Optimal Algorithmic Pricing for Interruptible Goods

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Abstract

I study selling mechanisms by a monopolist for imperfectly durable, interruptible and homogeneous goods. Having the infrastructure as a service public cloud computing market as a motivating example, I show that when market conditions are not transparent and under few certain additional conditions, by offering a randomized mechanism that incorporates a risk of interruption over consumption, the seller can improve her revenue in comparison to the standard deterministic mechanism proposed by Myerson (1981). In the optimal mechanism the risk of interruption can lead to differential pricing even if the valuation/type of the buyer is a private information. I find that when this mechanism is optimal it expands trade to more types of buyers than in the optimal deterministic case making low (high) valuation buyers better (worse) off. The optimal mechanism can be implemented by simultaneously allocating the good through a posted price and an auction where buyers face the risk of interruption. Auctioning the goods can be designed so as to incorporate the risk for the winners of losing access to their service while it is still in operation. The posted price mechanism can by construction eliminate that risk. Buyers of high valuations prefer to pay a risk premium and get the service through the posted price mechanism while buyers of low valuations unable to meet the price level of the risk premium and enter the auction.

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1 Introduction

This paper, inspired by the Infrastructure as a Service (IaaS) cloud computing market, studies selling mechanisms of (imperfectly) durable, homogeneous goods that can be interrupted during their consumption.

Users that consume such online goods, are usually able to observe their characteristics but they only have limited information about the market demand or supply conditions. They only observe the price they have to pay to consume them.

Algorithmic systems can observe in real time demand fluctuations and adjust pricing schedules accordingly at times that are of much smaller duration than the consumption period. Hence, the observed price fluctuations are often attributed to changes in market conditions and especially in changes in demand.

Changes in demand for online services have often been compared with the changes we observe in demand for electricity over time. While, both incorporate similar patterns, a crucial difference between the two sectors is that in the latter case, pricing is regulated. The electricity industry is structurally separated in generation, transmission and distribution of electricity. The transmission system operator is responsible for managing the flow of electricity and for choosing prices so that demand meets supply at real time (this is important given that electricity is a non-storable good). The indepedent operator does not have any profit maximizing motive. It just picks the pricing schedule in a way to ensure that the market clears.

In contrast, in digital markets, we see the prevelance of vertically integrated structures which both produce and manage the allocation of services over time. As a result, these structures can directly observe market conditions. But, they may not share such information with their clients.

In the public IaaS cloud computing market for example, vendors have built an online infrastructure that allows them to allocate computing power, data storage and computer system resources to their clients. The unit of computation and allocation is a virtual machine, known as instance. Clients observe the price of each instance at each point of time, but they do not know how many units are demanded and how demand fluctuates at real time.

Ride hailing is another example where the market has two sides and an intermediary matches each side with the other one. Drivers supply services. Passengers consume services. Intermediary's algorithm sets the price for the service. The two sides only observe this price but not the actual demand and supply conditions.

A first research question that this paper addresses is whether providers of online services have incentives to deviate from prices that equate supply and demand at real time by choosing price fluctuations that allow then to effectively price discriminate over their clients.

Algorithmic pricing has been linked to first-degree or some other form of behavioral price discrimination due to the ability of firms to infer users preferences and valuations through the analysis of their data. If users' valuations are revealed to the seller, the latter can perfectly price discriminate extracting all the user's surplus. However, such practices may also create reputational damages for firms once users realize these tactics. So, in the long-run there are also some costs involved.

So, the lack of transparency over market conditions is essential for the seller to avoid these costs. If the consumer only observes the price of a good, then she will not be able to assess if this price truly corresponds to the market conditions or if it is a part of an effective price discrimination strategy.

Price discrimination can also be the result of quality differentiation. Online sellers can offer a menu of services of different quality in order to be able to discriminate between users of different valuations. One way to do that is to stochastically interrupt the provision of the service (degrading in this way the quality of the service) on some offers in this menu and adjust the price accordingly. The resulting risk of interruption can be an attractive option for imperfect durable as well as homogeneous goods. It allows the seller to create different options of the same good by making future access to it stochastic.

This paper deals with such a class of market structures where goods are in principle interruptible: Let the seller be both in charge for the production and allocation of a homogeneous good that is consumed over time. The seller can make the provision of the good stochastically unavailable, while the consumers can credibly assess the risk of interruption over their consumption of the good before they buy it. Then, the seller can design a selling mechanism such that the risk of interruption is valuation specific.

But, when and under which conditions is such a mechanism optimal? Using a static mechanism design approach, for a homogeneous good, the optimal selling mechanism is found to be deterministic (Myerson, 1981).[1] Equilibrium price is unique and is set at an optimal level. Buyers with valuations below this price are excluded from the market and do not buy the good. Buyers with valuations above this price get the good and they can consume it with certainty.

