $A(k)$ and $\hat{A}(k)$, respectively. We then have $k \in \mathscr{K}$,

$$A(k) = \max\left\{\theta + \frac{c}{\phi^*(k)} - u, 0\right\} \leq \max\left\{\theta + \frac{c}{\hat{\phi}^*(k)} - u, 0\right\} = \hat{A}(k).$$

Let $D \geq 0$. We then have

$$\mathbb{P}_\mu\left(A\left(\tilde{k}\right) \leq D\right) = \mathbb{P}_\mu\left(\{k \in \mathscr{K} : A(k) \leq D\}\right) \geq \mathbb{P}_\mu\left(\left\{k \in \mathscr{K} : \hat{A}(k) \leq D\right\}\right) = \mathbb{P}_\mu\left(\hat{A}\left(\tilde{k}\right) \leq D\right),$$

which completes the proof. ■

A.12 Proof of Lemma 5

Denote an agency rent distribution that first-order stochastically dominates $A\left(\tilde{k}\right)$ by $\hat{A}\left(\tilde{k}\right)$. Then $\hat{R}^*$ solves

$$(1-\beta(1-\delta))((1-\delta)\kappa+\pi-c-u)-(1-\beta(1-\delta))^2 I+\beta(1-\delta)\mathbb{E}_\mu\left[\min\left\{\hat{A}\left(\tilde{k}\right),\hat{R}^*\right\}\right]-\hat{R}^*=0.$$

Since $A \mapsto \min\left\{A, \hat{R}^*\right\}$ is nondecreasing, we have

$$\mathbb{E}_\mu\left[\min\left\{\hat{A}\left(\tilde{k}\right), \hat{R}^*\right\}\right] \geq \mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), \hat{R}^*\right\}\right].$$

We have

$$(1-\beta(1-\delta))((1-\delta)\kappa+\pi-c-u)-(1-\beta(1-\delta))^2 I+\beta(1-\delta)\mathbb{E}_\mu\left[\min\left\{A\left(\tilde{k}\right), \hat{R}^*\right\}\right]-\hat{R}^* \leq 0.$$

As I show in Lemma A.1 in Appendix A.5, the function h is decreasing in R. The equilibrium threshold R^* therefore satisfies $R^* \leq \hat{R}^*$. ■

A.13 Proof of Proposition 7

Consider a search equilibrium with $\hat{R}^* = \hat{A}(\hat{k})$ for some $\hat{k} \in \mathscr{K}$. We can always construct a state space Ω and a partition $\hat{\Omega}$ of Ω, where $\hat{\Omega} \neq \{\Omega\}$ and $\hat{\Omega} \neq \{\{1\}, \ldots, \{n\}\}$ such that the agency rent distribution induced by the contracting environment with state space $\hat{\Omega}$, $\hat{A}\left(k\right)$, and the agency rent distribution induced by the contracting environment with state space Ω, $A\left(k\right)$, have the following property: $A\left(k\right) < \hat{A}\left(k\right)$ for all k such that $\hat{A}\left(k\right) < \hat{A}\left(\hat{k}\right)$, and $A\left(k\right) = \hat{A}\left(k\right)$ for all k such that $\hat{A}\left(k\right) \geq \hat{A}\left(\hat{k}\right)$. Then the inequalities in the proof of Lemma 5 become strict inequalities. In particular, $R^* < \hat{R}^*$. Specifically, the contractual innovation increases the probability of continued search. ■

A.14 Proof of Lemma 6

For each $(j,k) \in \{1.2\} \times \{1,2\}$, denote the measure of principal-agent matches of type (j,k) in period $t \in \mathbb{Z}$ by $h(j,k)$. Due to the equal proportions of principal and agent types, we have

$$h(1,1)+h(1,2) = h(2,1)+h(2,2) = h(1,1)+h(2,1) = h(1,2)+h(2,2) = \frac{1}{2}.$$

The inequalities imply that $h(1,1) = h(1,2) = h(2,1) = h(2,2)$. In particular, for all $(j,k) \in \{1,2\} \times \{1,2\}$, $h(j,k) = \frac{1}{4}$. ■

A.15 Proof of Proposition 8

I first show that the proposed allocation is an equilibrium. All contracts give agents the same expected payoff per period independent of their type. In addition, the expected payoff is equal to each agent's outside option. As a result, no agent has an incentive to deviate and reject a contract offer. Since all contracts give agents an expected payoff per period equal to their outside option, a principal cannot deviate and offer an incentive-compatible contract with a lower expected payoff. Since all contracts give agents the same expected payoff per period and since all agents generate the same expected surplus of effort $\pi - c - b > 0$, no principal has an incentive to deviate to not offering a contract.

Next, I show that the equilibrium is the unique full matching equilibrium. Consider a full matching equilibrium in which a type j principal offers a type k agent an incentive-compatible contract $s(j,k)$ with expected payoff $W(j,k) := \mathbb{E}_{p^{j,k}}[\tilde{s}(j,k)] - c$. Denote a type k agent's expected discounted payoff after contracting and before production in period $t \in \mathbb{Z}$ when matched with a type j principal by $M(j,k)$. For a type 1 agent, we have

$$M(1,1) = W(1,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)M(1,1)\right),$$
$$M(2,1) = W(2,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)M(2,1)\right),$$

which yields

$$M(1,1) = \frac{1}{1-\beta}\left[\xi W(1,1) + (1-\xi)W(2,1)\right],$$
$$M(2,1) = \frac{1}{1-\beta}\left[(1-\xi)W(1,1) + \xi W(2,1)\right],$$

where $\xi := \frac{2-\beta(2-\delta)}{2(1-\beta(1-\delta))}$. If a type 1 agent does not accept a contract offer, his expected discounted payoff is given by

$$Z(1) = b + \beta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) = b + \frac{\beta}{1-\beta}\frac{W(1,1)+W(2,1)}{2}.$$

Assume that there exists a full matching equilibrium with a (j,k) principal-agent pair such that

$W(j,k) > b$. Assume that $W(1,1) > b$ and $W(1,1) \geq W(2,1)$. The structure of the arguments for the other cases is identical. Consider a type 1 principal who meets a type 1 agent. The principal can deviate and offer a contract with an expected contractual payoff $\hat{W}(1,1)$. If the agent accepts, his expected discounted payoff is given by

$$\hat{M}(1,1) = \hat{W}(1,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)\hat{M}(1,1)\right),$$

which can be rewritten as

$$\hat{M}(1,1) = \frac{1}{1-\beta(1-\delta)}\left(\hat{W}(1,1) + \frac{\beta\delta}{2(1-\beta)}(W(1,1)+W(2,1))\right).$$

We have

$$\hat{M}(1,1) - Z(1) = \frac{1}{1-\beta(1-\delta)}\left(\hat{W}(1,1) - \beta(1-\delta)\frac{W(1,1)+W(2,1)}{2}\right) - b.$$

Simple algebra implies that $\hat{M}(1,1) - Z(1) > 0$ if $\hat{W}(1,1) = W(1,1) > b$ and $W(1,1) \geq W(2,1)$. As a result, the principal can deviate and offer a contract with an expected payoff $b < \hat{W}(1,1) < W(1,1)$, which the agent accepts. ■

A.16 Proof of Proposition 9

If $b \geq \bar{A}$, then the agents' outside option b determines the lower bound on agents' expected contractual payoffs as in Proposition 8. The proof therefore follows the same argument as the proof of Proposition 8. ■

A.17 Proof of Proposition 10

Simple algebra implies that $\theta \in (0,1)$. I first show that the proposed allocation is an equilibrium. Equilibrium requires that agents accept the contract offers by the principals. Denote the expected discounted payoff of a type k agent matched with a type j principal by $M(j,k)$. $M(1,1)$ and $M(2,1)$

satisfy

$$M(1,1) = \underline{A} + r(1,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)M(1,1)\right),$$
$$M(2,1) = \bar{A} + r(2,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)M(2,1)\right),$$

which yields

$$M(1,1) = \frac{1}{1-\beta}\left(\xi(\underline{A} + r(1,1)) + (1-\xi)(\bar{A} + r(2,1))\right),$$
$$M(2,1) = \frac{1}{1-\beta}\left((1-\xi)(\underline{A} + r(1,1)) + \xi(\bar{A} + r(2,1))\right),$$

where $\xi := \frac{2-\beta(2-\delta)}{2(1-\beta(1-\delta))}$.

Consider a $(1,1)$ principal-agent pair. If the agent deviates and does not accept the contract offer, his expected discounted payoff is given by

$$Z(1) = b + \beta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) = b + \frac{\beta}{1-\beta}\frac{\underline{A} + r(1,1) + \bar{A} + r(2,1)}{2}.$$

We have

$$M(1,1) - Z(1) = \frac{1}{1-\beta(1-\delta)}\left(\underline{A} + r(1,1) - \beta(1-\delta)\frac{\underline{A} + r(1,1) + \bar{A} + r(2,1)}{2}\right) - b.$$

Simple algebra implies that in Case (i) with $r(1,1) = 0$ and $r(2,1) = 0$, $M(1,1) - Z(1) \geq 0$ is equivalent to $(1-\theta)\bar{A} + \theta b \leq \underline{A}$. In Case (ii) with $r(1,1) = (1-\theta)\bar{A} + \theta b - \underline{A}$ and $r(2,1) = 0$, $M(1,1) - Z(1) \geq 0$ is satisfied with equality. It is clear that in a $(2,1)$ principal-agent pair, the agent has no incentive to deviate since the agent obtains the highest possible expected payoff in this match. The proof for the other principal-agent pairs follows the same argument.

Equilibrium requires that principals have no incentive to deviate to offering a different contract or not offering a contract. Denote the expected discounted payoff of a type j principal matched with a type k agent by $H(j,k)$. $H(1,1)$ and $H(1,2)$ satisfy

$$H(1,1) = \mathbb{E}_{q^1}[\tilde{x}] + \pi - c - \underline{A} - r(1,1) + \beta\left(\delta\left(\frac{1}{2}H(1,1) + \frac{1}{2}H(1,2)\right) + (1-\delta)H(1,1)\right),$$
$$H(1,2) = \mathbb{E}_{q^1}[\tilde{x}] + \pi - c - \bar{A} - r(1,2) + \beta\left(\delta\left(\frac{1}{2}H(1,1) + \frac{1}{2}H(1,2)\right) + (1-\delta)H(1,2)\right),$$

which yields

$$H(1,1) = \frac{\mathbb{E}_{q^1}[\tilde{x}] + \pi - c}{1-\beta} - \frac{1}{1-\beta}\left(\xi(\underline{A} + r(1,1)) + (1-\xi)(\bar{A} + r(1,2))\right),$$
$$H(1,2) = \frac{\mathbb{E}_{q^1}[\tilde{x}] + \pi - c}{1-\beta} - \frac{1}{1-\beta}\left((1-\xi)(\underline{A} + r(1,1)) + \xi(\bar{A} + r(1,2))\right).$$