The risk of interruption is already an important element in pricing mechanisms for the allocation of IaaS cloud services. A dominant provider is Amazon through its Elastic Compute Cloud (EC2) unit with numerous well known clients such as Expedia, Airbnb, Lyft, Netflix, Adope Systems and Zoom.[2] Its pricing schedule until recently[3] included two simulaneous options for the same virtual machine (same type of service)[4]: i) a posted price selling "on demand" mechanism, so that buyers could have guaranteed access to the virtual machines by paying a fixed non-discriminatory hourly rate[5]; ii) a spot market where virtual machines are auctioned and where the winners of the auctions get access to virtual machines but they also face a risk their access to be interrupted.[6]

[1]See also Maskin and Riley (1989), Riley and Zeckhauser (1983) and Skreta (2006).

[2]Amazon EC2 has been the undisputed leader in this line of business and the first vendor that enterred this market in August 2006. Users can specify certain parameters about the hardware and location and select among the available options.

[3]The description of Amazon's pricing strategies concerns the the period prior to 2019 when Amazon significantly changed its pricing strategy. Similar strategies (see the brief comment below) are followed by Google and Microsoft.

[4]So, the consideration of a homogeneous good which can be allocated though different pricing options is clearly relevant.

[5]Amazon also allows its clients to reserve virtual machines for a long period of time (e.g.,a year) at a cheaper posted per hour price.

[6]The other two leading vendors, Google and Microsoft also adopt pricing strategies which allow their clients to either get a virtual machine at a posted price and consume it over time without any risk, or get the virtual machine at a lower price facing such a risk of interruption.

Spot prices are set through a uniform price, sealed-bid auction. The bidders are neither aware about the number of auctioned units nor the total number of bidders or their identity. So, demand conditions and capacity constraints at each point of time are only known to the vendor. Bidders only observe the equilibrium price of the auction as well as the evolution of auction prices for a period up to the precending 90 days.

On the one side, spot instances allow the vendor to auction excess capacity (which is not allocated through the "on demand" posted price option). To use spot instances, buyers place a spot instance request, specifying, the number of spot instances they want to run, and the maximum price they are willing to pay per instance hour. Amazon changes the spot price periodically (e.g., hourly) based on supply and demand. When a user's bid is above the equilibrium spot price, her instances get scheduled, and run until either they complete, or until the equilibrium spot price for the given type of the virtual machine rises above the bid (e.g., due to the rising demand for that specific virtual machine by new bidders that enter the auction and bid above the equilibrium spot price), in which case the instances are automatically interrupted[7].

On the other side, the analysis of Agmon Ben-Yehuda et al. (2013) suggests that Amazon's pricing does not reflect market fluctuations but it is likely to be generated most of the time at random from within a tight price range via a dynamic hidden reserve price mechanism. This pricing schedule is made possible because there is not transparency over market conditions which are private information of the vendor.

Typical price fluctuations is the spot market are presented in Figure 1. The posted price is normalised to one. Spot market prices fluctuate over a wide range of values that are either smaller or even larger than the posted price. But, on average, the spot price for a given virtual machine is almost the half of its posted price. If a buyer bids 0.4, it faces a large risk of interruption, especially, if she wants to access the virtual machine for a long period of time. In contrast, a buyer with a bid close to 0.9 faces a significantly lower risk of interruption.

[7]Users are informed few seconds before the interruption takes place so that they can remove their data from the virtual machine before the interruption.

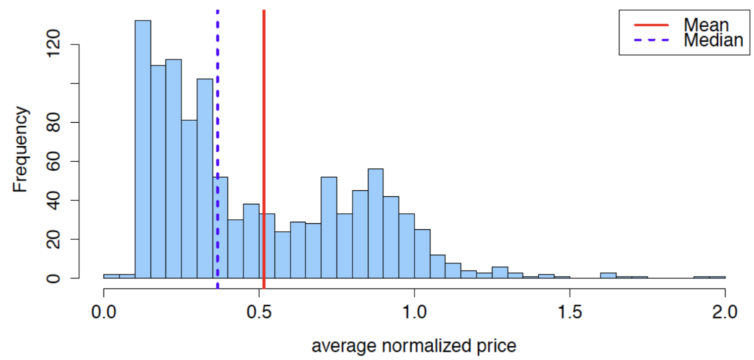

Figure 1: The frequency of different equilibrium prices in the spot market with respect to the posted price which is normalized to one. Source: Kilcioglu and Maglaras (2015).
Note: Data traces from March 1, 2015 to August 31, 2015 for 1,122 virtual machines. The figure plots the averages across all products.

A primary objective of this paper is to investigate if a seller of a homogeneous and interruptible good finds optimal to incorporate a risk of interruption in the selling mechanism not because of demand fluctuations but for price discrimination motives. To do that I consider a simple baseline model between one buyer and one seller where demand fluctuations are absent.

I develop a seller's optimal mechanism which is found to allow the monopolist to price discriminate between buyers of different private valuations through the imposed risk of interruption. The mechanism can be designed in a way that high valuation buyers prefer to pay a risk premium in order to avoid the risk of loosing their purchased goods while the low valuation buyers prefer to face the risk of interruption instead of paying the premium. The risk of interruption is essential for enabling an effective price discrimination and can maximize seller's revenue. Crucial for its optimality is that the duration for which a buyer wishes to consume the good or service depends on his valuation through a function that is common knowledge. Due to the duration dimension, the seller has more flexibility in offering pricing options that allows her to extract more rents out of buyers with different valuations.