First note that in pairs $(1,1)$ and $(2,2)$, principals have no incentive to deviate to not offering contracts since these matches give principals the highest possible expected payoff per period. In Case (i), principals cannot lower the fixed payoff since $r(1,1) = r(2,2) = r(1,2) = r(2,1) = 0$. Consider Case (ii) and a type 1 principal paired with a type 1 agent. The principal can deviate and offer a contract with a fixed payoff $\hat{r}(1,1)$. If the agent accepts, his expected discounted payoff is given by

$$\hat{M}(1,1) = \underline{A} + \hat{r}(1,1) + \beta\left(\delta\left(\frac{1}{2}M(1,1) + \frac{1}{2}M(2,1)\right) + (1-\delta)\hat{M}(1,1)\right),$$

which can be rewritten as

$$\hat{M}(1,1) = \frac{1}{1-\beta(1-\delta)}\left(\underline{A} + \hat{r}(1,1) + \frac{\beta\delta}{2(1-\beta)}\left(\underline{A} + r(1,1) + \bar{A} + r(2,1)\right)\right).$$

We have

$$\hat{M}(1,1) - Z(1) = \frac{1}{1-\beta(1-\delta)}\left(\underline{A} + \hat{r}(1,1) - \beta(1-\delta)\frac{\underline{A} + r(1,1) + \bar{A} + r(2,1)}{2}\right) - b.$$

Simple algebra implies that $\hat{M}(1,1) - Z(1) = 0$ if $\hat{r}(1,1) = r(1,1) = (1-\theta)\bar{A} - \underline{A} + \theta b$ and $r(2,1) = 0$. Hence, the principal cannot deviate and offer a contract with a lower fixed payoff $\hat{r}(1,1) < r(1,1)$, which the agent accepts. The proof for the other principal-agent pair follows the same argument.

Consider a type 1 principal paired with a type 2 agent. Note that the principal cannot offer a contract with a lower expected contractual payoff since $r(1,2) = r(2,1) = 0$. The principal can deviate to not offering a contract to the agent. The principal's expected discounted payoff is then given by

$$N(1) = \mathbb{E}_{q^1}[\tilde{x}] + \beta\left(\frac{1}{2}H(1,1) + \frac{1}{2}H(1,2)\right),$$

such that

$$H(1,2) - N(1) = \frac{\beta(1-\delta)}{2-2\beta(1-\delta)}(\underline{A} + r(1,1)) - \frac{(2-\beta(1-\delta))}{2-2\beta(1-\delta)}(\bar{A} + r(1,2)) + (\pi - c).$$

Simple algebra implies that in Case (i) with $r(1,1) = 0$ and $r(2,1) = 0$, $H(1,2) - N(1) \geq 0$ is equivalent to $\bar{A} \leq \theta(\pi - c) + (1-\theta)\underline{A}$. In Case (ii) with $r(1,1) = (1-\theta)\bar{A} + \theta b - \underline{A}$ and $r(2,1) = 0$, $H(1,2) - N(1) \geq 0$ is equivalent to $\bar{A} \leq \frac{1}{2-\theta}(\pi - c) + \frac{1-\theta}{2-\theta}b$. The proof for the other principal-agent pair follows the same argument.

Next, I show that the equilibrium is the unique full matching equilibrium. Consider a full matching equilibrium in which the highest expected contractual payoff is offered in a $(2,1)$ principal-agent pair, where $r(2,1) > 0$. In particular, we have $\bar{A} + r(2,1) \geq \underline{A} + r(1,1)$ and $\bar{A} + r(2,1) > b$. Consider a $(2,1)$ principal-agent pair. If the principal deviates and offers a contract with $\hat{r}(2,1)$, the agent accepts the offer if

$$\hat{M}(2,1) - Z(1) = \frac{1}{1-\beta(1-\delta)}\left(\bar{A} + \hat{r}(2,1) - \beta(1-\delta)\frac{\underline{A} + r(1,1) + \bar{A} + r(2,1)}{2}\right) - b \geq 0,$$

Simple algebra verifies $\hat{M}(2,1) - Z(1) > 0$ for $\hat{r}(2,1) = r(2,1)$. Thus, the principal has an incentive to deviate and offer fixed payoff $\hat{r}(2,1) < r(2,1)$. The proof for a $(1,2)$ principal-agent pair follows the same argument.

Consider a full matching equilibrium in which the highest expected contractual payoff is offered in a $(1,1)$ principal-agent pair. In particular, we have $\underline{A} + r(1,1) \geq \bar{A} + r(2,1) > b$. Consider a $(1,1)$ principal-agent pair. If the principal deviates and offers a contract with $\hat{r}(1,1)$, the agent accepts the offer if

$$\hat{M}(1,1)-Z(1)=\frac{1}{1-\beta(1-\delta)}\left(\underline{A}+\hat{r}(1,1)-\beta(1-\delta)\frac{\underline{A}+r(1,1)+\bar{A}+r(2,1)}{2}\right)-b\geq 0,$$

Simple algebra verifies $\hat{M}(1,1)-Z(1)>0$ for $\hat{r}(1,1)=r(1,1)$. Thus, the principal has an incentive to deviate and offer fixed payoff $\hat{r}(1,1)<r(1,1)$. The proof for a $(2,2)$ principal-agent pair follows the same argument. ■

A.18 Proof of Lemma 7

The steady-state measures of unmatched agents are given by the following system of equations:

$$\lambda_1=(1-\gamma)\left(\lambda_1+\delta\left(\frac{1}{2}-\lambda_1\right)\right)=\frac{\left((1-\delta)\lambda_1+\frac{\delta}{2}\right)\left((1-\delta)\lambda_2+\frac{\delta}{2}\right)}{(1-\delta)(\lambda_1+\lambda_2)+\delta},$$

$$\lambda_2=\gamma\left(\lambda_2+\delta\left(\frac{1}{2}-\lambda_2\right)\right)=\frac{\left((1-\delta)\lambda_1+\frac{\delta}{2}\right)\left((1-\delta)\lambda_2+\frac{\delta}{2}\right)}{(1-\delta)(\lambda_1+\lambda_2)+\delta},$$

which implies $\lambda_1=\lambda_2$. Simple algebra implies $\lambda_1=\lambda_2=\frac{\delta}{2(1+\delta)}$. It follows directly from $\lambda_1=\lambda_2$ that $\gamma=\frac{1}{2}$. We further have $\frac{1}{2}-\lambda_1=\frac{1}{2}-\lambda_2=\frac{1}{2(1+\delta)}$. ■

A.19 Proof of Proposition 11

Consider a partial matching equilibrium in which the highest expected contractual payoff is offered in a $(1,1)$ principal-agent match. Consider a $(1,2)$ principal-agent pair. The principal can deviate to offering the agent a contract with the same expected contractual payoff she offers to type 1 agents. Since this is not lower than the expected contractual payoff when the agent meets a type 2 principal, the agent would accept this contract offer. As a result, there is a profitable deviation for the principal. The symmetric argument applies if the highest expected contractual payoff is offered in a $(2,2)$ principal-agent match. ■

A.20 Proof of Proposition 12

I first show that the proposed allocation is an equilibrium. Equilibrium requires that agents accept the contract offers in matches $(1,1)$ and $(2,2)$. Accepting the contract offer gives an agent the expected payoff $\underline{A}+\max\{0,b-\underline{A}\}\geq b$ per period. Since this is the maximum expected payoff an agent can receive per period, agents have no incentive to deviate.

Equilibrium further requires that principals do not deviate and do not offer the contracts in matches $(1,1)$ and $(2,2)$. Since offering the contract gives a principal the maximum expected payoff she can receive per period, principals have no incentive to deviate to not offering contracts. Principals can also not lower the expected contractual payoff for agents as agents would either not accept the offers (i.e., if $b>\underline{A}$) or would not exert effort (i.e., if $b\leq\underline{A}$).

Equilibrium requires that principals do not deviate to offering contracts in matches $(1,2)$ and $(2,1)$. Denote the expected discounted payoff of a matched type 1 principal by $H(1)$ and the expected discounted payoff of an unmatched type 1 principal by $N(1)$. $H(1)$ and $N(1)$ satisfy

$$H(1)=\mathbb{E}_{q^1}\left[\tilde{x}\right]+\pi-c-\underline{A}-r(1,1)+\beta\left(\delta\left(\frac{1}{2}N(1)+\frac{1}{2}H(1)\right)+(1-\delta)H(1)\right),$$

and

$$N(1)=\mathbb{E}_{q^1}\left[\tilde{x}\right]+\beta\left(\frac{1}{2}N(1)+\frac{1}{2}H(1)\right),$$

which yields

$$\begin{aligned}H(1)&=\frac{\mathbb{E}_{q^1}\left[\tilde{x}\right]}{1-\beta}+\frac{2-\beta}{2-\beta(1-\delta)}\frac{\pi-c-\underline{A}-r(1,1)}{1-\beta},\\ N(1)&=\frac{\mathbb{E}_{q^1}\left[\tilde{x}\right]}{1-\beta}+\frac{\beta}{2-\beta(1-\delta)}\frac{\pi-c-\underline{A}-r(1,1)}{1-\beta}.\end{aligned}$$

Consider a $(1,2)$ principal-agent pair. The principal can deviate and offer a contract with expected contractual payoff $c+\bar{A}$, and the agent would accept the offer, since the expected contractual payoff is higher compared to a $(2,2)$ principal-agent match, that is, $\bar{A}>\max\{\underline{A},b\}=\underline{A}+r(2,2)$. Denote the expected discounted payoff of a type 1 principal matched with a type 2 agent who deviates and offers a contract to the agent by $\hat{H}(1,2)$. We have

$$\hat{H}(1,2) = \mathbb{E}_{q^1}[\tilde{x}] + \pi - c - \bar{A} + \beta\left(\delta\left(\frac{1}{2}H(1) + \frac{1}{2}N(1)\right) + (1-\delta)\hat{H}(1,2)\right),$$

and we get

$$N(1) - \hat{H}(1,2) = \frac{\bar{A}(2-\beta(1-\delta)) - 2(1-\beta(1-\delta))(\pi - c) - (\underline{A} + r(1,1))\beta(1-\delta)}{(1-\beta(1-\delta))(2-\beta(1-\delta))}.$$

Using $r(1,1) = \max\{b - \underline{A}, 0\}$, simple algebra implies that $N(1) - \hat{H}(1,2) \geq 0$ is equivalent to $\bar{A} \geq \theta(\pi - c) + (1-\theta)\max\{\underline{A}, b\}$. The proof for a $(2,1)$ principal-agent pair follows the same argument.