In other words, the risk of interruption introduces randomization in the selling mechanism. So, our results can be interpreted as a indication that when goods to sell are durable and can be interrupted, under certain conditions, stochastic mechanisms can dominate deterministic ones (evaluated at the optimal values of parameters).

Damaged good literature[8] provides motives for the monopolist to "damage" units of the good and sell two varieties of it (one damaged and one non-damaged) at two different posted prices. A relevant literature also deals with the versioning of information goods, where the selling contract incorporates various clauses in addition to price for each type of buyers (Varian, 1998, Beleflamme, 2005). As I show below, even if the good to sell is homogeneous and there is no any damaged units of it, the seller can still price discriminate in a profitable way. This is because the valuation specific optimal risk of interruption allows for generating different selling options (analogous to versioning), one for each buyer type, for the same good.

When a service is suddenly interrupted while it is still in use by the buyer, there is a termination cost which decreases the valuation of the buyer for the service. For example, buyers can be considered as downstream firms that use upstream services that are useful for their transactions with final consumers. Due to interruption of the upstream service[9], buyers may not be in the position to serve efficiently the market and they will incur some loses (e.g. damaged reputation, inability to meet commitments and deadlines which sometimes are enforced by contracts with the final consumers, inconvenience and working cost of transferring the data from virtual machines when they are about to shut down). Since the duration of consuming the good or service depends on the valuation a buyer, it also affects the expected risk of interruption. It is exactly this channel through which a static randomized mechanism can do better than the deterministic one proposed firstly by Myerson (1981).

We should not ignore that auction theory suggests that the seller of a good may decide to ignore bids below a hidden reserve price or equivalently to reduce the units offered in

[8] See for example, Mussa and Rosen (1978), Deneckere and McAfee (1996) and McAfee (2007).

[9] An example from the IaaS market: Consider a downstream firm that needs to have access to a cloud server (virtual machine) for running a service. If it looses the access to the server, it is unable to run the service anymore.

the spot market, to prevent the goods from being sold cheaply, or to give the impression of increased demand since bidders only observe the price and not actual demand conditions. Hence, the equilibrium price in the auction can be alternativelly considered as a choice variable of the auctioneer which does not fully reflect demand fluctuations. Hidden reserve prices have been analysed in the literature either in a common value environment (Vincent, 1995, Horstmann and Lacasse, 1997 and Nagarada, 2003) or under risk aversion (Li and Tan, 2017). Empirical studies on e-Bay auctions (Bajari and Hortacsu, 2003) reveal that high valuation sellers have incentives to implement a hidden reserve price mechanism. An alternative intuition for the stochasticity of the optimal mechanism could be it incorporates a hidden reserve price schedule through the extra degree of freedom introduced by the duration of consumption of the good.

For the implementation of this optimal mechanism, it is crucial that each buyer of a given valuation is able to assess the risk of interruption before the transaction takes place. This risk should be equal to the risk of interruption under the optimal mechanism which is decreasing in the valuation and ensures that a buyer of a given valuation will choose the specific service option that is designated for her.

One way is for the sellers to provide a signal for the price fluctuations over the good. For example, by being able to observe past price fluctuations, buyers can assess the risk of the price exceeding their valuation. The lower is their valuation, the greater this risk will be. Once, the price exceeds their valuation, the provision of the good is interrupted. Hence, the reported by the seller past price fluctuations allow the buyers to form some beliefs on the likelihood of interruption that depends on their private valuations. The seller can adjust its report so that the risk of interruption is the one that corresponds to the optimal mechanism for each valuation. However, this implies that price fluctuations as a part of the optimal mechanism will not necessary meet the actual market demand fluctuations as they also serve a price discrimination motive.

The model can easily be extended in the case of more than one buyer in order to investigate the role of capacity constraints when they are relevant. I find that when capacity is limited, it is less likely for the seller to employ a selling mechanism with the risk of interruption in the optimum. In another dimension, the paper also investigates how the optimal mechanism is impacted by the market position of the seller. I find that in the seller is more likely to choose a stochastic mechanism when it is the market leader.

I also discuss how the optimal mechanism can be implemented. On way can be the simultaneous use of a posted price and an auction where good can become stochastically unavailable. The probability of interruption is decreasing in bids and the bidders with bids close to the auction equilibrium price are more likely to see their services to be interrupted. Our novel approach provides a rationale for using posted price and auction selling mechanisms for homogeneous goods that are not only simultaneously offered but also simultaneously allocated (regardless the choice of the mechanism the buyer chooses), sharing in this way a main feature of IaaS market.

Computer science and information systems literature[10] has looked in depth on the rationale of using both a posted price and a spot market for allocating IaaS virtual machines, considering the design of selling mechanims as given. My approach here is to derive instead the optimal design of the selling mechanism and investigate whether its implementation can accommodate the use of both pricing options.

The rest of the paper is organized as follows: In section 2, I present the model environment between one buyer and one seller and I derive the optimal mechanism for selling a unit of durable and interruptible good when the buyer's valuation is her private information. I also compare the optimal mechanism with the deterministic one proposed by Myerson. Section 3 discussed the generalization of the results in the case of more than one buyer with unit demand and underlines the impact of capacity constraints on the optimal mechanism. The implementation of the mechanism through a posted parice and an auction is also discussed. Section 4 deals with the market position of the seller and how competitive pressure affects the optimal mechanism. Section 5 concludes.