Next, I show that the equilibrium is the unique partial matching equilibrium. Consider a partial matching equilibrium with fixed payoffs $r(1,1)$ and $r(2,2)$. Denote the expected discounted payoff of a matched type 1 agent by $M(1)$ and the expected discounted payoff of an unmatched type 1 agent by $Z(1)$. $M(1)$ and $Z(1)$ satisfy

$$M(1) = \underline{A} + r(1,1) + \beta\left(\delta\left(\frac{1}{2}Z(1) + \frac{1}{2}M(1)\right) + (1-\delta)M(1)\right),$$

and

$$Z(1) = b + \beta\left(\frac{1}{2}Z(1) + \frac{1}{2}M(1)\right),$$

which yields

$$\begin{aligned} M(1) &= \frac{2-\beta}{2-\beta(1-\delta)}\frac{\underline{A} + r(1,1)}{1-\beta} + \frac{\beta\delta}{2-\beta(1-\delta)}\frac{b}{1-\beta}, \\ Z(1) &= \frac{\beta}{2-\beta(1-\delta)}\frac{\underline{A} + r(1,1)}{1-\beta} + \frac{2-\beta(2-\delta)}{2-\beta(1-\delta)}\frac{b}{1-\beta}. \end{aligned}$$

Consider a partial matching equilibrium in which the highest expected contractual payoff is offered in a $(1,1)$ principal-agent pair. In particular $r(1,1) > \max\{b - \underline{A}, 0\}$. Consider a $(1,1)$ principal-agent pair. If the principal deviates and offers a fixed payoff $\hat{r}(1,1)$ and the agent accepts, the agent's expected discounted payoff is given by

$$\hat{M}(1) = \underline{A} + \hat{r}(1,1) + \beta\left(\delta\left(\frac{1}{2}Z(1) + \frac{1}{2}M(1)\right) + (1-\delta)\hat{M}(1)\right),$$

which yields

$$\hat{M}(1) = M(1) + \frac{1}{1-\beta(1-\delta)}\left(\hat{r}(1,1) - r(1,1)\right).$$

Thus,

$$\hat{M}(1) - Z(1) = \frac{1}{1-\beta(1-\delta)}\left(\hat{r}(1,1) - r(1,1)\right) + \frac{2}{2-\beta(1-\delta)}(\underline{A} + r(1,1) - b).$$

In particular, for $\hat{r}(1,1) = r(1,1)$, we have $\hat{M}(1) - Z(1) > 0$ such that the principal can offer a contract with $\hat{r}(1,1) < r(1,1)$, which the agent accepts. The proof for a $(2,2)$ principal-agent pair follows the same argument. ■

B Heterogeneity in Expected Value of Effort

This section discusses an extension of the partial equilibrium framework from Section 2 with heterogeneity in the expected value of effort. Specifically, relative to the model introduced in Section 2.1, I drop Assumption 1. Instead, I define for all $k \in \mathcal{K}$, $\pi(k) := \mathbb{E}_{p^k}[\tilde{x}] - \mathbb{E}_q[\tilde{x}] = \sum_{i=1}^n \Delta_i^k x_i$. Note that this implies that the expected value of effort is agent specific. The model can also be extended to a match-specific expected value of effort.

B.1 Benchmark Model without Contractual Constraints

In this section, I consider a benchmark model without contractual constraints from Assumption 3 as in Section 2.2. Note that the results regarding optimal contracts remain unchanged.

Consider an unmatched principal at the end of period $t \in \mathbb{N}$. Denote the principal's optimal expected discounted payoff in this case by V. If the principal does not search, her expected discounted payoff is given by $\beta(\mathbb{E}_q[\tilde{x}] + V)$. If the principal searches and does not contract with an agent at the beginning of the next period, her expected discounted payoff is given by $-\kappa + \beta(\mathbb{E}_q[\tilde{x}] + V)$.

If the principal searches and contracts with agent $k \in \mathscr{K}$ at the beginning of the next period, she incurs the fixed cost I and generates the expected payoff $\mathbb{E}_q[\tilde{x}] + \pi(k) - c - u$ in the same period. At the end of the period, the match breaks up with probability δ and the principal generates the expected discounted payoff V. With probability $1-\delta$, the match does not break up and the principal generates the expected payoff $\mathbb{E}_q[\tilde{x}] + \pi(k) - c - u$ again in the next period, and so on. Thus, the principal's expected discounted payoff is given by

$$-\kappa + \beta\left(\frac{\mathbb{E}_q[\tilde{x}] + \pi(k) - c - u + \delta V}{1-\beta(1-\delta)} - I\right).$$

Thus, V satisfies

$$V = \max\left\{\beta(\mathbb{E}_q[\tilde{x}] + V), -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}] + \pi\left(\tilde{k}\right) - c - u + \delta V}{1-\beta(1-\delta)} - I, \mathbb{E}_q[\tilde{x}] + V\right\}\right]\right\}. \tag{29}$$

Consider the case in which it is optimal for the principal to search, such that

$$V = -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}] + \pi\left(\tilde{k}\right) - c - u + \delta V}{1-\beta(1-\delta)} - I, \mathbb{E}_q[\tilde{x}] + V\right\}\right]. \tag{30}$$

In particular, the principal contracts with agent $k \in \mathscr{K}$ if

$$\frac{\mathbb{E}_q[\tilde{x}] + \pi(k) - c - u + \delta V}{1-\beta(1-\delta)} - I \geq \mathbb{E}_q[\tilde{x}] + V. \tag{31}$$

Notice that the right-hand side of (31) is independent of $\pi(k)$. Thus, there exists a threshold $\hat{H}$ such that (31) holds if and only if $\pi(k) \geq \hat{H}$. In particular, $\hat{H}$ satisfies

$$\frac{\mathbb{E}_q[\tilde{x}] + \hat{H} - c - u + \delta V}{1-\beta(1-\delta)} - I = \mathbb{E}_q[\tilde{x}] + V. \tag{32}$$

Substituting (32) into (30) and solving for V implies

$$V = -\frac{1-\beta(1-\delta)}{1-\beta}\kappa + \frac{\beta}{1-\beta}\left(\mathbb{E}_q[\tilde{x}] - c - u - (1-\beta(1-\delta))I\right) + \frac{\beta}{1-\beta}\mathbb{E}_\mu\left[\max\left\{\pi\left(\tilde{k}\right), \hat{H}\right\}\right]. \tag{33}$$

Substituting (33) into (32) and solving for $\hat{H}$ implies

$$\hat{H} = \hat{T}\left(\hat{H}\right) := (1-\beta(1-\delta))(c+u-(1-\delta)\kappa) + (1-\beta(1-\delta))^2 I + \beta(1-\delta)\mathbb{E}_\mu\left[\max\left\{\pi\left(\tilde{k}\right), \hat{H}\right\}\right]. \quad (34)$$

It is straightforward to verify that the function $\hat{T} : \mathbb{R} \to \mathbb{R}$ defined in (34) is a contraction mapping. The Banach fixed-point theorem then implies that the equilibrium condition (34) has a unique solution, that is, there exists a unique $\hat{H}^*$ such that $\hat{T}\left(\hat{H}^*\right) = \hat{H}^*$. Thus, if it is optimal for the principal to search, then the principal searches if she is not matched with an agent at the end of a period and contracts with the agent drawn at the beginning of the next period if $\pi(k) \geq \hat{H}^*$.

Notice that since the principal captures the whole expected surplus in this benchmark model, the equilibrium maximizes the sum of the principal's and agents' expected discounted payoffs and is first-best efficient. This implies that, in the absence of contractual constraints, the search frictions and moral hazard alone do not generate inefficiencies in my framework.

B.2 Equilibrium with Contractual Constraints

In this section, I consider the model with contractual constraints from Assumption 3 as in Section 2.3. Note that the results regarding optimal contracts remain unchanged.

Consider an unmatched principal at the end of period $t \in \mathbb{N}$. Following the same argument as in Section 2.3 and replacing π with $\pi(k)$, $k \in \mathscr{K}$, V is equal to

$$\max\left\{\beta(\mathbb{E}_q[\tilde{x}] + V), -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}] + \pi\left(\tilde{k}\right) - c - u - A\left(\tilde{k}\right) + \delta V}{1-\beta(1-\delta)} - I, \mathbb{E}_q[\tilde{x}] + V\right\}\right]\right\}. \quad (35)$$

Consider the case in which it is optimal for the principal to search, such that

$$V = -\kappa + \beta\mathbb{E}_\mu\left[\max\left\{\frac{\mathbb{E}_q[\tilde{x}] + \pi\left(\tilde{k}\right) - c - u - A\left(\tilde{k}\right) + \delta V}{1-\beta(1-\delta)} - I, \mathbb{E}_q[\tilde{x}] + V\right\}\right]. \quad (36)$$

In particular, the principal contracts with agent $k \in \mathscr{K}$ if

$$\frac{\mathbb{E}_q[\tilde{x}] + \pi(k) - c - u - A(k) + \delta V}{1 - \beta(1-\delta)} - I \geq \mathbb{E}_q[\tilde{x}] + V. \tag{37}$$

Notice that the right-hand side of (37) is independent of $\pi(k)$ and $A(k)$. Thus, there exists a threshold H such that (37) holds if and only if $\pi(k) - A(k) \geq H$. In particular, H satisfies

$$\frac{\mathbb{E}_q[\tilde{x}] + H - c - u + \delta V}{1 - \beta(1-\delta)} - I = \mathbb{E}_q[\tilde{x}] + V. \tag{38}$$

Substituting (38) into (36) and solving for V implies

$$V = -\frac{1-\beta(1-\delta)}{1-\beta}\kappa + \frac{\beta}{1-\beta}\left(\mathbb{E}_q[\tilde{x}] - c - u - (1-\beta(1-\delta))I\right) + \frac{\beta}{1-\beta}\mathbb{E}_\mu\left[\max\left\{\pi\left(\tilde{k}\right) - A\left(\tilde{k}\right), H\right\}\right]. \tag{39}$$

Substituting (39) into (38) and solving for R implies

$$H = T(H) := (1-\beta(1-\delta))(c + u - (1-\delta)\kappa) + (1-\beta(1-\delta))^2 I + \beta(1-\delta)\mathbb{E}_\mu\left[\max\left\{\pi\left(\tilde{k}\right) - A\left(\tilde{k}\right), H\right\}\right]. \tag{40}$$

It is straightforward to verify that the function $T : \mathbb{R} \to \mathbb{R}$ defined in (40) is a contraction mapping. The Banach fixed-point theorem then implies that the equilibrium condition (40) has a unique solution, that is, there exists a unique H^* such that $T(H^*) = H^*$. Thus, if it is optimal for the principal to search, then the principal searches if she is not matched with an agent at the end of a period and contracts with the agent drawn at the beginning of the next period if $\pi(k) - A(k) \geq H^*$.