[10]For example, Kicioglu and Maglaras (2015), Abhishek et al. (2012), Liu et al. (2014), Wang et al. (2013), Al-Roomi et al. (2013), Farooq and Zhu (2018).

2 One-buyer and One-seller

To begin with, consider one seller and one buyer. The seller has a durable indivisible and interruptible good for sale. She produces it as zero cost. The buyer has a private valuation for the good which is drawn randomly from distribution $F(v)$ with finite probability density function $f(v)$ which is positive in the support $[\underline{v}, \overline{v}]$, with $\overline{v} > \underline{v} \geq 0$ from which the valuation of the buyer is drawn. The distribution of valuations is common knowledge. The buyer with valuation v wants to consume the good for $b(v)+\varepsilon$ periods. The function $b(\cdot)$ is also common knowledge. However, at the time of the transaction, for a given valuation, the duration can only be assessed with a noice ε, a random variable with expected value of zero. So, the expected duration (i.e., expected number of periods that the good is consumed) is $b(v)$. We assume that the time difference between two concecutive periods is small enough so that we can treat $b(v)$ as a continuous and differentiable variable.[11]

The valuation v refers to consumption of the good for $b(v) + \varepsilon$ periods without any interruption. If the good is interrupted before $b(v)+\varepsilon$ periods are completed, then the buyer incurs a cost. In the baseline model, we consider for simplicitly this cost is disproprotionaly high[12].

So, the buyer's utility is[13]:

[11]The unit time in the IaaS cloud computing market is one hour (interrumption can in principle occur in each hour) even when clients prefer to consume the good for periods over several weeks or months.

[12]If an interruption occurs, then the buyer cannot derive any value out of the good or service. This is the case, for example, if the buyer uses the service to complete a specific task and she cannot derive any value before completion.

[13]We could alternatively consider a milder impact of the interruption, such that:

$$U(v) = \begin{cases} v & \text{, if interruption does not occur over } b(v) + \varepsilon. \\ \max\{v - c, 0\} & \text{, otherwise,} \end{cases}$$

where $\overline{v} > c > 0$ is the cost of interruption. The results we derive here would have been qualitatively the same with this alrernative specification for significant high cost c.

$$U(v) = \begin{cases} v & \text{, if interruption does not occur within } b(v)+\varepsilon \text{ periods} \\ 0 & \text{, otherwise} \end{cases} \quad (1)$$

Let the seller offer:

- an initial allocation rule $x(\cdot)$ which assigns the object to type v with probability $x(v)$ at the beginning of period 1.

- probabilities $\{\lambda_t(\cdot)\}_{t=1}^{b(v)+\varepsilon}$ that the consumption of good will not be interrupted for a type v and at each period t, from 1 up to $b(v)+\varepsilon$. So, at each period t the risk of interruption $1-\lambda_t(v)$ can have an impact only when type v gets the object with positive probability $x(v)$ (at the beginning of period 1) and the good has not been interrupted at any previous period $t' < t$.

- a payment $p(\cdot)$ from the buyer of type v to the seller for the allocation of the good under the rules $x(v)$ and $\{\lambda_t(v)\}_{t=1}^{b(v)+\varepsilon}$

A type of Myerson optimal deterministic mechanism is to define a threshold type $v^{my} \in [\underline{v}, \overline{v}]$ which is defined by the equation $v - \frac{1-F(v)}{f(v)} = 0$. In addition, there is no risk of interruption, as $\lambda_t(v) = 1, \forall t, v \in [\underline{v}, \overline{v}]$. Under the monotone likelihood property that $\frac{1-F(v)}{f(v)}$ is monotonously decreasing in $v, \forall v \in [\underline{v}, \overline{v}]$, buyer's types with $v \geq v^{my}$ get the object with probability $x(v) = 1$, while types of lower valuation, $v < v^{my}$ get the object with probability $x(v) = 0$. The expected payment for the seller is $\int_{\underline{v}}^{\overline{v}} x(v)(v - \frac{1-F(v)}{f(v)})dF(v)$.

We investigate whether the seller can do better by randomizing over probabilities $x(\cdot)$ and $\lambda(\cdot)$ in extended environments which incorporate a static trade relationship for consumption over time, as the one described above. Without loss of generality we restrict our attention to direct mechanisms following the revelation principle. Then, the mechanism we are looking for should be selected from the pool of mechanisms that satisfy the conditions of individual rationality and incentive compatibility. Buyer of type v should be willing to participate in the trade, so the expected benefit $r(v)$ from trade should be positive[14]:

$$r(v) = x(v)\tilde{\lambda}(v, b(v))v - p(v) \geq 0$$

where $\tilde{\lambda}(v, b(v)) = \prod_{t=1}^{t=b(v)} \lambda_t(v)$ is the overall probability of continuation within the expected number of periods $b(v)$ taking into account probabilities set by the seller at each period $\{\lambda_t(v)\}_{t=1}^{b(v)}$. Without a loss of generality, we can restrict ourselves to the case that $\tilde{\lambda}(v, b(v)) = \lambda^{b(v)}(v)$. Any path of probabilities $\{\lambda_t(v)\}_{t=1}^{b(v)}$ that result an overall probability $\tilde{\lambda}(v, b(v))$ can be replicated by adjusting the value of $\lambda_t(v)$ such that $\lambda_{t+1}(v) = \lambda_t(v), \forall t$. So, the individual rationality constraint becomes

$$x(v)\lambda^{b(v)}v - p(v) \geq 0 \tag{2}$$

The mechanism should also certify that the buyer of any true type v will not have any incentive to misrepresent himself by reporting a type v' and this should be true for every $v' \in [\underline{v}, \overline{v}]$ & $v' \neq v$:

$$x(v)\lambda^{b(v)}v - p(v) \geq x(v')\lambda^{b(v)}(v')v - p(v') \tag{3}$$

Incentive compatibility requires that

$$r(v) = \int_{\underline{v}}^{v} x(s)\lambda^{b(s)}(s)\left(1 + s\ln(\lambda)b'(s)\right) ds$$

The payment to seller will be

$$p(v) = x(v)\lambda^{b(v)}(v)v - \int_{\underline{v}}^{v} x(s)\lambda^{b(s)}(s)\left(1 + s\ln(\lambda)b'(s)\right) ds$$

so, the expected profit for the seller from the transaction will be:

$$\begin{aligned}
\Pi_s &= \int_{\underline{v}}^{\overline{v}} p(v)dF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \left[x(v)\lambda^{b(v)}(v)v - \int_{\underline{v}}^{v} x(s)\lambda^{b(s)}(s)\left(1 + s\ln(\lambda)b'(s)\right) ds\right] dF(v) \\
&= \int_{\underline{v}}^{\overline{v}} \left[x(v)\lambda^{b(v)}(v)(v - \frac{1-F(v)}{f(v)}) - \frac{1-F(v)}{f(v)}x(v)\lambda^{b(v)}(v)v\ln(\lambda)b'(v)\right] dF(v)
\end{aligned} \tag{4}$$

[14]The expectation operator here is with respect to the number of periods of consumption.

The maximization of seller's expected profit (4) requires to set optimally the choice variables $x(\cdot)$, $\lambda(\cdot)$. A necessary condition so that the maximization problem has an interior solution (in other words, the second order condition) is that the expected duration $b(v)$ is a monotonously increasing function of v, $\forall v \in [\underline{v}, \overline{v}]$:

$$b'(v) > 0$$

When the buyer expects to consume the good for a longer time, she has a greater valuation for it and vice verca. The optimal value $\lambda^*(v)$ is in this case:

$$\lambda^*(v) = e^{\frac{1}{\frac{1-F(v)}{f(v)}b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)}} \tag{5}$$

Note that $\lambda^*(v) < 1$ only if $\frac{1}{\frac{1-F(v)}{f(v)}b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)} < 0$.

The optimal allocation $x^*(v)$ depends on the virtual valuation:

$$A(v) = \begin{cases} (\lambda^*)^{b(v)}(v)\left(\frac{1-F(v)}{f(v)}\frac{b'(v)}{b(v)}\right) > 0 & \text{, if } \lambda^*(v) < 1 \\ v - \frac{1-F(v)}{f(v)} & \text{, if } \lambda^*(v) = 1 \end{cases}$$

Recall that the optimal risk of interruption is positive for a given buyer type v, if for that type: $\frac{1}{\frac{1-F(v)}{f(v)}b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)} < 0$.

But, since for $v \leq v^{my}$ we have $v \leq \frac{1-F(v)}{f(v)}$, given that $b(v) > 0$ and $b'(v) > 0$, we conclude that

$$\lambda^*(v) < 1, \forall v \in [\underline{v}, v^{my}].$$

In addition, for types $v > v^{my}$, it always is $A(v) > 0$ (for any value of λ^*). Hence, we conclude that $A(v) > 0\ \forall v \in [\underline{v}, \overline{v}]$.

To derive the optimal allocation, we also need to check whether the individual rationality constraint is satisfied for at least some types v. The expected benefit for type v under the optimal mechanism can be written as:

$$r^*(v) = \int_{\underline{v}}^{v} x(s)^*(\lambda^*)^{b(s)}(s)\left(1 + s\left(\frac{1}{\frac{1-F(s)}{f(s)}b'(s)} - \frac{1}{sb'(s)} - \frac{1}{b(s)}\right)b'(s)\right)ds$$

$$= \int_{\underline{v}}^{v} x(s)^*(\lambda^*)^{b(s)}(s)s\left(\frac{1}{\frac{1-F(s)}{f(s)}} - \frac{b'(s)}{b(s)}\right)ds.$$

So, type v will participate in the trade agreement if only if:

$$\frac{b(v)}{b'(v)} \geq \frac{1 - F(v)}{f(v)}.$$

We focus on smooth functions $b(v)$ that are neither too concave nor too convex such that

- There is a unique valuation v^L above which the participation constraint is satisfied: $\frac{b(v^L)}{b'(v^L)} = \frac{1-F(v^L)}{f(v^L)}$. We assume that $v^L > \underline{v}$.
- $\lambda^*(v)$ is monotonously increasing in valuation v, $\forall v \geq v^L$.

Then, the optimal allocation rule $x^*(\cdot)$ take the following form:

$$x^*(v) = \begin{cases} 1 & \text{, if } v \geq v^L \\ 0 & \text{, otherwise} \end{cases}$$

Let v^{cr} be defined as the unique threshold valuation such that

$$\frac{1}{\frac{1-F(v^{cr})}{f(v^{cr})} b'(v^{cr})} - \frac{1}{v^{cr} b'(v^{cr})} - \frac{1}{b(v^{cr})} = 0.$$

It is easy to see that $v^{cr} > v^{my}$. It is also easy to verify that $v^{cr} > v^L$. Under the optimal mechanism there are two possibilities:

- If $v^{cr} \geq \overline{v}$, the risk of interruption is strictly positive and monotonously decreasing for all $v \in [v^L, \overline{v}]$
- $v^{cr} \in (\underline{v}, \overline{v})$, the risk of interruption is decreasing for all $v \in [v^L, v^{cr})$ and equals to zero for all $v \in [v^{cr}, \overline{v}]$.

We will focus on the later case as it is more interesting (and better suits the examples wie will present below). Note that since $v^{cr} > v^L$, some stochasticity is always part of the optimal mechanism. Types $v \in [v^L, v^{cr})$ choose to consume the good with strictly positive risk of interruption. For these types, the risk of interruption allows the seller to price discriminate among buyers in an incentive compatible way and without any violation of individual rationality. High types above v^{cr} prefer to pay a risk premium to avoid that risk.

This effectively leads to a cross-subsidization of low types (between v^L and v^{my}) and we have a trade expansion towards lowe types that were excluded under the Myerson mechanism.

The following proposition establishes this:

Proposition 1. *Under the optimal mechanism ($x^*(\cdot)$, $\lambda^*(\cdot)$, $p^*(\cdot)$) trade expands with the participation of lower types than in the case of the Myerson mechanism, i.e.* $v^L < v^{my}$.

Proof. It suffices to show that there are some types v that do not trade under the Myerson mechanism, but, they still participate in the optimal mechanism. Hence, we should focus on types $v < v^{my} < v^{cr}$. For those types, the following inequality holds:

$$\frac{1}{\frac{1-F(v)}{f(v)}b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)} < 0$$

$$\Rightarrow \frac{b'(v)}{b(v)} > \frac{v\frac{1-F(v)}{f(v)}}{\frac{1-F(v)}{f(v)} - v}.$$

So, it suffices to show that there are some types $v < v^{my}$ such that

$$\frac{v\frac{1-F(v)}{f(v)}}{\frac{1-F(v)}{f(v)} - v} > \frac{1-F(v)}{f(v)} \Rightarrow v > \frac{1-F(v)}{2f(v)}.$$

But, this is true for the types v that are sufficiently close to v^{my}. □

The seller can do better than under the Myerson mechanism because she can exploit the extra degree of freedom introduced by the durability of the good. Since the exact duration is uncertain (at the time of the trade agreement), she prefers to consider a much lower unit of time over which it incorporates the risk for the good to be stochastically unavailable.

It is worth studying the welfare implications of the optimal mechanism considering the Myerson one as the reference point. Let CS_O and CS_M be the expected consumer surplus under the optimal and under the Myerson mechanism, respectively:

$$CS_O = \int_{v^L}^{v^{cr}} (\lambda^*)^{b(v)} v \left(1 - \frac{\frac{1-F(v)}{f(v)}b'(v)}{b(v)}\right) dF(v) + \int_{v^{cr}}^{\overline{v}} \frac{1-F(v)}{f(v)} dF(v),$$

and

$$CS_M = \int_{v^{my}}^{\overline{v}} \frac{1-F(v)}{f(v)} dF(v).$$

These two expressions, after some algebra lead to:

$$CS_O - CS_M = \int_{v^L}^{v^{my}} (\lambda^*)^{b(v)} v \left(1 - \frac{\frac{1-F(v)}{f(v)} b'(v)}{b(v)} \right) dF(v)$$

$$+ \int_{v^{my}}^{v^{cr}} (\lambda^*)^{b(v)} v \frac{1-F(v)}{f(v)} b'(v) \left(\frac{1}{\frac{1-F(v)}{f(v)} b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)} \right) dF(v).$$

The comparison of CS_O and CS_M can be decomposed in two effects. The first term refers to the trade expansion from using the optimal mechanism instead of the Myerson one and it is positive. Types between v^L and v^{my} can under the optimal mechanism participate in trade and derive some positive surplus. The second term captures the negative impact of price discrimination on consumer welfare. Types between v^{my} and v^{cr} would participate in both mechanisms but they are strictly worse off under the optimal mechanism because the seller now can extract a greater share of their surplus. Which of the two effects dominate depends on the exact form of distribution $F(v)$ and function $b(v)$. For functions that result to low threshold v^L the trade expansion effect becomes significant. In contrast, when v^L approaches v^{my}, it is the second negative effect that is more likely to dominate.