Comparing this result to the benchmark model from Section B.1, it is clear that the equilibrium in the presence of contractual constraints can be constrained inefficient if the set of agents the principal contracts with differ, that is, if the set of agents $k \in \mathscr{K}$ that satisfy $\pi(k) - A(k) \geq H^*$ differs from the set of agents $k \in \mathscr{K}$ that satisfy $\pi(k) \geq \hat{H}^*$, which will generally be the case.

C Long-Term Contracts

Consider long-term contracts of length $T \in \mathbb{N}$ and an agent $k \in \mathscr{K}$. Since I consider a single agent in this section, I omit the dependence of the notation on k. I denote the set of outputs by $\mathscr{X} := \{x_1, \ldots, x_n\}$ and the output in period $t \in \mathbb{N}$ by x^t. The random variable is denoted by $\tilde{x}^t$. A long-term contract of length T specifies, for all $t \in \{1, \ldots, T\}$ and all $\left(x^1, \ldots, x^t\right) \in \mathscr{X}^t$, contractual payments $s^t\left(x^1, \ldots, x^t\right)$ to the agent.

Since the principal-agent match breaks up with probability δ at the end of each period, the principal receives her contractual payoff in period t only with probability $(1-\delta)^{t-1}$. Let $\hat{\beta} := \beta(1-\delta)$. Since the principal's expected discounted payoff in the case of separation is independent of the contract s, the principal's objective is to choose a contract s to maximize

$$\mathbb{E}_{p^1,\ldots,p^T}\left[\sum_{t=1}^{T} \hat{\beta}^{t-1}\left(\tilde{x}^t - s^t\left(\tilde{x}^1, \ldots, \tilde{x}^t\right)\right)\right],$$

where p^t in the subscript of the expectation means that $\tilde{x}^t$ is sampled according to the probability distribution p. Equivalently, the principal's objective is to choose a contract s to minimize

$$\mathbb{E}_{p^1,\ldots,p^T}\left[\sum_{t=1}^{T} \hat{\beta}^{t-1} s^t\left(\tilde{x}^1, \ldots, \tilde{x}^t\right)\right]. \tag{41}$$

A long-term contract is incentive compatible if it induces effort in each period and each state. Since the principal-agent match breaks up with probability δ at the end of each period, the agent receives his contractual payoff in period t only with probability $(1-\delta)^{t-1}$. Since the agent's expected payoff in the case of separation is equal to u in each period and therefore independent of the contract s and independent of the agent's effort decisions, the agent's set of incentive constraints can be written as follows: For all $\hat{t} \in \{1, \ldots, T\}$ and all $\left(x^1, \ldots, x^{\hat{t}-1}\right) \in \mathscr{X}^{\hat{t}-1}$:

$$\mathbb{E}_{p^{\hat{t}},\ldots,p^T}\left[\sum_{t=\hat{t}}^{T} \hat{\beta}^{t-\hat{t}} s^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right] - c \geq \mathbb{E}_{q^{\hat{t}},p^{\hat{t}+1},\ldots,p^T}\left[\sum_{t=\hat{t}}^{T} \hat{\beta}^{t-\hat{t}} s^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right]. \tag{42}$$

Further, the contract needs to satisfy the agent's participation constraint:

$$\mathbb{E}_{p^1,\dots,p^T}\left[\sum_{t=1}^{T}\hat{\beta}^{t-1}\left(s^t\left(\tilde{x}^1,\dots,\tilde{x}^t\right)-c\right)\right]\geq\sum_{t=1}^{T}\hat{\beta}^{t-1}u. \tag{43}$$

Finally, the contract needs to satisfy contractual constraints, that is, for all $\hat{t}\in\{1,\dots,T\}$ and all $\left(x^1,\dots,x^{\hat{t}}\right)\in\mathscr{X}^{\hat{t}}$:

$$s^{\hat{t}}\left(x^1,\dots,x^{\hat{t}}\right)\geq\theta. \tag{44}$$

I denote the problem of minimizing (41) subject to the set of incentive constraints (42), the participation constraint (43), and the set of contractual constraints (44) by P_T, and its minimum by M_T.

An example of a feasible long-term contract is given by a sequence of optimal one-period contracts. Namely, for $\hat{t}\in\{1,\dots,T\}$ set $s^{\hat{t}}\left(x^1,\dots,x^{\hat{t}}\right)=s^*\left(x^{\hat{t}}\right)$, where s^* denotes an optimal contract from Lemma 2. We recall that the optimal one-period contract satisfies for all $\hat{t}\in\{1,\dots,T\}$

$$\mathbb{E}_{p^{\hat{t}}}\left[s^*\left(\tilde{x}^{\hat{t}}\right)\right]=\max\left(c+u,c+\theta+\frac{c}{\phi^*}\right),$$

where $\phi^*=\max_{1\leq i\leq n}\frac{p_i-q_i}{q_i}$. In particular, $M_1=\max\left(c+u,c+\theta+\frac{c}{\phi^*}\right)$. For such a contract we have

$$\mathbb{E}_{p^1,\dots,p^T}\left[\sum_{t=1}^{T}\hat{\beta}^{t-1}s^t\left(\tilde{x}^1,\dots,\tilde{x}^t\right)\right]=\sum_{t=1}^{T}\hat{\beta}^{t-1}\max\left(c+u,c+\theta+\frac{c}{\phi^*}\right)=\sum_{t=1}^{T}\hat{\beta}^{t-1}M_1.$$

The aim is to show that this value is actually a minimal one.

Proposition 13. *For all $T\in\mathbb{N}$,*

$$M_T\geq\sum_{t=1}^{T}\hat{\beta}^{t-1}M_1.$$

Proof. First, consider the case in which $M_1=c+u\geq c+\theta+\frac{c}{\phi^*}$. Then the claim follows immediately from the participation constraint (43).

Second, consider the case in which $M_1=c+\theta+\frac{c}{\phi^*}>c+u$. For $T=1$, the claim holds.

Inductive assumption: Assume that for $T \geq 2$,

$$M_{T-1} \geq \sum_{t=1}^{T-1} \hat{\beta}^{t-1} M_1.$$

Consider an optimal contract s for the problem P_T. As a first step, I construct a long-term contract $\hat{s}$ of length $T-1$. For any $\left(x^1, \ldots, x^{T-1}\right) \in \mathscr{X}^{T-1}$ the contract $x^T \rightarrow s^T\left(x^1, \ldots, x^{T-1}, x^T\right)$ is feasible for the problem P_1 since, if $c + \theta + \frac{c}{\phi^*} > c + u$, the incentive constraint implies the participation constraint. Hence,

$$\gamma\left(x^1, \ldots, x^{T-1}\right) := \mathbb{E}_{p^T}\left[s^T\left(x^1, \ldots, x^{T-1}, \tilde{x}^T\right)\right] - M_1 \geq 0.$$

Let

$$\hat{s}^t\left(x^1, \ldots, x^t\right) = \begin{cases} s^t\left(x^1, \ldots, x^t\right), \text{ if } 1 \leq t \leq T-2, \\ s^{T-1}\left(x^1, \ldots, x^{T-1}\right) + \hat{\beta}\gamma\left(x^1, \ldots, x^{T-1}\right), \text{ if } t = T-1. \end{cases}$$

The contract $\hat{s}$ satisfies the set of contractual constraints in P_{T-1}. In addition, it satisfies the set of incentive constraints in P_{T-1}. To show this, I make use of the relation

$$\mathbb{E}_{r^{\hat{t}}, \ldots, r^{T-1}}\left[\gamma\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^{T-1}\right)\right] = \mathbb{E}_{r^{\hat{t}}, \ldots, r^{T-1}, p^T}\left[s^T\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^T\right)\right] - M_1,$$

which is valid for any distributions $r^{\hat{t}}, \ldots, r^{T-1}$ of outputs $x^{\hat{t}}, \ldots, x^{T-1}$, respectively. Then

$$\mathbb{E}_{p^{\hat{t}}, \ldots, p^{T-1}}\left[\sum_{t=\hat{t}}^{T-1} \hat{\beta}^{t-\hat{t}} \hat{s}^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right] = \mathbb{E}_{p^{\hat{t}}, \ldots, p^T}\left[\sum_{t=\hat{t}}^{T} \hat{\beta}^{t-\hat{t}} s^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right] - \hat{\beta}^{T-\hat{t}} M_1$$

and

$$\mathbb{E}_{q^{\hat{t}}, p^{\hat{t}+1}, \ldots, p^{T-1}}\left[\sum_{t=\hat{t}}^{T-1} \hat{\beta}^{t-\hat{t}} \hat{s}^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right] = \mathbb{E}_{q^{\hat{t}}, p^{\hat{t}+1}, \ldots, p^T}\left[\sum_{t=\hat{t}}^{T} \hat{\beta}^{t-\hat{t}} s^t\left(x^1, \ldots, x^{\hat{t}-1}, \tilde{x}^{\hat{t}}, \ldots, \tilde{x}^t\right)\right] - \hat{\beta}^{T-\hat{t}} M_1.$$

In particular, $\hat{s}$ satisfies the set of incentive constraints in P_{T-1}. Using the inductive assumption, this implies that for $\hat{s}$, the participation constraint is slack. If this was not the case, then there would exist a feasible contract which satisfies the participation constraint with equality and imply

$$M_{T-1} = \sum_{t=1}^{T-1} \hat{\beta}^{t-1}(c+u) < \sum_{t=1}^{T-1} \hat{\beta}^{t-1} M_1,$$

which violates the inductive assumption. We conclude that $\hat{s}$ is feasible in P_{T-1}. Using the inductive assumption, this implies that

$$\mathbb{E}_{p^1,\ldots,p^{T-1}}\left[\sum_{t=1}^{T-1} \hat{\beta}^{t-1} \hat{s}^t\left(\tilde{x}^1,\ldots,\tilde{x}^t\right)\right] \geq M_{T-1} \geq \sum_{t=1}^{T-1} \hat{\beta}^{t-1} M_1.$$

Finally, for the original optimal contract s of length T, we have

$$M_T = \mathbb{E}_{p^1,\ldots,p^T}\left[\sum_{t=1}^{T} \hat{\beta}^{t-1} s^t\left(\tilde{x}^1,\ldots,\tilde{x}^t\right)\right] = \mathbb{E}_{p^1,\ldots,p^{T-1}}\left[\sum_{t=1}^{T-1} \hat{\beta}^{t-1} \hat{s}^t\left(\tilde{x}^1,\ldots,\tilde{x}^t\right)\right] + \hat{\beta}^{T-1} M_1 \geq \sum_{t=1}^{T-1} \hat{\beta}^{t-1} M_1 + \hat{\beta}^{T-1} M_1 = \sum_{t=1}^{T} \hat{\beta}^{t-1} M_1,$$

which proves the claim. ■