The ambiguity over the welfare impact of each mechanism caries over total welfare. Expected total welfare under Myerson is given by

$$\int_{v^{my}}^{\overline{v}} v dF(v).$$

The respective expected total welfare in the optimal mechanism is:

$$\int_{v^L}^{\overline{v}} v \left(\lambda^*(v)\right)^{b(v)} dF(v).$$

If the Myerson mechanism allows only a few types of the buyer to trade, then for a significant trade expansion (low threshold v^L) total welfare is expected to improve under the optimal mechanism. If on the other hand the expected duration is large enough or the risk of interruption is significant for all types, total welfare is expected to be higher under the Myerson mechanism.

Example: Let the valuation of the buyer be drawn from a uniform distribution over the support $[\alpha, 1]$, where α is sufficiently close to zero. Then, $F(v) = v$ and $f(v) = 1$. Let also

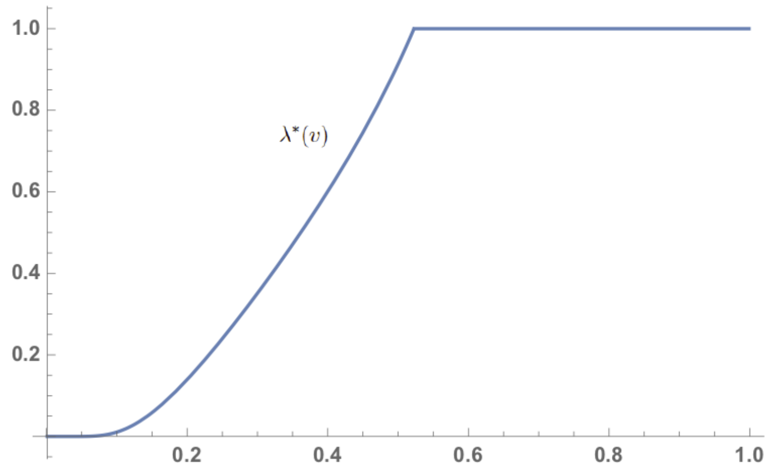

Figure 2: Probability of continuation of consumption at the optimal mechanism as a function of the valuation v.

the expected duration be $b(v) = cv + k$ (i.e. buyer of type v expects to consume the good for v periods in order to derive positive value v out of it), where $k > 1$ and $c > 0$. The critical threshold valuations become:

$$v^{my} = \frac{1}{2}, \qquad v^{cr} = \frac{c - k + \sqrt{c^2 + ck + k^2}}{3c} \qquad \& \qquad v^L = \max[\alpha, \frac{c-k}{2c}].$$

It is easy to see that these functions satisfy $v^{cr} > v^{my} > v^L$. Note that if $k > c$, then $v^L = \alpha$, so trade expansion covers all types of buyer.

Let $c = 1$, $k = 10$ and $\alpha = 0.0001$. Then $v^{cr} = 0.523$ and $v^L = \alpha$.

The optimal mechanism is

$$x(v) = 1, \qquad \lambda^*(v) = \min\{e^{\frac{1}{2(1-v)} - \frac{1}{2v} - \frac{1}{10+2v}}, 1\}, \forall v \in [\alpha, 1].$$

Buyer types with $v \geq v^{cr}$ get the good without any risk of interruption over its consumption ($\lambda^*(v) = 1$). Buyers with valuations below v^{cr} face a strictly positive risk of interruption which is strictrly decreasing in v. Figure 2 depicts $\lambda^*(v)$ between $[v^L, 1]$.

Consumer welfare under Myerson pricing is 0.125 and under the optimal mechanism is 0.124. Even if we have full trade expansion, consumer surplus is below the Myerson mechanism. However, buyers with $v \leq v^{my}$ are better off under optimal mechanism as they have gain a small but strictly positive payoff (with the exception of the lowest type).

3 N Buyers, Capacity Constraints and Implementation

Let's now consider a framework of one seller who can produce up to Q units of a homogeneous good or service for sale and N buyers with unit demand. The valuations of the buyers are drawn identically and independently from distribution $F(v)$ as it is defined above.

Intuitively, the seller wants to allocate as much of the Q units to the high valuation buyers because she can charge them a higher price. Depending on the capacity constraints, there is a marginal type $\hat{v}$ such that: $Q = N\left(F(\overline{v}) - F(\hat{v})\right)$. If capacity constraints are restrictive, $\hat{v}$ is expected to be high. Under the optimal mechanism the seller wishes to transact with as many types as possible provided that there are sufficient units Q.

The insights of the previous section still apply. However, now, the optimal mechanism will also depend on the capacity constraints Q, $\hat{v}$.

Proposition 2. *The optimal allocation rules are*

$$x^*(v) = \begin{cases} 1, & \text{if } v \geq \max[\hat{v}, v^L] \text{ and} \\ 0, & \text{otherwise,} \end{cases} \tag{6}$$

and

$$\lambda^*(v) = \left\{ \begin{array}{rl} 1 & \text{if } \hat{v} \geq v^{cr} \text{ or if } \hat{v} < v^{cr} \text{ and } v \geq v^{cr} \\ e^{\frac{1}{\frac{1-F(v)}{f(v)}b'(v)} - \frac{1}{vb'(v)} - \frac{1}{b(v)}} & \text{if if } \hat{v} < v^{cr} \text{ and } v < v^{cr} \end{array} \right\}. \tag{7}$$

Restrictive constraints over capacity increase the value of $\hat{v}$ and therefore it is less likely to have a strictly positive risk of interruption for some types of buyers in the optimal mechanism. If the seller has a small number of units for sale, then, she will allocate them to the high valuation buyers in a deterministic way, with certainty over consumption. The optimal mechanism in this case shifts towards the Myerson one. The risk of interruption can only arise in the optimal mechanism when capacity constraints are slack.

In the case that $\hat{v} < v^{cr}$, the implementation of the optimal mechanism can accommodate the simultaneous allocation through a posted price and an auction with the risk of interruption. For example, the posted price set at $p = v^{cr}$, and the auction that is efficient and is designed in a way that participants bid their true valuations can implement this mechanism (e.g., a generalised second price auction). Through the auction, the seller can link each bid (valuation) with a type specific optimal risk of interruption that is decreasing as types increase. So, buyers with valuations above v^{cr} choose the posted price mechanism. The seller has an incentive to set the reserve price of the auction at value equal to $\max[\hat{v}, v^L]$. So, buyers with valuations $v \in [\max[\hat{v}, v^L], v^{cr})$ can get a unit of the good in the auction but face a risk of interruption that is decreasing in v over its consumption. Buyers with valuations below the reserve price do not get the good.

4 Market Leadership and Price Discrimination

In this section we are discussing how the market position of the seller can affect her choices under the optimal mechanism. Many digital markets are characterized by market leaders who based on network effects, economies of scale and data driven economies of scope have an established their market leadership.

While mechanism design theory has provided great insights under a monopoly framework, it does not perform equally well in the case of an oligopoly. Introducing competition usually leads to complications regarding how to define model and how to solve it. These complications are in general difficult to overcome.

In this section I follow a reduced form reasoning of competition and market power, relying on the outside option of the buyer for a brief qualitative discussion. Let the expected benefit from transaction with the seller be defined as:

$$r(v) = x(v)\lambda^{b(v)}(v)v - p(v) \geq u(v), \tag{8}$$

where, $u(v)$ refers to the outside option of the buyer of type v, and it is monotonously increasing in v. If the seller is a market leader, this outside option is expected to be less attractive (small $u(\cdot)$). The opposite holds if the seller is a market follower.

The outside option is an additional constraint to consider when we solve the maximization problem. It essentially inflate threshold value of v^L increasing the importance of participation constraints in the optimal solution. In order words, it is more likely the market leaders for whom the outside option has a very small value to use the optimal mechanism as it was defined above.

However, when $u(\cdot)$ becomes significant, the seller is less able to extract surplus from the buyer since the latter has an attractive outside option to consider. Therefore, there is a competitive constraint that limits the ability of the seller to effectively price discriminate. This competitive constraint brings us back to a deterministic mechanism as the one of Myerson.

For example, in the extreme case, when $u(\cdot)$ is so high that under the unconstrained optimal mechanism the condition (8) is violated, we need to consider this constraint as binding when we solve the maximization problem.

The objective function in such a case becomes:

$$\Pi_s = \int_{\underline{v}}^{\overline{v}} \left[x(v)\lambda^{b(v)}(v)v - u(v) \right] dF(v).$$

The constrained optimal mechanism in this case requires $\lambda^*(v) = 1$. A stochastic mechanism that includes the risk of interruption as a way to effectively price discriminate between buyers of different valuations is more likely to be implemented by market leaders when they do not face significant outside threats.

5 Conclusion

In many digital markets, information over market conditions, demand and supply is private to the seller while buyers only observe prices. Algorithmic pricing incorporate price fluctuations over time and at frequency that is much smaller from the consumption period of the given good.

For information goods that are consumed over time, this lack of market transparency may provide incentives for the seller to adopt a stochastic pricing schedule in order to price discriminate across buyers of different valuations. I show that the seller finds optimal to introduce a risk of interruption over consumption which is specific to each type of the buyer.

Comparing derived mechanism with the deterministic one proposed by Myerson (1981), which would have been optimal under full transparency, I show that stochasticity leads to expansion of trade towards the low types of buyers who are subsidized by the high valuation clients. Buyers of different valuation consume different versions of the good. Each version differs from the other ones over the risk of interruption of consumption.

The welfare implications of the optimal mechanism are ambiguous. However, examples with specific linear functions for the duration of the consumption and the distribution of valuations suggest that consumer welfare is lower than in the case of the optimal deterministic mechanism. Low valuation buyers are better off as they participate in the market, while high valuation buyers have incentives to pay an increased risk premium to avoid their consumption being interrupted.

Stochasticity as well as price discrimination is less likely to emerge as a part of the optimal selling mechanism when capacity constraints are restrictive or when the seller faces competitive pressure in the market she operates.

The results of this paper suggest that we should look more thoroughly on the implications of algorithmic pricing as well as on the objectives of the algorithmic systems that set prices at real time in online ecosystems.

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